

# Logic for Applications

Ch 8: 1, 5c, 6c, 6d, 8a, 9a

Last 3 problems from the midterm

Ch10: 2,3

## 8.1

*For any clause C and formula S, there is a resolution deduction of C from S iff  $C \in R(S)$ . In particular, there is a resolution refutation of S iff  $\square \in R(S)$*

=>

*Basis:* the formula with one clause C: C is a member of  $R(S)$  (8.9.1). C is also a member of the proof sequence (8.4). There are no resolvents.

*Inductive Hypothesis:* for all formulas S with  $n$  clauses that have a resolution deduction of  $C_i$  from S,  $C_i$  is in  $R(S)$

*Induction:* for  $n+1$  clauses,  $C_{n+1}$  is the added clause. If  $C_{n+1}$  is an element of S, then there is a resolution deduction of  $C_{n+1}$  (8.4). Likewise,  $C_{n+1}$  is an element of  $R(S)$  according to 8.9.1. If  $C_{n+1}$  is a resolvent of  $C_i$  and  $C_j$  ( $j, i < n+1$ ), then by the inductive hypothesis,  $C_i$  and  $C_j$  are in  $R(S)$ . By rule 8.9.2,  $C_{n+1}$  is also a member of  $R(S)$ .

<=

*Basis:* the set  $R(S)$  where 8.9.1 has been applied consists of one clause C which is an element of  $R(S)$  and S. By 8.4, there is a resolution deduction of C because C is a member of S.

*Inductive Hypothesis:* for  $n$  applications of the recursive steps of 8.9, there are resolution deductions of all C in S.

*Induction:* for  $n+1$  applications of the recursive steps, we have either added a resolvent or a member of S to  $R(S)$ . If we have added a member of S, then there is a resolution deduction by 8.4. If we have added a resolvent of  $C_1$  and  $C_2$ , where  $C_1$  and  $C_2$  are already in  $R(S)$ , then by the inductive hypothesis, there are deductions of  $C_1$  and  $C_2$ . By 8.4, if C is a resolvent of  $C_1$  and  $C_2$ , then there is also a deduction of C.

## 8.5c

$$\sim((A \wedge B) \vee (B \vee C) \vee (A \wedge C))$$

In conjunctive normal form:

$$(\sim A \vee \sim B) \wedge \sim B \wedge \sim C \wedge (\sim A \vee \sim C)$$

In clausal form:

$$\{\{\sim A, \sim B\}, \{\sim B\}, \{\sim C\}, \{\sim A, \sim C\}\}$$

## 8.6

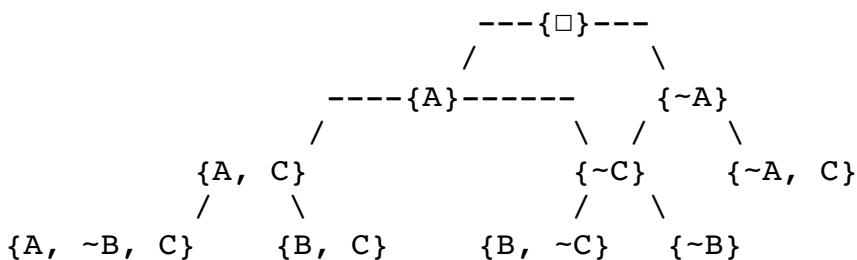
(c) Not satisfiable – the empty clause is always false, and if any of the clauses are false then S is false.

(d) Again, the empty clause is always false.

## 8.8a

$$R(\{\{A\}, \{B\}, \{A, B\}\}) = \{\{A\}, \{B\}, \{A, B\}\}$$

## 8.9a



## Last three problems from the midterm:

12.

- (a)  $(aa)^n b^m$   $n \geq 0, m \geq n$   
 (b)  $a^n c + (bb)^n$   $n \geq 0$   
 (c)  $(ab)^n (cd)^m (ba)^m (dc)^n$   $n \geq 0, m \geq 0$   
 (d)  $a^n c^m a^p b^q d^m b^n$   $n \geq 0, m > 0, p > 0$   
 (e)  $a^n b^m$   $n > 0, m \geq n, m \leq 2 * n$

13.

*Basis:* one production: 'b'.  $0 \leq 0 < 1$

*Inductive hypothesis:* All words from  $n$  productions are in  $L(G)$

*Induction:* A word of length  $n$  satisfies the IH. Adding one terminal to make  $n+1$  production cases includes the following cases, all of which satisfy the inequality  $0 \leq n < m$ :

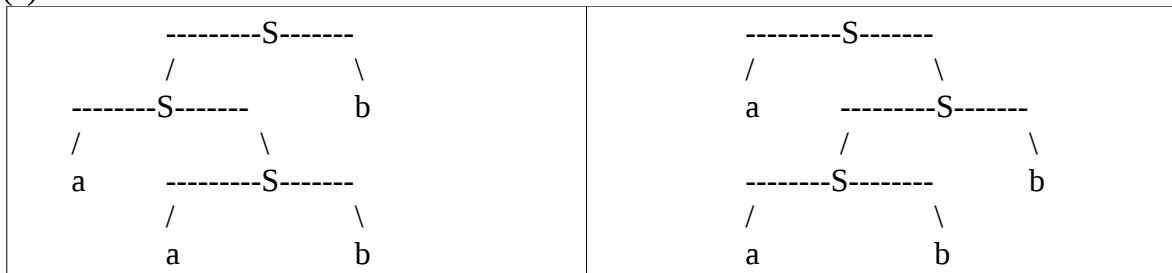
case 1:  $S \rightarrow aSb$   $a+1, b+1, 0 \leq n < m$

case 2:  $B \rightarrow bB \mid b$   $a+0, b+1, 0 \leq n < m$

14. Let  $G$  be the grammar
$$S \rightarrow aS \mid Sb \mid ab$$
(a)  $a^+b^+$ (b)  $S \Rightarrow Sb \Rightarrow aSb \Rightarrow aabb$ 

$$S \Rightarrow aS \Rightarrow aSb \Rightarrow aabb$$

(c)

(d)  $S \rightarrow aS \mid aZ$ 

$$Z \rightarrow bZ \mid b$$

## 10.2

(a)

(i)

A = "Congress refuses to enact new laws"

B = "The strike is over"

C = "The strike lasts more than one year"

D = "President of the firm resigns"

 $A \rightarrow (\sim(C \wedge D) \rightarrow \sim B)$ *or* $(A \wedge \sim C \wedge \sim D) \rightarrow \sim B$ 

(ii)

 $\sim(A \wedge \sim C \wedge \sim D) \vee \sim B$  $(\sim A \vee C \vee D \vee \sim B)$ 

(iii)