Logic ch 3.1

Sherri Shulman
TESC, CS

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Chapter 3.1: Esercises p 165: 1

1 Prove the following lemma: If G is an ordered goal clause, C an ordered program clause, G' and C' the unordered clauses corresponding to G and C, respectively (i.e., the union of the elements in the sequence) and the goal clause D' is an LI-resolvent of G' and C', then there is a sequence of LD-resolutions starting with G nd C and ending with an ordered goal clause D that corresponds to D'.

So suppose we have D', the LI-resolvent of G' and C'. The definition of LI-resolution is a sequence $\langle G_0, C_0\rangle, \cdots, \langle G_n, C_n\rangle, G_{n+1}$ where each G_i is a goal clause, and each C_i is a renaming clause of P, a program clause.

A LI-resolvent step then takes a goal clause and a program clause and produces a goal clause as the union of the literals in the two clauses.

An LI goal clause gives rise to an LD goal clause by specifying an order: $D' = \{\overline{l_1}, \cdots, \overline{l_k}\}$ gives us $D = [\overline{l_1}, \cdots, \overline{l_k}]$.

If the LI resolvent pair is a clause G' and C' where C' and G' each have 1 literal, then $G' = \{\bar{l}\}$ and $C' = \{l\}$ and the LI-resolution gives us $D' = \{\}$. The corresponding LD pair is $[\bar{l}], [l]$ and the empty sequence as the resolvent.

If the LI resolvent pair is a clause where G' has 1 literal and C' has 2. then $G' = \{\overline{l_1}\}, C' = \{\overline{l_2}, l_1\}$, and the resolvent is $D = \{\overline{l_2}\}$. The corresponding LD pair is $[\overline{l_1}], [l_1, \overline{l_2}]$ and the resolvent is $[\overline{l_2}]$ which is an LD goal clause.

If the LI resolvent pair is a clause where G' has 2 literal and C' has 1. then $G' = \{\overline{l_1}, \overline{l_2}\}, C' = \{l_1\}$, and the resolvent is $D = \{\overline{l_2}\}$. The corresponding LD pair is $[\overline{l_1}, \overline{l_2}], [\overline{l_1}]$ and the resolvent is $[\overline{l_2}]$ which is an LD goal clause.

Assume that for LI-resolvent pairs, each of which has k or fewer literals, that the property holds.

Now, we look at LI-resolvent pairs where either the goal clause, the program clause or both have $k+1$ clauses.

Case 1: Suppose that $G' = \{\overline{l_1}, \cdots, \overline{l_k}\} \cup \{\overline{l_{k+1}}\}$, and $C' = \{l'_1, \overline{l'_2}, \cdots, \overline{l'_k}\}$ Now there are 2 cases. Either the reolvent l'_1 resolves with one of the k literals OR it resolves with $\overline{l_{k+1}}$. If it resolves with one of the k, then this is the IH, and we get $D' = \{\overline{l_1}, \cdots, \overline{l'_2}, \cdots, \overline{l'_k}, \cdots \overline{l_k}\} \cup \{\overline{l_{k+1}}\}$ And D the LD resolvent as specified by the IH, with the new literal added at the end.

If l'_1 resolves with $\overline{l_{k+1}}$ then this resolvent is covered by te IH, and the resulting LD clause is the clause specified by G' and the resolution lf $\{\overline{l_{k+1}}\}$ and C'.

Case 2: Suppose that C' has $k+1$ literals: $C' = \{l'_1, \overline{l'_2}, \cdots, \overline{l'_k}\} \cup \{\overline{l'_{k+1}}\}$ OR $C' = \{\overline{l'_1}, \cdots, \overline{l'_k}\} \cup \{l_{k+1}\}$. In the first case, we just have the inductive hypothesis, one of the l_i literals in G' is the resolvent. In the second case, one of the l_i literal resolves with $\{l_{k+1}\}$ so we also have the inducxtsive hypothesis, with a clause of k literals and one of 1 literal.

Case 3: Both the goal and program clause have $k+1$ literals. Similar to above.

The only piece missing is that if we have an LI program clause and goal clause, giving rise to an LI clause as the resolution, which determines an LD clause, the union needs to gives rise to an LD clause. Ie, if we have G' resolving with $C' \cup \{l_{k+1}\} = D' \cup \{l_{k+1}\}$ which gives us the sequence $D ++ [l_{k+1}]$. Should demonstrate for all the cases. I'll leave that up to you.