Sources of capital growth

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ABSTRACT

Data from national accounts show no effect of change in net saving or consumption, in ratio to market-value capital, on change in growth rate of market-value capital (capital acceleration). Thus it appears that capital growth and acceleration arrive without help from net saving or consumption restraint. We explore ways in which this is possible, and discuss implications for economic teaching and public policy.

1. Introduction and overview

Many economists over the centuries have reasoned that net saving, or equivalently net investment[1], should tend to give equal capital growth. Economists since the early nineteenth century have added the proviso that net saving cannot safely outpace innovation; more capital must mean capital redesigned for greater productivity if economies are to escape risk of capital glut and diminishing returns (West (1815), Ricardo (1815), Malthus (1815)). Roy Harrod (1939) described that limit for safe net saving, meaning the rate of imagining and developing new ideas for more productive forms of capital, as the "warranted rate". Harrod, and many other economists of his time and since, have focused on growth of output rather than of capital, but have modeled growth of output by first assuming the equivalence of net saving and capital growth, within the warranted rate, and then looking for effects of that capital growth on later output growth.

Some other economists, including John Rae (1834) and John Stuart Mill (1848) (see Appendix C), argued that capital growth might also be explained by a rise in productivity of capital and labor already extant. Ways might found for existing factors to produce more, that is, and so to allow more consumption, or more capital growth, or any mix of the two, without inputs of net saving. Robert Solow (1957) allowed that possibility for "disembodied" growth, where plant and products already existing are repurposed or redeployed in more productive ways.

We test between those two explanations of capital growth, by net saving or by increase in productivity of capital and labor already in existence, by comparing net saving to concurrent change in market-value capital in 86 countries. As changes in net saving are expected to be associated with opposite changes in consumption, we also compare change in consumption to concurrent change in capital growth (capital acceleration). All data are drawn from national accounts of those countries as collated on the free website World Inequality Database.

Tests show no effect of net saving or of change in consumption on growth or acceleration of market-value capital. These findings support the views of Rae and Mill, and of Solow as to disembodied growth. They suggest that capital growth, even in acceleration, arrives without help from net saving or consumption restraint. Net saving, if so, raises the physical quantity of capital, but not the aggregate value, and so reduces the value per unit.

Our findings are most easily explained by the present value principle, and by production efficiencies enabled through innovation. Value is created in the mind of the market at the moment when prospective cash flows are discounted. It is created only if the market sees a path, step by step, from the start, to practical realization of those prospective cash flows. Then capital growth arrives when the market first evaluates prospective cash flows, and is realized eventually in physical outcomes insofar as the market has predicted correctly. Meanwhile the innovator acquires materials and plant capacity and labor skills at market prices determined by their uses in current technology, but applies them more productively until competition catches up. It is that temporary market advantage to the innovator which explains capital

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[1] As reported in national accounts; they differ only by statistical discrepancy. Also see Appendix A.

growth without net saving in a practical and mechanical sense, while the present value principle gives the explanation in terms of market valuation. This idea will be called "free growth theory" for easy reference.

It predicts only at the largest scales, and only for the private sector. Individuals and groups and even small economies can grow through investment from outside. That possibility is foreclosed only at the scale of all capital and all economies together. The public sector, meanwhile, responds to political rather than market choices, and grows or shrinks accordingly.

If free growth theory is right, tax policy and other policy to encourage saving over consumption should be reviewed. These policies include the higher tax on ordinary income than on capital gains, and the double tax on corporate dividends.

Inferences for economic teaching include the obvious ones for growth theory and for net saving in general. They include others as well. One of the central doctrines of the marginalist revolution has held that market realization converges to producer cost, when that cost includes imputed interest on assets owned. Net saving gives producer cost, and falls short of market realization in the presence of technological growth from new ideas. Meanwhile the doctrine that net income equals consumption plus net saving is put into question by evidence offered here suggesting that net saving increases the physical quantity of capital, but not the aggregate value. In general, economics might consider relying less on book value and net investment, and more on market value and on the power of ideas.

2. Net saving, capital growth and capital acceleration

This study compares net saving S_{net} to concurrent growth in market-value capital K from data in national accounts. Capital growth ΔK_i in each year i for each reporting country is found as $K_i - K_{i-1}$, and compared to $S_{net,i}$ reported for that year and country. As we will be testing for differences between ΔK and $S_{net,i}$, we begin by writing

$$\Delta K_i = S_{net,i} + Q_{s,i} \ . \tag{1}$$

Here Q means the sum of market noise, which may prove positive or negative or zero, plus any part of capital growth explained by concurrent productivity gain as described by Rae and Mill, and by Solow as to disembodied growth. We call this sum of noise and productivity gain "free growth" in that it costs no net saving. $Q_{s,i}$ means its value as measured from data for saving in the i^{th} year.

The object of testing is to find effects of S_{net} on concurrent ΔK, and so to help evaluate historic and current teachings as to those effects. We submitted above that most teaching, with exceptions noted as to Rae, Mill, and Solow, and within the warranted rate, predicts net saving to differ from concurrent growth in market-value capital only by market noise which tends to balance out over scale and time. Free growth Q in Eq. (1), in that case, will give the market noise converging to zero. That consensus prediction, which we will challenge, will here be called "thrift theory"; S_{net}, in thrift theory, is expected to converge to ΔK if held within the warranted rate. That is,

$$E(\Delta K) = S_{net} \quad \text{or equivalently} \quad E(Q_s) = 0, \quad \text{if} \quad \frac{\Sigma S_{net}}{\Sigma K} \leq u \ , \quad \text{in thrift theory,} \tag{2}$$

where:

1. ΣS_{net} is collective net saving over the economy
2. ΣK is collective capital over the economy before current ΣS_{net}, and
3. u is the warranted rate.

