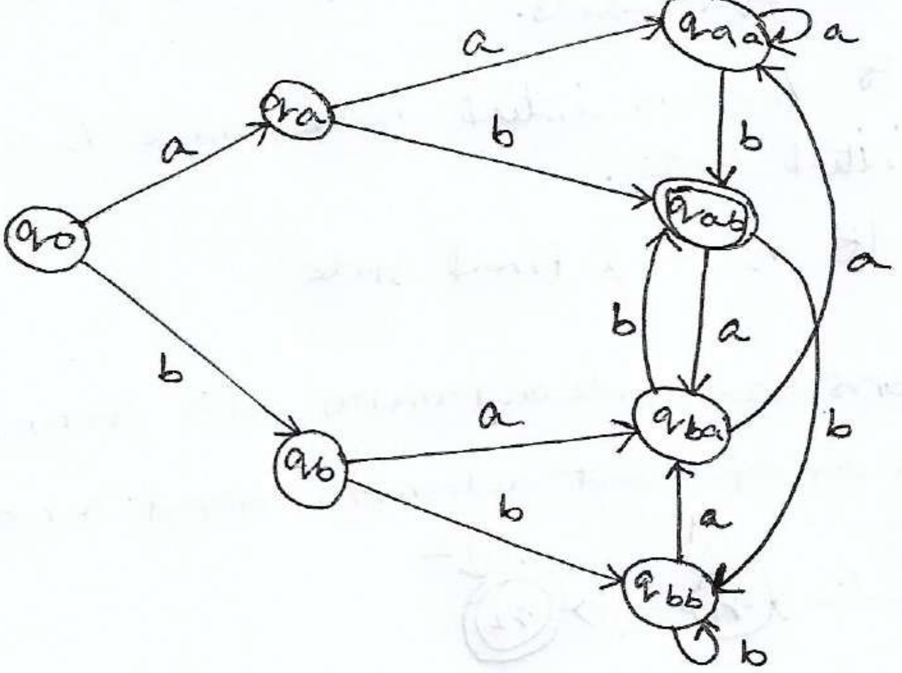
Date 1-02-2021

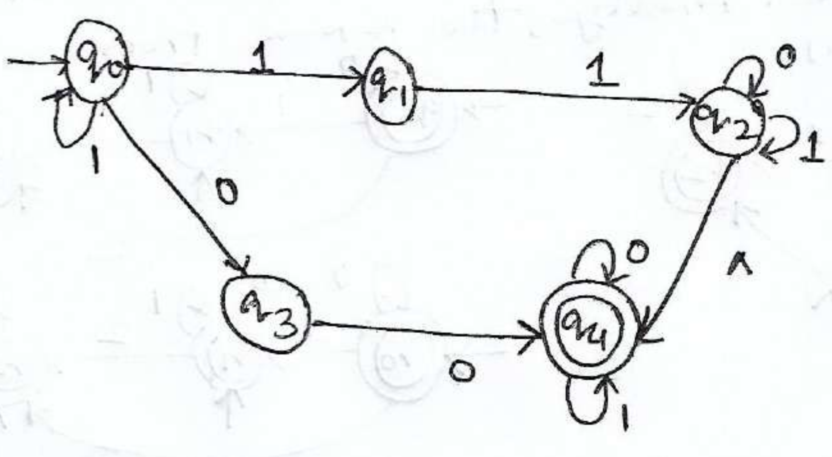
Automata Theory

Ex:- Construct a DFA accepting all strings over {a,b} ending in ab.

Sol^n:- we require two transition for accepting the string ab. If the symbol b is processed after aa or ba, then also we end in ab. So, we can have states for remembering aa, ab, ba, bb. The state corresponding to ab can be final state in our DFA. Keeping these in mind, we construct the required DFA. Its Transition diagram is described by fig.



* Λ-NFA : Transition system for a Nondeterministic automaton which contains Λ-Transition



* Conversion of Λ-NFA into NFA.

