Languages and Machines

Ch 13: 1,2,3,4,7,8ab, 10, 15, 19, 23, 26

1.

f(1,0) = g(1) = 12 = 1
f(1,1) = h(1,0,f(1,0)) = h(1,0,1) = 2
f(1,2) = h(1,1,f(1,1)) = h(1,1,h(1,0,f(1,0))) = h(1,1,h(1,0,1)) = h(1,1,2) = 4
f(5,0) = g(5) = 52 = 25
f(5,1) = h(5,0,f(5,0)) = h(5,0,25) = 30
f(5,2) = h(5,1,f(5,1)) = h(5,1,h(5,0,f(5,0))) = h(5,1,h(5,0,25)) = h(5,1,30) = 36

2.

a) What does the superscript mean?
The constant “2” can be derived from: s.s.z
b) pred(s(x)) = x
I assume they let us do pattern matching like that.
c) f(x) = add . (mult . (2, p1) , 2)   (we know add, mult, and 2 to be primitive recursive)
Not totally sure on the syntax when it comes to partial application and composition in this system. In haskell it would like like: (+2) . (*2)

3.

a) sg(0) = 0
sg(s(y)) = y
sg is not recursive, so I don't see why would we have to give g and h
b) sub(x, 0) = g(x) = x
sub(x, s(y)) = h(x, y, sub(x,y))
    where h(x,y,z) = pred(z)
c) exp(x,0) = g(0) = 1
exp(x,s(y)) = h(x,y,e(x,y))
    where h(x,y,z) = z * x

4.

a) Prove that total functions are closed under composition, where f is defined by the composition of h and g1 ... gn.

By the definition of composition, the domain of f is the same as the domain of h, which we know is total, so then f must be total. (Did I state this right? Is it just closure of total functions over composition? Is there more to it than this?) The domain of gx is the range of g(x-1), each of which is total.

b) The definition of primitive recursion is f(x1..xn,0) = g(x1..xn) and f(x1..xn,s(y)) = h(x1..xn,y,f(x1..xn,y)). We can prove this by induction. For the base case, f(x1..xn,0) = g(x1..xn), we can see that f, when its last parameter is 0, is equal to the total function g. In the inductive step, where y <= n, we can see that f(x1..xn,s(k)) is equal to the total function h(x1..xn,k,f(x1..xn,k)). (I'm guessing there's probably more to it on this one too, but I don't know where to go with it. The proof seems obvious.)

7.

a) 1/1 L
1/1 R 1/1L
B/B R 1/1 R
B/B L B/B L
B/B L
B/B L

B/B L

b)

f(x) = gt(x,0)*x

8.

a) max(x,y) = gt(x,y)*x + eq(x,y)*x + gt(y,x)*y

b) min(x,y) = lt(x,y)*x + eq(x,y)*x + lt(y,x)*y

15.

[3,0] = 2431

[0,0,1]

[1,0,1,2] = 22315273

[0,1,1,2,0] = 21325273111

19.

f(4) = 6

f(5) = 9

f(6) = 11

(I did these on the white board)

f(0) = gn(1,2,3,4)

f(s(n)) = h(n,gn(n))

h = gn(dec(p2, 2), dec(p2,3), dec(p2,4), (dec(p2,3)+dec(p2,1)))

f' = dec(f(n),1)

23.

a) sg (mz[eq(x,z*z*z)])

b) r(x) = sg (mi[eq(g(i), g(i+x))])

c) l(x) = cosg (mi[gt(x,(g(i)-h(i))]

d) f(x) = sg (mz [eq(x,dec(1,2) + dec(2,2)])

e) f(x) = sg (mz [gt(dec(1,z),x)*gt(dec(2,z),x)*(g(dec(1,z))==h(dec(2,z))])

26.

a) pred

b)

gn(0,0,$2^1 3^2$)

gn(1,1,$2^1 3^2$)

gn(2,2,”)

gn(3,1,”)

gn(4,0,$2^1 3^2$)

gn(0,0,$2^1 3^2 5^2 7^2$)

gn(1,1,$2^1 3^2 5^2 7^2$)

gn(2,2,$2^1 3^2 5^2 7^2$)

gn(5,3,$2^1 3^2 5^2 7^2$)

gn(6,2,$2^1 3^2 5^2 7^2$)

gn(4,1,$2^1 3^2 5^2$)

gn(4,0,$2^1 3^2 5^2$)