Analysis of Algorithms Final

Jay R Bolton

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1 ***(a)*** (Each inner node has 7 children)

```
                                  cn^2                          ...7^0 * (cn^2/3^(2*0))
     h=log_3 7                  /    \
                              /        \
                        c(n/3)^2   ... c(n/3)^2             ...7^1 * (cn^2/3^(2*1)
                        /       \        /       \
                      /           \    /           \
                c(n/9)^2  ... c(n/9)^2  c(n/9)^2 ... c(n/9)^2     ...7^2 * (cn^2/3^(2*2))
                   ...           ...     ...     ...        ...leaves=n^(log_3 7)
```

Width of leaves, each with $T(1) = \Theta(1)$, is $n^{log_3 7}$. Height of the tree is $log_3 n$.
The summation form is $T(n) = \sum_{i=0}^{h} 7^i \frac{n}{3^i}^2$.

(b)

$$\sum_{i=0}^{log_3 n} \frac{7^i cn^2}{3^{i2}} + n^{log_3 7}$$
$$< cn^2 \sum_{i=0}^{\infty} \frac{7^i}{9^i} + n^{log_3 7}$$
$$= cn^2 \frac{1}{1 - 7/9} + n^{log_3 7}$$
$$= 3cn^2 + n^{log_3 7}$$
$$< 4cn^2$$
$$= \mathcal{O}(n^2)$$

(c) Induction for $\mathcal{O}$

Inductive Hypothesis: $T(n) \geq cn^2$

Induction: $T(n) \geq 7c\frac{n^2}{3^2} + dn^2$

$$= cn^2 \frac{7}{9} + dn^2$$

$$\geq cn^2 \quad \text{with } c \leq 4 \text{ and } n \geq 0$$

Induction for Ω

Inductive Hypothesis: $T(n) \leq cn^2$

Induction: $T(n) \leq 7c\frac{n^2}{3^2} + dn^2$

$$= cn^2 \frac{7}{9} + dn^2$$

$$\leq cn^2 \quad \text{with } c \geq \frac{9}{3}d \text{ and } n \geq 0$$

(d)

$a = 7,\ b = 3$

$f(n) = n^2 = \Omega(n^{log_3 7 + \epsilon})$

$lim_{n\to\infty} \dfrac{n^2}{n^{log_3 7+\epsilon}} < \infty$ (it is polynomially larger)

Regularity condition:

$7\dfrac{n^2}{3^2} \leq cn^2$

$\dfrac{7}{9}n^2 \leq cn^2 \qquad$ for $c \geq 7/9$ and $c \leq 1$ and $n \geq 0$

(Passes)

Case 3: $\Theta(n^2)$