Sources of capital growth

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ABSTRACT

Data from national accounts show no effect of change in net saving or consumption, in ratio to market-value capital, on change in growth rate of market-value capital (capital acceleration). Thus it appears that capital growth and acceleration arrive without help from net saving or consumption restraint. We explore ways in which this is possible, and discuss implications for economic teaching and public policy.

1. Introduction and overview

Many economists over the centuries have reasoned that net saving, or equivalently net investment[1], should tend to give equal capital growth. Economists since the early nineteenth century have added the proviso that net saving cannot safely outpace innovation; more capital must mean capital redesigned for greater productivity if economies are to escape risk of capital glut and diminishing returns (West (1815), Ricardo (1815), Malthus (1815)). Roy Harrod (1939) described that limit for safe net saving, meaning the rate of imagining and developing new ideas for more productive forms of capital, as the "warranted rate". Harrod, and many other economists of his time and since, have focused on growth of output rather than of capital, but have modeled growth of output by first assuming the equivalence of net saving and capital growth, within the warranted rate, and then looking for effects of that capital growth on later output growth.

Some other economists, including John Rae (1834) and John Stuart Mill (1848) (see Appendix C), argued that capital growth might also be explained by a rise in productivity of capital and labor already extant. Ways might be found for existing factors to produce more, that is, and so to allow more consumption, or more capital growth, or any mix of the two, without inputs of net saving. Robert Solow (1957) allowed that possibility for "disembodied" growth, where plant and products already existing are repurposed or redeployed in more productive ways.

We test between those two explanations of capital growth, by net saving or by increase in productivity of capital and labor already in existence, by comparing net saving to concurrent change in market-value capital in 86 countries. As changes in net saving are expected to be associated with opposite changes in consumption, we also compare change in consumption to concurrent change in capital growth (capital acceleration). All data are drawn from national accounts of those countries as collated on the free website World Inequality Database.

Those test results from data for net saving/capital ratio and for consumption/capital ratio are easiest to reconcile in an interpretation that change in neither affects change in capital growth rate.[2] This finding would support the views of Rae and Mill, and of Solow as to disembodied growth. It would suggest that capital growth, even in acceleration, arrives without help from net saving or consumption restraint. Net saving, if so, raises the physical quantity of capital, but not the aggregate value, and so reduces the value per unit.

The data are most easily explained by the present value principle, and by production efficiencies enabled through innovation. Value is created in the mind of the market at the moment when prospective cash flows are discounted. It is created only if the market sees a path, step by step, from the start, to practical realization of those prospective cash flows. Then capital growth arrives when the market first evaluates prospective cash flows, and is realized eventually in physical outcomes insofar as the market has predicted correctly. That is to say that capital growth, in the mind of the market, precedes and pays for any needed investment in new plant and equipment.

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[1]As reported in national accounts; they differ only by statistical discrepancy. Also see Appendix A.
[2]Conditional on investment of replacement saving to offset depreciation first. See Section ?? below.

Meanwhile the innovator acquires materials and plant capacity and labor skills at market prices determined by their uses in current technology, but applies them more productively until competition catches up. It is that temporary market advantage to the innovator which explains capital growth without net saving in a practical and mechanical sense, while the present value principle gives the explanation in terms of market valuation. This idea will be called "free growth theory" for easy reference.

It predicts only at the largest scales, and only for the private sector. Individuals and groups and even small economies can grow through investment from outside. That possibility is foreclosed only at the scale of all capital and all economies together. The public sector, meanwhile, responds to political rather than market choices, and grows or shrinks accordingly.

If free growth theory is right, tax policy and other policy to encourage saving over consumption should be reviewed. These policies include the higher tax on ordinary income than on capital gains, and the double tax on corporate dividends.

Inferences for economic teaching include the obvious ones for growth theory and for net saving in general. They include others as well. One of the central doctrines of the marginalist revolution has held that market realization converges to producer cost, when that cost includes imputed interest on assets owned. Net saving gives producer cost, and falls short of market realization in the presence of technological growth from new ideas. Meanwhile the doctrine that net income equals consumption plus net saving is put into question by evidence offered here suggesting that net saving increases the physical quantity of capital, but not the aggregate value. In general, economics might consider relying less on book value and net investment, and more on market value and on the power of ideas.

2. Net saving, capital growth and capital acceleration

Thrift theory will mean the combined ideas that all net saving S_{net} is realized in equal growth of capital K, if S_{net} holds within the warranted rate, and that there is no other source of capital growth except market noise regressing to zero. "Thrift assumptions" will mean those two plus the assumptions that current S_{net} holds within the warranted rate. The acronym uta will mean "under thrift assumptions." Then

$$\Delta K = S_{net} , \quad \text{uta.} \tag{1}$$

Define "thrift" s^* by $s^* = \frac{S_{net}}{K}$, and divide Eq. (1) by capital to predict capital growth rate $g(K)$ as

$$g(K) = s^* , \quad \text{uta, where } s^* = \frac{S_{net}}{K}. \text{ Then} \tag{2}$$

$$\Delta g(K) = \Delta s^* , \quad \text{uta.} \tag{3}$$

$\Delta g(K)$ and Δs^* respectively will be called "capital acceleration" and "thrift change," either of which may be positive or negative or zero. Division of Eq. (3) by capital acceleration, with rearrangement, gives

$$\frac{\Delta s^*}{\Delta g(K)} = \frac{\Delta g(K)}{\Delta g(K)} = 1 , \quad \text{uta.} \tag{4}$$

The argument has continued from growth in Eq. (2) to acceleration in Eqs. (3) and (4) on the reasoning that causal relationships among variables, in this case between thrift and capital growth rate, tend to be revealed most reliably in their concurrent changes.[3] Eq. (4) was derived from Eq. (3) so that success of predictions from growth assumptions can be measured against a standard of unity. For notational convenience, then, we define the "thrift index" θ by $\theta = \frac{\Delta s^*}{\Delta g(K)}$, and restate Eq. (4) as

$$\theta = 1 , \quad \text{uta, where } \theta = \frac{\Delta s^*}{\Delta g(K)} . \tag{5}$$

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[3]The ratio $\frac{\Delta A}{\Delta B}$, that is, compares changes in A to concurrent changes in B, and so suggests a causal relationship between A and B insofar as that ratio holds constant. This is the principle on which the concepts of correlation, covariance and regression are founded.

