Logic ch 2.13

Sherri Shulman
TESC, CS

Mar 9, 2011

Chapter 2.13: Esercises p 151: 1,5

1 a) $\{P(x,y), P(y,z)\}, \{\neg P(u, f(u))\}$ We'll resolve on the first term: $P(x,y)$ and $\neg P(u, f(u))$. So we'll substitute $[x/u, y/f(u)]$ giving us $\{P(f(u), z)\}$. There are no further resolvents since u cannot resolve with $f(u)$.

b) $\{P(x,x), \neg R(x, f(x))\}, \{R(x,y), Q(y,z)\}$ We'll resolve on $R(x,y)$ and $\neg R(x, f(x))$. This gives the substitution $[y/f(x)]$. Applying the substitution gives us $\{P(x,x), Q(f(x), z)\}$. There are no further resolvents.

c) $\{P(x,y), \neg P(x,x), Q(x, f(x), z)\}, \{\neg Q(f(x), x, z), P(x,z)\}$. We'll resolve on $Q(x, f(x), z)$ and $\neg Q(f(x), x, z)$. The only substitution would require us to have x substituted for by $f(x)$, which fails the occurs check. What if we resolve on $\neg P(x,x)$ and $P(x,z)$. Then we have te substitution $[x/z]$ giving us: $\{P(z,y), Q(z, f(z), z), \neg Q(f(z), z, z)\}$. There are no further resolvents because of the occurs check. Note that even if we change the variables that we still run into the occurs check. So if we change the first clause to $P(x_1, y), \neg P(x_1, x_1), Q(x_1, f(x_1), z)\}$. In that case we get the substitution $[x_1/f(x)][x/f(x_1)] = [x_1/f(f(x_1))]$.

5 Now we have 5 clauses in our set:

(i) $\{P(a, x, f(y)), P(a, z, f(h(b))), \neg Q(y,z)\}$

(ii) $\{\neg Q(h(b), w), (w,a)\}$

(iii) $\{\neg P(a, w, f((b))), H(x,a)\}$

(iv) $\{P(a, u, f(h(u))), H(u,a), Q(h(b), b)\}$

(v) $\{\neg H(v,a)\}$

We'll start by choosing clause (i) and (iii) with the substitution $[x/w, y/h(b)]$. After we apply the substitution we have the clause: (vi) $\{P(a, z, f(h(b))), \neg Q(h(b), z), H(w,a)\}$. Now we'll choose (iv) and (vi). This gives us a substitution $[z/u][u/b]$ and then applying that we get: (vii) $\{H(b,a), H(w,a)\}$. Now we can resolve (vii) and (v) which gives us the substitution $[v/b]$ and applying that we get : (viii) $\{H(w,a)\}$. Now we can resolve (viii) and (v) again to get the substitution $[w/v]$, giving us the empty clause. (Note that I wiped out two terms in the second resolution step: $\neg Q(h(b), b)$ and $Q(h(b), b)$ as well as the two terms that we explicitly resolved on.