$E(\Delta K)$ and $E(Q_s)$ here give the expected values of ΔK and Q_s respectively. Expected value means predicted average of outcomes over all observations. We will test those predictions against data for saving and capital growth taken from national accounts. As secular economic growth has tended to make later stocks and flows larger than earlier ones, we first divide by K (normalize) to avoid overweighting of more recent years in finding that average. Division of Eq. (1) by K_{i-1} gives

$$\frac{\Delta K_i}{K_{i-1}} = \frac{S_{net,i}}{K_{i-1}} + \frac{Q_{s,i}}{K_{i-1}} \ . \tag{3}$$

The first term in Eq. (3) gives capital growth rate $g(K)$. The second term is a variant of the Keynesian net saving rate s_{net} where capital rather than output becomes the denominator. This flow will show as s^*, with the subscript "net" left implicit, and with the understanding that the denominator shows market-value capital rather than output. The third term in Eq. (3) will be called free growth rate and shown as q_s. Then

$$g(K)_i = \frac{\Delta K_i}{K_{i-1}} , \quad s_i^* = \frac{S_{net,i}}{K_{i-1}} , \quad \text{and} \quad q_i = \frac{Q_{s,i}}{K_{i-1}} ,$$

so that Eq. (3) can be shown more compactly as

$$g(K)_i = s_i^* + q_{s,i} \, . \tag{4}$$

By the definition $q_s = \frac{Q_s}{K}$, an expected value $E(q_s) = 0$ implies $E\left(q_s\right) = 0$. Application of Eq. (2) to Eq. (4) now gives

$$E\left(g(K)\right) = s^* \quad \text{and} \quad E(q_s) = 0 \, , \quad \text{under thrift assumptions,} \tag{5}$$

where "thrift assumptions" are that thrift theory is correct and that the warranted rate is not exceeded. We will test Eq. (5) directly in Section 5 below. Meanwhile we will test it indirectly by testing its implications for changes in capital growth, which we call capital acceleration and show as $\Delta g(K)$. First note that Eq. (4) allows

$$\Delta g(K)_i = \Delta s_i^* + \Delta q_{s,i} \, , \quad \text{where} \tag{6}$$

$$\Delta g(K)_i = g(K)_i - g(K)_{i-1} \, , \quad \Delta s_i^* = s_i^* - s_{i-1}^* \, , \quad \text{and} \quad \Delta q_{s,i} = q_{s,i} - q_{s,i-1} \, .$$

For any variables a and b, we may reason $E(a - b) = E(a) - E(b)$. By this and by Eqs. (5) and (6), then,

$$E\left(\Delta g(K)\right) = \Delta s^* \quad \text{and} \quad E(\Delta q_s) = 0 \, , \quad \text{under thrift assumptions.} \tag{7}$$

Division of Eq. (6) by capital acceleration, and rearrangement, gives

$$\frac{\Delta s_i^*}{\Delta g(K)_i} + \frac{\Delta q_{s,i}}{\Delta g(K)_i} = 1 \, . \tag{8}$$

Define $\theta_{s,i} = \frac{\Delta s_i^*}{\Delta g(K)_i}$ and $\varphi_{s,i} = \frac{\Delta q_{s,i}}{\Delta g(K)_i}$ to restate Eq. (8) as

$$\theta_{s,i} + \varphi_{s,i} = 1 \, . \tag{9}$$

Next define $\theta_s = E(\theta_{s,i})$ and $\varphi_s = E(\varphi_{s,i})$. By Eq. (9), then,

$$\theta_s + \varphi_s = 1 \, . \quad \text{Eq. (7) now implies} \tag{10}$$

$$\theta_s = 1 \quad \text{and} \quad \varphi_s = 0 \, , \quad \text{under thrift assumptions.} \tag{11}$$

θ and φ will be called the "thrift index" and "free growth index" respectively. θ_s and φ_s give their values as found from data for change in net saving. Expected values, again, are predicted averages of outcomes. Thus Eq. (11) and thrift theory can be tested by finding average values of $\theta_{s,i}$ and comparing findings to the expected value $\theta = 1$. First we find

$$\overline{\theta_{s,i}} = \frac{1}{m} \sum^m \theta_{s,i} \quad \text{and} \quad \overline{\varphi_{s,i}} = \frac{1}{m} \sum^m \varphi_{s,i} \, ,$$

where m is the number of observed values of $\theta_{s,i}$ and $\varphi_{s,i}$, and test the predictions

$$\overline{\theta_{s,i}} \cong 1 \quad \text{and} \quad \overline{\varphi_{s,i}} \cong 0 \, , \quad \text{under thrift assumptions.}$$

Calculations of $\overline{\theta_{s,i}}$ and $\overline{\varphi_{s,i}}$ are not expected to show 1 and 0 exactly, under thrift assumptions, so that $\overline{\theta_{s,i}}$ and $\overline{\varphi_{s,i}}$ are not exactly equal to θ_s and φ_s, because the number of samples m is finite.

Fig. 1 shows average values of $\theta_{s,i}$ and $\varphi_{s,i}$ for 86 countries, both unweighted and weighted to GDP, over the period 1980-2022. To control distortions brought by small absolute denominators, years were screened out where $|\Delta g(K)|$ was found at less than 0.01 (see Section 10). Results show $\overline{\varphi_{s,i}} \cong 1$ and $\overline{\theta_{s,i}} \cong 0$. These findings appear to refute thrift theory, and to support free growth theory as defined earlier. Q_s, predicted in thrift theory to describe effects of market noise converging to zero, is revealed to include also the effects of productivity gain as described by Rae, Mill and Solow. We will now test thrift theory from a different approach.

