ENAE441 Fall 2022

Orbit determination from a remote telescope

Due December 12, 2022 at 11:59PM

1 Objective

You will determine the orbit of a satellite and verify the validity of your solution based on data collected by a commercial service. This is telescope data that has been reduced to a pair of angles (supplied in both right ascension/declination and azimuth/elevation).

2 Data

The data comes from a commercial service that operates a network of telescopes, and the satellites are all in deep space at the time of observation, usually geosynchronous equatorial orbit. There are two consecutive nights of observation at "299E" (i.e., GEO belt longitude of $-61°$), October 29 and 30, 2020. The file names for the first night start with `opt2` and those of the second night `opt3`. You will load the files into Matlab, which will create two tables with the data, one for each night; see ELMS for more information. You should reserve the second night for validation (see section 3.5) and use only the first night for orbit determination.

It is possible that the satellites in your data set maneuvered over the timespan of the data sets, which will affect the results you get in the orbit determination.

3 Analysis

3.1 Initial orbit determination

Start with initial orbit determination from three observations from the first night using the Gauss angles-only method to find the position vector as described in section 7.3.4 of the book and then solving for the orbit with the Gibbs method on the three vectors. You should pick several sets of three vectors, some close together early in the first night data set, some spread throughout the set, and some close together late in the set. By computing the RMS of the propagation residuals to the second set (see section 3.5), determine which is the best set to use to start the orbit estimation.

3.2 Estimation

Once the best initial orbit is determined (section 3.1), you are ready to do an orbit estimation (chapter 9 of the book) using the first data set. SNaG-app provides the capability for doing this and the validation. The Orbit determination documentation describes the functions to do the orbit estimation once you have the initial orbit, and the Observations documentation describes steps to get subsets of the observations. You will need version 14 (or higher) as shown by `constants.appversion`.

Important functions in SnaG-app that you will need include the following

1. `make_station()` Makes an observation station structure to use in `determine_orbit`.
2. `determine_orbit()` Solves the orbit deterimation problem.
3. `propagate()` To propagate an initial or estimated orbit.
4. `ephemeris_interp()` To interpolate the propagated orbits to selected observation times.

See “Orbits/Orbit determination” part of the documentation and links therein for more details.

3.3 Example

The main function you will use is `determine_orbit`. An example of how this is called is as follows, assuming that `initial_est` is set to the results of the initial orbit determination.

```
% Load the observation set
load("opt2satAset1.mat");
uninteresting = ["right_ascension_deg", "declination_deg",...
          "site_latitude_deg", "site_longitude_deg", "site_altitude_m"];
myobs = select_columns(opt2satAset1, uninteresting, true);
% Define a force model
force_model_4x4_1m2_cr1 = force_model(4, 4, 0, 0, 1, 1, 1000);
% Define the observation site
oapchile = make_station("OAP-Chile",...
     -30.1428030000, -70.6945280000, 1500.000000);
% Make a subset of the original data
night1_early_25pts = myobs(1:4:100,:);
od_night1_early = determine_orbit(initial_est, oapchile,...
           night1_early_25pts, force_model_4x4_1m2_cr1)
```

In this example, 25 points spread evenly in the early part (12% or so) of the data set for the first night are used, the force model is fairly simple with a 4×4 geopotential model and solar radiation pressure set with a coefficient of reflectively of 1 and area of $1\,\mathrm{m}^2$; see section 3.4 for guidance on choosing realistic values. The output structure has several different fields; see the documentation for interpretation.

```
od_night1_early =
  struct with fields:

             details: {10x1 cell}
           estimated: [1x1 struct]
           residuals: [25x3 timetable]
          run_number: 3
             station: [1x1 struct]
         force_model: [1x1 struct]
    initial_estimate: [1x1 struct]
        observations: [25x4 table]
```

The main output is the state determined from the estimation in `estimated`, but the other output has useful information as well; for example, `details` shows the RMS residuals of the observations at each iteration.

```
>> determine_night1_early.estimated.position_m
ans =
          38697129.3776636
         -16786880.0008237
         -74237.6725961468
>> determine_night1_early.estimated.velocity_ms
ans =
          1223.68767868804
          2819.38968020644
         -4.16631251112905
>> epoch = determine_night1_early.estimated.epoch);
>> datestr(epoch,constants.iso8601_fmt_w)
ans =
    '2020-10-29T00:00:00.000'
```

3.4 Study effects of different models

Your task will be to determine the effect of different observation data sets and different force models in the propagator. You should set up a procedure to do the orbit estimation varying these quantities as follows.

1. Vary the observation data set, holding the propagator constant.
2. Vary the force model, holding the observation data set constant.

You will assess the relative importance of the quantities being varied first by the RMS of the fit which is shown in the last iteration `details` field of the solution, and more importantly by the propagated residual RMS, section 3.5. Other factors that come into consideration are the ability to converge and the time of computation. The conclusions you draw from your results should assess the benefit of the quantities varied. Which observation set distribution or force model yields the best result, or is the difference not significant?

Gravity forces to test include two body, 2×0, 2×2, and 20×20. Do your comparison with no other perturbations. Find which are the best and whether the difference is significant, and which are significantly worse. The comparison of 2×0, 2×2 will show whether the tesseral harmonics of the second order are significant vs. just the zonal harmonic. Once you have decided on a good gavity force model, use that for the remaining tests with other perturbations with luni-solar (third body) and solar radiation pressure forces.

