

Number System

- » Number :- It is group of symbols or digits.
- » maximum digit in any number system whose radix/base is $R = R-1$
minimum digit is always '0'.
- » Radix / base :- Total number of digits in any number system is called radix or base of that number system.
 - For Eg:- in base 10 there are 10 digits from 0-9.
 - in base 7 there are 7 digits from 0-6
 - in hexadecimal system , there are digits from 0-15_(F) because base is 16
- » Most significant digit :-

Positional weight \rightarrow for any number system whose radix is R in that number system positional weight = R^i

$$\text{Eg:- } (124.12)_{10} \quad \begin{matrix} 3 & 2 & 1 & 0 & -1 & -2 \\ \leftarrow i \end{matrix}$$

$\uparrow R$

In above positional weight of digit 1 is 10^2 . It is highest and so it is called MSD.

MSD → The digit which has highest positional weight.

LSD → The digit which has lowest positional weight is called least significant digit.

In above example 2 is LSD because it has lowest positional weight i.e. 10^{-2} . In binary number system, digit is called bit.

D Determine LSD, MSD and positional weight of digit 5 if it is represented in base 7.

$$(235.14)_7$$

↓ ↓ ↓
MSD LSD

7^0

26/07/17

» Representation of integers.

$$N = \sum_{i=0}^{n-1} a_i r^i$$

$$\begin{array}{r} 3 2 1 0 \\ \frac{2}{a_3} \frac{2}{a_2} \frac{2}{a_1} a_0 \\ \downarrow \\ 1 0 1 1 \end{array}$$

$$1 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$$

$$= a_0 + a_1 r^1 + a_2 r^2 + a_3 r^3 + \dots$$

a_i = value of digit at position i .

r^i = value of positional weight at position i .

n = total no. of integer digits.

→ Conversion of any number system whose radix is r into decimal number system
i.e. $(.)_r \rightarrow (.)_{10}$

Apply the formula $N = \sum_{i=0}^{n-1} a_i \cdot r^i$

convert the following

$$(1) (123)_4 \rightarrow (.)_{10} \rightarrow 1 \times 4^2 + 2 \times 4^1 + 3 \times 4^0 \\ \rightarrow 16 + 8 + 3 = (27)_{10}$$

$$(2) (AB)_{16} \rightarrow (.)_{10} \rightarrow 10 \times 16^1 + 11 \times 16^0 \\ \rightarrow (176)_{10}$$

$$(3) (271)_8 \rightarrow (.)_{10} \rightarrow 2 \times 8^2 + 7 \times 8^1 + 1 \times 8^0 \\ \rightarrow 128 + 56 + 1 = (185)_{10}$$

→ For fractional part

Apply the formula $N_{10} = \sum_{i=-1}^{-M} a_i r^i$

$$(N)_{10} = a_{-1} r^{-1} + a_{-2} r^{-2} + a_{-3} r^{-3} + \dots$$

$$\text{convert } (23)_9 = (.)_{10}$$

$$= 2 \times 9^{-1} + 3 \times 9^{-2} \\ = (.32)_{10}$$

Q convert $(121.32)_4 \rightarrow (?)_{10}$

$$1 \times 4^2 + 2 \times 4^1 + 1 \times 4^0 + . 3 \times 4^{-1} + 2 \times 4^{-2}$$

$$(16 + 8 + 1), (\frac{3}{4} + \frac{1}{16})$$

$$\rightarrow (25.875)_{10}$$

$$(N)_{10} = \sum (\text{value of digit at position } i) \times (\text{P.W. at position } i)$$

Conversion from decimal number system into any number system whose radix is r .

Let $N = a_0 + a_1 r^1 + a_2 r^2$

divide N by r

| N/r | quotient (q) | remainder |
|-------|---------------|-----------|
| q/r | $a_1 + a_2 r$ | a_0 |
| q/r | a_2 | a_1 |
| q/r | 0 | a_2 |

For integer part divide decimal no. by r in which number system we want to convert.

and separate quotient and remainder part. Again divide quotient by r until quotient becomes '0' and take the answer from bottom to top. (for integer part only).

Convert $(185)_{10} \rightarrow (?)_8$

| | | | |
|---|-----|---|----------------------------|
| 8 | 185 | 1 | \uparrow $(271)_{10}$ |
| 8 | 23 | 1 | |
| 8 | 2 | 7 | |
| 0 | 2 | 2 | |

$(72)_{10} \rightarrow (?)_2$

| | | | |
|---|----|---|-----------------------------|
| 2 | 72 | 0 | \uparrow $(1001000)_2$ |
| 2 | 36 | 0 | |
| 2 | 18 | 0 | |
| 2 | 9 | 0 | |
| 2 | 4 | 1 | |
| 2 | 2 | 0 | |
| 2 | 1 | 0 | |
| 0 | 1 | 1 | |

$(43)_{10} \rightarrow (?)_{16}$

| | | | |
|----|----|----|---------------------------|
| 16 | 43 | 11 | \uparrow $(2B)_{16}$ |
| 16 | 2 | 11 | |
| 0 | 2 | 2 | |
| 0 | 2 | 2 | |

>> for fractional part

To convert from base 10 to any radix number system, multiply by R & in which we want to convert and separate integer and fractional part. Again repeat above process until fractional part is zero or any above step repeats. Take ans in top to bottom.