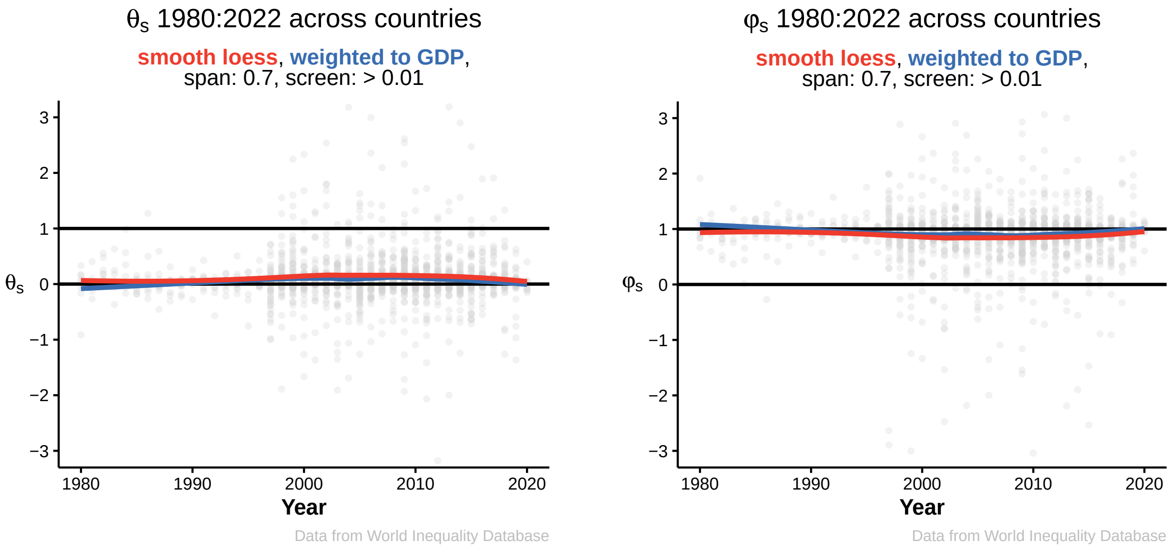

Fig. 1: Thrift and free growth indexes derived from net saving (86 countries).

Table 1
Regression of Δs^* and Δq_s on $\Delta g(K)$ (Screen = 0.01). H_0 per thrift theory: $\Delta g(K) \cong \Delta s^*$ & $\Delta q_s \cong 0$

	Δs^*	Δq_s
Regression of value shown on $\Delta g(K)$	0.1092***	0.8908***
	(0.0163)	(0.0163)
Observations	1,412	1,412
R^2	0.32023	0.95280
Within R^2	0.22472	0.95076
year fixed effects	✓	✓
country fixed effects	✓	✓

3. Consumption and capital growth

In national accounts and most of the literature, saving equals consumption forgone in that a fixed source of spending is spent on the sum of the two. Replacement saving to offset depreciation costs an equal potential gain in consumption while making up for losses in capital, that is, and net saving costs an equal actual decrease in consumption. We accept these meanings as definitions of saving, while disputing only the doctrine that investment of net saving brings actual increase in capital. Keynes (1936) expressed this definition in the equation $c + s = 1$, where c and s are the ratios of consumption and gross saving to gross output, or consumption and net saving to net output. We have been dividing by capital, rather than output, and will let c^* show the ratio of consumption to capital. We then adapt the Keynesian equation to show

$$s_i^* + c_i^* = 1\ , \quad \text{from which } s_i^* = 1 - c_i^*\ , \quad \text{where } c_i^* = \frac{C_i}{K_{i-1}}\ .$$

We now define q_c as free growth rate q found from data for consumption rather than from saving. By analogy to Eqs. (4) and (5), then,

$$g(K)_i = 1 - c_i^* + q_{c,i}\ , \quad \text{and} \tag{12}$$

$$E(g(K)) = 1 - c^* \quad \text{and} \quad E(q_c) = 0 \quad \text{under thrift assumptions.} \tag{13}$$

Meanwhile Eq. (12) allows

$$\Delta g(K)_i = \Delta(1 - c_i^* + q_{c,i}) = \Delta q_{c,i} - \Delta c_i^*\ . \quad \text{By Eq. (13), then,} \tag{14}$$

$$E(\Delta g(K)) = -\Delta c^* \quad \text{and} \quad E(\Delta q_c) = 0 \quad \text{under thrift assumptions.} \tag{15}$$

As with Eqs. (5) and (7), we will put off direct tests of Eq. (13) until Section 5, and meanwhile test it indirectly by testing its implications from Eq. (14) forward. As with Eq. (6), we first divide Eq. (14) by capital acceleration $\Delta g(K)$ to find

$$\frac{\Delta q_c}{\Delta g(K)_i} + \frac{-\Delta c_i^*}{\Delta g(K)_i} = 1\ . \tag{16}$$

Define $\theta_{c,i} = \frac{-\Delta c_i^*}{\Delta g(K)_i}$ and $\varphi_{c,i} = \frac{\Delta q_{c,i}}{\Delta g(K)_i}$ to re-express Eq. (16) as

$$\theta_{c,i} + \varphi_{c,i} = 1\ . \tag{17}$$

By Eqs. (12) through (15), further,

$$E\left(\theta_{c,i}\right) = 1 \quad \text{and} \quad E\left(\varphi_{c,i}\right) = 0\ , \quad \text{under thrift assumptions.} \tag{18}$$

Define $\theta_c = E\left(\theta_{c,i}\right)$ and $\varphi_c = E\left(\varphi_{c,i}\right)$ to re-express Eqs. (17) and (18) respectively as

$$\theta_c + \varphi_c = 1\ , \quad \text{and} \tag{19}$$

$$\theta_c = 1 \quad \text{and} \quad \varphi_c = 0\ , \quad \text{under thrift assumptions.} \tag{20}$$

We infer $\theta_c \cong \overline{\theta_{c,i}}$ and $\varphi_c \cong \overline{\varphi_{c,i}}$ as before, and test thrift theory by comparing average yearly values of $\theta_{c,i}$ and $\varphi_{c,i}$ to its predictions $\overline{\theta_{c,i}} \cong 1$ and $\overline{\varphi_{c,i}} \cong 0$.

Fig. 2 shows results of tests of these predictions from data for consumption reported in national accounts. Consumption was measured as the sum of personal consumption expenditure PCE and government consumption expenditure GCE. Capital K was again measured at market value. Again, years showing $|\Delta g(K)| < .01$ were screened out to control small denominator effects. Test results show $\varphi_c \cong 1$ and $\theta_c \cong 0$, as with tests for φ_s and θ_s from net saving. Table 2, which shows φ_s and φ_c for the same 86 countries separately, likewise finds $\varphi_s \cong 1$ and $\varphi_c \cong 1$. Thus it appears that capital acceleration arrives without help from either net saving or consumption restraint. Next we will see how these findings for φ_s and φ_c might be explained.

