J Bolton Set 4

Logic for Applications

Section 2, ch 1-3 Ch 2: 1, 2, 3, 4, 6, 8, 9
Ch 3: 1a, 2ab, 3, 4, 6

2.2.1

a, c, e, g

2.2.2

b, e, f

2.2.3

(b) none
(e) R(z, f(w))
(f) R(x, y), P(z), ((Ay)P(z)), (((Ay)P(z)) → R(x, y))

2.2.4

free(R(z, f(w))) = z, w
free(R(x,y)) = x, y
free(P(z)) = z
free(((Ay)P(z))) = z
free((((Ay)P(z)) → R(x, y))) = z, x, y
bound(R(z, f(w))) = nil
bound(R(x,y)) = nil
bound(P(z)) = nil
bound(((Ay)P(z))) = y
bound((((Ay)P(z)) → R(x, y))) = y (only in first subformula)

2.2.6

Every term has the same number of left and right parentheses. Also, every initial segment of a term has greater or equal left parentheses than right. If the length of the initial segment is greater than two, then the number of left parentheses is greater than the right.

<u>Basis:</u>
2.2(i) – a variable has no parentheses. 0 = 0 and 0 <= 0
2.2(ii) – a constant also has no parentheses
<u>Inductive hypothesis:</u>
All terms created from *n* applications of the recursive step (2.2(iii)) have the same number of left and right parentheses
<u>Induction:</u>
A term having n+1 applications of the recursive step is a function f with terms t1...tn as parameters. By the inductive hypothesis, each term t1...tn has an equal number of right and left parentheses – this number we can call x. By 2.2(iii) , one left parenthesis and one right parenthesis has been added in the n+1st recursive step. Therefore, the number of right parentheses is x+1 and the number of left parentheses is x+1, which are equal.

In the n+1 recursive step, a new right parenthesis is added to the end of the string (2.2(iii)). Since each term has equal parens and a left paren has been added before the end, the initial segment with the greatest number of right parens is always equal to to the number of left parens minus one.

2.2.8 and 2.2.9

Every formula has the same number of left and right parentheses. Also, every initial segment of a formula has greater left than right parentheses.

Basis:

The atomic formula has one left paren and one right paren. By 2.2.6, each term in the atomic formula has an equal number of left and right parens.

Also, the intial segments of each term have their left parens greater than or equal to their right parens. Therefore, the initial segment of the atomic formula with the greatest number of right parens compared to left is the segment that includes the whole string except the last right paren. Thus, all initial segments of an atomic formula have less right parens than left.

IH:

The formulas created with n applications of the recursive steps (2.5(ii) and 2.5(iii)) have equal left and right parens and all initial segments have greater left parens than right.

Induction:

By the IH, A and B have equal left and right parens. The application of each of the cases in 2.5(ii) adds one left paren and one right paren, which maintains the equality. The application of the two cases in 2.5(iii) adds two left parens and two right parens, which also maintains the equality.

By the IH, the initial segments of A and B have greater left than right parens. The n+1 application adds either one or two right parens to the end of the string, equal to the number of left parens added before them. Each initial segment of length x adds one left paren before it adds a right paren, owing to that the right parens always come after the left. Therefore, the initial segment with the greatest right parens compared to left will be the segment that includes the whole formula except the last right paren. Since the parens are equal in number, the right parens will always be less by at least one than the left in all initial segments for all formulas.

2.3.3

free variables in each formula, respectively:

c, d

x, y, d

c, d, x, y, d

x, c, d

x, y, second z

2.3.4

Every term t has a unique formation tree associated with it.

Basis:

A term tree of a constant or variable has the root node as that constant or variable and is therefore unique.

IH:

All terms created with n applications of the recursive step have a unique tree.

Induction:

The term t created from n+1 applications has parameters t1...tq, each of which have a unique tree by the inductive hypothesis. The root node is labeled with t and must be unique (2.12). The root node has q children based on its number of parameters, each of which are unique trees.

2.3.6

Every atomic formula is associated with a unique formation tree.
(Note: since atomic formula are not recursively defined, I assume there is no need for induction)
By 2.14, all atomic formula are unique. Thus, the label of the root node of the formulation tree for an atomic formula must be unique. Each of the children are terms, and each have unique trees by 2.3.4