Convert $(.175)_{10}$ into $(\)_4$

$$\begin{array}{rcl} .175 \times 4 & = & .700 & 0 \\ .700 \times 4 & = & 2.800 & 2 \\ .800 \times 4 & = & 3.200 & 3 \\ .200 \times 4 & = & .800 & 0 \\ .800 \times 4 & = & 3.200 & 3 \end{array} \quad \downarrow$$

$(.02303)_4$

Convert $(412.35)_{10}$ into $(\)_{16}$

| | | |
|----|-----|----|
| 16 | 412 | |
| 16 | 25 | 12 |
| 16 | 1 | 9 |
| | 0 | 1 |

$$\begin{array}{l} .35 \times 16 = 5.6 \quad 5 \\ .6 \times 16 = 9.6 \quad 9 \\ .6 \times 16 = 9.6 \quad 9 \end{array} \quad \downarrow$$

$(19C:599)_{16}$

» To convert
Step \rightarrow $(\)_{b_1} \rightarrow (\)_{b_2}$
 $(\)_{b_1} \rightarrow (\)_{10} \rightarrow (\)_{b_2}$

$\underset{Q}{=}$ $(124)_5 \rightarrow (\)_4$

$$1 \times 5^2 + 2 \times 5^1 + 4 \times 5^0 = 25 + 10 + 4 \\ = (39)_{10}$$

| | | | |
|---|---|---|-----------|
| 4 | 3 | 9 | |
| 4 | 9 | 3 | ↑ |
| 2 | 2 | 1 | |
| 0 | 2 | | $(213)_4$ |

» If base is in 2^l , then

Eg:- $(\)_2 \xrightarrow[\text{group}]{\text{make } 3 \text{ bit}} (\)_8 \quad \{ 8 = 2^3 \}$

$(\)_8 \xrightarrow[\text{digit by 3 bit}]{\text{Replace every}} (\)_2$
421 code

$\underset{Q}{=} (1011011)_2 \rightarrow (\)_8$

$$\begin{array}{ccccccc} 4 & 2 & 1 & 4 & 2 & 1 & 4 & 2 & 1 \\ \underline{0} & 0 & 1 & \underline{0} & 1 & 1 & \underline{0} & 1 & 1 \end{array} \rightarrow (133)_8$$

Q Convert $(10101, 10101)_2 \rightarrow (\)_2$

$$\begin{array}{r} 421 \\ 010,101 \cdot 101,010 \end{array} \rightarrow (25, 50)_2$$

$\gg ()_2 \xrightarrow[\text{Replace every digit with 8421 code.}]{\substack{\text{make 4 bit group}}} ()_{16}$

Q Convert $(AB \cdot 9)_{10} \rightarrow ()_2$

$$\begin{array}{r} 8421 \quad 8421 \quad 8421 \\ A \quad B \quad : \quad 9 \\ (1010 \ 1011 \cdot 1001)_2 \end{array}$$

Convert $(55)_8 \rightarrow ()_{16}$

$$\begin{array}{r} 421 \quad 421 \\ 5 \quad 5 \\ (101101)_2 \rightarrow \underbrace{0010}_{8421} \underbrace{1101}_{8421} \\ . \quad (20)_{16} \end{array}$$

Q $\sqrt{41} = (5)_{10}$ in which base system it is possible?

$$41 = 25$$

$$1 \times b^0 + 4 \times b^1 = 25$$

$$4b + 1 = 25$$

$$4b = 24$$

$$\boxed{b = 6}$$

Q If $\frac{55}{5} = 11$ in which base system it is possible?

$$\frac{55}{5} = 11$$

$$1+ = +1$$

$$1 \times b_1^0 + 1 \times b_1^1 = 1 \times b_2^0 + 1 \times b_2^1$$

$$\cancel{b_1^1} = \cancel{b_2^1}$$

$$\frac{5 \times b^0 + 5 \times b^1}{5 \times b^0} = 1 \times b^0 + 1 \times b^1$$

$$\frac{5+5b}{5} = b+1$$

$$5+5b = 5(b+1)$$

$$b+1 = b+1$$

$$\Rightarrow \boxed{b > 16}$$

$$\frac{302}{20} = 12.1$$

Find N.S.

$$\frac{3 \times b^2 + 0 \times b^1 + 2 \times b^0}{2 \times b^1 + 0 \times 2^0} = \frac{1 \times b^1 + 2 \times b^0 + 1 \times b^{-1}}{1}$$

$$\frac{3b^2 + 2}{2b} = b + 2 + \frac{1}{b}$$

$$\frac{3b^2 + 2}{2b} = \frac{b^2 + 2b + 1}{b}$$

$$3b^2 + 2 = 2b^2 + 4b + 1$$

$$b^2 - 4b = 0$$

$$b(b-4) = 0$$

$$b=0 \quad X \quad \boxed{b=4}$$

11/02/17

& in any no. system $ny = 20$ and $yx = 31$
 and if y is successive digit of x . what will
 be value of x, y and base b in which no.sys.
 is represented.

