J Bolton Week 3

Languages and Machines

Chap 2 (pp58-60): 1,5,6,10,12,13,14,23

1.
l(empty) = 0
l(wa) = 1 + l(w)

5.
(a)
L_0 = {b}
L_1 = L_0 U {bb, bab, bba}
L_2 = L_0 U L_1 U {bbb, babab, bbaba, bbbaa}
(b)
No – you can never have the same number of a's as b's
(c)
No – for the same reason above

6.
Basis
b is an element of L
Rec.
if u is an element of L
then aau, ua, ub are elements of L

10.
Basis
0 >= 0
Inductive Hypothesis
na(u) >= nb(u)
Induction
case 1: uab
In this case, we have added one a and one b. thus:
na(u) + 1 >= nb(u) + 1
case 2: ua
In this case, wehave added one a and zero b's. Thus:
na(u) + 1 >= nb(u)
In both cases, the equality holds.

12.
All elements from the set defined by the first definition are elements in the set defined by the second definition, and vice versa.
First proof: all elements of first def are elements of second def
Basis:
lambda = lambda^R and a=a^R by the def. of reversal.
IH:

$w = w^R$
Induction:
$(awa)^R =$
$(a(wa))^R =$ [associativity]
$((wa)^R a^R) =$
$a^R w^R a^R =$
$aw^R a =$ [theorem 2.1.6]
awa [Inductive hypthesis]

Second proof: all elements of second def are elements of first def
Basis
lambda =lambda^R is the first element of the second definition
lambda is also in the base case of the first definition
IH
$w = w^R = aua$ where length(w) = n
Induction
length(wa) = n+1
n = head(w)
$(nwa)^R = nwa$ [2nd definition]
w = nun [IH]
un = w'
$(nw'a)^R =$
$(n(wa))^R =$
$(wa)^R n^R =$
$a^R w^R n^R =$
awn
thus, nwa = awn
a = n

13.
L_2: Four characters, each either an a or b
L_3: multiples of four characters, with the characters any order
L_1 U L_3: any number of sequences of three a's interspersed with any number of sequences of length four with any pattern of characters

14.
a*b*c*

23.
abbcc