This inference from thrift assumptions can be tested directly through the argument above from data for net saving, and indirectly from an argument from consumption to net saving in Section 3. θ_s and θ_c will show the expressions for testing the prediction $\theta = 1$ from data for net saving and consumption respectively. Then θ_s becomes $\frac{\Delta s^*}{\Delta g(K)}$, from Eq. (5), while θ_s^* and θ_c^*, when θ_c is defined, give test results for those predictions.

Tests Eq. (2), however, which predicts $\frac{s^*}{g(K)} = 1$, uta, as a step in the argument to Eq. (4), will be taken first. Fig. 1 shows $\frac{s^*}{g(K)} = 0.481$ as average over all years and countries, GDP-weighted. Regression of s^* on $g(K)$ shows as 0.1857 in Table 1.

Tests of the prediction $\theta_s^* = 1$ in Eq. (4) should give a clearer picture, again, as it tests in accelerations. Fig. 2 shows findings for θ_s^* at 0.064 over the same period and countries. Table 3 shows regression of Δs^* on $\Delta g(K)$ at 0.1000.

Discussion of these findings, and of ways to predict them other than by thrift theory, will follow tests of the prediction for θ, from data for concurrent rather than net saving.
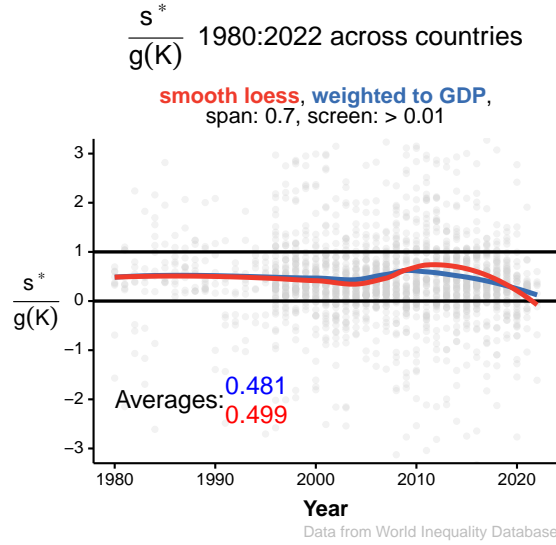
Fig. 1: $\frac{s^*}{g(K)}$ (86 countries)

Table 1
Regression of s^* on $g(K)$ (Screen = 0.01). H_0 per thrift theory: $g(K) = s^*$ & $\frac{s^*}{g(K)} = 1$.

	s^*
Regression of s^* on $g(K)$	0.1857***
	(0.0423)
Observations	1,826
R^2	0.77165
Within R^2	0.13962
Year fixed effects	✓
Country fixed effects	✓

Table 2

Average $\frac{s^*}{g(K)}$ in 86 countries (screen = 0.01). Number of years clearing screen shown in ()

