Date
15-2-2021

"FINITE AUTOMATA WITH OUTPUT"

Mealy and Moore Machine

* An Automaton in which the output depends only on the input is called an automaton without a Memory. An
* Automaton in which the output depends on the states as well, is called automaton with a finite memory.
* An automaton in which the output depends only on the states of the machine is called a Moore machine.
* An automaton in which the output depends on the state as well as on the I/P at any instance of time is called a Mealy machine.

<u>Mealy machine</u>:→ The value of the output function $z(t)$ in the most general case is a function of the present state $q(t)$ and the present input $x(t)$, i.e.

$$z(t) = \lambda(q(t), x(t))$$

where λ is called the output function. This generalized model is usually called the Mealy machine.

<u>Moore machine</u>:→ If the output function $z(t)$ depends only on the present state and is independent of the current input, the output function may be written as

$$Z(t) = \lambda(q(t))$$

This restricted model is called the Moore machine. It is more convenient to use Moore machine in automata Theory.

Most General Definitions of Moose and Mealy machine

Definitions :→

Moose Machine :→ A Moose machine is a six tuple $(Q, \Sigma, \Delta, \delta, \lambda, q_0)$, where

(i) Q is a finite set of states;
(ii) Σ is the input alphabet
(iii) Δ is the output alphabet
(iv) δ is the Transition function $\Sigma \times Q$ into Q;
(v) λ is the output function mapping Q into Δ;
(vi) q_0 is the initial state.

Mealy Machine :→ A Mealy machine is a six tuple $(Q, \Sigma, \Delta, \delta, \lambda, q_0)$, where all the symbol except λ have the same meaning as in the Moose machine.

λ is the output function mapping $\Sigma \times Q$ into Δ

For Example, Table 1 describes a Moose machine. The initial state q_0 is marked with an arrow. The table defines δ and λ.

Table :- A Moose machine

Present state	Next state δ		Output
	$a=0$	$a=1$	λ
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

For the Input String 0111, the transition of states is given by $q_0 \rightarrow q_3 \rightarrow q_0 \rightarrow q_1 \rightarrow q_2$. The output string is 00010.
For the input string Λ, the output is $\lambda(q_0) = 0$.

Transition Table 2 describes a Mealy machine.

A Mealy machine

Present state	Next state			
	$a=0$		$a=1$	
	state	output	state	output
$\to q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Note :→ For the input string 0011, the transition of states is given by $q_1 \to q_3 \to q_2 \to q_4 \to q_3$, and the output string is 0100.

In this case of a Mealy machine, we get an output only on the application of an input symbol. So for the input string Λ, the output is only Λ. It may be observed that in the case of a Moore machine, we get $\lambda(q_0)$ for the input string Λ.

* For a Moore machine if the input string is of length n, the output string is of length $n+1$. The first output is $\lambda(q_0)$ for all output strings.

* In the case of a ~~mealy~~ machine if the input string is of length n, the output string is also of the same length n.

* Procedure for Transforming A Mealy machine into a moore machine :→

We develop procedure for transforming a Mealy machine into a Moore machine and vice-versa so that for a given input string the output strings are the same (except for the first symbol) in both the machines.

Example:- Consider the Mealy Machine described by the transition table given by following table. Construct a Moore machine which is equivalent to the Mealy machine.

Present state	Next state			
	Input a=0		Input a=1	
	State	output	State	output
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Soln:- At the first stage we develop the procedure so that both machines accept exactly the same set of input sequences. We look into the next state column for any state, say q_i, and determine the number of different outputs associated with q_i in that column.

we split q_i into several different states, the number of such states being equal to the number of different outputs associated with q_i. For Example, in this Problem, q_1 is associated with one output 1 and q_2 is associated with Two different outputs 0 and 1. Similarly, q_3 and q_4 are associated with the outputs 0 and 0, 1, respectively. So, we split q_2 into q_{20} and q_{21}. Similarly, q_4 is split into q_{40} and q_{41}. Now table (above) can be reconstructed for the new states as given by following table.

State table for Example:

Present state	Next state			
	Input a=0		input a=1	
	State	output	State	output
$\rightarrow q_1$	q_3	0	q_{20}	0
q_{20}	q_1	1	q_{40}	0
q_{21}	q_1	1	q_{40}	0
q_3	q_{21}	1	q_1	1
q_{40}	q_{41}	1	q_3	0
q_{41}	q_{41}	1	q_3	0

The pair of states and outputs in the next state column can be rearranged as given by table following table.