$$y = x+1$$

$$x \times b^1 + y \times b^0 = 20$$

$$xb + y = 20 \rightarrow ① \rightarrow xb + x+1 = 20$$

$$y \times b^1 + x \times b^0 = 31$$

$$yb + x = 31 \rightarrow ② \rightarrow (x+1)b + x = 31$$

$$\text{eqn } ① = (x+1)b + 1 = 20$$

$$\text{eqn } ② = (x+1)b + x = 31$$

$$① - ②$$

$$-b + 1 = -11$$

$$-b = -12$$

$$\boxed{b = 12}$$

$$\begin{aligned} xy &= 20 \\ x \times 12^1 + y \times 12^0 &= 20 \\ 12x + y &= 20 \quad \rightarrow \textcircled{3} \end{aligned}$$

$$\begin{aligned} yx &= 31 \\ y \times 12^0 + x \times 12^1 &= 31 \\ 12y + x &= 31 \rightarrow \textcircled{4} \end{aligned}$$

on solving $\textcircled{3}$ & $\textcircled{4}$

$$x = 19/13$$

$$y = 22/13$$

» Arithmetical operations in different base.

$$\begin{array}{r} \text{Addition :- } \quad y_1 \quad x_1 \\ \hline y_2 \quad x_2 \end{array}$$

if $x_1 + x_2 > b-1 \Rightarrow x_1 + x_2 - b \cdot 1$ forward carry

$$\begin{array}{r} \text{Eg :- } \quad (24)_8 \\ + (55)_8 \\ \hline 8 \quad 9 \\ -8 -8 \\ \hline 101 = (101)_8 \end{array}$$

Eg :

$$\begin{array}{r}
 & 1 & 1 \\
 & (3 & A & B)_{16} \\
 + (9 & 6 & 5)_{16} \\
 \hline
 13 & 17 & 16 \\
 -16 & \underline{-16} \\
 1 & 0
 \end{array}$$

$$= (D10)_{16}$$

Subtraction :-

$$\begin{array}{ll}
 y_1 & x_1 \\
 y_2 & x_2
 \end{array}$$

if $x_1 < x_2$ borrow base from y_1

Q Perform the following opn.

$$\begin{array}{r}
 & 3 & 5 \\
 (1) & (3 & 4 & 1)_5 & + \\
 - (1 & 2 & 3)_5 \\
 \hline
 (2 & 1 & 3)_5
 \end{array}$$

$$\begin{array}{r}
 & 2 & 5 & + 8 \\
 (2) & (8 & 8 & 5)_8 & \\
 - (1 & 7 & 6)_8 \\
 \hline
 (1 & 6 & 7)_8
 \end{array}$$

Q If $35 + 53 = 110$ then how much $137 + 731 = ?$

$$(3 \times b^1 + 5 \times b^0) + (5 \times b^1 + 3 \times b^0) = 1 \times b^2 + 1 \times b^1 + 0 \times b^0$$

$$3b + 5 + 5b + 3 = b^2 + b + 0$$

$$8b + 8 = b^2 + b + 0$$

$$b^2 - 7b - 8 = 0$$

$$b^2 - 8b + b - 8 = 0$$

$$b(b-8) + (b-8) = 0$$

$$b = -1, 8$$

$b = 8$

$$\begin{array}{r}
 & 1 & 3 & 7 \\
 + & 7 & 3 & 1 \\
 \hline
 & 8 & 8 & 8 \\
 - & 8 & & 8 \\
 \hline
 & 0 & & 0
 \end{array}$$

$$(1080)_8$$

multiplication :-

Eg :-

$$\begin{array}{r}
 & +3 \\
 \times & (2 & 5) \\
 \times & (1 & 4) \\
 \hline
 & 1 & 8 & 20 \\
 - & 6 & & -6 \\
 \hline
 & 5 & & 14 \\
 - & 6 & & -6 \\
 \hline
 & & 8 & 2
 \end{array}
 \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{1}$$

$$\begin{array}{r}
 (25)_6 \\
 (14)_6 \\
 \hline
 \textcircled{1} 152 \\
 + 25 \\
 \hline
 10 & 2 \\
 - 6 \\
 \hline
 4 & 4 & 2
 \end{array}$$

$$= (442)_6$$

$$(25)_6 \rightarrow ()_{10} \rightarrow 2 \times 6^1 + 5 \times 6^0 \\ = 12 + 5 = (17)_{10}$$

$$(14)_6 \rightarrow ()_{10} \rightarrow 1 \times 6^1 + 4 \times 6^0 = (10)_{10}$$

$$\begin{array}{r} 17 \\ \times 10 \\ \hline = (170)_{10} \end{array}$$

$$(170)_{10} \rightarrow ()_6$$

$$\begin{array}{r} 6 | 170 | 0 \\ \hline 6 | 28 | 2 \\ \hline 6 | 4 | 4 \\ \hline 0 | 4 \end{array} \quad (442)_6$$

$$\stackrel{0}{=} (15)_6 \times (23)_4 = ()_4$$

$$1 \times 6^1 + 5 \times 6^0 = 6 + 5 = (11)_{10}$$

$$\begin{array}{r} 4 | 11 | 3 \\ \hline 4 | 2 | 3 \\ \hline 0 \end{array} \quad (23)_4$$

$$\begin{array}{r} (23)_4 \\ \times (23)_4 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ 23 \\ \hline 201 \\ 56 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 89 \\ -44 \\ \hline 45 \\ -44 \\ \hline 1 \end{array} \quad \begin{array}{r} 201 \\ -44 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 201 \\ 192 \\ \hline 321 \end{array} \Rightarrow (1321)_4$$

