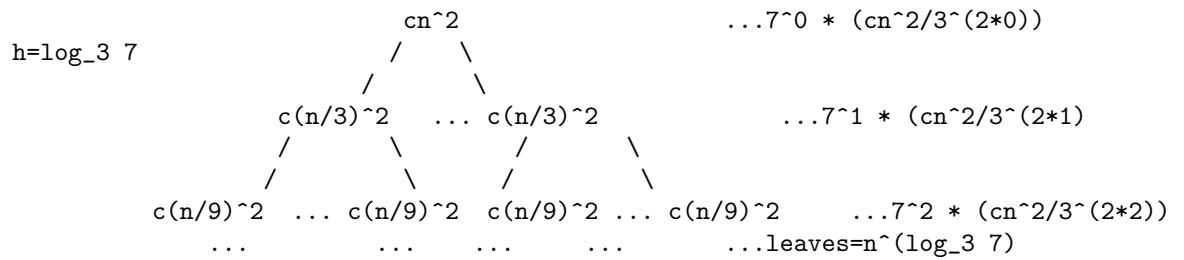


Analysis of Algorithms Final

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- 1 (a) (Each inner node has 7 children)



Width of leaves, each with $T(1) = \Theta(1)$, is $n^{\log_3 7}$. Height of the tree is $\log_3 n$.

The summation form is $T(n) = \sum_{i=0}^h 7^i \frac{n^2}{3^i}$.

(b)

$$\begin{aligned}
 & \sum_{i=0}^{\log_3 n} \frac{7^i cn^2}{3^{i2}} + n^{\log_3 7} \\
 & < cn^2 \sum_{i=0}^{\infty} \frac{7^i}{9^i} + n^{\log_3 7} \\
 & = cn^2 \frac{1}{1 - 7/9} + n^{\log_3 7} \\
 & = 3cn^2 + n^{\log_3 7} \\
 & < 4cn^2 \\
 & = \mathcal{O}(n^2)
 \end{aligned}$$

(c) Induction for \mathcal{O}

Inductive Hypothesis: $T(n) \geq cn^2$

Induction: $T(n) \geq 7c \frac{n^2}{3^2} + dn^2$

$$= cn^2 \frac{7}{9} + dn^2$$

$$\geq cn^2 \quad \text{with } c \leq 4 \text{ and } n \geq 0$$

Induction for Ω

Inductive Hypothesis: $T(n) \leq cn^2$

Induction: $T(n) \leq 7c \frac{n^2}{3^2} + dn^2$

$$= cn^2 \frac{7}{9} + dn^2$$

$$\leq cn^2 \quad \text{with } c \geq \frac{9}{3}d \text{ and } n \geq 0$$

(d)

$$a = 7, \ b = 3$$

$$f(n) = n^2 = \Omega(n^{\log_3 7 + \epsilon})$$

$$\lim_{n \rightarrow \infty} \frac{n^2}{n^{\log_3 7 + \epsilon}} < \infty \text{ (it is polynomially larger)}$$

Regularity condition:

$$7 \frac{n^2}{3^2} \leq cn^2$$

$$\frac{7}{9} n^2 \leq cn^2 \quad \text{for } c \geq 7/9 \text{ and } c \leq 1 \text{ and } n \geq 0$$

(Passes)

Case 3: $\Theta(n^2)$