Table:- Revised State table

Present state	Next state		output
	$a=0$	$a=1$	
$\rightarrow q_1$	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

Table "above" gives the Moore machine. Here we observe that the initial state q_1 is associated with 1. This means that with input $\wedge$ we get an output of 1, if the machine starts at state q_1. Thus this machine accepts a zero length sequence (null sequence) which is not accepted by the Mealy machine.

To overcome this situation, either we must neglect the response of a Moore machine to input $\wedge$, or we must add a new string state q_0, whose state transition are identical with those of q_1 but whose output is 0.

So, table "above" is transformed to table "below"

Table:- Moore machine of Example "above"

Present state	Next state $a=0$	Next state $a=1$	Output
$\rightarrow q_0$	q_3	q_{20}	0
q_1	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

From the foregoing procedure it is clear that if we have an m-output, n-state Mealy machine, the corresponding m-output Moore machine has no more than $mn+1$ states.

~~Procedure for transforming a Mealy machine into a Moore machine:-~~ We develop procedure for transforming

Procedure for Transforming a Moore Machine into a Mealy machine: →

We modify the acceptability of input string by a Moore machine by neglecting the response of the Moore machine to input $\wedge$. We thus define that Mealy machine M and Moore machine M' are equivalent if for all input strings ω, $bZ_M(\omega) = Z_{M'}(\omega)$, where b is the output of the Moore machine for its initial state. We give the following result: Let $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ be a Moore machine. Then the following procedure may be adopted to construct an equivalent Mealy machine M_2.

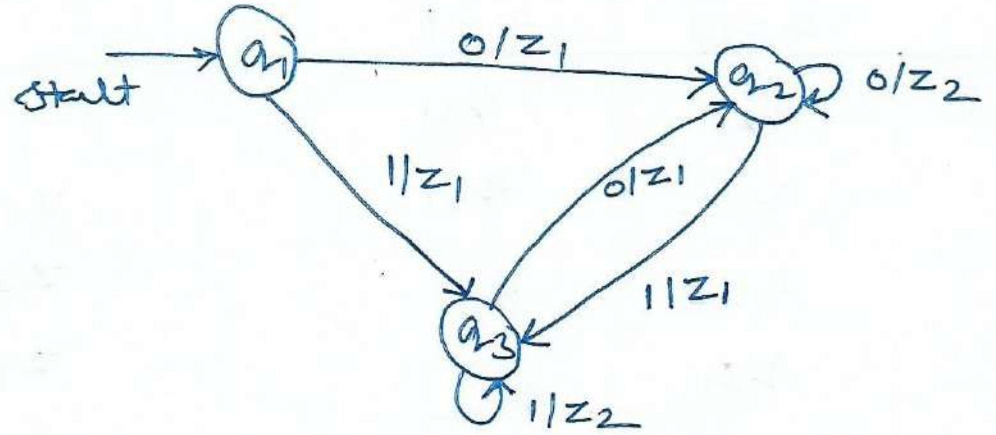
Construction: → (i) We have to define the output function λ' for the Mealy machine as a function of the present state and the input symbol. We define λ' by

$$\lambda'(q, a) = \lambda(\delta(q, a))$$ for all states q and input symbol a.

(ii) The transition function is the same as that of the given Moore machine.

Example: — (9)

Consider a Mealy machine represented by Fig. Construct a Moore machine equivalent to this Mealy machine.



Mealy machine of Ex.

Sol? — Let us convert transition diagram into transition table

Transition table for Ex.

Present state	Next state			
	$a=0$		$a=1$	
	state	output	state	output
$\to q_1$	q_2	z_1	q_3	z_1
q_2	q_2	z_2	q_3	z_1
q_3	q_2	z_1	q_3	z_2

For the given problem: q_1 is not associated with any output; q_2 is associated with two different outputs z_1 and z_2; q_3 is associated with two different ~~states~~ outputs z_1 and z_2. Thus we must split q_2 into q_{21} and q_{22} with outputs z_1 and z_2, respectively and q_3 into q_{31} and q_{32} with output z_1 and z_2, respectively.

following table shows The Transition table of Moore machine For Example.

