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Boolean Algebra

» Boolean Algebra - Ordinary Algebra

Boolean Algebra	Ordinary Algebra
• In B·A $1+1=1$	• $1+1=2$
• $a \cdot a = a$	• $a \cdot a = a^2$
• $a+a=a$	• $a+a=2a$.
• only •, + are available	• •, +, %, x, - is possible.
• square root is not possible, cube root	• square root, cube root is possible.

» variables : a to z, A to Z
variable is available in complemented or in uncomplemented form.

» Operator : operation to be performed on variables
operators here :-
AND •
OR +
NOT ‾ / '

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>> Boolean laws and theorems

* Duality theorem :- Replace • by + and vice versa and 1 with 0 and vice-versa.

1. OR Law

$a + 0 = a$

$a + 1 = 1$

(Idempotent Law) $a + a = a$

$a + \bar{a} = 1$

$1 + 0 = 1$

$0 + 0 = 0$

$1 + 1 = 1$

NOTE :- Two constants are applicable in boolean algebra i.e 0 and 1.

2. AND Law

$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot a = a$ (Idempotent law)

$a \cdot \bar{a} = 0$

$1 \cdot 0 = 0$

$0 \cdot 0 = 0$

$1 \cdot 1 = 1$

3. Commutative law

$A + B = B + A$

$A \cdot B = B \cdot A$

4. Associative law

$A + (B + C) = (A + B) + C$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

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5. <u>Distributive Law</u>

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

