Homework 1

Autumn 2023

WRITE YOUR NAME HERE

2023-10-02

Instructions

- This homework is due in Gradescope on Wednesday Oct 11 by midnight PST. There is a 15 minute grace period and submissions made during this time will not be marked as late. Any work submitted past this period is considered late.
- Please answer the following questions in the order in which they are posed.
- Don't forget to (i) make a local copy this document for your work and to (ii) knit the document frequently to make sure there are no compilation errors.
- When you are done, download the PDF file as instructed in section and submit it in Gradescope.

Exercises

1. The crew of Apollo 17 consisted of two pilots and one geologist. Suppose that NASA had actually trained three pilots and two geologists. How many possible Apollo 17 crews could have been formed

   a. if the two pilot positions have identical duties?

   b. if there is a pilot and a co-pilot?

   Write the sample space in each case. You may denote the three pilots as P_1, P_2, P_3 and the two geologists as G_1, G_2. So (P_1, P_2, G_1) represents the outcome that pilots 1 and 2 and geologist 1 were selected.

2. For two events A and B with $P(A) = 0.5$ and $P(B) = 0.8$, what are the largest and smallest possible values for $P(A \cap B)$? Be sure to explain each answer.

   Hint you will need to use the Bonferroni inequality and also the subset inequality you learned in section.

3. If $P(A) = \frac{1}{3}$ and $P(B^c) = \frac{1}{4}$, can A and B be disjoint? Support your answer.

4. Three events A, B and C are defined in a sample space. Show that

$$P(A \cup B \cup C) \leq P(A) + P(B) + P(C).$$

   Hint Define $E = B \cup C$ and apply the union bound to $P(A \cup E)$ first. Then apply the union bound again to $P(E)$.