Jay Bolton Week 2

Logic for Applications

Ch 4: 1,4,6
Ch 5: 1,2
Ch 6: 6

4.1

(a)

```
     F((a v a) ↔ a)
     /            \
T(a v a)          F(a v a)
 |                 |
F(a)              T(a)
 |                 |
T(a v a)          F(a v a)
 |    \            |
T(a)  T(a)        F(a)
 |     |           |
 X     X          F(b)
                   |
                   X
```

(b)

```
     F((a ^ a) ↔ a)
     /            \
T(a ^ a)          F(a ^ a)
 |                 |
F(a)              T(a)
 |                 |
T(a ^ a)          F(a ^ a)
 |                 |     \
T(a)              F(a)  F(a)
 |                 |     |
T(a)               X     X
 |
 X
```

(c)

```
  F((a ^ b) ↔ (b ^ a))
  /                 \
 T(a ^ b)           F(a ^ b)
  |                  |
 F(b ^ a)           T(b ^ a)
  |                  |
 T(a ^ b)           T(a)
  |                  |
 T(a)               T(b)
  |                  |
 T(b)               F(a ^ b)
  |                  |     \
 F(b ^ a)           F(a)   F(b)
 /     \             |      |
F(b)   F(a)          X      X
|       |
X       X
```

(d)

```
  F((a v b) ↔ (b v a))
  /                 \
 T(a v b)           F(a v b)
  |                  |
 F(b v a)           T(b v a)
  |                  |
 F(b)               T(a)
  |                  |
 F(a)               T(b)
  |                  |
 T(a v b)           F(a ^ b)
  |     \            |     \
 T(a)  T(b)        F(a)    F(b)
  |      |           |      |
  X      X           X      X
```

4.4
(a)

```
  F((a → (b → c)) → ((a ^ b) → c))
   |
  T((a → (b → c))
   |
  F((a ^ b) → c))
   |
  T(a ^ b)
   |
  F(c)
   |
  T(a ^ b)
   |
  T(a)
   |
  T(b)
   |
  T((a → (b → c))
  /               \
 F(a)              T(b → c)
  |                /     \
  X              F(b)  T(c)
                  |      |
                  X      X
```

(b)

```
F(((a ^ b) → c) → (a → (b → c)))
              |
           F((a → (b → c))
              |
           T((a ^ b) → c))
              |
           F((a → (b → c))
              |
           T(a)
              |
           F(b → c)
              |
           T(b)
              |
           F(c)
              |
           T((a ^ b) → c))
           /                    \
       F(a ^ b)               T(c)
        |       \               |
       F(a)    F(b)             X
        |        |
        X        X
```

4.6

```
      F((a → b) ↔ (~b → ~a))
      /                          \
  T(a → b)                   F(a → b)
   |                              |
  F(~b → ~a)                 T(~b → ~a)
   |                              |
  T(~b)                      F(a → b)
   |                                |
  F(~a)                            T(a)
   |                                |
  T(a)                             F(b)
   |                                |
  T(~b)                            T(~b → ~a)
   |                                |           \
  F(b)                             F(~b)      T(~a)
  T(a → b)                          |            |
   |        \                      T(b)        F(a)
  F(a)    T(b)                      |            |
   |        |                       X            X
   X        X
```

Jay Bolton Week 2

5.1

$V(a \wedge b)=T \rightarrow V(a) = T \wedge V(b) = T$ (definition of valuation/truth tables)
Which corresponds to *Figure 9, 2a*, defining the atomic tableaux

$V(a \wedge b)=F \rightarrow V(a)=F \vee V(b)=F$ (def. of valuation/truth tables)
Corresponding to *2b*

$V(\sim a)=T \rightarrow V(a) = F$ (def. of valuation/truth tables)
Corresponding to *3a*

$V(\sim a)=F \rightarrow V(a) = T$ (def. of valuation/truth tables)
Corresponding to *3b*

$V(a \vee b)=T \rightarrow V(a) = T \vee V(b) = T$ (def. of valuation/truth tables)
Corresponding to 4*a*

$V(a \vee b)=F \rightarrow V(a) = F \wedge V(b) = F$ (def. of valuation/truth tables)
Corresponding to 4*b*

$V(a \rightarrow b)=T \rightarrow V(a) = F \vee V(b) = T$ (def. of valuation/truth tables)
Corresponding to *5a*

$V(a \rightarrow b)=F \rightarrow V(a) = T \wedge V(b) = F$ (def. of valuation/truth tables)
Corresponding to *5b*

$V(a \rightarrow b)=F \rightarrow V(a) = T \wedge V(b) = F$ (def. of valuation/truth tables)
Corresponding to *5b*

$V(a \leftrightarrow b) = F \rightarrow (V(a) = T \wedge V(b) = F) \vee (V(a) = F \vee V(b) = T)$ (def. of valuation/truth tables)
Corresponding to *6b*

5.2

$T(A \rightarrow B)$ occurs on P implies FA or TB occurs on P (since it is a finished tableau)
Thus, either $V(A) = F$ or $V(B) = T$ by the induc. hyp.
In either case, $V(A \rightarrow B) = T$ by the definition of valuation (truth tables)

$F(A \rightarrow B)$ occurs on P implies TA and FB occur on P
Thus, $V(A) = T$ and $V(B) = F$ by the induc. hyp.
Thus, $V(A \rightarrow B) = F$ by the definition of valuation (truth tables)

$F(\sim A)$ occurs on P implies TA occurs on P
Thus, $V(A) = T$ by the induc. hyp.

$T(\sim A)$ occurs on P implies FA occurs on P
Thus, $V(A) = F$ by the induc. hyp.

T(A v B) occurs on P implies TA or TB occurs on P
Thus, either V(A) = T or V(B) = T by the induc. hyp.
Thus, V(A v B) = T by the definition of valuation (truth tables)

F(A v B) occurs on P implies FA and FB occur on P
Thus, V(A) = F and V(B) = F by the induc. hyp.
Thus, V(A v B) = F by the definition of valuation (truth tables)

T(A ↔ B) occurs on P implies TA and TB occur on P or FA and FB occur on P
Thus, V(A) = T and V(B) = T or V(A) = F and V(A) = F by the induc. hyp.
Thus, V(A ↔ B) = T by the definition of valuation (truth tables)

F(A ↔ B) occurs on P implies TA and FB occur on P or FA and TB occur on P
Thus, V(A) = T and V(B) = F or V(A) = F and V(B) = T by the induc. hyp.
Thus, V(A ↔ B) = F by the definition of valuation (truth tables)

⊢

6.6
(a) → (c) [6.8]
(c) → (a) [6.6]
(a) ↔ (c)

(b) → (d) [5.3]
(d) → (b) [5.1]
(b) ↔ (d)

proof of (a) ↔ (b)
By definition 6.2, a tableaux proof of “S|=s” is:

Fs
|
T(^S)

While a tableau proof of |=^S → s is:

F(^S → s)
|
Fs
|
T(^S)

Which are equivalent. The rest are equivalent by transitivity.