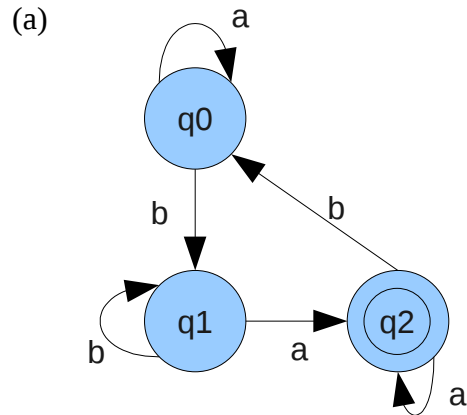


Languages and Machines

Chap 5 (pp 184-186): 1abcd, 12, 13, 22d, 23, 25d.

1.



(b)

| | | | |
|--|---|---|---|
| $[q0, abaa]$ $\vdash [q0, baa]$ $\vdash [q1, aa]$ $\vdash [q2, a]$ $\vdash [q2, \lambda]$ accepts | $[q0, bbbabb]$ $\vdash [q1, bbabb]$ $\vdash [q1, babb]$ $\vdash [q1, abb]$ $\vdash [q2, bb]$ $\vdash [q0, b]$ $\vdash [q1, \lambda]$ rejects | $[q0, bababa]$ $\vdash [q1, ababa]$ $\vdash [q2, baba]$ $\vdash [q0, aba]$ $\vdash [q0, ba]$ $\vdash [q1, a]$ $\vdash [q2, \lambda]$ accepts | $[q0, bbbaa]$ $\vdash [q1, bbbaa]$ $\vdash [q1, baa]$ $\vdash [q1, aa]$ $\vdash [q2, a]$ $\vdash [q2, \lambda]$ accepts |
|--|---|---|---|

(c)
See above.

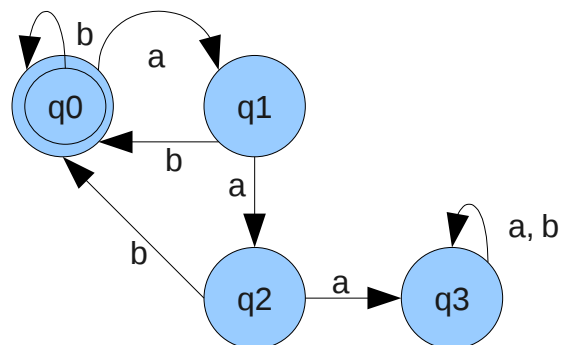
(d)
 $(a^*b+a^+)(ba^*b+a^+)^*$

12.

$Q = \{q0, q1, q2, q3\}$

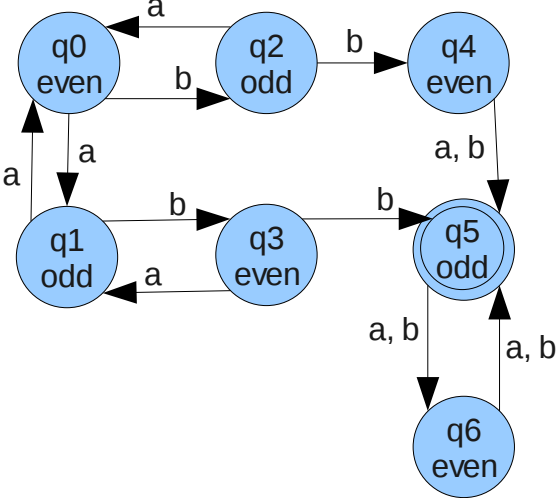
$\Sigma = \{a, b\}$

$F = \{q0\}$



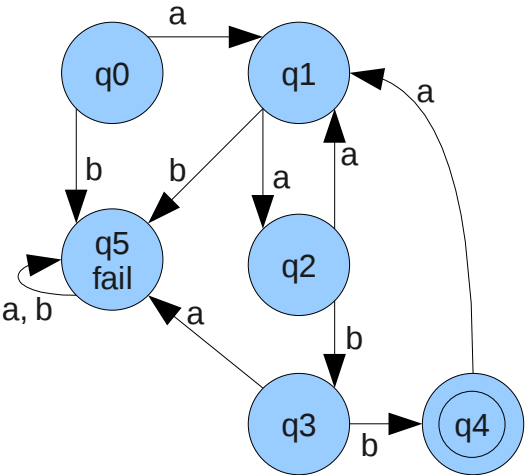
13.

