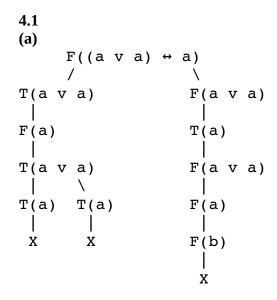
Jay Bolton Week 2

Logic for Applications

Ch 4: 1,4,6 Ch 5: 1,2 Ch 6: 6



Jay Bolton Week 2

(d)
$$F((a \ v \ b) \leftrightarrow (b \ v \ a)) \\ / \\ T(a \ v \ b) & F(a \ v \ b) \\ | \\ F(a \ v \ b) & | \\ F(b \ v \ a) & T(b \ v \ a) \\ | \\ F(b) & T(a) & | \\ | \\ F(a) & T(b) & | \\ | \\ T(a \ v \ b) & F(a \ ^b) \\ | \\ T(a) & T(b) & F(a) & F(b) \\ | \\ X & X & X & X & X \\$$

T(b) F(a)

X

X

F(a) T(b)

X X

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5.1
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 $V(a \land b)=T \rightarrow V(a) = T \land V(b) = T$ (definition of valuation/truth tables) Which corresponds to *Figure 9*, *2a*, defining the atomic tableaux

 $V(a \land b)=F \rightarrow V(a)=F \lor V(b)=F$ (def. of valuation/truth tables) Corresponding to 2b

 $V(\sim a)=T \rightarrow V(a)=F$ (def. of valuation/truth tables) Corresponding to 3a

 $V(\sim a)=F \rightarrow V(a)=T$ (def. of valuation/truth tables)

Corresponding to 3b

 $V(a \ v \ b)=T \rightarrow V(a)=T \ v \ V(b)=T$ (def. of valuation/truth tables) Corresponding to 4a

 $V(a \ v \ b)=F \rightarrow V(a)=F \land V(b)=F$ (def. of valuation/truth tables) Corresponding to 4b

 $V(a \rightarrow b)=T \rightarrow V(a)=F \ v \ V(b)=T$ (def. of valuation/truth tables) Corresponding to 5a

 $V(a \rightarrow b)=F \rightarrow V(a)=T \wedge V(b)=F$ (def. of valuation/truth tables) Corresponding to 5b

 $V(a \rightarrow b)=F \rightarrow V(a)=T \land V(b)=F$ (def. of valuation/truth tables) Corresponding to 5b

 $V(a \leftrightarrow b) = F \rightarrow (V(a) = T \land V(b) = F) \lor (V(a) = F \lor V(b) = T)$ (def. of valuation/truth tables) Corresponding to 6b

5.2

 $T(A \rightarrow B)$ occurs on P implies FA or TB occurs on P (since it is a finished tableau) Thus, either V(A) = F or V(B) = T by the induc. hyp. In either case, $V(A \rightarrow B) = T$ by the definition of valuation (truth tables)

 $F(A \rightarrow B)$ occurs on P implies TA and FB occur on P Thus, V(A) = T and V(B) = F by the induc. hyp. Thus, $V(A \rightarrow B) = F$ by the definition of valuation (truth tables)

 $F(\sim A)$ occurs on P implies TA occurs on P Thus, V(A) = T by the induc. hyp.

 $T(\sim A)$ occurs on P implies FA occurs on P Thus, V(A) = F by the induc. hyp.

T(A v B) occurs on P implies TA or TB occurs on P

Thus, either V(A) = T or V(B) = T by the induc. hyp.

Thus, $V(A \lor B) = T$ by the definition of valuation (truth tables)

F(A v B) occurs on P implies FA and FB occur on P

Thus, V(A) = F and V(B) = F by the induc. hyp.

Thus, $V(A \lor B) = F$ by the definition of valuation (truth tables)

 $T(A \leftrightarrow B)$ occurs on P implies TA and TB occur on P or FA and FB occur on P

Thus, V(A) = T and V(B) = T or V(A) = F and V(A) = F by the induc. hyp.

Thus, $V(A \leftrightarrow B) = T$ by the definition of valuation (truth tables)

 $F(A \leftrightarrow B)$ occurs on P implies TA and FB occur on P or FA and TB occur on P

Thus, V(A) = T and V(B) = F or V(A) = F and V(B) = T by the induc. hyp.

Thus, $V(A \leftrightarrow B) = F$ by the definition of valuation (truth tables)

 \vdash

6.6

- (a) \rightarrow (c) [6.8]
- (c) \rightarrow (a) [6.6]
- (a) \leftrightarrow (c)
- (b) \rightarrow (d) [5.3]
- (d) \rightarrow (b) [5.1]
- (b) \leftrightarrow (d)

proof of $(a) \leftrightarrow (b)$

By definition 6.2, a tableaux proof of "S|=s" is:

While a tableau proof of $|= \land S \rightarrow s$ is:

$$F(^{S} \rightarrow s)$$

$$|$$

$$Fs$$

$$|$$

$$T(^{S})$$

Which are equivalent. The rest are equivalent by transitivity.