Table 2
Average $\varphi_{s,i}$ and $\varphi_{c,i}$ in 86 countries (screen = 0.01). Number of years clearing screen shown in ()

Country	Period	$\overline{\varphi_{s,}}$	$\overline{\varphi_{c,i}}$	Country	Period	$\overline{\varphi_{s,}}$	$\overline{\varphi_{c,i}}$
Armenia	1997 - 2018 (17)	0.94	1.30	Israel	1997 - 2017 (18)	0.95	1.68
Aruba	1997 - 2001 (5)	-0.38	8.98	Italy	1982 - 2017 (21)	1.01	0.98
Australia	1962 - 2019 (41)	0.97	1.09	Japan	1981 - 2017 (28)	0.95	1.13
Austria	1997 - 2013 (11)	0.93	1.11	Kazakhstan	1997 - 2018 (18)	0.90	1.09
Azerbaijan	1997 - 2018 (22)	0.39	1.85	Kuwait	2005 - 2017 (11)	0.98	1.12
Bahrain	2010 - 2013 (4)	0.59	1.32	Kyrgyzstan	1998 - 2019 (20)	0.73	1.19
Belgium	1997 - 2014 (11)	0.90	1.06	Latvia	1998 - 2015 (16)	1.00	1.13
Bolivia	1999 - 2013 (12)	1.03	1.28	Lithuania	1997 - 2016 (16)	0.93	1.14
Botswana	1997 - 2000 (4)	0.73	1.27	Luxembourg	1997 - 2018 (18)	0.99	1.09
Brazil	1998 - 2018 (20)	0.86	1.21	Malaysia	2008 - 2015 (7)	1.04	1.27
British Virgin Islands	1997 - 1999 (3)	0.35	3.63	Malta	1997 - 2019 (21)	0.80	1.03
Bulgaria	1997 - 2017 (15)	0.99	1.21	Mexico	1997 - 2019 (21)	0.95	1.14
Burkina Faso	2001 - 2018 (15)	0.94	1.23	Moldova	1997 - 2018 (21)	0.74	1.19
Cameroon	1998 - 2003 (6)	0.35	4.59	Mongolia	2008 - 2019 (10)	0.92	1.07
Canada	1974 - 2020 (37)	0.91	1.11	Morocco	2000 - 2019 (16)	0.84	1.38
Cape Verde	2009 - 2017 (9)	0.56	2.01	Netherlands	1997 - 2019 (14)	0.94	1.12
Chile	1998 - 2018 (18)	0.93	1.37	New Zealand	1997 - 2019 (21)	0.92	1.12
China	1993 - 2014 (19)	0.91	1.04	Nicaragua	2008 - 2018 (11)	0.93	1.13
Colombia	1997 - 2019 (17)	0.98	2.26	Niger	1997 - 2019 (20)	0.82	1.95
Costa Rica	2014 - 2017 (4)	0.97	1.09	Norway	1983 - 2020 (32)	0.86	1.12
Croatia	1997 - 2018 (21)	0.83	2.09	Peru	2009 - 2019 (9)	0.96	1.33
Curaçao	2002 - 2016 (12)	1.13	1.18	Philippines	1997 - 2019 (19)	0.68	2.20
Cyprus	1997 - 2018 (19)	0.94	1.11	Poland	1997 - 2019 (15)	0.91	1.23
Czechia	1995 - 2018 (14)	0.99	1.04	Portugal	1997 - 2020 (17)	0.98	1.08
Côte d'Ivoire	1997 - 2000 (4)	1.03	1.10	Qatar	2004 - 2018 (12)	0.72	0.87
Denmark	1999 - 2020 (20)	0.92	1.00	Romania	1997 - 2019 (16)	1.02	1.11
Dominican Republic	2007 - 2015 (8)	0.76	1.33	Russia	1997 - 2018 (14)	0.89	1.11
Ecuador	2009 - 2018 (10)	0.85	1.87	Saudi Arabia	2005 - 2009 (3)	-0.92	0.21
Egypt	1998 - 2015 (17)	1.04	1.75	Serbia	1999 - 2019 (16)	0.59	2.04
Estonia	1997 - 2017 (15)	0.93	1.11	Slovakia	1997 - 2020 (19)	0.93	1.20
Finland	1997 - 2020 (19)	0.97	1.11	Slovenia	1997 - 2019 (17)	0.92	1.08
France	1952 - 2019 (41)	0.87	1.13	South Africa	1997 - 2019 (17)	0.95	1.14
Germany	1971 - 2013 (22)	1.01	1.21	South Korea	1997 - 2018 (14)	0.96	1.04
Greece	1997 - 2019 (21)	0.90	1.27	Spain	1997 - 2017 (20)	0.95	1.15
Guatemala	2007 - 2019 (9)	0.96	0.93	Sweden	1951 - 2020 (57)	0.98	1.05
Guinea	2005 - 2010 (6)	0.94	1.27	Switzerland	1993 - 2018 (22)	0.94	1.03
Honduras	2002 - 2015 (14)	0.70	1.56	Tunisia	1997 - 2011 (12)	0.69	1.06
Hong Kong SAR China	1997 - 2020 (21)	0.91	1.12	Turkey	2011 - 2017 (7)	0.83	1.24
Hungary	1997 - 2018 (17)	0.93	1.11	Ukraine	1997 - 2019 (22)	0.93	1.16
Iceland	2002 - 2014 (13)	0.85	1.07	United Kingdom	1971 - 2018 (38)	1.00	1.14
India	2000 - 2017 (15)	0.88	1.08	United States	1972 - 2018 (40)	1.00	1.14
Iran	1998 - 2018 (19)	0.15	1.09	Vanuatu	2003 - 2007 (4)	0.72	0.92
Ireland	1997 - 2019 (20)	0.96	0.96	Venezuela	1999 - 2019 (19)	0.05	1.61

Note: Thrift theory predicts $\overline{\varphi_{s,i}} \cong \overline{\varphi_{c,i}} \cong 0$. Free growth theory predicts $\overline{\varphi_{s,i}} \cong \overline{\varphi_{c,i}} \cong 1$.

