Chapter 2.9

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- 5 I'm only going to do one of these. These are very error prone (!) easy to make a transcription mistake. Notice that I assume that the scope of a quantifier was just one atomic formula (so in $\forall y\varphi \vee \psi$ the $\forall$ only applies to the φ. Otherwise it would have been written $\forall y(\varphi \vee \psi)$ or even more verbosely $((\forall y)\varphi)$.)

  – a

$$\begin{aligned}
\forall y(\exists x P(x,y) \to Q(y,z)) \wedge \exists y(\forall x R(x,y) \vee Q(x,y)) &\quad \text{given}\\
\forall w(\exists x P(x,w) \to Q(w,z)) \wedge \exists y(\forall x R(x,y) \vee Q(x,y)) &\quad \text{3a}\\
\forall w(\exists x P(x,w) \to Q(w,z)) \wedge \exists y(\forall u(R(u,y) \vee Q(x,y))) &\quad \text{3a}\\
\forall w(\forall v(P(v,w) \to Q(w,z))) \wedge \exists y(\forall u(R(u,y) \vee Q(x,y))) &\quad \text{4b}\\
\exists t\,[\forall w(\forall v(P(v,w) \to Q(w,z))) \wedge (\forall u(R(u,t) \vee Q(x,t)))] &\quad \text{3b'}\\
\exists t \forall s \forall w(\forall v(P(v,w) \to Q(w,z))) \wedge (R(s,t) \vee Q(x,t)) &\quad \text{3a'}\\
\forall s \forall w(\forall v(P(v,w) \to Q(w,z))) \wedge (R(s,f_t) \vee Q(x,f_t)) &\quad \text{skolem}\\
(P(v,w) \to Q(w,z)) \wedge (R(s,f_t) \vee Q(x,f_t)) &\quad \text{final universal closure}
\end{aligned}$$