Country	Period	$\frac{s^*}{g(K)}$	Country	Period	$\frac{s^*}{g(K)}$
Armenia	1996 - 2020 (23)	-0.63	Italy	1980 - 2022 (36)	0.13
Aruba	1996 - 2001 (6)	1.25	Japan	1980 - 2021 (29)	0.19
Australia	1980 - 2018 (33)	0.24	Kazakhstan	1997 - 2022 (25)	0.60
Austria	1996 - 2021 (23)	0.83	Kuwait	2003 - 2017 (14)	1.11
Azerbaijan	1996 - 2020 (23)	1.57	Kyrgyzstan	1996 - 2021 (24)	0.46
Bahrain	2009 - 2013 (5)	-2.43	Latvia	1996 - 2021 (24)	-0.23
Belgium	1996 - 2021 (21)	0.61	Lithuania	1996 - 2021 (24)	0.13
Bolivia	1998 - 2015 (17)	0.40	Luxembourg	1996 - 2021 (22)	0.03
Botswana	1996 - 1999 (4)	3.54	Malaysia	2007 - 2015 (9)	2.52
Brazil	1996 - 2019 (21)	0.25	Malta	1996 - 2021 (23)	0.16
British Virgin Islands	1996 - 1999 (4)	2.45	Mauritius	2014 - 2018 (5)	0.04
Bulgaria	1996 - 2016 (16)	0.14	Mexico	1996 - 2021 (22)	0.15
Burkina Faso	2000 - 2019 (20)	0.23	Moldova	1996 - 2019 (22)	-0.91
Cameroon	1996 - 2019 (24)	1.10	Mongolia	2006 - 2020 (15)	0.10
Canada	1980 - 2022 (38)	0.37	Morocco	1999 - 2021 (23)	1.13
Cape Verde	2008 - 2017 (9)	0.53	Netherlands	1996 - 2021 (23)	0.51
Chile	1997 - 2021 (19)	0.84	New Zealand	1996 - 2019 (21)	0.78
China	1992 - 2020 (29)	0.81	Nicaragua	2007 - 2018 (12)	0.08
Colombia	1996 - 2022 (27)	0.60	Niger	1996 - 2019 (24)	0.90
Costa Rica	2013 - 2020 (6)	0.21	Norway	1981 - 2021 (38)	0.95
Croatia	1996 - 2021 (24)	0.43	Peru	2008 - 2021 (14)	0.76
Curaçao	2001 - 2016 (14)	1.26	Philippines	1996 - 2022 (27)	1.76
Cyprus	1996 - 2021 (25)	0.07	Poland	1996 - 2021 (25)	0.71
Czechia	1994 - 2020 (20)	0.17	Portugal	1996 - 2022 (24)	0.00
Côte d'Ivoire	1996 - 2000 (5)	0.29	Qatar	2002 - 2017 (14)	1.89
Denmark	1996 - 2022 (26)	0.22	Romania	1996 - 2020 (22)	0.03
Dominican Republic	1996 - 2016 (10)	1.52	Russia	1996 - 2017 (13)	-0.19
Ecuador	2008 - 2020 (13)	0.53	Saudi Arabia	2003 - 2009 (7)	2.65
Egypt	1997 - 2015 (19)	0.92	Senegal	2015 - 2021 (7)	0.24
El Salvador	2015 - 2019 (5)	-0.62	Serbia	1998 - 2021 (23)	0.61
Estonia	1996 - 2021 (24)	0.45	Slovakia	1996 - 2022 (24)	0.15
Finland	1996 - 2021 (23)	0.17	Slovenia	1996 - 2021 (20)	0.28
France	1980 - 2021 (35)	0.26	South Africa	1996 - 2022 (24)	0.28
Germany	1980 - 2022 (38)	0.70	South Korea	1996 - 2020 (24)	0.63
Greece	1996 - 2021 (26)	0.17	Spain	1995 - 2021 (25)	0.32
Guatemala	2002 - 2021 (20)	-0.72	Sweden	1980 - 2021 (37)	0.41
Guinea	2004 - 2010 (5)	0.72	Switzerland	1993 - 2021 (25)	0.69
Honduras	2001 - 2015 (14)	-0.57	Tunisia	1996 - 2011 (16)	0.29
Hong Kong	1997 - 2011 (15)	0.90	Turkey	2010 - 2017 (8)	1.32
Hungary	1996 - 2021 (21)	0.14	USA	1980 - 2021 (38)	0.31
Iceland	2001 - 2014 (14)	0.22	Ukraine	1996 - 2019 (21)	0.17
India	1999 - 2019 (21)	0.57	United Kingdom	1981 - 2021 (36)	0.16
Indonesia	2017 - 2019 (3)	0.94	Uruguay	2016 - 2017 (2)	1.16
Iran	1997 - 2017 (20)	4.96	Uzbekistan	2011 - 2021 (10)	0.21
Ireland	1996 - 2021 (22)	0.26	Vanuatu	2002 - 2007 (6)	0.29
Israel	1996 - 2019 (24)	0.46	Venezuela	1998 - 2019 (20)	0.55

Note: Thrift theory predicts $\frac{s^*}{g(K)}$. Free growth theory makes no prediction for these data.
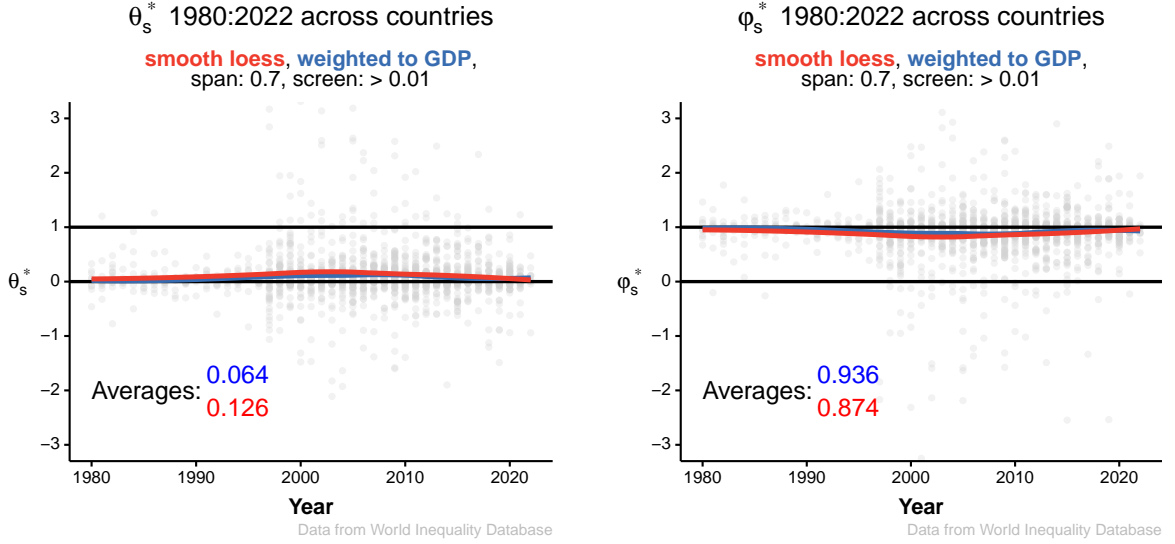
Fig. 2: Thrift and free growth indexes derived from net saving (86 countries).