It is Possible to convert a Transition system with Λ-moves into an Equivalent Transition system without Λ-moves. we shall give a Simple method of doing it with the help of an example

Suppose we want to replace a Λ-move from vertex V_1 to vertex V_2. Then we proceed as follows;

Step 1 find all the edges starting from V_2

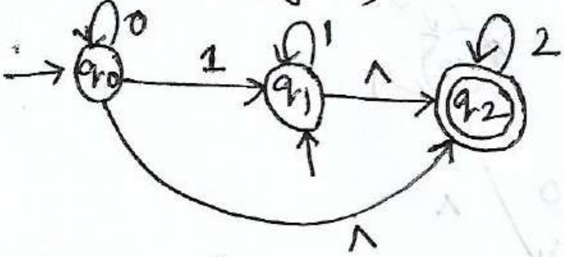
Step 2 Duplicate all these edges starting from V_1, without changing the edge labels.

Step3:- If V_1 is an initial state, make V_2 also as initial state.

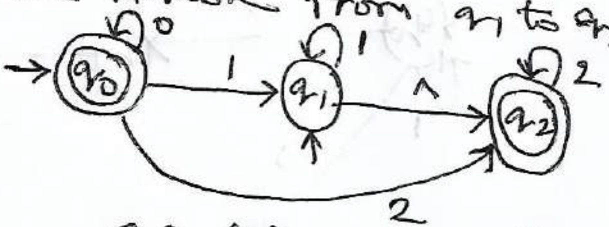
Step 4:- If V_2 is a final state

Example:- Consider a finite automaton, with Λ-moves, given in figure. obtain an equivalent automaton without Λ moves.

Soln:- We first eliminate the Λ-move from q_0 to q_1 to get fig ⓐ. q_1 is made an initial state. Then we eliminate the Λ-move from q_0 to q_2 in fig ⓐ to get fig (b). As q_2 is a final state, q_0 is also made a final state. Finally, the Λ-move from q_1 to q_2 is eliminated in fig (b).



(a)



(b)

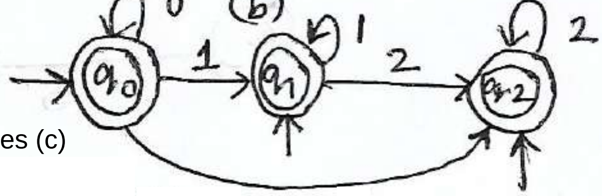


Figure:- Transition System for Example without Empty-Moves (c)

Minimization of FINITE Automata :→

Here, we construct an automaton with the minimum number of states equivalent to a given Automaton M.

Definition :- Two states q_1 and q_2 are equivalent (denoted by $(q_1 \equiv q_2)$) if both $\delta(q_1, x)$ and $\delta(q_2, x)$ are final states, or both of them are non final states for all. $x \in \Sigma^*$

As it is difficult to construct $\delta(q_1, x)$ and $\delta(q_2, x)$ for all $x \in \Sigma^*$ (there are an infinite number of strings in Σ^*), we give one more definition.

Definition :→ Two states q_1 and q_2 are K-equivalent $(K \geq 0)$ if both $\delta(q_1, x)$ and $\delta(q_2, x)$ are final states or both non final states for all strings x of length K or less. In particular, any two final states are 0-equivalent and any two non final states are also 0-equivalent.

We mention some of the properties of these relations.

Property 1:- The relations we have defined, i.e. equivalence and K-equivalence are equivalence relations, i.e. they are reflexive, symmetric and transitive.

Property 2:- We have to partitions of Q in two disjoint class/set. These partitions can be denoted by π and π_K, respectively. The elements of π_K are K-equivalence classes.

Property 3:- If q_1 and q_2 are K-equivalent for all $K \geq 0$, then they are equivalent.

Property 4:- If q_1 and q_2 are $(K+1)$-equivalent, then they are equivalent.

Property 5:- $\pi_n = \pi_{n+1}$ for some n. (π_n denotes The set of equivalence classes under n-equivalence)

Construction of Minimum Automation :→

Step 1! (Construction of π_0). By definition of 0-equivalence, $\pi_0 = \{Q_1^0, Q_2^0\}$ where Q_1^0 is The set of all final states and $Q_2^0 = Q - Q_1^0$.