2) $(2k-1)$'s complement

$\mu \rightarrow 2, 4, 6, 8, 10, 16, \dots$

μ is always even

$(2k-1)$'s comp \rightarrow subtract every digit from $(2k-1)$

Q Determine 1's complement of 1001

$$\begin{array}{r} 1001 \\ - 1001 \\ \hline 0110 \end{array}$$

Q Determine 9's complement of 257

$$\begin{array}{r} 999 \\ - 257 \\ \hline 742 \end{array}$$

Q Determine 7's complement of 156

$$\begin{array}{r} 777 \\ - 156 \\ \hline 621 \end{array}$$

Q Determine F's complement of A27

$$\begin{array}{r} F F F \\ A 2 7 \\ \hline 5 D 8 \end{array}$$

∴ r 's complement

$r \rightarrow$ always even

$$r's \text{ comp} = (r-1)'s \text{ comp} + 1$$

Q determine 2's complement of 1011

$$\begin{array}{r} (r-1)'s \\ \hline 1111 \\ 1011 \\ \hline 0100 \\ + 1 \\ \hline r's \quad 0101 \end{array}$$

Q determine 8's complement of 7000

$$\begin{array}{r} 7777 \\ - 7000 \\ \hline 0777 \\ \begin{array}{l} 8 \\ 8 \\ 8 \\ - 8 \\ \hline 0000 \end{array} \end{array}$$

$$x^8 \rightarrow (1000)_8$$

Q determine 16's complement of 25F0

$$\begin{array}{r} FFFF \\ - 25F0 \\ \hline DAD0 \\ \begin{array}{l} F \\ - F \\ \hline 0 \end{array} \end{array} \quad \begin{array}{r} DA10F \\ + 1 \\ \hline DA10 \end{array}$$

» $(r-1)$'s complement of $N = r^m - r^n - N$
 where $m = \text{total no. of integer digit}$
 $n = \text{total no. of fractional digit}$

$$\begin{aligned} 9\text{'s complement of } 34 &= 10^2 - 10^0 - 34 \\ &= 100 - 1 - 34 \\ &= 99 - 34 \end{aligned}$$

» subtraction by using $(r-1)$'s complement method.
 (1 or 9's method) $A - B$

Step 1: Determine $(r-1)$'s complement (1's or 9's) of B and add this to A .

Step 2: If $A > B$, in this case always carry is generated and add this carry to LSD of result obtained from step 1.

If $A < B$, in this case carry is not generated and final answer will be again $(r-1)$'s complement of result obtained from step 1. with negative sign.

Q Perform the following operations by using 1's complement method.

$$\begin{array}{r} 1011 \text{ (A)} \\ - 1010 \text{ (B)} \\ \hline ? \end{array} \quad \begin{array}{l} 1\text{'s comp. of } B = 0101 \\ \begin{array}{r} 1011 \\ 0101 \\ \hline 1110 \end{array} \\ \begin{array}{r} 2 2 2 2 \\ -2 -2 -2 -2 \\ \hline 0000 \end{array} \\ \begin{array}{r} +1 \\ \hline 0001 \end{array} \end{array}$$

Answer: 0001

$$\begin{array}{r}
 0110 \\
 -1110 \\
 \hline
 -1000
 \end{array}
 \quad
 \begin{array}{r}
 1's \text{ comp of } B = 0001 \\
 0110 \\
 +0001 \\
 \hline
 0111
 \end{array}$$

$$\text{comp of } (0111) = -1000 \text{ (Answer)}$$

Q Perform the following operation by using
9's complement method

$$\begin{array}{r}
 35 \\
 -85 \\
 \hline
 -50
 \end{array}
 \quad
 \begin{array}{r}
 9's \text{ comp of } B = 99 \\
 85 \\
 \hline
 14
 \end{array}$$

$$\begin{array}{r}
 14 \\
 35 \\
 \hline
 49 \\
 -9 \\
 \hline
 -50
 \end{array}$$

Ans = -50

$$\begin{array}{r}
 -235 \\
 -814 \\
 \hline
 221
 \end{array}
 \quad
 \begin{array}{r}
 9's \text{ comp of } B = 999 \\
 014 \\
 \hline
 985
 \end{array}$$

$$\begin{array}{r}
 ① 85 \\
 235 \\
 \hline
 320
 \end{array}$$

$$\begin{array}{r}
 220 \\
 -14 \\
 \hline
 86
 \end{array}$$

$$\begin{array}{r}
 86 \\
 -220 \\
 \hline
 649
 \end{array}$$

$$\begin{array}{r}
 320 \\
 -14 \\
 \hline
 220
 \end{array}$$

$$9's \text{ comp of } 320$$

$$\begin{array}{r}
 985 \\
 +235 \\
 \hline
 2200
 \end{array}$$

$$\begin{array}{r}
 +1 \\
 \hline
 \end{array}$$

» subtraction by using 2's complement method
(2's & 10's method) A - B
Determine 2's complement of B and add this to A.

Step 1: If $A > B$, in this case carry is always generated and discard / remove carry from result obtained in above step.

If $A < B$, in this case carry is not generated and final answer will be again 1's complement of result obtained from steps with negative sign.