Table 3

Regression of Δs^* and Δq_s on $\Delta g(K)$ (Screen = 0.01). H_0 per thrift theory: $\Delta g(K) \cong \Delta s^*$ & $\Delta q_s \cong 0$

	Δs^*	Δq_s
Regression of value shown on $\Delta g(K)$	0.1000***	0.9000***
	(0.0145)	(0.0145)
Observations	1,574	1,574
R^2	0.32867	0.96233
Within R^2	0.22713	0.95965
Year fixed effects	✓	✓
Country fixed effects	✓	✓

3. Consumption and capital growth

Thrift theory and most teaching reason that saving is drawn from potential consumption, in that skimping on replacement saving would have afforded more consumption, and that net saving, after replacement saving has offset depreciation, is drawn from actual consumption as its only possible source. Both conclusions follow from definitions. Then $\Delta S_{net} = -\Delta C$. Divide by capital, and define $c^* = \frac{C}{K}$, to find

$$\Delta s^* = -\Delta c^*, \quad \text{where } c^* = \frac{C}{K}, \tag{6}$$

independently of thrift assumptions.[4] Divide by capital acceleration to see

$$\frac{\Delta s^*}{\Delta g(K)} = -\frac{\Delta c^*}{\Delta g(K)}, \tag{7}$$

again without dependence on assumptions. Substitution of this in Eq. (5) shows

$$\theta = 1 \quad \text{uta}, \quad \text{where } \theta = -\frac{\Delta c^*}{\Delta g(K)}. \tag{8}$$

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[4]The asterisk, again, distinguishes $c^* = \frac{C}{K}$ from the Keynesean $c = \frac{C}{Y}$.

Then let θ_c, the value of θ reasoned from effects of changes on consumption, show as $\theta_c = -\frac{\Delta c^*}{\Delta g(K)}$, and let θ_c^* mean values of θ_c found in tests.

Test results appear in Fig. (3). θ_c^*, predicted to find $\theta_c^* = 1$, under thrift assumptions, shows at -0.142. As it tests of θ_s, findings were averaged over all years and countries weighted to GDP. Table 1 shows regression of $-\Delta c^*$ on $\Delta g(K)$ as -0.3760.
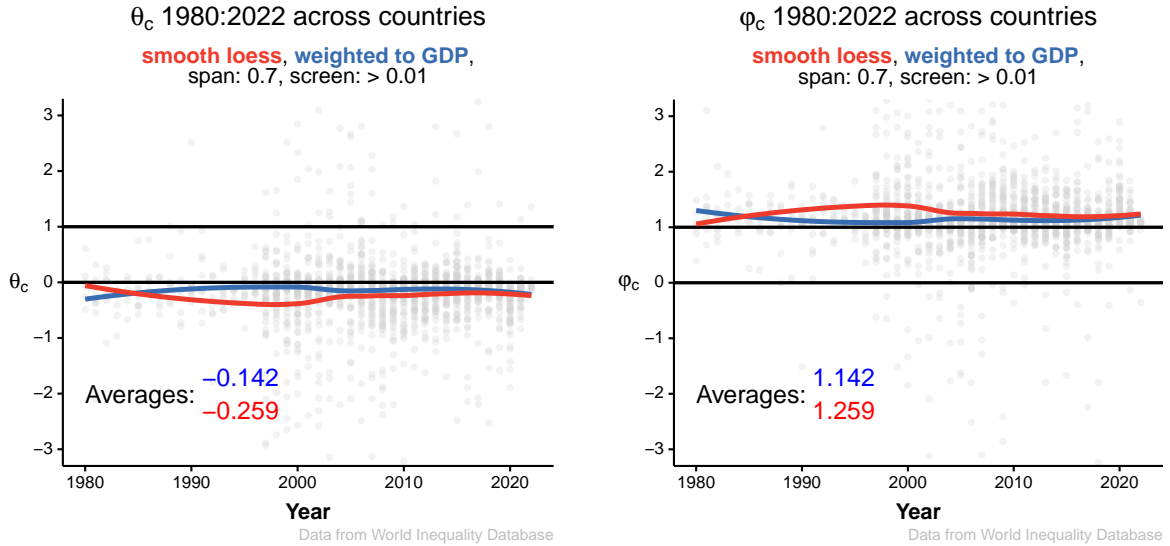

Fig. 3: Thrift and free growth indexes derived from consumption (86 countries).

Table 4

Regression of $-\Delta c^*$ and Δq_c on $\Delta g(K)$ (Screen = 0.01). H_0 per thrift theory: $\Delta g(K) \cong -\Delta c^* \& \Delta q_c \cong 0$

	$-\Delta c^*$	Δq_c
Regression of value shown on $\Delta g(K)$	-0.3760***	1.376***
	(0.1055)	(0.1055)
Observations	1,548	1,548
R^2	0.53993	0.88294
Within R^2	0.31853	0.86226
Year fixed effects	✓	✓
Country fixed effects	✓	✓

4. Interpretation of test results

Any sustainable theory of capital growth must account for test results, including those found in Sections 2 and 3. A search for explanations other than thrift theory can begin with study of implications of the view of Rae and Mill, and of Solow in part as shown.

Free growth theory expects that capital growth is explained by a gain in productivity of capital and labor already present, with no help from net saving, but with the proviso that replacement saving offsets depreciation first. A shortfall in replacement saving is called dissaving, or negative net investment. Let "free growth" Q show whatever part of capital growth is explained by productivity gain after allowance for that exception for dissaving. Then free growth theory

predicts

$$\Delta K = Q , \quad \text{if } S_{net} \geq 0 . \tag{9}$$

Let Eq. (8) define "free growth assumptions," and let ufga mean "under free growth assumptions."