Step 2:- (Construction of π_{K+1} from π_K). Let Q_i^K be any subset in π_K. if q_1 and q_2 are in Q_i^K, They are $(K+1)$-equivalent provided $\delta(q_1, a)$ and $\delta(q_2, a)$ are K-equivalent. Find out whether $\delta(q_1, a)$ and $\delta(q_2, a)$ are in the same equivalence class in π_K for every $a \in \Sigma$. If so, q_1 and q_2 are $(k+1)$-equivalent. In this way, Q_i^K is further divided into $(K+1)$-equivalence classes. Repeat this for every Q_i^K in π_K to get all The elements of π_{K+1}.

Step 3:→ Construct π_n for $n = 1, 2, ---$ until $\pi_n = \pi_{n+1}$.

Step 4:→ (Construction of minimum automaton). For The required minimum state automaton, The states are the equivalence classes obtained in step 3. i.e, The elements of π_n. The state table is obtained by replacing a state q by the corresponding equivalence class $[q]$.

Ex:- construct a minimum state automaton equivalent to the finite automaton described by fig

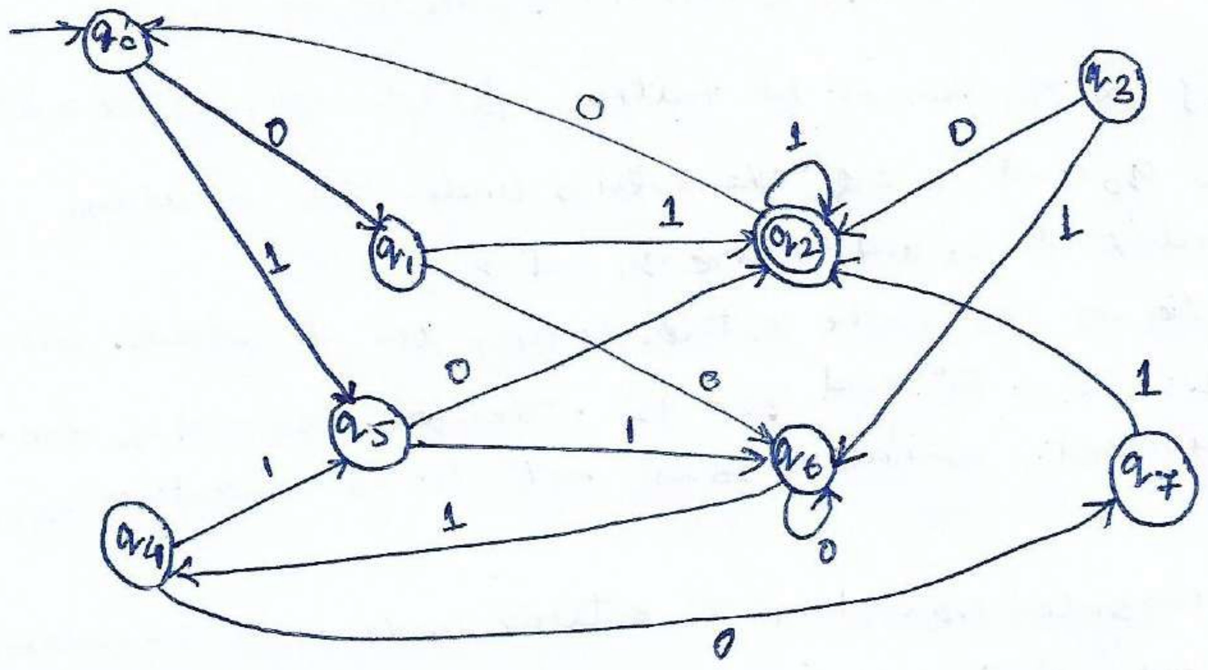


fig:- finite automaton

Solution:- It will be easier if we construct the transition table as shown in ~~fig~~ Table

Table:- transition table for example/above DFA

State/Σ	0	1
→ q_0	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

By applying step 1, we get

$Q_1^0 = F = \{q_2\}$, $Q_2^0 = Q - Q_1^0$

so $\pi_0 = \{\{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\}$

The $\{q_2\}$ in π_0 cannot be further partitioned. So $Q_1' = \{q_2\}$.