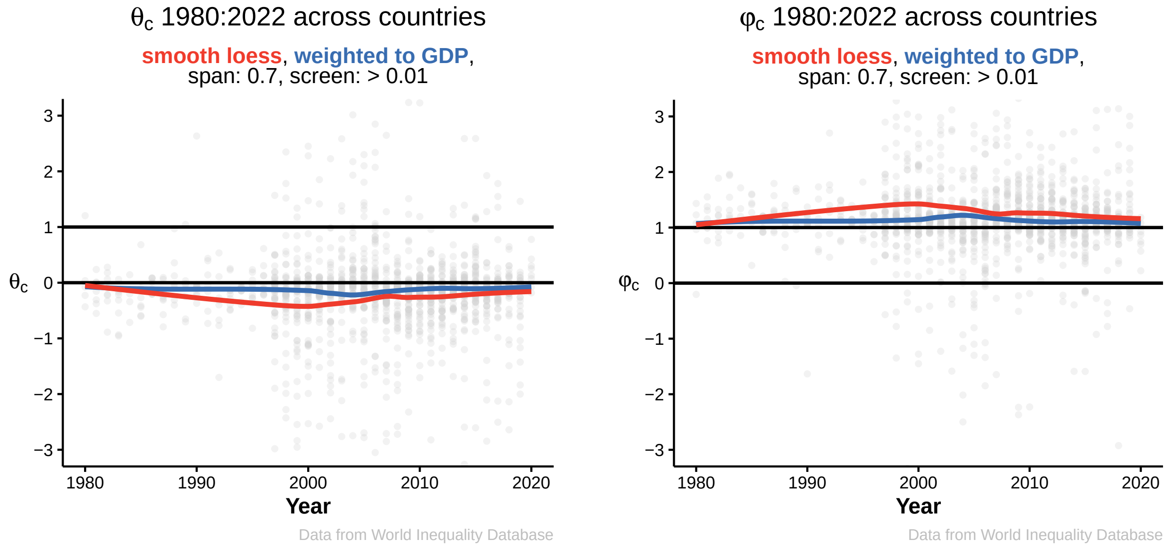

Fig. 2: Thrift and free growth indexes derived from consumption (86 countries).

Table 3
Regression of $-\Delta c^*$ and Δq_c on $\Delta g(K)$ (Screen = 0.01). H_0 per thrift theory: $\Delta g(K) \cong -\Delta c^* \& \Delta q_c \cong 0$

	Δc^*	Δq_c
Regression of value shown on $\Delta g(K)$	-0.4006***	1.401***
	(0.1090)	(0.1090)
Observations	1,387	1,387
R^2	0.57351	0.88607
Within R^2	0.34762	0.86689
year fixed effects	✓	✓
country fixed effects	✓	✓

4. Mechanics of free growth

Some growth is capital widening, where structures and implements increase in number but do not change in design. Capital widening, however, is practical only so far before glut and diminishing returns set in. Further growth from that point must come from capital deepening, meaning improvements in the design of capital. Solow (1956) noted a kind of middle ground between capital widening and capital deepening in the disembodied growth mentioned earlier; ships carrying coals to Newcastle can raise prospective cash flows, and hence present value, by reversing the business plan. But Solow, who came to conclusions similar to ours from different evidence, puzzled as to how capital growth without net saving could be possible for capital deepening through "embodied" growth, where products of new design are made from plant of new design.[2]

The solution, we suggest, is that embodied growth is disembodied growth on a finer scale. At each step toward realization of the new plant and products, raw materials and products and labor skills and plant capacity currently available on the market are adapted to new uses. The innovator pays for these inputs at a market price determined by their value in established productive uses, but applies them innovatively to realize higher prospective cash flows, and hence higher present values, to the innovator (Marshall (1890), Schumpeter and Opie (1934)). This difference in

[2]The terms capital deepening, capital widening, embodied growth and disembodied growth are all Solow's.

present value realized less price paid will here be called the "innovator's reserve", meaning reserve price for inputs of capital and labor.[3] The innovator's reserve quantifies the part of free growth explained by productivity gain as distinct from random market noise. As such, it is the quantity added to depreciation saving to enable embodied growth, so that net saving is never needed.

Our findings support those of Piketty and Zucman (2014) and Kurz (2023) as to the market power of innovators to explain capital growth beyond net saving. Again, we go farther by questioning the assumption that net saving contributes even a part of capital growth. Data shown in the Tables and Figures here suggest that it does not. Hence we attribute all capital growth and acceleration to the innovator's reserve, aside from market noise, and none to net saving.

5. Testing directly from capital growth

The essential tenet of thrift theory shows in Eq. (5), which expects capital growth to equal net investment within the authorized rate. We have first tested it, and its counterpart in Eq. (13), from their implications for capital acceleration, rather than directly, because causalities connecting variables tend to be most clearly revealed in simultaneous changes in those variables. In this case, causalities connected to $g(K)$ are likeliest to be revealed in changes to $g(K)$, meaning capital acceleration $\Delta g(K)$.

Results of those tests were vivid. Tables and figures above seem to show that change in net investment, expected in thrift theory to explain all of capital acceleration, explains none of it. We will now test Eqs. (5) and (13) directly, but we cannot expect equal clarity in test results. Free growth theory predicts that capital growth is explained by the innovator's reserve alone, and is not a function of s^* or c^*. For this reason, it makes no predictions as to the relationships among $g(K)$, s^* and c^* at any time. It expects rather that Eq. (13) might predict correctly in occasional coincidence, but not in general. By Eq. (12), versions of the thrift index applying to capital growth directly may be given as

$$\theta^*_{s,i} = \frac{s^*_i}{g(K)_i}\ , \quad \theta^*_{c,i} = \frac{(1-c^*)_i}{g(K)_i}\ , \quad \theta^*_s = E(\theta^*_{s,i}) \quad \text{and} \quad \theta^*_c = E(\theta^*_{c,i})$$

Eq. (13) now implies

$$E(\theta^*_s) = E(\theta^*_c) = 1 \quad \text{under thrift assumptions.} \tag{21}$$

Again, free growth theory makes no prediction as to the relationship of capital growth to net saving or to consumption.

Fig. 3 plots average θ^*_s and θ^*_c, GDP-weighted, for 86 countries over the period 1980-2022. Table 4 shows associated regressions. Table 5 shows θ^*_s and θ^*_c for each of these 86 countries for which data were reported. As with tests from capital acceleration, these findings do not seem to support thrift theory.