Let "free growth rate" q be defined by $q = \frac{Q}{K}$. Now divide Eq. (8) by capital, as before, to get

$$g(K) = q , \quad \text{ufga}, \quad \text{where } q = \frac{Q}{K} . \text{ Then} \tag{10}$$

$$\Delta g(K) = \Delta q \quad \text{ufga.} \tag{11}$$

Δq is "productivity change," which may be positive or negative. Divide by capital acceleration and rearrange, again, to arrive at

$$\frac{\Delta q}{\Delta g(K)} = 1 , \quad \text{ufga.} \tag{12}$$

"Free growth index" φ will be defined by $\varphi = \frac{\Delta q}{\Delta g(K)}$. Then Eq. (12) may also show as

$$\varphi = 1 , \quad \text{ufga} , \quad \text{where } \varphi = \frac{\Delta q}{\Delta g(K)} . \tag{13}$$

We know no way to measure q or Δq or φ directly, but can test these predictions of free theory, to a point, from results in tests of thrift theory. First came the finding $\frac{s^*}{g(K)} = 0.481$, with regression of s^* on $g(K)$ found at 0.1857. Free growth theory makes no prediction as to proportions between s^* and $g(K)$ when $s^* \geq 0$. The regression 0.1857 suggests causal relationship between s^* and $g(K)$, but does not reveal the direction of causality. It may be that income growth brought by capital growth $g(K)$ motivates households to save more, and conversely as $g(K)$ drops, in order to keep consumption steady, as expected in the permanent income hypothesis (Friedman 1957). A given household may not know the effect of its saving on net saving of all households together, so that combined saving or dissaving of all together, so that combined saving or dissaving of all together might or might not push the total into net saving or dissaving over the whole economy. This effect might explain the observed regression of s^* on $g(K)$ at 0.1857.

$\frac{\Delta s^*}{\Delta g(K)}$ appeared at 0.064, with regression of Δs^* on $\Delta g(K)$ shown as 0.1000. Both are measures of causal relatedness. Free growth theory, in Eq. (10), expects positive or negative productivity change Δq to explain positive or negative $\Delta g(K)$ if S_{net} is positive. A fall of S_{net} from positive to negative, with a shortfall in replacement investment, at the collective scale, is most probable in recessions or depressions. In a Keynesian interpretation, a drop in investment through misjudgment, rather than compelled by events,[5] was a chief cause of the slump, and deficit investment will bring part of the recovery to norms. This interpretation would fit thrift theory. Other views might see the cause as overestimated, as thought in the Austrian school, or an unrelated disruption as in ways or trade wars or unlucky bets, so that drops in income and saving would be effects.

The possibilities that growth may be caused by thrift, and that thrift may be caused by growth, may be called "thrift causality" and "growth causality." Then as much as 48.1% of $g(K)$ might be explained by thrift causality from data for $\frac{s^*}{g(K)}$, although other results suggest less. All these data for net saving might be explained by growth causality, pending analysis of tests from consumption.

The finding for θ_c is particularly informative in that the equations $\Delta s^* = -\Delta c^*$ and $\frac{\Delta s^*}{\Delta g(K)} = -\frac{\Delta c^*}{\Delta g(K)}$ were not derived from any assumptions, and because causal connections show most reliably in accelerations. Measurements of $-\frac{\Delta c^*}{\Delta g(K)}$, that is, should give the causal relatedness between s^* and $g(K)$ exactly if data were exact, and with high confidence if mismeasurements tend to offset.

The result $\theta_c^* = -\frac{\Delta c^*}{\Delta g(K)} = \frac{\Delta s^*}{\Delta g(K)} = -0.142$, or equivalently $\frac{\Delta c^*}{\Delta g(K)} = 0.142$, shows significant positive causal connection between c^* and $g(K)$ in that they tend to vary in the same direction, and so discourages an expectation that all or any net saving is recovered in capital growth.[6]

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[5]Firms and households held rainy day funds in cash, rather than spending on new goods and services.
[6]Any part so recovered would add the same fraction to θ_c.

Free growth theory, which expects productivity change Δq to explain and equal capital acceleration $\Delta g(K)$, could explain the finding $\frac{\Delta c^*}{\Delta g(K)} = 0.142$ in the reasoning that capital added by Δq would imply output added, in proportion to the output/capital ratio, and consumption added in proportion to the consumption/capital ratio.

While tests of $\frac{s^*}{g(K)}$ and θ_s allowed some recovery of net saving in capital growth, test results for these and θ_c together are easiest to reconcile with free growth theory, where all capital growth is explained by productivity gain. Pending further study, we will proceed on this interpretation.

Table 5

Average $\varphi_{s,i}$ and $\varphi_{c,i}$ in 86 countries (screen = 0.01). Number of years clearing screen shown in ()