Q Perform the following opn using 2's comp.

$$\begin{array}{r} 1011 \\ - 1010 \\ \hline ? \end{array}$$

2's comp of B = 2's comp + 1
= 01010 + 1
= 0110

$$\begin{array}{r} 0110 \\ + 1011 \\ \hline \times 10001 \end{array}$$

= (0001)

Answer = 0001

» Advantage of 2's complement over 1's complement

Computer always uses 2's complement method instead of 1's complement method because in 2's complement method we discard carry but in 1's complement method we add again carry to LSD so only one adder circuit

is required in 2's complement method. Hence, less hardware, high speed.

(i) Perform the following operation using 10's comp.

$$\begin{array}{r} 298 \\ - 19 \\ \hline 289 \end{array} \quad \begin{array}{r} 10's \text{ comp of } 98 = 991 \\ 999 \\ - 009 \\ \hline 990 \\ + 1 \\ \hline 991 \end{array}$$

$$\begin{array}{r} 991 \\ + 298 \\ \hline \text{discard } \textcircled{1} 289 \end{array}$$

$$\text{Answer} = 289$$

» Signed Number

There are three methods to represent any signed number

(i) Sign Magnitude

MSB bit represents sign bit & remaining bits represent magnitude of that number. If sign bit (MSB) is '1', it indicates no. is negative and if its value is '0', it indicates no. is positive.

Eg: $+5 \Rightarrow$ $\begin{array}{r} 0 \\ | \\ 101 \end{array}$ $\xrightarrow{\text{Binary representation}}$
 $-5 \Rightarrow$ $\begin{array}{r} 1 \\ | \\ 101 \end{array}$ $\xrightarrow{\text{sign}}$

128 64 32 16 8 4 2 1

8 bit +57 \Rightarrow 00111001 Total should be
 -57 \Rightarrow 11011100 8 bit

(1) 1's complement method

In this positive number is represented same as in case of sign magnitude method and for negative number determine 1's complement of N.

Eg: +5 \Rightarrow 0101
 -5 \Rightarrow 1010

Range of Number in 1's complement method is
 $+ (2^{n-1} - 1)$ to $- (2^{n-1} - 1)$

(2) 2's complement method

For positive number representation is same as in case of above two method and for negative number determine 2's complement of N.

Eg: +5 \Rightarrow 0101
 -5 \Rightarrow 1010
 $\underline{+ 1}$
 1011
 $-5 \Rightarrow 1011$

Range:- $+ (2^{n-1} - 1)$ to $- (2^{n-1})$

Q How many bits are required to represent -64 by using 1's complement method and 2's complement method?

$$+64 = 01000000$$

$$\begin{array}{r} -64 = 10111111 \\ +1 \end{array} \quad (1\text{'s complement})$$

$$-64 = 11000000 \quad (2\text{'s complement})$$

8 bits required.

$$(1\text{'s}) \text{ range} = + (2^{n-1} - 1) \text{ to } -(2^{n-1} - 1)$$

$n=7 \Rightarrow -63 \times$

$n=8 \Rightarrow -127 \times$

$$(2\text{'s}) \text{ range} = + (2^{n-1} - 1) \text{ to } -(2^{n-1})$$

$n=7 = -64 \checkmark$

$$\text{For } 1\text{'s} = 8 \text{ bits}$$

$$2\text{'s} = 7 \text{ bits}$$

- Q Assuming all numbers in 2's complement form which number is divisible by 11111011
- 11108100
 - both a & b
 - 11100111
 - Name of these.

$$\begin{array}{r}
 11111011 \\
 \xrightarrow{\text{2's comp}}
 \end{array}
 \begin{array}{r}
 00000100 \\
 + 1 \\
 \hline
 00000101 \\
 = -5
 \end{array}$$

$$\begin{array}{r}
 11100100 \\
 \xrightarrow{\text{2's comp}}
 \end{array}
 \begin{array}{r}
 00011011 \\
 + 1 \\
 \hline
 00011100 \\
 = -28
 \end{array}$$

$$\begin{array}{r}
 11100111 \\
 \xrightarrow{\text{2's comp}}
 \end{array}
 \begin{array}{r}
 00011000 \\
 + 1 \\
 \hline
 00011001 \\
 = -25
 \end{array}$$

» Arithmetical Operations in signed number

- +5 - 6
- 5 - 6
- 6 - 5

} by using 2's comp. method.

(i) $+5 + (-6)$

$$\begin{array}{r} +5 \rightarrow 0101 \\ +6 \rightarrow 0110 \\ -6 \rightarrow 1001 \\ \hline & +1 \\ & \overbrace{\quad\quad\quad}^{\text{+1}} \\ & 0110 \\ \hline & 1111 \end{array}$$

(ii) $-5 - 6$

$$\begin{array}{r} +5 \rightarrow 0101 \\ -5 \rightarrow 1010 \\ \hline & +1 \\ & \overbrace{\quad\quad\quad}^{\text{+1}} \\ & 1011 \\ -6 \rightarrow & 1010 \end{array}$$

$$= 1011$$

$$\begin{array}{r} +1010 \\ \hline \times 10101 \\ \hline +1 \\ \hline 0101 \end{array} \Rightarrow 0110$$

$\begin{array}{r} 1001 \\ \times 1 \\ \hline 1010 \end{array}$

(iii) $6 - 5$

$$+6 \rightarrow 0110$$

$$\begin{array}{r} -5 \rightarrow 1010 \\ \hline \times 1000 \end{array}$$

$$= 0000$$

8

Perform the following operation using 2's complement & make 8 bit result.

