J Bolton Set 3

Logic for Applications

Ch 8: 1, 5c, 6c, 6d, 8a, 9a
Last 3 problems from the midterm
Ch10: 2,3

8.1

For any clause C and formula S, there is a resolution deduction of C from S iff C e R(S). In particular, there is a resolution refutation of S iff ☐ e R(S)

=>

Basis: the formula with one clause C: C is a member of R(S) (8.9.1). C is also a member of the proof sequence (8.4). There are no resolvents.
Inductive Hypothesis: for all formulas S with *n* clauses that have a resolution deduction of C_i from S, C_i is in R(S)
Induction: for *n+1* clauses, C_{n+1} is the added clause. If C_{n+1} is an element of S, then there is a resolution deduction of C_{n+1} (8.4). Likewise, C_{n+1} is an element of R(S) according to 8.9.1. If C_{n+1} is a resolvent of C_i and C_j (j, i < n+1), then by the inductive hypothesis, C_i and C_j are in R(S). By rule 8.9.2, C_{n+1} is also a member of R(S) .

<=

Basis: the set R(S) where 8.9.1 has been applied consists of one clause C which is an element of R(S) and S. By 8.4, there is a resolution deduction of C because C is a member of S.
Inductive Hypothesis: for *n* applications of the recursive steps of 8.9, there are resolution deductions of all C in S.
Induction: for n+1 applications of the recursive steps, we have either added a resolvent or a member of S to R(S). If we have added a member of S, then there is a resolution deduction by 8.4. If we have added a resolvent of C_1 and C_2, where C_1 and C_2 are already in R(S), then by the inductive hypothesis, there are deductions of C_1 and C_2. By 8.4, if C is a resolvent of C_1 and C_2,then there is also a deduction of C.

8.5c

~((A ^ B) v (B v C) v (A ^ C))
In conjunctive normal form:
(~A v ~B) ^ ~B ^ ~C ^ (~A v ~C)
In clausal form:
{{~A, ~B}, {~B}, {~C}, {~A, ~C}}

8.6

(c) Not satisfiable – the empty clause is always false, and if any of the clauses are false then S is false.
(d) Again, the empty clause is always false.

8.8a

R({{A}, {B}, {A,B}}) = {{A}, {B}, {A,B}}

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8.9a

```
                         ---{□}---
                       /           \
            ----{A}------      {~A}
          /               \   /     \
   {A, C}               {~C}     {~A, C}
   /     \              /    \
{A, ~B, C}    {B, C}     {B, ~C}   {~B}
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Last three problems from the midterm:

12.

(a) $(aa)^n b^m$ n >= 0, m >= n

(b) $a^n c+(bb)^n$ n >= 0

(c) $(ab)^n(cd)^m(ba)^m(dc)^n$ n >= 0, m>= 0

(d) $a^n c^m a^p b^p d^m b^n$ n >= 0, m > 0, p > 0

(e) $a^n b^m$ n > 0, m >= n, m <= 2*n

13.

Basis: one production: 'b'. 0 <= 0 < 1

Inductive hypthothesis: All words from n productions are in L(G)

Induction: A word of length n satisfies the IH. Adding one terminal to make n+1 production cases includes the following cases, all of which satisfy the inequality 0 <= n < m:

case 1: S → aSb a+1, b+1, 0<=n<m

case 2: B-> bB | b a+0, b+1, 0<=n<m

14. Let G be the grammar

S → aS|Sb|ab

(a) a+b+

(b) S => Sb => aSb => aabb

S => aS => aSb => aabb

(c)

<pre>   ---------S-------
         /   \\
--------S-------   b
/   \\
a   ---------S-------
          /   \\
          a   b</pre>
	<pre>---------S-------
	/ \\
	a ---------S-------
	         / \\
	--------S-------- b
	/ \\
	a b</pre>

(d) S → aS | aZ

Z → bZ | b

10.2

(a)

(i)

A = “Congress refuses to enact new laws”
B = “The strike is over”
C = “The strike lasts more than one year”
D = “President of the firm resigns”

$A \rightarrow (\sim(C \wedge D) \rightarrow \sim B)$

or

$(A \wedge \sim C \wedge \sim D) \rightarrow \sim B$

(ii)

$\sim(A \wedge \sim C \wedge \sim D) \vee \sim B$

$(\sim A \vee C \vee D \vee \sim B)$

(iii)