Problem Set 1a
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Intro

Welcome to my first RAI problem set, and certainly one of my first LaTeX documents. **See the next page for proofs!**

The problems:

1) For the union of a set and another set to be equivalent to one of the sets, that set must be the superset of (i.e. must contain) the other set.

Proof. Let A, B be sets.
i. $A \cup B = A$
ii. $A \subseteq B$

WTS: (i) $\iff$ (ii)

a) (i) $\implies$ (ii)

Let: $x \in A, y \in B, C = \{z \mid z \in A \vee z \in B\} = A \cup B$
wts $A = C \wedge \forall z \in C \quad \exists x \in A \mid z = x$
For all z to be in C and still maintain membership in B while C=A,
B must also be a subset of or equal to A (i.e. $y \in B \subseteq A$).

b) (ii) $\implies$ (i)

Let: $x \in A, \ \ y \in B, \ \ B \subseteq A$
$A \setminus B = \emptyset \ \ \vee D$ where $D = \{z \mid z \notin B \ \ \wedge \ \ z \in A\}$
case 1 ($B = A$): $A \cup B = A \cup A = \{\alpha \mid \alpha \in A\} = A$
case 2 ($B \subset A$): $A \cup B = \{\alpha \mid \alpha \in A \ \ \vee \ \ \alpha \in B\}$
Since all α in B are also in A, $A \cup B = A$.

□

2) The composition of a function on its inverse on its codomain maps to a subset of or equivalent of its codomain.

Proof. Let X, Y be sets, $A \subseteq X$, $B \subseteq Y$ be subsets.

□

3) The multiplicative identity in a field is unique.

Proof. Let: F is a field, i.e. nonempty, closed in addition and multiplication.
$a, b, x \in F$.

For contradiction, assume $\forall x \;\; \exists a \neq b$:

$\forall x \;\; \exists a \;\; | \; a \cdot x = x \quad \wedge \quad \forall x \;\; \exists b \;\; | \; b \cdot x = x$

(without invoking division or a need for closure under division:)

$a \cdot x = b \cdot x \iff (a - b) \cdot x = 0$. If $x \neq 0$, $\; a = b$. $\square$