i) $+32 - 49 \rightarrow -17$

$$+32 = 001\ 00000.$$

$$+49 = 001\ 10001$$

$$\begin{array}{r} \\ + \\ \hline 010 \end{array}$$

$$\begin{array}{r} -49 = 1100\ 1110 \\ +1 \\ \hline 1100\ 1111 \end{array}$$

$$\begin{array}{r} 001\ 00000 \\ + 11001111 \\ \hline 11101111 \end{array}$$

You cannot see
ye wahi haat

$$00010001 = -17$$

Verify

$$\begin{array}{r} 11101111 \\ 00010000 \\ +1 \\ \hline 00010001 \end{array}$$

» BCD Number (8421 code) (Decimal code)
There are 10 digits from 0 to 9
Binary coded Decimal \rightarrow BCD.
Replace every digit by equivalent 8421 code

Represent 276.4 into BCD

$$\begin{array}{cccc} 2 & 7 & 5 & . & 4 \\ 8421 & 8421 & 8421 & & 8421 \\ (0010\ 0111\ 0101\ .\ 0100) \end{array}$$

» Addition of two BCD number
Add two digits of BCD number if sum is greater than 9, add 6 otherwise add 0.

Q Perform the following operation of BCD number

$$\begin{array}{r} 37 \\ + 25 \\ \hline 62 \\ - 16 \\ \hline 62 \end{array} \quad \begin{array}{r} 37 \rightarrow 100101 \\ 25 \rightarrow 011001 \\ 0011 \quad 0111 \\ 0010 \quad 0101 \\ 1100 \\ 0110 \quad 0010 \end{array}$$

» ~~Access~~ XS-3 Code

Add every three in every digit and convert with 8421 code.

Q Convert 1) 125.9 into XS-3 2) 97.5 into XS-3

$$\begin{array}{r} 125,9 \\ + 333,3 \\ \hline 458.12 \end{array} \quad \begin{array}{r} 010001011000.1000 \end{array}$$

$$\begin{array}{r} 97.5 \\ 33.3 \\ \hline 1210.8 \end{array} \quad \begin{array}{r} (\text{for } 10100110000.1000) \\ 10001010.1000 \end{array}$$

» BCD subtraction
It is performed by using either 9's or 10's complement method.

Subtract

$$\begin{array}{r} 235 \\ - 187 \\ \hline \end{array}$$

$$\begin{array}{r} 87 \\ - 6 \\ \hline \end{array}$$

999

-187

812 → 9's comp.

+ 235

① 047

+ 1

48

99

- 06

93

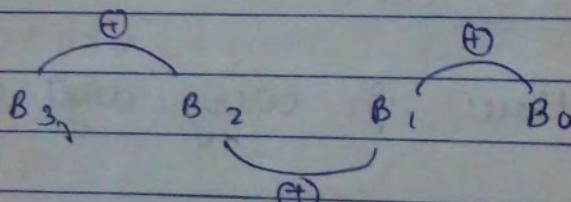
+ 1

94

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× ① 081

» Conversion from Binary to Gray code



$$G_3 = B_3$$

$$G_2 = B_3 \oplus B_2$$

$$G_1 = B_2 \oplus B_1$$

$$G_0 = B_1 \oplus B_0$$

Convert 1011 into Gray code

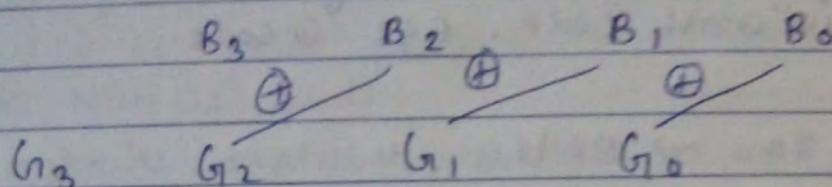
11100 ← Ans in gray code

Q Convert $(32)_8 \rightarrow (?)_9$

$$(32)_8 \rightarrow (011010)_2$$

$$\text{Grey code} \rightarrow 010111$$

» conversion from Grey to Binary code



$$G_3 = B_3$$

$$G_2 = G_3 \oplus B_2$$

$$G_1 = G_2 \oplus B_1$$

$$G_0 = G_1 \oplus B_0$$

Q Convert $(1010)_6 \rightarrow (?)_2$

1010

Aps $\rightarrow 1100$

| No | 8421 | X5-3 | 5421 | 2421 |
|----|------|------|------|------|
| 0 | 0000 | 0011 | 0000 | 0000 |
| 1 | 0001 | 0100 | 0001 | 0001 |
| 2 | 0010 | 0101 | 0010 | 0010 |
| 3 | 0011 | 0110 | 0011 | 0011 |
| 4 | 0100 | 0111 | 0100 | 0100 |
| 5 | 0101 | 1000 | 1000 | 1011 |
| 6 | 0110 | 1001 | 1001 | 1100 |
| 7 | 0111 | 1010 | 1010 | 1101 |
| 8 | 1000 | 1011 | 1011 | 1110 |
| 9 | 1001 | 1100 | 1100 | 1111 |

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» sequential code

If we add 1 to n^{th} code and we get next code of $n+1$. Ex: 8421, XS-3

» Unit distance code (cyclic code)

From going previous number to next number if only 1 variable changes that code is called unit distance code. Eg: 6 code

$$3 \rightarrow 0010 \rightarrow 0011$$

$$4 \rightarrow 0100 \rightarrow 0110$$

$$6 \rightarrow 0110 \rightarrow 0101$$

$$7 \rightarrow 0111 \rightarrow 0100$$

It is used in K map.