6. Optimum investment policy

Data and arguments adduced suggest that the optimum amount of saving, at the global scale, is replacement saving to offset depreciation, and nothing more. That would not mean book depreciation, as this study has stressed differences between book and market values. Up to a point, it should be possible to analyze the composition of market capital, and to model depreciation of the whole. A better plan, as Solow (1956) wrote in response to Harrod's knife edge argument (Harrod, 1939), is to trust the market to maximize rate of return, and to sense the point where glut begins and returns fall.[4]

[3] i.e., capital and labor inputs are worth more to the innovator in that the innovator applies them in ways to realize greater returns. The present value of additional cash flow enabled by this advantage in return quantifies the innovator's reserve and equivalently the non-random component of free growth.

[4] Harrod had argued that saving must hit the warranted rate exactly or risk positive feedback through the operation of the output/capital ratio (accelerator).

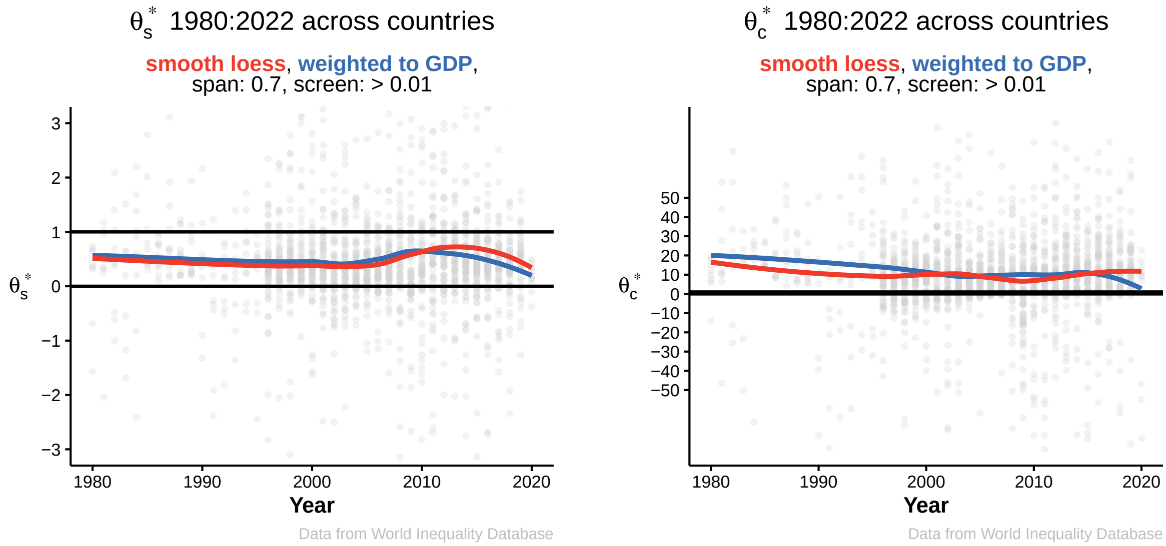

Fig. 3: θ_s^* and θ_c^* (86 countries)

Table 4
Regression of s^* and $1 - c^*$ on $g(K)$ (Screen = 0.01). H_0 per thrift theory: $g(K) \cong s^*$ and $g(K) = 1 - c^*$.

	s^*	$1 - c^*$
Regression of value shown on $g(K)$	0.2103***	-0.6760***
	(0.0541)	(0.1480)
Observations	1,691	1,665
R^2	0.76527	0.83560
Within R^2	0.14579	0.10122
year fixed effects	✓	✓
country fixed effects	✓	✓

Markets do so imperfectly when tax and other public policy reward saving over distributions and consumption. Findings in this paper suggest review of such policies. These include the double tax on dividends, and the greater tax rate on ordinary income than on capital gains. Effects of removing the double tax, and removing the difference between tax rates on ordinary income and on capital gains, could be revenue-neutral and non-partisan if the corporate tax were raised to match, if the tax rates on ordinary income and on capital gains met somewhere between, and if thoughtful grandfathering eased the transition.

7. Net output

The concept of net output is found in our discussions, but not in our equations. Net output means value added. Appendix B will show an argument that to equate value added to the sum of consumption and capital growth, and neither less nor more, would miss two components, one negative and one positive, in value added to human capital, and that both of these components are unmeasurable by known means. Since we work here from measurable data for consumption and net saving and market value capital, we did our best to reason from those three quantities alone.

Table 5
θ_s^* and θ_c^* in 86 countries (screen = 0.01). Number of years clearing screen shown in ()

