Languages and Machines

Chap 3: 9, 11, 37

9.

$S \to aSc \mid Z$
$Z \to bZc \mid Zc \mid \lambda$

11.

$S \to aSB \mid \lambda$
$B \to b \mid bBa$

37.

L_1	L_2	$L_1 \cup L_2$
$S \to WX$	$S \to YZ$	$S \to WX \mid YZ$
$W \to aWb \mid ab$	$Y \to aY \mid a$	$W \to aWb \mid ab$
$X \to cX \mid c$	$Z \to bZc \mid bc$	$X \to cX \mid c$
		$Y \to aY \mid a$
		$Z \to bYc \mid bc$

$L_1 \cup L_2$ will always be ambiguous because the string *abc* can be generated by either language:

$S \to WX$ $S \to abX$ $S \to abc$	$S \to YZ$ $S \to aZ$ $S \to abc$

Find a CFG over {a,b} that generates the language consisting of strings that have twice as many a's as b's and prove your grammar correct.

$S \to SaSaSbS \mid SaSbSaS \mid SbSaSaS \mid \lambda$

Basis

aab, aba, and baa all clearly have $n_a = 2 * n_b$

Inductive Hypothesis

$n_a = 2 * n_b$ for up to z derivations

Induction

- Suppose we have derived the word *w* with *z* derivations, which satisfies what we are trying to prove by the inductive hypothesis.
- The following rules are possibly derived from *w* for a total of z+1 derivations*:*

Rule	$\mathbf{n_a}$	$\mathbf{n_b}$
$S \to SaSaSbS$	2	1
$S \to SaSbSaS$	2	1
$S \to SbSaSaS$	2	1
$S \to \lambda$	0	0

- In each case, the number of a's is 2* the number of b's as desired.