Country	Period	$\overline{\varphi_{s,i}}$	$\overline{\varphi_{c,i}}$	Country	Period	$\overline{\varphi_{s,i}}$	$\overline{\varphi_{c,i}}$
Armenia	1997 - 2020 (20)	0.88	1.18	Italy	1981 - 2021 (22)	0.94	1.12
Aruba	1997 - 2001 (5)	-0.22	8.44	Japan	1981 - 2021 (30)	0.95	1.13
Australia	1974 - 2019 (35)	0.97	1.08	Kazakhstan	1997 - 2022 (23)	0.92	1.16
Austria	1997 - 2021 (16)	0.90	1.13	Kuwait	2005 - 2017 (11)	0.96	1.13
Azerbaijan	1997 - 2020 (23)	0.25	2.20	Kyrgyzstan	1998 - 2021 (22)	0.77	1.29
Bahrain	2010 - 2013 (4)	0.59	1.32	Latvia	1998 - 2021 (20)	0.99	1.15
Belgium	1997 - 2021 (19)	0.95	1.06	Lithuania	1997 - 2021 (18)	0.90	1.19
Bolivia	1999 - 2014 (12)	0.96	1.12	Luxembourg	1997 - 2021 (25)	1.00	1.02
Botswana	1997 - 2000 (4)	0.76	1.27	Malaysia	2008 - 2015 (7)	0.99	1.19
Brazil	1998 - 2020 (22)	0.90	1.24	Malta	1997 - 2021 (23)	0.92	1.04
British Virgin Islands	1997 - 1998 (2)	-1.24	-2.20	Mauritius	2015 - 2018 (4)	0.98	0.99
Bulgaria	1997 - 2017 (15)	1.03	1.19	Mexico	1997 - 2021 (22)	0.94	1.16
Burkina Faso	2001 - 2019 (16)	0.96	1.28	Moldova	1997 - 2019 (21)	0.89	1.27
Cameroon	1997 - 2019 (19)	0.82	2.71	Mongolia	2008 - 2021 (11)	0.90	1.06
Canada	1974 - 2022 (41)	0.91	1.08	Morocco	2000 - 2021 (17)	0.81	1.27
Cape Verde	2009 - 2016 (8)	0.73	2.02	Netherlands	1997 - 2021 (20)	0.97	1.08
Chile	1998 - 2021 (22)	0.94	1.48	New Zealand	1997 - 2019 (18)	0.86	1.02
China	1993 - 2020 (23)	0.91	1.05	Nicaragua	2008 - 2019 (12)	0.94	1.13
Colombia	1997 - 2022 (18)	1.03	2.39	Niger	1997 - 2019 (19)	0.78	1.87
Costa Rica	2014 - 2020 (6)	0.96	1.25	Norway	1982 - 2021 (34)	0.88	1.08
Croatia	1997 - 2021 (22)	0.81	2.09	Peru	2009 - 2021 (11)	0.96	1.30
Curaçao	2002 - 2016 (14)	0.59	0.51	Philippines	1997 - 2022 (20)	0.63	1.20
Cyprus	1997 - 2021 (23)	1.06	1.16	Poland	1997 - 2021 (15)	0.80	1.07
Czechia	1995 - 2021 (19)	1.03	1.08	Portugal	1997 - 2022 (17)	0.98	1.08
Côte d'Ivoire	1997 - 2000 (4)	1.03	1.05	Qatar	2004 - 2018 (12)	0.71	0.86
Denmark	1997 - 2022 (22)	0.95	1.14	Romania	1997 - 2020 (14)	0.98	1.16
Dominican Republic	2007 - 2015 (9)	0.85	1.41	Russia	1997 - 2018 (14)	0.91	1.11
Ecuador	2009 - 2020 (12)	0.83	1.91	Saudi Arabia	2005 - 2009 (3)	-1.23	0.02
Egypt	1998 - 2015 (16)	0.87	2.14	Senegal	2018 - 2021 (3)	0.81	2.31
El Salvador	2016 - 2019 (4)	0.02	-2.32	Serbia	1999 - 2021 (22)	0.78	1.67
Estonia	1997 - 2021 (19)	0.94	1.11	Slovakia	1997 - 2021 (18)	0.91	1.18
Finland	1997 - 2022 (21)	1.02	1.20	Slovenia	1997 - 2021 (20)	0.90	1.07
France	1972 - 2021 (33)	0.91	1.16	South Africa	1997 - 2022 (21)	0.90	1.13
Germany	1972 - 2022 (31)	0.95	1.14	South Korea	1997 - 2020 (15)	0.96	1.05
Greece	1997 - 2021 (24)	0.76	1.47	Spain	1996 - 2021 (23)	0.96	1.11
Guatemala	2003 - 2021 (11)	0.85	1.32	Sweden	1974 - 2022 (41)	0.98	1.15
Guinea	2005 - 2010 (6)	0.92	1.23	Switzerland	1993 - 2020 (24)	0.92	1.09
Honduras	2003 - 2015 (10)	0.60	0.85	Tunisia	1997 - 2011 (12)	0.68	1.17
Hong Kong	1997 - 2011 (13)	0.96	1.25	Turkey	2011 - 2015 (5)	1.16	1.54
Hungary	1997 - 2021 (21)	0.93	1.10	USA	1972 - 2021 (44)	0.98	1.13
Iceland	2002 - 2014 (12)	0.90	1.12	Ukraine	1997 - 2020 (23)	0.95	1.18
India	2000 - 2019 (14)	0.81	0.99	United Kingdom	1972 - 2021 (42)	0.98	1.21
Indonesia	2019 - 2019 (1)	1.41	0.98	Uruguay	2017 - 2017 (1)	0.44	0.98
Iran	1998 - 2018 (19)	-0.09	1.09	Uzbekistan	2012 - 2021 (9)	0.95	1.14
Ireland	1997 - 2021 (23)	0.98	1.23	Vanuatu	2003 - 2007 (3)	0.85	1.19
Israel	1997 - 2017 (18)	0.93	1.72	Venezuela	1999 - 2019 (20)	0.69	1.13

Note: Thrift theory predicts $\overline{\varphi_{s,i}} \cong \overline{\varphi_{c,i}} \cong 0$. Free growth theory predicts $\overline{\varphi_{s,i}} \cong \overline{\varphi_{c,i}} \cong 1$.

