

**Logic for Applications**

p 165: 1

p 171: 1, 3, 5(abcdef), 8(abc), 10

**Ch 1****1.**

Using induction:

Basis: if  $G' = \{\sim l\}$  and  $C' = \{l\}$  then  $D' = \{\}$ , which corresponds to  $D$  as desiredInductive hypothesis: assume  $D$  and  $D'$  correspond for all  $G'$  and  $C'$  with literals  $\leq n$ 

Suppose our goal clause  $G'$  has  $n+1$  literals, with the  $n+1$ st literal  $x$  resolving against some literal  $y$  in  $C'$ . We would then get  $D' = G' \cup C' - (n+1 \cup l)$ , which corresponds to  $D$  by the IH. Suppose instead that  $C'$  has  $n+1$  literals -- we would then use the same reasoning to find the correspondence between  $D$  and  $D'$ . Last, suppose that both  $G'$  and  $C'$  have  $n+1$  literals. In this case, we can still use the IH.

This proof is pretty awful, I know, but I'm running out of time in the quarter and will try to return to it later to do a better job.

**Ch 2****1**

Using an indentation tree:

 $\sim eq(c,a)$ 

Path 1:

 $\sim eq(c,a)$ 

Path 1.1:

 $\sim eq(c,Y)$ 

failure

 $\sim eq(Y,a)$ 

failure

failure

Path 2:

 $\sim eq(a,c)$  $\sim eq(a,Y)$  $\sim eq(a,b)$  $\sim eq(b,c)$ 

box!

**3**Goal:  $\sim p(X,Y)$ 

Path 1:

 $\sim q(X,Y)$  $\sim q(a,b)$  $\sim p(b,Z)$  $\sim q(b,Y)$ 

failure

Path 1.2:

 $\sim p(b,b)$  box! ( $X/a$ ,  $Y/b$ )

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**a**

second\_element(X,[\_,X|\_]).

**b**

substitute\_for\_second\_element(Y,[X|Xs],[X,Y|Xs]).

**c**

switch([X|Xs], [L,Zs]) :- get\_last(L,Ls,Xs), append(Ls, [X], Zs).

% first param = the last item

% second param = given list without the last item

% third param = given list

get\_last(L,[],[L]).

get\_last(L,[\_|Xs],[\_|Ys]) :- get\_last(L,Xs,Ys).

**d**

?- second\_element(b,[a,[b,c],d,e]).

false.

?- second\_element(b,[a,b,c]).

true.

?- second\_element([b,c],[a,[b,c],d,e]).

true.

?- substitute\_for\_second\_element([a,[b,c],d],[a,b,c],[a,[b,c],d]).

false.

?- switch([a,b,c],[a,c,b]).

false.

?- switch([a,b,c],[c,b,a]).

true .

**e**

?- second\_element(Z,[a,[b,c],d,e]).

Z = [b, c].

?- second\_element(Z,[a,b,c]).

Z = b.

?- substitute\_for\_second\_element([a,[b,c],d],[a,b,c],Z).

Z = [a, [a, [b, c], d], c].

?- substitute\_for\_second\_element(b,[a,[a,[b,c],d],c],Z).

Z = [a, b, c].

?- switch([a,b,c],Z).

Z = [c, b, a] .

?- switch([c,b,a],Z).

Z = [a, b, c] .

?- second\_element(b,Zs), substitute\_for\_second\_element(b,Zs,[a,b,c]).

Zs = [a, b, c].

?- second\_element(b,Zs), switch(Zs,[a,b,c]).

Zs = [c, b, a] .

**f**

laws:

Substituting a second element for its own second element results in the same list

?- substitute\_for\_second\_element(b,[a,b,c],[a,b,c]).

true.

Switching a list twice yields the original list

?- switch([a,b,c],[c,b,a]), switch([c,b,a],[a,b,c]).

true .

**8****a**

Basis: zero added to any number is zero

IH: assume the recursive step to be true where the second parameter has length  $\leq n$ .

If the second parameter has length  $n+1$  -- that is:  $\text{add}(X, s(Y), s(Z))$  -- then our subgoal becomes  $\text{add}(x, Y, Z)$ , which we know to be addition by the IH.

**b**

$\text{mul}(\_, 0, 0)$ .

$\text{mul}(X, 1, X)$ .

$\text{mul}(A, s(B), C) :- \text{add}(D, A, C), \text{mul}(A, B, D)$ .

**c**

Basis: 0 multiplied with anything is 0. 1 multiplied with anything is that thing.

IH: assume the recursive step to be equivalent to  $mn=r$  where the second param has length  $\leq n$ .

If the second param has length  $n+1$  ( $\text{mul}(A, s(B), C)$ ), then that is true if  $\text{add}(D, A, C)$  and  $\text{mul}(A, B, D)$  are true. Since  $x(y+1) = xy+x$  (or  $A(s(B)) = \text{mul}(A, B, C), \text{add}(C, A)$ ), and by the inductive hypothesis we know that  $\text{mul}(A, B, D) \Leftrightarrow a*b=d$ , then we know  $\text{mul}(A, s(B), C) \Leftrightarrow A*s(B)=C$

**10.**

$\text{fltn}([], [])$ .

$\text{fltn}([X|F], [X|Xs]) :- \text{atomic}(X), \text{fltn}(F, Xs)$ .

$\text{fltn}(F3, [X|Xs]) :- \text{fltn}(F1, X), \text{fltn}(F2, Xs), \text{append}(F1, F2, F3)$ .