Consider q_0 and $q_1 \in Q_2^0$. The entries under the 0-column corresponding to q_0 and q_1 are q_1 and q_6; they lie in Q_2^0. The entries under the 1-column are q_5 and q_2. $q_2 \in Q_1^0$ and $q_5 \in q_2^0$. Therefore, q_0 and q_1 are not 1-equivalent. Similarly, q_0 is not 1-equivalent to q_3, q_5 and q_7.

Now, consider q_0 and q_4. The entries under the 0-column are q_1 and q_7. Both are in Q_2^0. The entries under the 1-column are q_5, q_5. So q_4 and q_0 are 1-equivalent. Similarly q_0 is 1-equivalent to q_6. $\{q_0, q_4, q_6\}$ is a subset in π_1. So,

$Q_2' = \{q_0, q_4, q_6\}$.

Repeat The construction by considering q_1 and any one of The states q_3, q_5, q_7. Now, q_1 is not 1-equivalent to q_3 or q_5 but 1-equivalent to q_7.

Hence, $Q_3' = \{q_1, q_7\}$. The elements left over in Q_2^0 are q_3 and q_5.

By considering the entries under the 0-column and The 1-column, we see that q_3 and q_5 are 1-equivalent, so $Q_4' = \{q_3, q_5\}$. Therefore,

$$\pi_1 = \{\{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}.$$

The $\{q_2\}$ is also in π_2 as it cannot be further Partitioned Now, The entries under The 0-column correspondig to q_0 and q_4 are q_1 and q_7, and These lie in The same equivalence class in π_1, The entries under the 1-column are q_5, q_5.

So q_0 and q_4 are 2-equivalent. But q_0 and q_6 are not 2-equivalent. Hence, $\{q_0, q_4, q_6\}$ is Partitired into $\{q_0, q_4\}$ and $\{q_6\}$. q_1 and q_7 are 2-equivalent, q_3 and q_5 are also 2-equivalent. Thus $\pi_2 = \{\{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$. q_0 and q_4 are 3-equivalent. The q_1 and q_7 are 3-equivalent. Also q_3 and q_5 are 3-equivalent. Therefore,

$$\pi_3 = \{\{q_2\}. \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\}\}$$

As $\pi_2 = \pi_3$, π_2 gives us The equivalentce classes, The minimum state automaton is

$$M' = (Q', \{0,1\}, \delta', q_0', F')$$

where

$$Q' = \{[q_2], [q_0, q_4], [q_6], [q_1, q_7], [q_3, q_5]\}$$

$$q_0' = [q_0, q_4], \quad F' = [q_2]$$

and δ' is defined by table below

State/Σ	0	1
$[q_0, q_4]$	$[q_1, q_7]$	$[q_3, q_5]$
$[q_1, q_7]$	$[q_6]$	$[q_2]$
$[q_2]$	$[q_0, q_4]$	$[q_2]$
$[q_3, q_5]$	$[q_2]$	$[q_6]$
$[q_6]$	$[q_6]$	$[q_0, q_4]$