5. Mechanics of free growth

Some growth is capital widening, where structures and implements increase in number but do not change in design. Capital widening, however, is practical only so far before glut and diminishing returns set in. Further growth from that point must come from capital deepening, meaning improvements in the design of capital. Solow (1956) noted a kind of middle ground between capital widening and capital deepening in the disembodied growth mentioned earlier; ships carrying coals to Newcastle can raise prospective cash flows, and hence present value, by reversing the business plan. But Solow, who came to conclusions similar to ours from different evidence, puzzled as to how capital growth without net saving could be possible for capital deepening through "embodied" growth, where products of new design are made from plant of new design.[7]

The solution, we suggest, is that embodied growth is disembodied growth on a finer scale. At each step toward realization of the new plant and products, raw materials and products and labor skills and plant capacity currently available on the market are adapted to new uses. The innovator pays for these inputs at a market price determined by their value in established productive uses, but applies them innovatively to realize higher prospective cash flows, and hence higher present values, to the innovator (Marshall (1890), Schumpeter and Opie (1934)). This difference in present value realized less price paid will here be called the "innovator's reserve", meaning reserve price for inputs of capital and labor.[8] The innovator's reserve quantifies the part of free growth explained by productivity gain as distinct from random market noise. As such, it is the quantity added to depreciation saving to enable embodied growth, so that net saving is never needed.

Our findings support those of Piketty and Zucman (2014) and Kurz (2023) as to the market power of innovators to explain capital growth beyond net saving. Again, we go farther by questioning the assumption that net saving contributes even a part of capital growth.

6. Optimum investment policy

Data and arguments suggest that the optimum amount of saving, at the global scale, is replacement saving to offset depreciation, and nothing more. That would not mean book depreciation, as this study has stressed differences between book and market values. Up to a point, it should be possible to analyze the composition of market capital, and to model depreciation of the whole. A better plan, as Solow (1956) wrote in response to Harrod's knife edge argument (Harrod, 1939), is to trust the market to maximize rate of return, and to sense the point where glut begins and returns fall.[9] Markets do so imperfectly when tax and other public policy reward saving over distributions and consumption. Findings in this paper suggest review of such policies. These include the double tax on dividends, and the greater tax rate on ordinary income than on capital gains. Effects of removing the double tax, and removing the difference between tax rates on ordinary income and on capital gains, could be revenue-neutral and non-partisan if the corporate tax were raised to match, if the tax rates on ordinary income and on capital gains met somewhere between, and if thoughtful grandfathering eased the transition.

7. Inferences for national accounts and macroeconomics

We have attempted to show that net saving does not tend to equate to capital growth, and indeed contributes no part of capital growth. If our argument proves correct, then national accounts, and the teachings of macroeconomics (macro) which they express, are constructed on a fundamental misperception. We would therefore welcome any critiques of our findings as a matter of academic and public interest.

8. Inferences for microeconomics

Microeconomics ("micro") studies the mutual effects of supply, demand and price. It teaches that producer cost, including interest as the cost of time, tends to equilibrate to market value realized. We have proposed an exception in

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[7]The terms capital deepening, capital widening, embodied growth and disembodied growth are all Solow's.

[8]i.e., capital and labor inputs are worth more to the innovator in that the innovator applies them in ways to realize greater returns. The present value of additional cash flow enabled by this advantage in return quantifies the innovator's reserve and equivalently the non-random component of free growth.

[9]Harrod had argued that saving must hit the warranted rate exactly or risk positive feedback through the operation of the output/capital ratio (accelerator).

the innovator's reserve; the innovator pays the going rates for skills and materials needed, but applies them to make products of greater market value.

9. Inferences for inflation

If free growth theory is correct, then net investment adds a superfluous cost to capital growth, while decreasing market value per unit through capital glut. Both effects would tend to contribute directly to capital inflation, and indirectly to inflation of consumer goods by adding depreciation costs. This consideration adds to the case for urgent attention to our argument, and for more study to confirm or refute it.

10. Net output

The concept of net output is found in our discussions, but not in our equations. Net output means value added. Appendix B will show an argument that to equate value added to the sum of consumption and capital growth, and neither less nor more, would miss two components, one negative and one positive, in value added to human capital, and that both of these components are unmeasurable by known means. Since we work here from measurable data for consumption and net saving and market value capital, we did our best to reason from those three quantities alone.

11. Data sources

All our data are drawn from Distributional National Accounts (DINA) from the free online database World Inequality Database (WID). This source collates data from national accounts and tax data of 105 countries in constant currency units, and adjusts them where needed to conform to current standards of the System of National Accounts (SNA) published by the United Nations. We show results for the 86 of those countries which report all three of the factors, namely net saving, consumption and market-value capital, needed for deriving the thrift and free growth indexes. WID's source for these data is national accounts.

Consumption C in our text and equations is reproduced from the sum of Government Final Consumption Expenditure (mcongo)[10] and Private Expenditures of Households and NPISH (mconhn). This sums government expenditure GCE and personal consumption expenditure PCE. Net saving S_{net} and market-value K are taken from net national saving (msavin) and market-value Capital Wealth (mnweal) respectively. GDP, which we use only for weighting purposes in Figs. 1 and 2, is reproduced from GDP (mgdpro).

12. Accessing our results and methods

Tables and other displays of our findings for each country, and showing our methods of calculation, can be accessed at the web appendix (https://web-appendix.shinyapps.io/Sources_of_Capital_Growth).

13. Displays

The web appendix includes displays of θ_s^* and θ_c^* for each year in each country over the report period. These tend to show upward and downward spikes in values of θ_s^* and θ_c^* in some years. Those spikes tend to be associated with small absolute values of denominators, in these cases $\Delta g(K)$, in those countries and years. Small denominators magnify errors in measurements of numerators. Worse, when denominators are small, small mismeasurements in them might reverse the term in sign.

To maximize reliability of test results, we apply a range of screens to omit years where absolute denominators fall below a given threshold. Some displays show data for all years, regardless of denominator size. Others screen out all years where absolute denominators are less then 0.01, then 0.025, then continuing upward in increments of 0.025 to a maximum screen of 0.15. θ_s^* and θ_c^* are plotted for each country unscreened and at each of the seven successive levels of screening. All displays applied a screen of 0.01. The denominator whose absolute value is screened is capital acceleration $\Delta g(K)$ or capital growth rate $g(K)$ in all displays.