Including solar radiation pressure could be helpful in determining the orbit. Doing so requires mass, surface area, and coefficient of reflectivity of the satellite. Geostationary communication satellites are large in order to carry the transimitters needed to reach the earth's surface, so their cross sectional area is tens of square meters and masses are on the order of several metric tons. Lacking any specific knowledge of particular satellite, it is common to take a coefficient of reflectively of 1.2 for a satellite. Try three different sets of solar radiation pressure parameters. You will not need to include atmospheric drag for geostationary satellites.

3.5 Validation of solution with propagation residuals

The accuracy of an orbit estimate may be determined by propagating the estimated orbit and then computing the residuals of observations at observation times that were not used in the original orbit determination. To help you do this, a second night of observations for the same satellite you analyzed on the first night is provided. The name of the file and variable starts with `opt3`. Any orbit estimate, including that resulting from the initial orbit determination, can be propagated to the observation times of the second night, the Cartesian state converted to azimuth and elevation, and the residual computed for each by taking the difference of the observed and propagated. This set of residuals can be reduced to a single number by computing the root mean square (RMS), that is,

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^{\text{obs}} - \zeta_i^{\text{pred}}\right)^2 + \left(\epsilon_i^{\text{obs}} - \epsilon_i^{\text{pred}}\right)^2}, \tag{1}$$

where where ζ_i^{obs}, ϵ_i^{obs} are the azimuth and elevation at time t_i as obtained from your data set, and ζ_i^{pred}, ϵ_i^{pred} are the azimuth and elevation obtained by propagating the estimate to the observation time t_i and transforming the coordinates from ECI to azimuth, elevation and range as viewed from the ground station at that time. Pick a set of N observations that are spread roughly evenly through the second data set; recommended values $15 \leq N \leq 50$.

You should use caution when computing the RMS on angles, because angles lie on a circle and not the real line, so the difference could appear to be large when it is not. If the two values being subtracted lie on opposite sides of the angle reset line, for example $1°$ and $359°$, then the difference is small, in this case $2°$, and not $\pm 358°$. If you obtain very large residuals, more than a few degrees, check for this problem.

The RMS computed here is different from the RMS obtained during the estimation process, shown at each step in the `details` output, which is the RMS of the observations used in that estimation. This RMS is based on an entirely new set of observations, at a later time. It is an unweighted RMS calculation, that is, each of the different observations and member of the angle pairs have the same weight as every other.

The force model used to propagate for validation should have all forces except atmospheric drag turned on, with geopotential degree and order maximum (20). This way, the predicted location of the satellite is as close as possible to what would be the actual location if the estimated orbit was correct. Where a force needs a parameter (such as area), you should use the same value as in the estimation. See the documentation "Propagation" and "Force model" for use of the function `force_model()` which includes sun and moon perturbation.

The function `propagate` will only create an ephemeris at regular time steps, and the data set you have been given will likely not have regular time steps. Therefore, you should use the function `ephemeris_interp` to interpolate the ephemeris; it takes two arguments, the first is the ephemeris, and the second is a vector of datetimes to interpolate to.

While the RMS gives a single number “figure of merit” to the results, it is sometimes useful to see details. One way to do that is to plot residuals for each observation, as residual value vs. time. To do so, plot the residuals as two curves, one for each angle $\zeta_i^{\text{obs}} - \zeta_i^{\text{pred}}$ and $\epsilon_i^{\text{obs}} - \epsilon_i^{\text{pred}}$ vs. t_i. You can also compute separate aziuth and elevation RMS,

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^{\text{obs}} - \zeta_i^{\text{pred}}\right)^2} \tag{2}$$

and

$$\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\epsilon_i^{\text{obs}} - \epsilon_i^{\text{pred}}\right)^2}. \tag{3}$$

You might find that two sets of results that have a similar RMS have notably different residual results early or late in the propagated time span.

3.6 Maneuver detection

After completing the above analysis, repeat the calculation with the initial orbit determination on the second night’s set, then do an estimation with more of the second night’s data. You do not need to experiment with different sets for the initial orbit determination; use the same distribution pattern that worked on the first night’s data (e.g., early, middle, late).

To see if there was a maneuver, propagate the solution from the first night forward and solution from the second night backward and compute the magnitude of the position difference and and magnitude of the velocity difference between the two at regular time steps between the two epochs. If the difference position distance exceeds a few kilometers along this range, find the approximate time at which the position difference is a minimum. That is possibly the time of a maneuver, and the magnitude of the maneuver will be velocity difference at that time.

4 Report format

Write a report that includes the following:

1. A brief statement and summary of the problem: what you set out to do and what you found.

2. For each of the parts described in the study, show the procedures and calculations used.

3. Conclusion, interpreting your results and an explanation on the sources of errors and limitations of your analysis.

One member of the group should upload the report to Gradescope as a PDF. You should include

1. A synopsis of your data; you need not include all your data in the report, but you should make clear the data set(s) and observation numbers of subsets used. For small sets of data, such as for initial orbit determination, you can include all the relevant information.

2. Any software files used in the computation. Indicate their function and origin (who wrote it and where you got it from). Do not include code that you downloaded (e.g. SNaG-app), but do reference it and show how you used it.

3. A brief statement (paragraph or two) describing what *each member* of the group did. This should include all the work on the project, even work that didn't lead to anything in the final report. If a member started participation late or not at all, or had significant absences, please note that as well.

Your whole group should collaborate on one report. Please coordinate within your group to make sure you are all in agreement when the report is finished and should be submitted, and agree on one person who will submit the finished report.

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