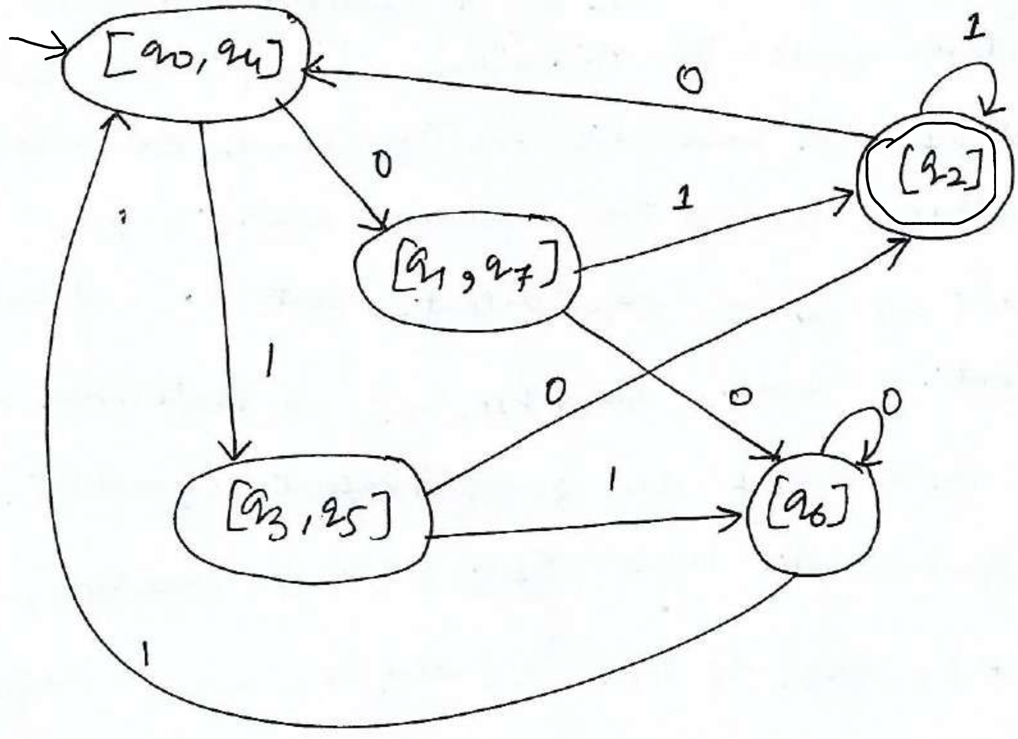


fig:- Minimum state automaton of Example.

Ex:- Construct the minimum state automaton equivalent to the Transition diagram given by fig.

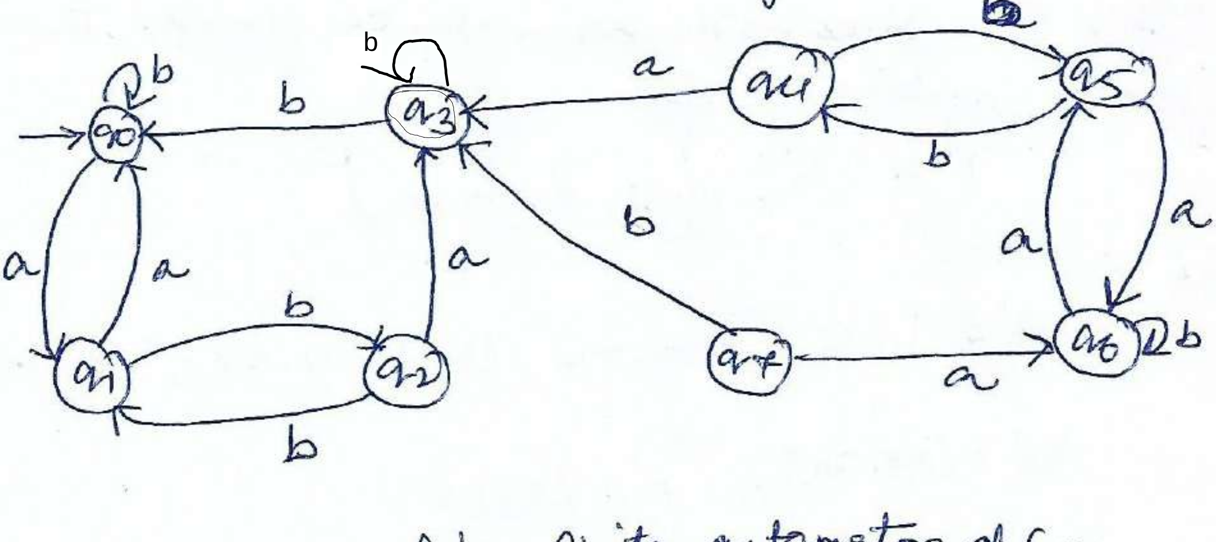


fig:- finite automaton of Ex.

Soln:- we construct the Transition table as given by Following Table.

Table :- Transition Table for Ex.