» self complementing code

$n^{\text{'s}}$ code = 1's comp. of ($9^{\text{'s}}$ comp. of n)

Eg: for 2421 code

$$4 \rightarrow 0100$$

1's comp. of ($9^{\text{'s}}$ comp. of 4)

1's comp. of (5 code)

1's comp. of 1011

$$= 0100$$

Eg: 2421 code, XS-3 code

» weighted code and Non-weighted code.
weighted codes are those code which follows positional weight rule and non-weighted code are those which do not follow non-weighted code.

Weighted code \rightarrow 8421, 2421, 5921

Non-weighted code \rightarrow XS-3, Gray code

» Alpha Numeric code.

It contains alphabet, letters as well as number.

Eg:

ASCII \rightarrow American Standard Code for Information Interchange. It is 7 bit code.

EBCDIC \rightarrow Extended Binary Coded Decimal Information Code. It is 8 bit code.

Hollerith \rightarrow it is used in punch machine.

| | | | | | | |
|----|---|---|---|----|---|--|
| \$ | 5 | - | - | -- | 0 | |
| a | 5 | - | - | -- | 0 | |
| o | 5 | - | - | -- | 0 | |
| l | 5 | - | - | -- | 0 | |
| l | 5 | - | - | -- | 0 | |

» Error detecting and correcting code.
when message signal (data) is transmitted at receiver from transmitter noise (error) may introduce in the system, due to this bits are changed either from 0 to 1 or 1 to 0.

one (1) parity bit is added along with message signal and at receiver we check whether there is even parity or odd parity.

EVEN PARITY → if total no. of 1's are even at receiver end.

ODD PARITY → if total no. of 1's are odd at receiver end.

ASCII

0 → 30H (0110000)

A → 41H (G50)(1000001)

1 → 31H

B → 42H (1000010)

2 → 32H

9 → 39H

- Q ASCII digit D is transmitted from transmitter along with parity bit. what will be code at receiver end for
A) even parity
B) odd parity

D → 44H → 1000 100

i) 01000100 → even parity

ii) 11000100 → odd parity

Q An 8 bit code 11000111 is received at receiver end check whether this code is correct or not for even parity?

Total no. of 1's are odd so for even parity code & not correct.

2) Hamming code (single bit error detecting and correcting code)

$$n = m + p$$

$$m \leq 2^p - p - 1$$

n: total no. of bits received at receiver end

m: total no. of bits of message signal

p: total no. of parity bits

Parity bits are added along with message signal and transmitted to receiver.

Q If total no. of received bits are 7 determine value of m and p

$$\text{Let } m = 4 \quad 4 \leq 2^p - p - 1$$

$$\text{if } p=2$$

$$4 \leq 2^2 - 2 - 1$$

$$4 \leq 1 \times$$

$$\text{if } p=3$$

$$4 \leq 2^3 - 3 - 1$$

$$4 \leq 4 \leq$$

$$n = 7, m = 4, p = 3$$

Total message bits = 4
Total parity bits = 3

NOTE: Parity bits are placed at 2^i position
i.e.

| | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| M ₇ | M ₆ | M ₅ | P ₄ | M ₃ | P ₂ | P ₁ |

For 15 bit hamming code

if message bits are 4 determine sequence of all the bits

~~shortest~~ maximum value = 15 and to represent 15 we require 4 bits, so total parity bits are 4
 $\therefore p = 4$, Hence, $m = 11$

| | | | | | | | | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| M ₁₅ | M ₁₄ | M ₁₃ | M ₁₂ | M ₁₁ | M ₁₀ | M ₉ | P ₈ | M ₇ | M ₆ | M ₅ | P ₄ | H ₃ | P ₂ | P ₁ |

To check parity bit corresponds to which digit

P₁ \rightarrow 1, 3, 5, 7 (LSB 1)

1 \rightarrow 001

P₂ \rightarrow 2, 3, 6, 7 (Middle 1)

2 \rightarrow 010

P₃ \rightarrow 4, 5, 6, 7 (MSB 1)

3 \rightarrow 011

same for 15 bit

4 \rightarrow 100

5 \rightarrow 101

6 \rightarrow 110

7 \rightarrow 111

?