Imp :- $A + BC = (A + B) \cdot (A + C)$

6. <u>Involution Law</u>

$$\overline{\overline{A}} = A$$

7. <u>Absorption Law</u>

$$A + AB = A$$

$$A + A'B = (A + B)$$

$$A \cdot (A' + B) = A \cdot B$$

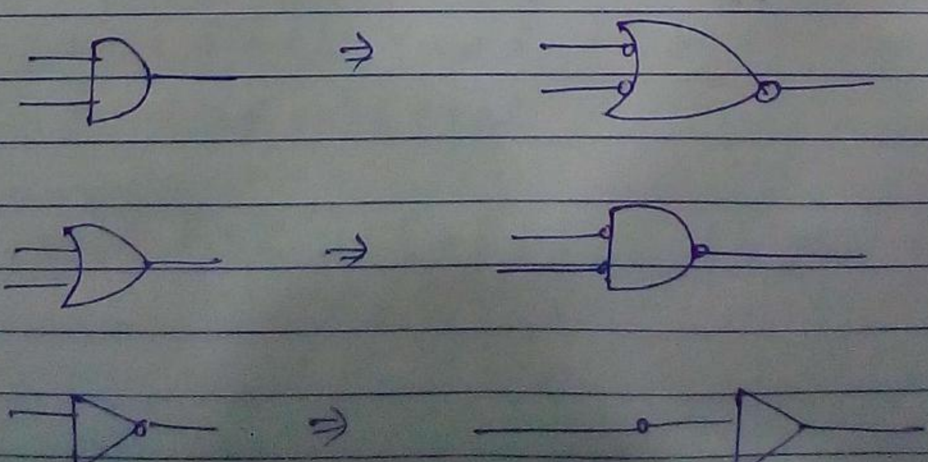
8. <u>De Morgan's Theorem</u>

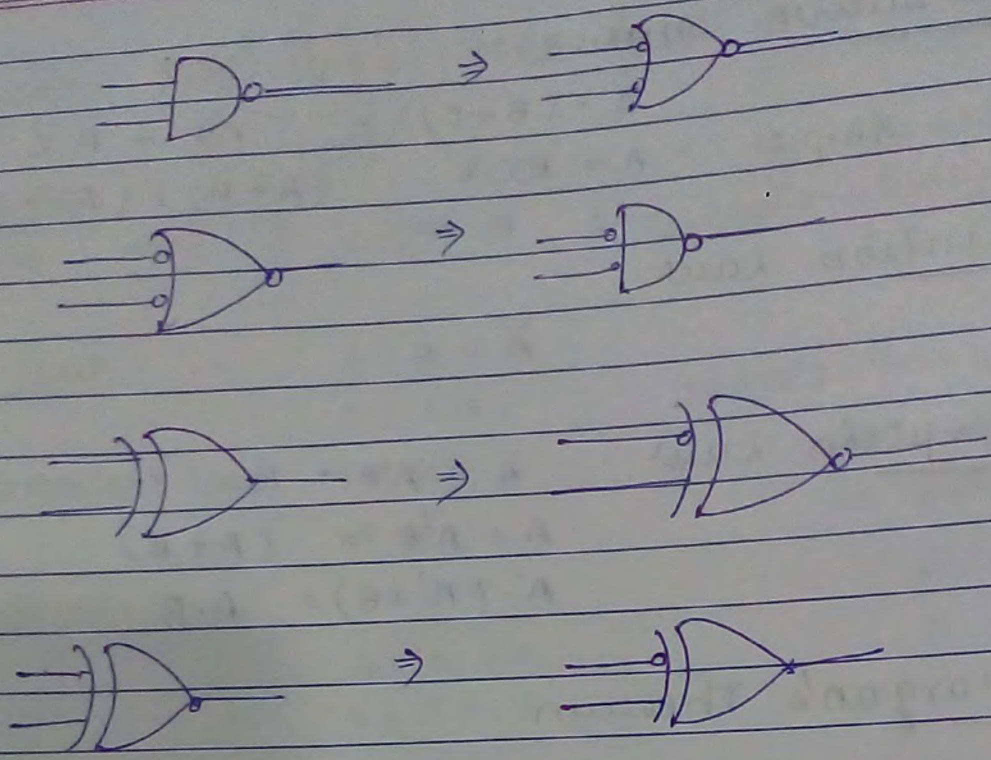
<u>Ist thm</u> :- complement of sum is equal to the complement of product.

$$\overline{a + b} = \bar{a} \cdot \bar{b} \rightarrow \overline{a \cdot b} = \bar{a} + \bar{b}$$

★ NOR gate is equal to bubbled AND gate.
NAND gate is equal to bubbled OR gate

» Alternate symbols of logic gates :-





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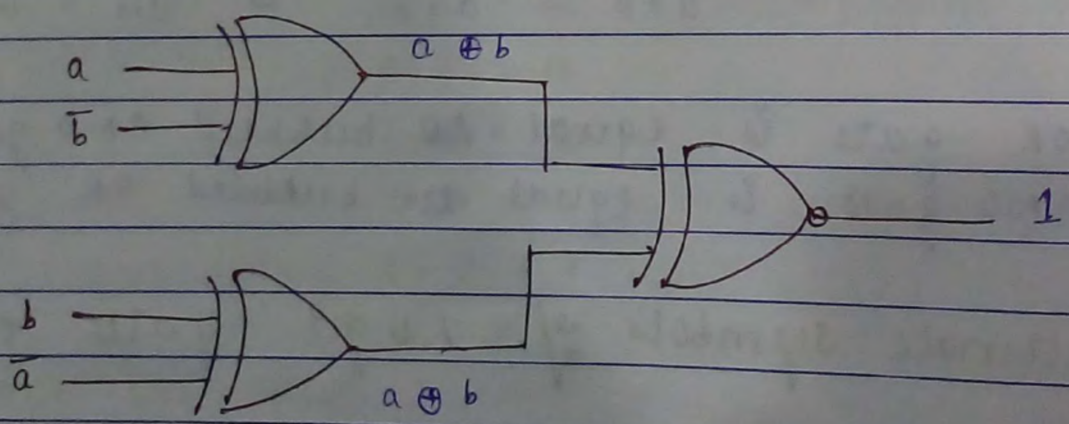
Q what will be output of following circuit



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>> 2mP Consensus Theorem

In this, there are three variables and in it one variable is in complemented form and remaining two variables are in uncomplemented form and answer is taken through those terms where complemented variable is present.
This thm is also called redundant theorem.

Ex $ab + bc + c\bar{a} = ab + c\bar{a}$ (SOP)

Proof → LHS → $ab + bc \cdot 1 + c\bar{a}$
$= ab + bc(a + \bar{a}) + c\bar{a}$
$= ab + abc + \bar{a}bc + \bar{a}c$
$= a \cdot b(1 + c) + \bar{a}c(1 + b)$
$= ab + \bar{a}c$
$=$ RHS

$(a+b) \cdot (b+c) \cdot (c+\bar{a}) = (a+b) \cdot (c+\bar{a})$ (POS)

