

Homework 2

Autumn 2023

WRITE YOUR NAME HERE

2023-10-09

Instructions

- This homework is due in Gradescope on Wednesday Oct 18 by midnight PST. There is a 15 minute grace period and submissions made during this time will not be marked as late. Any work submitted past this period is considered late.
- Please answer the following questions in the order in which they are posed.
- Don't forget to (i) make a local copy this document for your work and to (ii) knit the document frequently to make sure there are no compilation errors.
- When you are done, download the PDF file as instructed in section and submit it in Gradescope.

Exercises

1. Suppose 12 coins are tossed and the outcome (head or tail) is recorded for each.
 - a. The sample space S corresponding to this “experiment” consists of all possible sequences of heads and tails that result from tossing 12 coins. How many elements are in S ? Calculate this number in a code chunk, report the answer using inline code and also explain your answer very briefly.

Hint: Refer to Braille alphabet example 3.2. This problem is similar but with a 12 dot matrix.
 - b. Let E denote the event that 7 of the 12 coins land on heads. How many elements are in E ? Calculate this number in a code chunk, report the answer using inline code and also explain your answer very briefly.

Hint: Referring to the Braille alphabet example again, suppose we now want to count all the letters we could form with 3 raised dots. All you need to do is decide which three of the six dots will be raised. How many ways can you make this decision?
 - c. Calculate $P(E)$ assuming all the elements in S are equally likely. Report your final answer in a sentence using inline code.
2. To estimate the number N of goldfish in a pond, $r = 25$ fish were caught, tagged and released. Later, a second sample of $n = 20$ fish were caught and 5 fish in this sample were noted to be tagged.
 - a. How many possible samples of size $n = 20$ can be formed from the N fish in the pond? (Leave your answer in terms of a binomial coefficient - you cannot calculate it because you don't know N)
 - b. The event E contains all the samples which have 5 tagged and 15 untagged fish. How many elements are in the event E ? (Leave your answer in terms of N)

- c. Assuming each possible sample is equally likely, give an expression for $P(E)$. (Leave your answer in terms of N)
- d. In this part, we will examine visually how $P(E)$ varies as a function of N . Fill in the blanks in the R code provided and run it in R to create the plot. You should remove the `eval = FALSE` chunk option before knitting. (Note: You DO NOT need to know **tidyverse** or the **ggplot** function to answer this question)

```
fishes <- tibble( #data frame
  N = 40:200,    #possible values for N: 40,41, ...,200
  prob = ___ ) ) #write expression for P(E) in terms of N

ggplot(data = fishes,
  mapping = aes(x = ___,          #x and y variables
                y = ___ ) ) +
  geom_line() +                  #type of plot to draw
  labs( title = " ___" , #labels for plot
        x = "___",
        y = "___")
```

3. Among all students seeking treatment at Hall Health, 0.5% are eventually diagnosed as having mononucleosis (event A). Of those who do have mono, 90% complain of a sore throat (event B). But 30% of those not having mono also have sore throats.
- a. Make a tree diagram of the probabilities relating presenting with a sore throat and a diagnosis of mononucleosis. Read the notes below:
- Don't forget to include the **openintro** package in the setup chunk
 - Create a new code chunk, give it a name. Then see problem 2a in **Problem2.Rmd** for the code to create the tree and add that to your code chunk with the numbers appropriately changed.
- b. If a student comes to Hall Health and says that she has a sore throat, what is the probability that she has mono? Report your answer rounded to 4 decimal places, using inline code. Be sure to show your steps carefully. (Hint: please see problem 2b in **Problem2.Rmd** for help, We will deduct points for not defining events and carefully showing your work step by step.)
4. If A and B are independent events, then the following pairs are also independent.
- A and B^c
 - A^c and B
 - A^c and B^c .

Prove result c. only. You may use the results from a. and b. without proof in your answer.