Present state	Next state		output
	$a=0$	$a=1$	
$\rightarrow q_1$	q_{21}	q_{31}	
q_{21}	q_{22}	q_{31}	z_1
q_{22}	q_{22}	q_{31}	z_2
q_{31}	q_{21}	q_{32}	z_1
q_{32}	q_{21}	q_{32}	z_2

figure gives The Transition diagram of The required moore machine

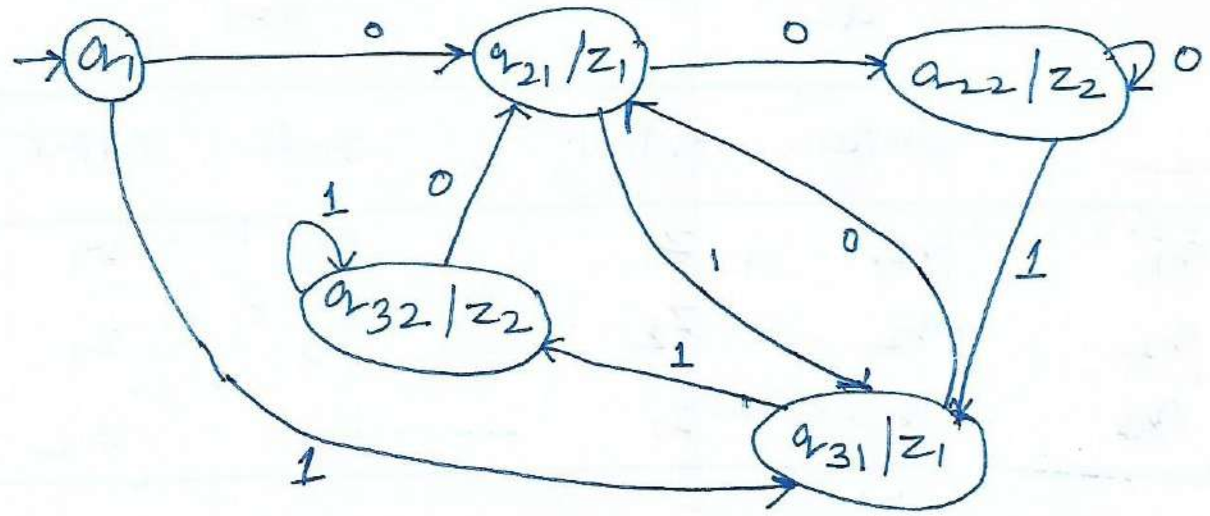


fig :- Moore Machine of Ex.

Ex:- Construct a Mealy Machine which is equivalent to the Moore machine given by table given below

Present state	Next state		output
	$a=0$	$a=1$	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Solⁿ:- we must follow the reverse procedure of converting a Mealy machine into a Moore machine. In the case of the Moore machine, for every I/P symbol we form the pair consisting of the next state and the corresponding output and reconstruct the table for the Mealy Machine.

For Example, the state q_3 and q_1 in the next state columns should be associated with output 0 and 1, respectively. The transition table for the Mealy machine is given by table below

Present state	Next state			
	$a=0$		$a=1$	
	State	output	State	output
$\rightarrow q_0$	q_3	0	q_1	1
q_1	q_1	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Note:- we can reduce the number of states in any model by considering states with identical transitions.

If two states have identical transitions (i.e the rows corresponding to these two states are identical), then we can delete one of them.

Example:- Consider the Moore Machine described by the transition table given by table "below" construct the corresponding Mealy machine.

Present state	Next state		Output
	$a=0$	$a=1$	
$\rightarrow q_1$	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

Solution:-> We construct the transition table as in in following table by associating the output with the transitions. In table below, the rows corresponding to q_2 and q_3 are identical. So we can delete one of the two states, i.e q_2 or q_3. we delete q_3.

Present state	Next state			
	$a=0$		$a=1$	
	State	output	State	output
$\rightarrow q_1$	q_1	0	q_2	0
q_2	q_1	0	q_3	1
q_3	q_1	0	q_3	1

The table next to it; gives the reconstructed table.

final Mealy Machine of Example

Present state	Next state			
	$a=0$		$a=1$	
	State	output	State	output
$\rightarrow q_1$	q_1	0	q_2	0
q_2	q_1	0	q_2	1

in the above table, we have deleted the q_3-row and replaced q_3 by q_2 in the other rows.