LHS $= (a+b) \cdot (b+c+0) \cdot (c+\bar{a})$
$= (a+b) \cdot (b+c+a\bar{a}) \cdot (c+\bar{a})$
$= (a+b) \cdot (b+c+a) \cdot (b+c+\bar{a}) \cdot (c+\bar{a})$
$= (\underset{x}{a}+\underset{y}{b}+0) \cdot (\underbrace{a+b}_{x}+\underset{z}{c}) (\bar{a}+b+c) \cdot (c+\bar{a}+0)$
$(a+b+0 \cdot c) \quad (c+\bar{a}+b \cdot 0)$
$= (a+b)(c+\bar{a})$

Q2 $(a+b)(\bar{b}+\bar{c})(\bar{c}+\bar{a}) = (a+b)(\bar{c}+\bar{a})$

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Q3. Minimise the following function

$$Y = \overline{\bar{a}bc + \bar{b}\bar{c}}$$

$$\overline{a + \bar{b}\bar{c} + \bar{b}\bar{c}}$$

$$= \overline{a + \bar{b} + \bar{c} + \bar{b}\bar{c}}$$

$$\times \ \overline{bc}\,(1+\bar{a})$$

$$= \times \ \cancel{\overline{b}\,\overline{c}\cdot 1}$$

$$= \times \ \cancel{bc}$$

$$= \overline{(a + \bar{b} + \bar{c} + \bar{b})\cdot(a + \bar{b} + \bar{c} + \bar{c})}$$

$$= \overline{(a + \bar{b} + \bar{c})\cdot(a + \bar{b} + \bar{c})}$$

$$= \overline{a + \bar{b} + \bar{c}}$$

$$= \bar{a}bc$$

Q4 $$Y = \overline{\bar{b}\bar{c} + \bar{c}\bar{a} + a\bar{b} + b}$$

$$= \bar{b}\bar{c} + \bar{c}\bar{a} + (b + a\bar{b}) + (b + a\bar{b})$$

$$= \bar{b}\bar{c} + \bar{c}\bar{a} + b + a + b + \bar{b}$$

$$= a + b + \bar{b} + \bar{c}\bar{a}$$

$$= a + \bar{c}\bar{a}$$

$$= \overline{a\bar{b} + \bar{c}\bar{a} + b}$$

$$= \overline{\bar{c}\bar{a} + (a+b)(b+\bar{b})}$$

$$= \overline{\bar{c}\bar{a} + (a+b)}$$

$$= \overline{\bar{a}\bar{c} + a + b}$$

$$= \overline{(a+\bar{a})\cdot(\bar{c}+a) + b}$$

$$= \overline{\bar{c} + a + b}$$

$$= \bar{a}\bar{b}c$$

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05 $Y = AB + A\bar{B}C + A\bar{B}\bar{C}$

$= AB + A\bar{B}(C+\bar{C})$

$= AB + A\bar{B} = A$

If inputs are n total combinations equal to 2^n and total logical expressions equal to 2^{2^n}

Q If no of inputs are 2, then total logical exps are

$2^{2^2} = 2^4 \Rightarrow 16$ logical expressions

Q If n is 4 then total no of logical expressions are.

$2^{2^4} = 2^{16} = 65536$

$\rightarrow 64K$

$1K = \cancel{20}\ 2^{10} B$

$2^{16} = 2^6 \cdot 2^{10}$

$= 2^6 K = 64K$

Q Determine whether NAND gate follows commutative or ~~distributive~~ Associative law or not.

Truth Table

A	B	A.B	$\overline{A.B}$	B.A	$\overline{B.A}$
0	0	0	1	0	1
0	1	0	1	0	1
1	0	0	1	0	1
1	1	1	0	1	0

Commutative law: A . B = B . A
Verified for NAND gate.

distributive law.

A	B	C	B+C	A.(B+C)	$\overline{A.(B+C)}$	A.B	AC
0	0	0	0	0	1	0	0
0	0	1	1	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	0
1	0	1	1	1	0	0	1
1	1	0	1	1	0	1	0
1	1	1	1	1	0	1	1

AB + AC	$\overline{AB + AC}$
0	1
0	1
0	1
0	1
0	1
1	0
1	0
1	0

NAND gate is commutative but not associative and distributive

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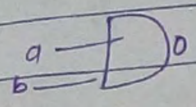
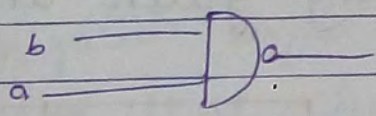
① commutative :-

