Assignment 1, Data Structures

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1.6 a.

$$S = 1 + \frac{1}{4} + \frac{1}{4^2} \cdots \frac{1}{4^i} \cdots \tag{1}$$

$$4S = 4 + 1 + \frac{4}{4^2} + \frac{4}{4^3} \cdots \frac{4}{4^i} \cdots \tag{2}$$

$$= 4 + 1 + \frac{1}{4} + \frac{1}{4^2} \cdots \frac{1}{4^i} \cdots \tag{3}$$

$$4S - S = 4 = 3S \tag{4}$$

$$\frac{3S}{3} = S = \frac{4}{3} \tag{5}$$

b.

$$S = \frac{1}{4^1} + \frac{2}{4^2} \cdots \frac{i}{4^i} \cdots \tag{6}$$

$$4S = 1 + \frac{8}{4^2} + \frac{12}{4^3} \cdots \frac{4 * i}{4^i} \cdots \tag{7}$$

$$= 1 + \frac{1}{2} + \frac{3}{4^2} + \frac{4}{4^3} \cdots \frac{i}{4^{i-1}} \cdots \tag{8}$$

$$4S - S = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} \cdots \frac{1}{4^i} \cdots \tag{9}$$

$$= \frac{4}{3} \tag{10}$$

$$\frac{3S}{3} = S = \frac{4}{9} \tag{11}$$

c.

$$S = \frac{1^2}{4^1} + \frac{2^2}{4^2} \cdots \tag{12}$$

$$4S = 2 + \frac{3^2}{4^2} + \frac{4^2}{4^3} \cdots \tag{13}$$

$$4S - S = 3S = \frac{7}{4} + \frac{5}{4^2} + \frac{7}{4^3} \cdots \tag{14}$$

$$4 * 3S = \frac{33}{4} + \frac{7}{4^2} + \frac{9}{4^3} \cdots \tag{15}$$

$$4 * 3S - 3S = \frac{13}{2} + \frac{2}{4^2} + \frac{2}{4^3} \cdots \tag{16}$$

$$= \frac{13}{2} + 2(\frac{1}{4^2} + \frac{1}{4^3} \cdots) \tag{17}$$

$$= \frac{13}{2} + 2(\frac{4}{3} - \frac{5}{4}) \tag{18}$$

$$= \frac{13}{2} + 2(\frac{1}{12}) \tag{19}$$

$$= \frac{13}{2} + \frac{1}{6} \tag{20}$$

$$= \frac{39}{6} + \frac{1}{6} \tag{21}$$

$$= \frac{20}{3} \tag{22}$$

$$9S = \frac{20}{3} \tag{23}$$

$$S = \frac{20}{27} \tag{24}$$

d. This one was a real monkey of a problem. I did not really get a solution. I noted that with the powers, if we take the difference of their difference n times (where n is the power), then we end up with n! in the numerators. To take the difference of the differences of the numerators of S, we do $4S - S$ and then $3(4S) - 3S$. We continue to do that n times.

$$f(x, 1) = 4x - x \tag{25}$$

$$f(x, n) = f(4x - x, n - 1) \tag{26}$$

$$f(S) = ? \tag{27}$$

1.10 **a.** Prove:

$$\sum_{i=1}^{n} 2i - 1 = n^2 \tag{28}$$

Inductive hypothesis:

$$\sum_{i=1}^{n} 2i - 1 = n^2 \rightarrow \sum_{i=1}^{n+1} 2i - 1 = (n+1)^2 \tag{29}$$

$$\tag{30}$$

Induction:

$$n^2 + 2(n+1) - 1 = (n+1)^2 \tag{31}$$
$$n^2 + 2n + 1 = (n+1)^2 \tag{32}$$
$$(n+1)^2 = (n+1)^2 \tag{33}$$
$$\tag{34}$$

b. Prove:

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2 \tag{35}$$
$$= \left(\frac{n(n+1)}{2}\right)^2 \tag{36}$$
$$= \frac{n^2(n+1)^2}{4} \tag{37}$$

Base case:

$$\frac{1^2(1+1)^2}{4} = 1^3 \tag{38}$$
$$\frac{1^2(1+1)^2}{4} = 1 \tag{39}$$
$$\frac{1(2)^2}{4} = 1 \tag{40}$$
$$\frac{4}{4} = 1 \tag{41}$$
$$1 = 1 \tag{42}$$
$$\tag{43}$$

Inductive hypothesis:

$$\frac{n^2(n+1)^2}{4} \rightarrow \frac{(n+1)^2(n+1)+1)^2}{4} \tag{44}$$
$$= \frac{(n+1)^2(n+2)^2}{4} \tag{45}$$

Induction:

$$\frac{n^2(n+1)^2}{4} + (n+1)^3 = \frac{(n+1)^2(n+2)^2}{4} \tag{46}$$
$$\frac{n^2(n+1)^2 + 4(n+1)^3}{4} = \frac{(n+1)^2(n+2)^2}{4} \tag{47}$$
$$\frac{(n+1)^2(n^2+4n+4)}{4} = \frac{(n+1)^2(n+2)^2}{4} \tag{48}$$
$$\frac{(n+1)^2(n+2)^2}{4} = \frac{(n+1)^2(n+2)^2}{4} \tag{49}$$