

(To the prompt, I enjoy the solo struggle and find in-person makes it difficult to focus, particularly with the fear of "being wrong" or "looking dumb" kicks in. But, that's also probably a part to maturing as a (person) mathematician?)

Way of Analysis introduces Newton's Method on p. 28 (without explicitly calling it "Newton's Method"), and I am reminded of the power I felt the first time I used it to calculate an irrational number, a Promethean feeling. Prior to that, the closest I had come was in measuring a circle's circumference and diameter with string, recording the values and dividing by them to approximate pi. Calculating by hand feels, in a sense, like blasphemy against your calculator, and it was no less a rush today while reading WoA than the first time I did it in Calculus I an arbitrary number of years ago.

Since it's fun to type and very useful in my career (engineering), I'll just reiterate it:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Now, to get $\sqrt{2}$, we need to do a little shifting to make $\sqrt{2}$ a root:

$$x^2 = 2$$

$$\text{Let: } f(x) = x^2 - 2$$

$$\therefore f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right)$$

(Note: WoA uses the negative $\sqrt{2}$, which makes the above relation a sum; either way works!)

Anyway, hard to describe, but I am beginning to see the power of the reals and just how limited the rationals are; the rationals being a set I have internally (without realizing it) held to be equivalent to the reals because the reals can be represented by the rationals. This is a similar logical fallacy to assuming that a word and its meaning are equivalent (that words somehow have magical power); that code free of syntax errors cannot hold semantic errors.

Is this what it feels like to follow the "Way?"