Chapter 11

Thursday, November 16, 2023 8:12 PM

Chapter 11

The Exponential Distribution

>0

$\rightarrow$ rate parameter

Is $f(y)$ a valid density function?

Check 2 conditions:

- $f(y) \overset{?}{\geq} 0 \quad \forall\, y$ ✓
- $\int^{\infty} f(y)\, dy \overset{?}{=} 1$

$$-\int_{-\infty}^{\infty} f(y)\,dy \overset{?}{=} 1$$

$$\int_{-\infty}^{\infty} f(y)\,dy = \int_{0}^{\infty} \lambda e^{-\lambda y}\,dy = \cancel{\lambda}\,\frac{e^{-\lambda y}}{-\cancel{\lambda}}\Bigg|_0^{\infty}$$

$$= -\left(\cancel{e^{-\lambda \cdot \infty}}^{\,0} - e^{-\lambda \cdot 0}\right)$$

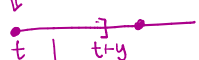
$$= 1$$

To prove thm 1.1, we are going to find the CDF of Y.

$$F_Y(y) = P(Y \leq y)$$

We can differentiate F to get the PDF f

event has happened here



X = # times the event happens in this time interval.

$$f_X(x) = e^{-\lambda y}(\lambda y)^x / x!$$

easier to calculate

easier to calculate

$= 1 - P(Y > y)$

Let X = # breakdowns in 1 month $\left(\frac{1}{12} \text{ yr}\right)$

$\sim \text{Pois}\left(6 \cdot \frac{1}{12} = \frac{1}{2}\right)$

$f(x) = \dfrac{e^{-1/2} (1/2)^x}{x!}$ $\quad x = 0, 1, 2, \ldots$

want $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-1/2} = \boxed{.3935}$

$\boxed{\text{In R}}$

> 1 - dpois(x=0, lambda = 1/2) $\quad (1 - P(X=0))$

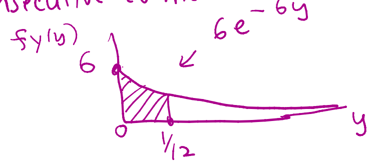
> ppois(q=0, lambda=1/2, lower.tail=F) $\quad (P(X > 0) = P(X \geq 1))$

> 1 - exp(1/2)

Lets define Y = time (in years) between 2 consecutive events

$Y \sim \text{Exp}(6)$

$f_Y(y) = 6e^{-6y} \quad y \geq 0$



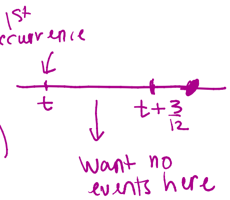
want $P\left(Y \leq \frac{1}{12}\right) = F_Y(1/12)$

$= 1 - e^{-6 \cdot 1/12}$

want $P(Y \leq \frac{1}{12}) = F_Y(1/12)$

$= 1 - e^{-6 \cdot 1/12}$

$= 1 - e^{-1/2} = \boxed{0.3935}$



X = # breakdowns in a three month period $(t = \frac{3}{12}\text{ yr})$

$\sim$ Pois $(3/2)$

$f(x) = e^{-3/2} \frac{(3/2)^x}{x!}$ $x = 0, 1, 2, \ldots$

want $P(X=0) = e^{-3/2}$

Y = time (in years) between 2 consecutive events

$\sim$ Exp(6)

$f_Y(y) = 6e^{-6y}$ $y \geq 0$

want $P(Y \geq \frac{3}{12}) = 1 - P(Y < 3/12) = 1 - P(Y \leq 3/12)$

$= 1 - F_Y(3/12)$

$= 1 - (1 - e^{-6 \cdot 3/12}) = \boxed{e^{-3/2}}$

Y = time (in years) between 2 consecutive events

$Y \sim$ Exp(6)

$f_Y(y) = 6e^{-6y}$ $y \geq 0$

Given: $Y \geq 2/12 = Y \geq 1/6$

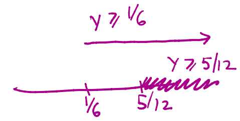
want: $P(Y \geq 5/12 \mid Y \geq 1/6)$

$\left[\text{Def of } P(A|B) = \frac{P(A \cap B)}{P(B)}\right]$

$P(Y \geq 5/12 \mid Y \geq 1/6) = \frac{P(Y \geq 5/12 \cap Y \geq 1/6)}{}$

$$P(Y \geq 5/12 \mid Y \geq 1/6) = \frac{P(Y \geq 5/12 \cap Y \geq 1/6)}{P(Y \geq 1/6)}$$

Def of $\left[P(A|B) = \frac{P(A \cap B)}{P(B)}\right]$



$Y \geq \frac{5}{12} \subset Y \geq \frac{1}{6}$

$\therefore \left(Y \geq \frac{5}{12}\right) \cap \left(Y \geq \frac{1}{6}\right) = \left(Y \geq \frac{5}{12}\right)$

$$= \frac{P(Y \geq 5/12)}{P(Y \geq 1/6)}$$

$$= \frac{e^{-6 \cdot 5/12}}{e^{-6 \cdot 1/6}}$$

$$= \frac{e^{-5/2}}{e^{-1}} = e^{-\frac{5}{2}} \cdot e^{1}$$

$$= e^{-\frac{5}{2}+1} = \boxed{e^{-3/2}}$$

$$= P(Y \geq 3/12)$$

(from ⓑ)

3 months

2 months

$$f(x) = \lambda e^{-\lambda x} \quad x \geq 0$$

As λ increases the mean of an exponential random variable decreases