Chapter 2.6

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I had planned to put the entries in my documents and then cut and paste them into xfig. But xfig did not have cut and paste. So, I'm not doing a figure for the CST. Instead, each path is represented as a sequence.

- 1

$$F(\exists x)(\varphi(x) \vee \psi(x)) \leftrightarrow (\exists x)\varphi(x)\psi(x) \vee (\exists x)\psi(x)$$
$$T(\exists x)(\varphi(x) \vee \psi(x))$$
$$F(\exists x)\varphi(x) \vee (\exists x)\psi(x)$$
$$T(\exists x)(\varphi(x) \vee \psi(x))$$
$$T\varphi(c1)\text{8a: a new constant}$$
$$F(\exists x)\varphi(x) \vee (\exists x)\psi(x)$$
$$F(\exists x)\varphi(x)$$
$$F(\exists x)\psi(x)$$
$$F(\exists x)\varphi(x)\text{8b: any constant}$$
$$F\varphi(c_1)$$
$$*$$

$$F(\exists x)(\varphi(x) \vee \psi(x))$$
$$T(\exists x)\varphi(x) \vee (\exists x)\psi(x)$$
$$F(\exists x)(\varphi(x) \vee \psi(x))$$
$$F(\varphi(c_1) \vee \psi(c_1))\text{any constant}$$
$$F\varphi(c_1)$$
$$F\psi(c_1)$$
$$T(\exists x)\varphi(x) \vee (\exists x)\psi(x)$$
$$\text{this is a branch using atomic tableau 4a}$$
$$T(\exists x)\varphi(x)$$
$$T\varphi(c_2)\text{new constant, unify with any above } c_1 = c_2$$
$$*$$
$$T(\exists x)\psi(x)$$
$$T\psi(c_3)\text{new constant, unify with any above } c_1 = c_3$$
$$*$$

- 12a

$$F\forall x\exists y\neg\forall z\varphi(x,y,z) \leftrightarrow \forall x\exists y\exists z\neg\varphi(x,y,z)$$
this is branch using atomic tableau 6b
$$T\forall x\exists y\neg\forall z\varphi(x,y,z)$$
$$F\forall x\exists y\exists z\neg\varphi(x,y,z)$$
$$T\forall x\exists y\neg\forall z\varphi(x,y,z)$$
$$T\exists y\neg\forall z\varphi(c_1,y,z) \text{ any } c_1$$
$$T\neg\forall z\varphi(c_1,c_2,z) \text{ new } c_2$$
$$F\forall z\varphi(c_1,c_2,z)$$
$$F\varphi(c_1,c_2,c_3) \text{ new } c_3$$
$$F\forall x\exists y\exists z\neg\varphi(x,y,z)$$
$$F\exists y\exists z\neg\varphi(c_4,y,z) \text{ new } c_4$$
$$F\exists z\neg\varphi(c_4,c_5,z) \text{ any } c_5$$
$$F\neg\varphi(c_4,c_5,c_6) \text{ any } c_6$$
$$T\varphi(c_4,c_5,c_6)$$
c_1 any , c_4 new
c_2 new , c_5 any
c_3 new , c_6 any
$*$

$$F\forall x\exists y\neg\forall z\varphi(x,y,z)$$
$$T\forall x\exists y\exists z\neg\varphi(x,y,z)$$
$$F\forall x\exists y\neg\forall z\varphi(x,y,z)$$
$$F\exists y\neg\forall z\varphi(c_1,y,z) \text{ new } c_1$$
$$F\neg\forall z\varphi(c_1,c_2,z) \text{ any } c_2$$
$$T\forall z\varphi(c_1,c_2,z)$$
$$T\varphi(c_1,c_2,c_3) \text{ any } c_3$$
$$T\forall x\exists y\exists z\neg\varphi(x,y,z)$$
$$T\exists y\exists z\neg\varphi(c_4,y,z) \text{ any } c_4$$
$$T\exists z\neg\varphi(c_4,c_5,z) \text{ new } c_5$$
$$T\neg\varphi(c_4,c_5,c_6) \text{ new } c_6$$
$$F\varphi(c_4,c_5,c_6)$$
c_1 new , c_4 any
c_5 new , c_2 any
c_6 new , c_3 any $*$

- 15 This is an infinite tableau, because the entry $T(\forall x)R(x,x)$ will have an i'th occurrence. For every i'th occurrence, we will reduce (finish) the i'th occurence by introducing an $i+1$'th occurrence, as well as a new individual(for any ground term). Using rule 2 from definition 6.7 on p 116, the i'th occurrence is finished when we insert the entry with the ground term as well as the $i+1$'th entry. So we have an infinite tableau, meaning that we have an infinite sequence of finite tableau. If there are entries of the form $T\forall$ or $F\exists$, then each time we reduce such an entry, we introduce a new unreduced entry. However, for any unreduced entry, we will reduce it in a finite number of steps. So we can consider a tableau finished (if infinite) if the only unreduced entries remaining are ones that are of the above form.