\therefore A 4 bit message signal 1011 is transmitted at receiver end. What will be hamming code at receiver end for
 A) even parity
 B) odd parity

$$4 \leq 2P - b - 1$$

if $p = 2$ \times

$4 \leq 4$ if $p = 3$ \checkmark

$$\Rightarrow m = 4$$

$$p = 3$$

$$n = 7$$

| M_7 | M_6 | M_5 | P_4 | M_3 | P_2 | P_1 |
|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |

For even parity

$$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (1, 1, 1, 1)$$

$$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (0, 1, 0, 1)$$

$$P_4 \rightarrow 4, 5, 6, 7 \rightarrow (0, 1, 0, 1)$$

For odd parity

$$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 1, 1)$$

$$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (1, 1, 0, 1)$$

$$P_4 \rightarrow 4, 5, 6, 7 \rightarrow (1, 1, 0, 1)$$

| M_7 | M_6 | M_5 | P_4 | M_3 | P_2 | P_1 |
|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |

Ans \rightarrow For even parity = 101010
 For odd parity = 1011110

Q 4 bit message signal is 0101 for odd parity.
 will be 7 bit hamming code at receiver end.

| | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| M ₇ | M ₆ | M ₅ | P ₄ | M ₃ | P ₂ | P ₁ |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 |

$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 0, 0)$
 $P_2 \rightarrow 2, 3, 6, 7 \rightarrow (1, 1, 1, 0)$
 $P_4 \rightarrow 4, 5, 6, 7 \rightarrow (0, 0, 1, 0)$
 Ans $\rightarrow 0100110$

10/08/17

Q Message signal 110011000110 is transmitted from transmitter. what will be 15 bit hamming code at receiver end for even parity.

$$n = 15$$

$$m = 11$$

$$p = 4$$

| | | | | | | | | | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---|
| M ₁₅ | M ₁₄ | M ₁₃ | M ₁₂ | M ₁₁ | M ₁₀ | M ₉ | P ₈ | M ₇ | M ₆ | M ₅ | P ₄ | M ₃ | P ₂ | P ₁ | |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |

$$P_1 \rightarrow 1, 3, 5, 7, 9, 11, 13, 15$$

$$P_2 \rightarrow 2, 3, 6, 7, 10, 11, 14, 15$$

$$P_4 \rightarrow 4, 5, 6, 7, 12, 13, 14, 15$$

$$P_8 \rightarrow 8, 9, 10, 11, 12, 13, 14, 15$$

Message signal $\rightarrow 110011000110011$

for even parity
for 8-10 marks do for both.

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- Q A 7 bit hamming code received at receiver is 1011011.
Determine whether this code is correct or not, if not correct; correct it for even parity

$$n = 7$$

$$p = 3$$

$$m = 4$$

| | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| M ₇ | M ₆ | M ₅ | P ₄ | M ₃ | P ₂ | P ₁ |

$$\begin{aligned}P_1 &\rightarrow 1, 3, 5, 7 \text{ (odd)} \rightarrow (1, 0, 1, 1) \rightarrow \text{NUT } E \cdot P \neq 1 \\P_2 &\rightarrow 2, 3, 6, 7 \rightarrow (1, 0, 0, 1) \rightarrow \text{EP} \neq 0 \\P_3 &\rightarrow 4, 5, 6, 7 \rightarrow (1, 1, 0, 1) \rightarrow \text{NUT } E \cdot P \neq 1\end{aligned}$$

$$(101) \Rightarrow 5$$

Hence there is error at 5th bit
and change that bit

Correct code $\rightarrow 1001011$

- Q A 7 bit hamming code received at receiver 1010110
Determine this code is correct or not if not correct it
for odd parity

| | | | | | | |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| M ₇ | M ₆ | M ₅ | P ₄ | M ₃ | P ₂ | P ₁ |

$$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 1, 1) \rightarrow 0$$

$$P_2 \rightarrow (2, 3, 6, 7) \rightarrow (1, 1, 0, 1) \rightarrow 0$$

$$P_3 \rightarrow (4, 5, 6, 7) \rightarrow (0, 1, 0, 1) \rightarrow 1$$

$$(100) \rightarrow 4$$

Error at 4th bit, correct code $\rightarrow 1011110$

Q) 15 bit hamming code at receiver end is
 $01\ 00\ 11\ 000\ 11\ 0011$. determine whether this code is
 correct or not for even parity. if not correct
 it

| | | | | | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | |
| M_{15} | M_{14} | M_{13} | M_{12} | M_{11} | M_{10} | M_9 | P_8 | M_7 | M_6 | M_5 | P_4 | M_3 | P_2 | P_1 |

$$P_1 \rightarrow 1, 3, 5, 7, 9, 11, 13, 15 \rightarrow (1, 0, 1, 0, 0, 1, 0, 0) \rightarrow 1 \uparrow$$

$$P_2 \rightarrow 2, 3, 6, 7, 10, 11, 14, 15 \rightarrow (1, 0, 1, 0, 1, 1, 1, 0) \rightarrow 1$$

$$P_3 \rightarrow 4, 5, 6, 7, 12, 13, 14, 15 \rightarrow (0, 1, 1, 0, 0, 0, 1, 0) \rightarrow 1$$

$$P_4 \rightarrow 8, 9, 10, 11, 12, 13, 14, 15 \rightarrow (0, 0, 1, 1, 0, 0, 1, 0) \rightarrow 1$$

$$(1\ 1\ 1\ 1) \rightarrow 15$$

error at 15th bit

correct code $\rightarrow 11\ 0011\ 000\ 11\ 0011$

Q) A 7 bit hamming code is received as 1011110.
 determine whether this code is correct for odd parity

| | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| M_7 | M_6 | M_5 | P_4 | M_3 | P_2 | P_1 |

$$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 1, 1) \rightarrow 0$$

$$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (1, 1, 0, 1) \rightarrow 0$$

$$P_4 \rightarrow 4, 5, 6, 7 \rightarrow (1, 1, 0, 1) \rightarrow 0$$

$$(0\ 0\ 0) \rightarrow 0$$

NO error in this code