Country	Period	$\overline{\theta_s^*}$	$\overline{\theta_c^*}$	Country	Period	$\overline{\theta_s^*}$	$\overline{\theta_c^*}$
Armenia	1996 - 2018 (21)	-0.64	8.04	Israel	1996 - 2019 (24)	0.44	7.08
Aruba	1996 - 2001 (6)	1.21	-4.56	Italy	1980 - 2020 (35)	0.11	-3.71
Australia	1980 - 2018 (33)	0.24	15.71	Japan	1980 - 2017 (27)	0.18	7.45
Austria	1996 - 2019 (22)	0.82	27.43	Kazakhstan	1997 - 2019 (22)	0.50	6.01
Azerbaijan	1996 - 2018 (21)	1.20	4.94	Kuwait	2003 - 2017 (14)	1.14	8.78
Bahrain	2009 - 2013 (5)	-6.03	-14.91	Kyrgyzstan	1996 - 2019 (23)	0.73	-1.01
Belgium	1996 - 2019 (20)	0.61	40.43	Latvia	1996 - 2019 (23)	-0.21	15.09
Bolivia	1998 - 2015 (17)	0.35	17.13	Lithuania	1996 - 2019 (22)	0.11	12.21
Botswana	1996 - 1999 (4)	3.14	9.53	Luxembourg	1996 - 2017 (20)	0.51	13.46
Brazil	1996 - 2018 (20)	0.24	6.77	Malaysia	2007 - 2015 (9)	2.27	1.18
British Virgin Islands	1996 - 1999 (4)	2.39	-1.95	Malta	1996 - 2019 (23)	0.35	25.23
Bulgaria	1996 - 2016 (17)	0.08	20.13	Mexico	1996 - 2019 (20)	0.27	12.74
Burkina Faso	2000 - 2018 (19)	0.21	11.07	Moldova	1996 - 2019 (21)	-0.62	2.00
Cameroon	1997 - 2003 (7)	0.94	-3.99	Mongolia	2006 - 2019 (14)	-0.28	-9.53
Canada	1980 - 2020 (36)	0.39	11.42	Morocco	1999 - 2019 (21)	1.69	7.54
Cape Verde	2008 - 2017 (9)	0.55	-0.71	Netherlands	1996 - 2019 (23)	0.88	23.79
Chile	1997 - 2018 (18)	1.06	8.96	New Zealand	1996 - 2019 (21)	0.66	14.13
China	1992 - 2016 (25)	0.84	11.32	Nicaragua	2007 - 2018 (12)	0.08	7.53
Colombia	1996 - 2019 (24)	0.71	-0.93	Niger	1996 - 2019 (24)	0.90	2.10
Costa Rica	2013 - 2016 (4)	0.62	14.08	Norway	1981 - 2020 (37)	0.87	13.94
Croatia	1996 - 2019 (21)	0.73	-1.55	Peru	2008 - 2019 (12)	0.83	15.55
Curaçao	2001 - 2016 (15)	1.00	5.15	Philippines	1996 - 2019 (24)	1.53	-2.02
Cyprus	1996 - 2019 (23)	0.12	4.81	Poland	1996 - 2019 (23)	0.61	10.15
Czechia	1994 - 2019 (18)	0.22	11.08	Portugal	1996 - 2020 (22)	0.01	16.81
Côte d'Ivoire	1996 - 2000 (5)	0.36	-5.14	Qatar	2002 - 2017 (14)	1.85	2.18
Denmark	1996 - 2020 (24)	0.26	2.83	Romania	1996 - 2019 (22)	0.11	12.56
Dominican Republic	1996 - 2016 (10)	1.43	1.55	Russia	1996 - 2017 (12)	0.02	-2.70
Ecuador	2008 - 2018 (11)	0.59	-0.60	Saudi Arabia	2003 - 2009 (7)	2.69	13.89
Egypt	1997 - 2015 (19)	0.95	-1.80	Serbia	1998 - 2019 (17)	-0.45	-0.28
Estonia	1996 - 2019 (22)	0.34	11.35	Slovakia	1996 - 2019 (22)	0.20	17.95
Finland	1996 - 2020 (19)	0.29	11.85	Slovenia	1996 - 2019 (20)	0.32	13.84
France	1980 - 2019 (31)	0.23	6.70	South Africa	1996 - 2020 (22)	0.31	18.12
Germany	1980 - 2020 (36)	0.70	21.93	South Korea	1996 - 2018 (22)	0.66	19.05
Greece	1996 - 2019 (23)	0.10	0.95	Spain	1995 - 2019 (22)	0.38	17.37
Guatemala	2002 - 2019 (18)	-0.70	8.22	Sweden	1980 - 2020 (37)	0.25	5.03
Guinea	2004 - 2010 (5)	0.75	14.84	Switzerland	1993 - 2019 (23)	0.79	21.89
Honduras	2001 - 2015 (14)	-0.40	5.61	Tunisia	1996 - 2011 (16)	0.29	-1.58
Hong Kong SAR China	1997 - 2020 (23)	0.79	13.54	Turkey	2010 - 2017 (8)	1.23	2.47
Hungary	1996 - 2019 (21)	0.11	11.65	Ukraine	1996 - 2019 (21)	0.20	2.38
Iceland	2001 - 2014 (14)	0.20	7.65	United Kingdom	1982 - 2017 (32)	0.17	15.72
India	1999 - 2017 (19)	0.57	10.09	United States	1980 - 2018 (34)	0.34	17.53
Iran	1997 - 2018 (21)	5.03	4.03	Vanuatu	2002 - 2007 (6)	0.33	9.95
Ireland	1996 - 2019 (23)	0.44	8.00	Venezuela	1998 - 2019 (21)	0.65	0.41

Note: Thrift theory predicts $\overline{\theta_{s,i}^*} \cong 1$ and $\overline{\theta_{c,i}^*} \cong 1$. Free growth theory makes no prediction for these data.

8. Data sources

All our data are drawn from Distributional National Accounts (DINA) from the free online database World Inequality Database (WID). This source collates data from national accounts and tax data of 105 countries in constant currency units, and adjusts them where needed to conform to current standards of the System of National Accounts (SNA) published by the United Nations. We show results for the 86 of those countries which report all three of the factors, namely net saving, consumption and market-value capital, needed for deriving the thrift and free growth indexes. WID's source for these data is national accounts.

Consumption C in our text and equations is reproduced from the sum of Government Final Consumption Expenditure (mcongo)[5] and Private Expenditures of Households and NPISH (mconhn). This sums government expenditure GCE and personal consumption expenditure PCE. Net saving S_{net} and market-value K are taken from net national saving (msavin) and market-value Capital Wealth (mnweal) respectively. GDP, which we use only for weighting purposes in Figs. 1 and 2, is reproduced from GDP (mgdpro).

9. Accessing our results and methods

Tables and other displays of our findings for each country, and showing our methods of calculation, can be accessed at the web appendix (https://3woilz-0-0.shinyapps.io/RhinoApplication/).

10. Displays

Eqs. (10) and (19) show $\theta_s + \varphi_s = 1$ and $\theta_c + \varphi_c = 1$. Some displays here and in the web appendix save space by showing φ_s and φ_c only, leaving θ_s and θ_c implicit as their complements to unity. The web appendix includes displays of φ_s and φ_c for each year in each country over the report period. These tend to show upward and downward spikes in values of φ_s and φ_c in some years. Those spikes tend to be associated with small absolute values of denominators, in these cases $\Delta g(K)$, in those countries and years. Small denominators magnify errors in measurements of numerators. Worse, when denominators are small, small mismeasurements in them might reverse the term in sign.

