J Bolton Set 6

Logic for Applications
(Ch 12) P 144: 1
(Ch 13) P 151: 1, 5
(Ch 14) P 157: 2,3

Ch 12
1.
(a)
S0 = {P(x,y), P(y,f(z))}
d1 = {x,y}
s1 ={x/y}
S1 = {P(y,y), P(y,f(z))}
d2 = {y, f(z)}
s2 = {f(z), y}
S2 = {P(y,f(z)}

(b)
S0 = {P(a,y,f(y)), P(z,z,u)}
d1 = {a,z}
s1 = {a/z}
S1 = {P(z,y,f(y)), P(z,z,u)}
d2 = {y,z}
s2 = {y/z}
S2 = {P(z,z,f(y)), P(z,z,u)}
d3 = {f(y), u}
s3 = {u/f(y)}
S3 = {P(z,z,f(y))}

And so on.

Ch 13
1.
(a)
{P(x,y), P(y,z)}, {~P(u, f(u))}
z/f(u), y/u
{P(x,u), P(u,f(u))}, {~P(u, f(u))}
unify P(u, f(u))
{P(x,u)}

(b)
{P(x,x), ~R(x,f(x))}, {R(x,y), Q(y,z)}
y/f(x)
{P(x,x), ~R(x,f(x))}, {R(x,f(x)), Q(f(x),z)}
unify R(x,f(x))
{P(x,x)}, {Q(f(x),z)}

And so on.

5. v, ii, iv, i
Did these in a pretty much random order; I'm sure the steps can be much shortened.
1. resolve (v) and (ii) to get:
(vi) = {~Q(h(b),v)}
2. resolve (vi) and (iv) to get:
(vii) = {P(a,u,f(h(u))), H(u,a), Q(h(b),b)}
3. resolve (vii) and (v) to get:
(viii) = {P(a,v,f(h(v))), Q(h(b),b)}
4. resolve (viii) and (i) to get:
(ix) = {P(a,v,f(h(v))), P(a,x,f(h(b))), P(a, f(h(b))}
5. resolve (ix) and (iii) to get:
(x) = {P(a,x,f(h(v))), P(a, f(h(v)), H(x,a)}
We can then resolve (x) with (iii) repeatedly until our only terms are the predicates H(a,b), and then resolve that with (v) repeatedly. I don't think I need to type it out.

Ch 14
2.
By 13.6 and 13.10, we already have the soundness and completeness of resolution (that is not necessarily linear). For completeness, we must then show that if S is unsatisfiable, there is a linear resolution refutation of S, which we are given by theorem 14.4. For soundness, we assume that S produces box through linear resolution, and we must show that S must also be unsatisfiable. Suppose S is satisfiable. Then, by 13.6, box cannot be an element of the resolution of S, but it is, so this is the desired contradiction. (This probably isn't very correct, but I don't currently have time to wrestle with it more and will try to return to it later).

3.
This doesn't use the exact same syntactic rules from our book
i.
Ex. Dragon(x)
ii.
Ex. Dragon(x) & (Sleeping(x) | Hunting(x))
iii.
Ex. Dragon(x) & (Hungry(x) -> ~Sleeping(x))
iv.
Ex. Dragon(x) & (Tired(x) -> ~Hunting(x))
(a).
Hunts in the forest (rules iii and ii)
(b).
Sleeping (rules iv and ii)