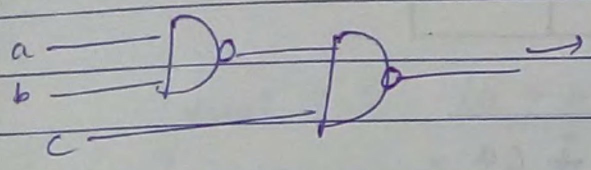


$= \overline{a\cdot b} \Rightarrow$



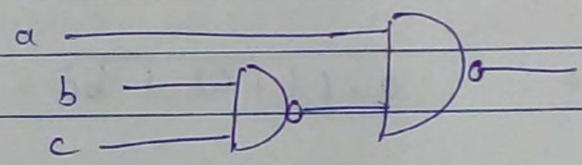
$\overline{ab}$

② Associative



$\rightarrow \overline{\overline{a\cdot b}\cdot C}$

$\neq$



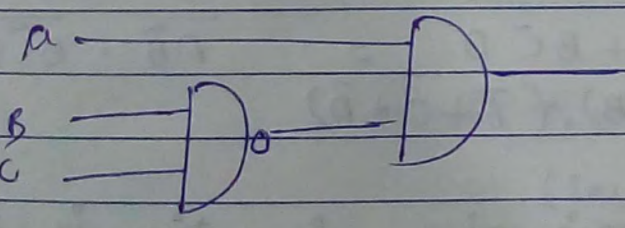
$= \overline{a\cdot \overline{b\cdot c}}$

③ Distributive

$$A\cdot(B+C) = AB + AC$$

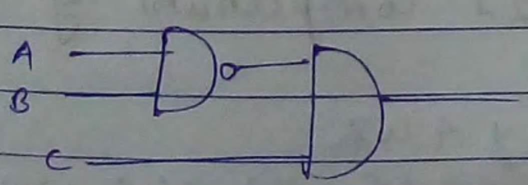
$$\Rightarrow A\cdot(\bar{B}+\bar{C}) = A\bar{B} + A\bar{C}$$

$$\Rightarrow A\cdot(\overline{B\cdot C})$$



$(\overline{B\cdot C})\cdot A$

$\neq$



$(\overline{A\cdot B})\cdot C$

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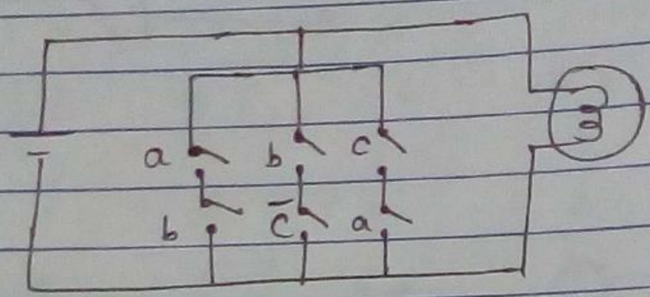
Q what will be output expression in the following given circuit.



$Y = \overline{a.b + b\bar{c} + ca}$

$Y = \overline{a.(b+c) + b\bar{c}} \Rightarrow \overline{b\bar{c} + ca}$.

$Y = \overline{b\bar{c}} \cdot \overline{ca} \Rightarrow (\bar{b} + c).(\bar{c} + \bar{a})$

» complement of a function

Q Determine complement of $Y = A\bar{B} + B\bar{C}D$

$Y = \overline{A\bar{B} + B\bar{C}D} = \overline{A\bar{B}} \cdot \overline{B\bar{C}D}$

$Y = (\bar{A} + B).(\bar{B} + C + \bar{D})$

Q If negative logic (dual) of a function is $\bar{x}y + wz$ what will be 1's complement of that function.

f dual $= \bar{x}y + wz$

~~f complement~~ $= \cancel{(\bar{x} + \bar{y}) \cdot (\bar{w} + \bar{z})}$

$= x\bar{y} + w\bar{z}$

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>> To determine 1's complement of any logic function

or negative logic

first determine dual of that function and then complement each variable.

Q Determine 1's complement $Y = a \cdot \bar{b} + b \cdot \bar{c} \cdot d$

$$Y_{dual} = (\bar{a} + b) \cdot (\bar{b} + c + \bar{d})$$

$$Y_{\cancel{complement}} = (a + \bar{b}) \cdot (b + \bar{c} + d)$$