To maximize reliability of test results, we apply a range of screens to omit years where absolute denominators fall below a given threshold. Some displays show data for all years, regardless of denominator size. Others screen out all years where absolute denominators are less then .01, then .025, then continuing upward in increments of .025 to a maximum screen of .15. φ_s and φ_c are plotted for each country unscreened and at each of the seven successive levels of screening. All displays applied a screen of .01. The denominator whose absolute value is screened is capital acceleration $\Delta g(K)$ or capital growth rate $g(K)$ in all displays.

Screening out years where absolute $\Delta g(K)$ or $g(K)$ is small would cost little in informative value even if measurements were exact. In those years, there is little capital acceleration or capital growth, positive or negative, for either thrift theory or free growth theory to explain. Market noise alone might account for $\Delta g(K)$ or $g(K)$ in such years. Screening reduces the number of observations, but increases the reliability and informative value of each.

11. Disclaimers

We accept that capital growth is impossible without capital replacement first, to make up losses to depreciation, and that the only source of capital replacement is replacement investment. Thus we accept the necessity and efficacy of replacement investment. We dispute only the efficacy of net investment in realizing capital growth after replacement investment has made up for depreciation.

12. Discussion and conclusions

Capital glut is the condition warned against by West, Ricardo, Malthus and Harrod. It is loosely defined as oversupply of capital at the current state of technology. We will not attempt a more exact definition here. Findings shown in our displays, anyhow, suggest that net saving raises the physical quantity of capital, say in number of shops, manufacturing plants or finished goods of similar design, without raising aggregate value of capital, and so contributes to capital glut.

[5] WID code

These findings challenge the teachings that capital growth is effected by net saving enabled by consumption restraint, and that producer cost, including imputed interest as the opportunity cost of capital, converges to market realization. Evidence showing $\overline{\varphi_{s,i}} \cong 1$ and $\overline{\varphi_{c,i}} \cong 1$ suggests that all capital growth is free, and consequently that market realization, in the presence of innovation, exceeds producer cost by the entirety of capital growth. Meanwhile the same evidence, which indicates that net saving adds no capital value, suggests a review of the teachings that consumption plus net saving gives net income, and that consumption plus net investment gives net output. Appendix B will also question the latter teachings.

Embodied growth is disembodied growth on a finer scale. It redeploys or repurposes existing labor skills, raw materials, and plant capacity, as well as existing finished goods, to achieve higher returns than available from the customary uses which determine their prices. The present value of yields from this advantage in return, or equivalently the innovator's reserve, defines the non-random component in free growth.

Appendix A. Equivalence of saving and investment

Any usage which treats saving and investment as equal must deal with the fact that saving held in cash is not investment in the usual sense of spending on productive capital, and contributes nothing to output. Keynes (1936) defined that unspent saving as intended saving, and actual saving as the part so spent.

We suggest that investors seek to maximize return, within risk tolerance, and will sometimes hold saving in cash or equivalents at zero return in recessions and depressions when positive returns cannot be found, and so when investing in the usual sense would tend to reduce capital and output rather than increase them. Thus investment could have the usual meaning in usual times, when prospective returns bring animal spirits, and could include saving in cash when not. It is in this sense that we equate saving to investment.

Appendix B. Net output with human capital

Human capital is impractical to measure, as it leaves little market record other than for its rental income in pay and investment cost in schooling. Thus national accounts leave it implicit, and allow us to infer what we can from data for pay and schooling. Those accounts are founded on the principle, sound when terms are appropriately redefined, that net output, or value added, is expressed in the sum of capital growth and net outflow from the value-added chain. In national accounts, then, where physical capital is the whole of capital while net outflow of the chain is the whole of consumption, the reasoning is

$$Y = \Delta K + C \,, \quad \text{neglecting human capital}^{6}. \tag{B.1}$$

It is possible in principle to model a value-added chain which includes human capital, and to compare findings with those shown in Eq. (B.1). Let human capital H, in that new model, stand as the last link in the value-added chain. Adapting the classic illustration of the value added principle, say that farms produce wheat, mills convert the wheat to flour, bakeries convert the flour into bread, and humans convert some of the bread, called invested consumption, into human capital. The net outflow from this extended value-added chain is not all of consumption, but only pure consumption, meaning the part remaining after the part invested in human capital (invested consumption) is subtracted[7]. By this reasoning, the principle that net output is expressed in capital growth plus net outflow gives

$$Y = \Delta K + \Delta H + C_p \,, \quad \text{allowing human capital,} \tag{B.2}$$

where C_p gives pure consumption.

Yoram Ben-Porath (1967) reasoned that growth in human capital equals invested consumption plus self-invested work less human depreciation[8]. Let C_s, W_s and $D(H)$ show these flows respectively. Thus the combined arguments of Schultz and Ben-Porath arrive at

$$C = C_s + C_p \quad \text{and} \quad \Delta H = C_s + W_s - D(H) \,, \quad \text{allowing human capital.} \tag{B.3}$$

[6] Where ΔK, mistakenly, we argue, is measured as S_{net}.

[7] The concepts of invested consumption, pure consumption, self-invested work and human depreciation were introduced in Schultz (1961).

[8] Equation 4 in Ben-Porath's paper, summarizing his first three equations. His terms and notation differ from ours. The concepts of invested consumption were also introduced by Schultz (1961).

Substitution of these equations into Eq. (B.2) finds

$$Y = \Delta K + C_s + W_s - D(H) + C_p \quad \text{and consequently}$$

$$Y = \Delta K + C + W_s - D(H)\,, \quad \text{allowing human capital,} \tag{B.4}$$

if Schultz and Ben-Porath and the reasoning here are right.

Appendix C. Mill's statement of the free growth idea

Mill (1848), book 1, chapter 5, section 4, includes:

> If it were said, for instance, that the only way to accelerate the increase of capital is by increase of saving, the idea would probably be suggested of greater abstinence, and increased privation. But it is obvious that whatever increases the productive power of labour creates an additional fund to make savings from, and enables capital to be enlarged not only without additional privation, but concurrently with an increase of personal consumption.

This passage may be the first clear statement of what we call free growth theory. His use of the words "accelerate" and "concurrently" suggest that his path of reasoning was something like ours, and that he meant what we call free growth rather than the alternating phases of higher and lower saving rates pictured in thrift theory.

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