State/Σ	a	b
→q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$\textcircled{q_3}$	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

Since There is only one final state q_3, $Q_1^0 = \{q_3\}$, $Q_2^0 = Q - Q_1^0$. Hence, $\pi_0 = \{\{q_3\}, \{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}\}$. As $\{q_3\}$ cannot be Partitioned further, $Q_1^0 = \{q_3\}$.

Now q_0 is 1-equivalent to q_1, q_5, q_6, but not to q_2, q_4, q and so

$$Q_2^1 = \{q_0, q_1, q_5, q_6\},$$ q_2 is 1-equivalent to q_4. Hence, $Q_3^1 = \{q_2, q_4\}$. The only element remaining in Q_2^0 is q_7. Therefore $Q_4^1 = \{q_7\}$. Thus,

$$\pi_1 = \{\{q_3\}, \{q_0, q_1, q_5, q_6\}, \{q_2, q_4\}, \{q_7\}\}$$

$$Q_1^2 = \{q_3\}$$

q_0 is 2-equivalent to q_6 but not to q_1 or q_5. so,

$$Q_2^2 = \{q_0, q_6\}$$

As q_1 is 2-equivalent to q_5,

$$Q_3^2 = \{q_1, q_5\}$$

As q_2 is 2-equivalent to q_4,

$Q_4^2 = \{q_2, q_4\}$, $Q_5^2 = \{q_7\}$

Thus, $\pi_2 = \{\{q_3\}, \{q_0, q_6\}, \{q_1, q_5\}, \{q_2, q_4\}, \{q_7\}$

$Q_1^3 = \{q_3\}$

As q_0 is 3-equivalent to q_6,

$Q_2^3 = \{q_0, q_6\}$

As q_1 is 3-equivalent to q_5

$Q_3^3 = \{q_1, q_5\}$

As q_2 is 3-equivalent to q_4,

$Q_4^3 = \{q_2, q_4\}$, $Q_5^3 = \{q_7\}$

Therefore, $\pi_3 = \{\{q_3\}; \{q_0, q_6\}, \{q_1, q_5\}, \{q_2, q_4\}, \{q_7\}$

As $\pi_3 = \pi_2$, π_2 gives us the equivalent classes, The minimum state automaton is $M' = (Q', \{a, b\}, \delta', q_0', F')$

where $Q' = \{[q_3], [q_0, q_6], [q_1, q_5], [q_2, q_4], [q_7]\}$

$q_0' = [q_0, q_6]$, $F' = [q_3]$ and δ' is defined in table

State/Σ	a	b
$[q_0, q_6]$	$[q_1, q_5]$	$[q_0, q_6]$
$[q_1, q_5]$	$[q_0, q_6]$	$[q_2, q_4]$
$[q_2, q_4]$	$[q_3]$	$[q_1, q_5]$
$[q_3]$	$[q_3]$	$[q_0, q_6]$
$[q_7]$	$[q_0, q_6]$	$[q_3]$

Fig:- Transition table of minimum state Automaton

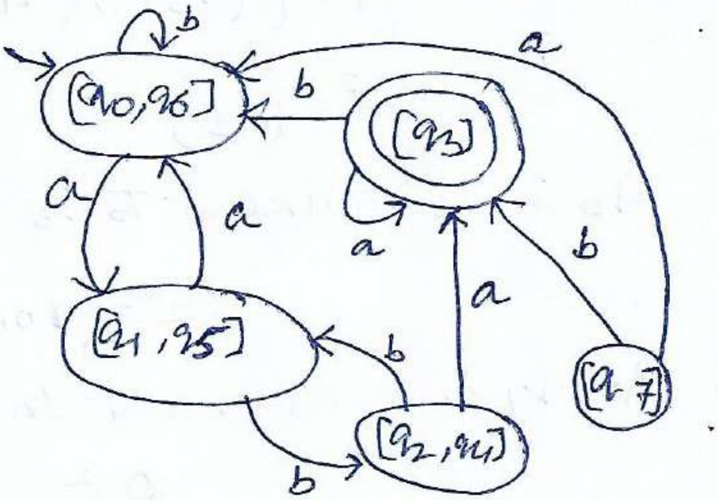


Fig:- Minimum state Automaton of Example