

# Homework 2

Autumn 2023

WRITE YOUR NAME HERE

2023-10-09

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## Instructions

- This homework is due in Gradescope on Wednesday Oct 18 by midnight PST. There is a 15 minute grace period and submissions made during this time will not be marked as late. Any work submitted past this period is considered late.
- Please answer the following questions in the order in which they are posed.
- Don't forget to (i) make a local copy this document for your work and to (ii) knit the document frequently to make sure there are no compilation errors.
- When you are done, download the PDF file as instructed in section and submit it in Gradescope.

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## Exercises

1. Suppose 12 coins are tossed and the outcome (head or tail) is recorded for each.
  - a. The sample space  $S$  corresponding to this “experiment” consists of all possible sequences of heads and tails that result from tossing 12 coins. How many elements are in  $S$ ? Calculate this number in a code chunk, report the answer using inline code and also explain your answer very briefly.  
*Hint: Refer to Braille alphabet example 3.2. This problem is similar but with a 12 dot matrix.*
  - b. Let  $E$  denote the event that 7 of the 12 coins land on heads. How many elements are in  $E$ ? Calculate this number in a code chunk, report the answer using inline code and also explain your answer very briefly.  
*Hint: Referring to the Braille alphabet example again, suppose we now want to count all the letters we could form with 3 raised dots. All you need to do is decide which three of the six dots will be raised. How many ways can you make this decision?*
  - c. Calculate  $P(E)$  assuming all the elements in  $S$  are equally likely. Report your final answer in a sentence using inline code.
2. To estimate the number  $N$  of goldfish in a pond,  $r = 25$  fish were caught, tagged and released. Later, a second sample of  $n = 20$  fish were caught and 5 fish in this sample were noted to be tagged.
  - a. How many possible samples of size  $n = 20$  can be formed from the  $N$  fish in the pond? (Leave your answer in terms of a binomial coefficient - you cannot calculate it because you don't know  $N$ )
  - b. The event  $E$  contains all the samples which have 5 tagged and 15 untagged fish. How many elements are in the event  $E$ ? (Leave your answer in terms of  $N$ )

- c. Assuming each possible sample is equally likely, give an expression for  $P(E)$ . (Leave your answer in terms of  $N$ )
- d. In this part, we will examine visually how  $P(E)$  varies as a function of  $N$ . Fill in the blanks in the R code provided and run it in R to create the plot. You should remove the `eval = FALSE` chunk option before knitting. (Note: You DO NOT need to know **tidyverse** or the **ggplot** function to answer this question)

```
fishes <- tibble( #data frame
  N = 40:200,    #possible values for N: 40,41, ...,200
  prob = ___ ) ) #write expression for P(E) in terms of N

ggplot(data = fishes,
  mapping = aes(x = ___,          #x and y variables
                y = ___) ) +
  geom_line() +                  #type of plot to draw
  labs( title = " ___" , #labels for plot
        x = "___",
        y = "___")
```

3. Among all students seeking treatment at Hall Health, 0.5% are eventually diagnosed as having mononucleosis (event  $A$ ). Of those who do have mono, 90% complain of a sore throat (event  $B$ ). But 30% of those not having mono also have sore throats.
- a. Make a tree diagram of the probabilities relating presenting with a sore throat and a diagnosis of mononucleosis. Read the notes below:
- Don't forget to include the **openintro** package in the setup chunk
  - Create a new code chunk, give it a name. Then see problem 2a in **Problem2.Rmd** for the code to create the tree and add that to your code chunk with the numbers appropriately changed.
- b. If a student comes to Hall Health and says that she has a sore throat, what is the probability that she has mono? Report your answer rounded to 4 decimal places, using inline code. Be sure to show your steps carefully. (Hint: please see problem 2b in **Problem2.Rmd** for help, We will deduct points for not defining events and carefully showing your work step by step. )
4. If  $A$  and  $B$  are independent events, then the following pairs are also independent.
- $A$  and  $B^c$
  - $A^c$  and  $B$
  - $A^c$  and  $B^c$ .

Prove result c. only. Show your work carefully with justification.