Screening out years where absolute $\Delta g(K)$ or $g(K)$ is small would cost little in informative value even if measurements were exact. In those years, there is little capital acceleration or capital growth, positive or negative,

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[10]WID code

for either thrift theory or free growth theory to explain. Market noise alone might account for $\Delta g(K)$ or $g(K)$ in such years. Screening reduces the number of observations, but increases the reliability and informative value of each.

14. Disclaimers

We accept that capital growth is impossible without capital replacement first, to make up losses to depreciation, and that the only source of capital replacement is replacement investment. Thus we accept the necessity and efficacy of replacement investment. We dispute only the efficacy of net investment in realizing capital growth after replacement investment has made up for depreciation.

15. Discussion and conclusions

Capital glut is the condition warned against by West, Ricardo, Malthus and Harrod. It is loosely defined as oversupply of capital at the current state of technology. We will not attempt a more exact definition here. Findings shown in our displays, anyhow, suggest that net saving raises the physical quantity of capital, say in number of shops, manufacturing plants or finished goods of similar design, without raising aggregate value of capital, and so contributes to capital glut.

These findings challenge the teachings that capital growth is effected by net saving enabled by consumption restraint, and that producer cost, including imputed interest as the opportunity cost of capital, converges to market realization. Evidence suggests that all capital growth is free, and consequently that market realization, in the presence of innovation, exceeds producer cost by the entirety of capital growth. Meanwhile the same evidence, which indicates that net saving adds no capital value, suggests a review of the teachings that consumption plus net saving gives net income, and that consumption plus net investment gives net output. Appendix B will also question the latter teachings.

Embodied growth is disembodied growth on a finer scale. It redeploys or repurposes existing labor skills, raw materials, and plant capacity, as well as existing finished goods, to achieve higher returns than available from the customary uses which determine their prices. The present value of yields from this advantage in return, or equivalently the innovator's reserve, defines the non-random component in free growth.

Appendix A. Equivalence of saving and investment

Any usage which treats saving and investment as equal must deal with the fact that saving held in cash is not investment in the usual sense of spending on productive capital, and contributes nothing to output. Keynes (1936) defined that unspent saving as intended saving, and actual saving as the part so spent.

We suggest that investors seek to maximize return, within risk tolerance, and will sometimes hold saving in cash or equivalents at zero return in recessions and depressions when positive returns cannot be found, and so when investing in the usual sense would tend to reduce capital and output rather than increase them. Thus investment could have the usual meaning in usual times, when prospective returns bring animal spirits, and could include saving in cash when not. It is in this sense that we equate saving to investment.

Appendix B. Net output with human capital

Human capital is impractical to measure, as it leaves little market record other than for its rental income in pay and investment cost in schooling. Thus national accounts leave it implicit, and allow us to infer what we can from data for pay and schooling. Those accounts are founded on the principle, sound when terms are appropriately redefined, that net output, or value added, is expressed in the sum of capital growth and net outflow from the value-added chain. In national accounts, then, where physical capital is the whole of capital while net outflow of the chain is the whole of consumption, the reasoning is

$$Y = \Delta K + C , \quad \text{neglecting human capital.} \tag{B.1}$$

It is possible in principle to model a value-added chain which includes human capital, and to compare findings with those shown in Eq. (B.1). Let human capital H, in that new model, stand as the last link in the value-added chain. Adapting the classic illustration of the value added principle, say that farms produce wheat, mills convert the wheat to flour, bakeries convert the flour into bread, and humans convert some of the bread, called invested

consumption, into human capital. The net outflow from this extended value-added chain is not all of consumption, but only pure consumption, meaning the part remaining after the part invested in human capital (invested consumption) is subtracted[11]. By this reasoning, the principle that net output is expressed in capital growth plus net outflow gives

$$Y = \Delta K + \Delta H + C_p \, , \quad \text{allowing human capital,} \tag{B.2}$$

where C_p gives pure consumption.

Yoram Ben-Porath (1967) reasoned that growth in human capital equals invested consumption plus self-invested work less human depreciation[12]. Let C_s, W_s and $D(H)$ show these flows respectively. Thus the combined arguments of Schultz and Ben-Porath arrive at

$$C = C_s + C_p \quad \text{and} \quad \Delta H = C_s + W_s - D(H) \, , \quad \text{allowing human capital.} \tag{B.3}$$

Substitution of these equations into Eq. (B.2) finds

$$Y = \Delta K + C_s + W_s - D(H) + C_p \quad \text{and consequently}$$
$$Y = \Delta K + C + W_s - D(H) \, , \quad \text{allowing human capital,} \tag{B.4}$$

if Schultz and Ben-Porath and the reasoning here are right.

Appendix C. Mill's statement of the free growth idea

Mill (1848), book 1, chapter 5, section 4, includes:

> If it were said, for instance, that the only way to accelerate the increase of capital is by increase of saving, the idea would probably be suggested of greater abstinence, and increased privation. But it is obvious that whatever increases the productive power of labour creates an additional fund to make savings from, and enables capital to be enlarged not only without additional privation, but concurrently with an increase of personal consumption.

This passage may be the first clear statement of what we call free growth theory. His use of the words "accelerate" and "concurrently" suggest that his path of reasoning was something like ours, and that he meant what we call free growth rather than the alternating phases of higher and lower saving rates pictured in thrift theory.

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[11]The concepts of invested consumption, pure consumption, self-invested work and human depreciation were introduced in Schultz (1961).

[12]Equation 4 in Ben-Porath's paper, summarizing his first three equations. His terms and notation differ from ours. The concepts of invested consumption were also introduced by Schultz (1961).