Set 4 Homework, Analysis of Algorithms

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- p 194: 8.1-4, p 197: 8.2-4, p 200: 8.3-2, p 204: 8.4-2, p 206: 8.3 or 8.4
- p 215: 9.1-1, p 219: 9.2-1, p 223: 9.3-8 , 9.3-9, p 224: 9-2

Chapter 8

8.1-4 To sort each k sublist, we will use an efficient comparison sort ($\Omega(n\ lg\ n)$).

$$\begin{aligned} T(n) &= k\Omega(n/k\ lg\ n/k) + \Theta(1) \\ &= kc(n/k\ lg\ n/k) + d \\ &= cn\ lg\ n/k + d \\ &\geq cn\ lg\ n/k \\ &\geq cn\ lg\ k \qquad \text{because k is a constant} \end{aligned}$$

Really not sure if I did that right.

8.2-4 First do counting sort up to line 9 ($\mathcal{O}(n+k)$) to get C. Then we get our output with:

$$\begin{aligned} &result := C[a + (a - b)] \\ &result := result - C[a - 1] \quad if\ (a - 1) \geq 0 \end{aligned}$$

Which is $\Theta(1)$. That was an interesting/challenging puzzle.

8.3-2 Insertion sort is stable: if we scan left-to-right, we will insert elements their rightmost insertion slot, meaning that leftmost equal elements remain leftmost and rightmost remain rightmost.

Merge sort is also stable. The merge step scans left to right, and will insert the left element first between two equal elements.

Heap sort is not a stable sort. If you are performing BuildMaxHeap, and your current node is equal to the first left child, and you swap the current node with the parent node, then the child will move to the current position, thus reversing the order of the two equal nodes.

Quicksort is also not stable because of the partition function, which can swap elements out of order.

You can make any sort stable by pairing every element with its unique index. After the sort you can sort the indices of equal elements, for a worst case of also $n\ lg\ n$, which just adds a constant factor.

8.4-2 The worst case occurs when all n the elements go into a single buckeet.

If we simply used n buckets, it basically becomes counting sort and avoids the quadratic worst case.

8.3

8.4

Chapter 9

9.1-1

9.2-1

9.3-8

9.3-9

9.2