$Q = \{q_0 \dots q_6\}$ $\Sigma = \{a, b\}$ $F = \{q_5\}$



22.

(d)
 $((aa)+bb)^*$
 $Q = \{q_0 \dots q_5\}$
 $\Sigma = \{a, b\}$
 $F = \{q_4\}$



23.

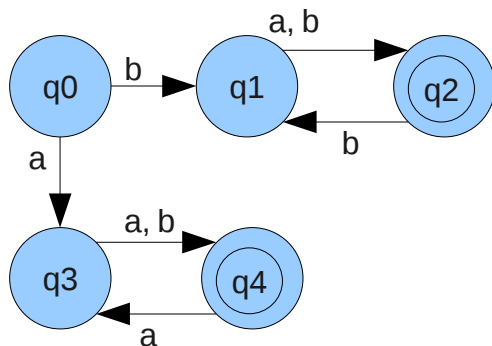
(a)

| t | a | b |
|----|----------|----------|
| q0 | {q0, q1} | 0 |
| q1 | 0 | {q1, q2} |
| q2 | {q0, q1} | 0 |

- (b) $[q_0, aaabb] \vdash [q_0, aabb] \vdash [q_0, abb] \vdash [q_1, bb] \vdash [q_1, b] \vdash [q_2, \text{lambda}]$
- (c) Yes.
- (d) $(a+b)^+$

25.

(d)



Prove that for any DFA M , any two strings u and v , and any state q in M : $\delta^{\wedge}(q, uv) = \delta^{\wedge}(\delta^{\wedge}(q, u), v)$. Hint. Use induction on v .

Basis:

$u = \text{lambda}, v = \text{lambda}$

$\delta^{\wedge}(q, \text{lambda}\text{lambda}) = \delta^{\wedge}(q, \text{lambda}) = q$

$\delta^{\wedge}(\delta^{\wedge}(q, u), v) = \delta^{\wedge}(\delta^{\wedge}(q, \text{lambda}), \text{lambda}) = \delta^{\wedge}(q, \text{lambda}) = q$

$u = \text{lambda}, v$ is an element of Σ and $\text{length}(v) = 1$

$\delta^{\wedge}(q, \text{lambda}v) = \delta^{\wedge}(q, v) = \delta(q, v)$

$\delta^{\wedge}(\delta^{\wedge}(q, \text{lambda}), v) = \delta^{\wedge}(q, v) = \delta(q, v)$

u is an element of Σ and $\text{length}(u) = 1$ and $v = \text{lambda}$

$\delta^{\wedge}(q, u\text{lambda}) = \delta^{\wedge}(q, u) = \delta^{\wedge}(q, u)$

$\delta^{\wedge}(\delta^{\wedge}(q, u), \text{lambda}) = \delta^{\wedge}(q, u) = \delta(q, u)$

Inductive hypothesis:

$\delta^{\wedge}(q, uv) = \delta^{\wedge}(\delta^{\wedge}(q, u), v)$ for all for all strings where the length of v is less or equal to n .

Induction:

if v is length $n+1$, we can call it wa where $\text{length}(a) = 1$. Thus:

$\delta^{\wedge}(q, uwa) = \delta(\delta^{\wedge}(q, uw), a) = \delta(\delta^{\wedge}(\delta^{\wedge}(q, u), w), a)$

$\delta^{\wedge}(q, uwa)$

$=$

$\delta(\delta^{\wedge}(q, uw), a)$ [def. 5.2.4]

$=$

$\delta(\delta^{\wedge}(\delta^{\wedge}(q, u), w), a)$ [Inductive hypothesis]

$\delta^{\wedge}(\delta^{\wedge}(q, u), wa)$

$=$

$\delta(\delta^{\wedge}(\delta^{\wedge}(q, u), w), a)$ [def. 5.2.4]