Simplify the following expression using Boolean algebra

(1) $Z = AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$ | $AB + AB = AB$, $B.B = B$

$= AB + AC + B + BC$

$= AB + AC + B(1+C)$ | $1 + C = 1$

$= AB + AC + B$

$= B + AB + AC$

$= B(1+A) + AC$ | $1 + A = 1$

$\boxed{Z = B + AC}$ ANS

(2) $Y = ABC + A\bar{B}C + AB\bar{C}$

$= AC(B + \bar{B}) + AB\bar{C}$ | $B + \bar{B} = 1$

$= AC + AB\bar{C}$

$= A(C + B\bar{C})$ | $C + B\bar{C} = C + B$

$= AC(C+B)$

$\boxed{Y = A(B+C)}$ ANS

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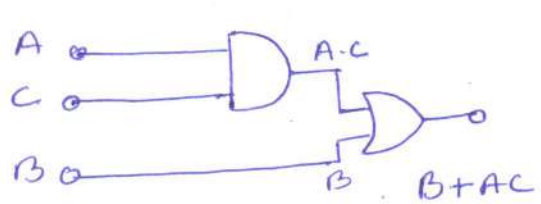
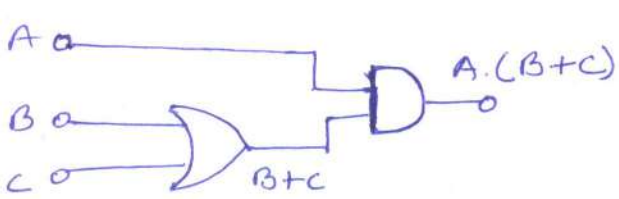
(1)



(2)



(3) $Z = AB + ABC + \bar{A}B + A\bar{B}C$

$= AB + \bar{A}B + ABC + A\bar{B}C$

$= B(A+\bar{A}) + AC(B+\bar{B})$

$\boxed{Z = B + AC}$

$A+\bar{A} = 1$
$B+\bar{B} = 1$

(4) $Z = AB + AC + B\bar{C}$

$= AB(C+\bar{C}) + AC + B\bar{C}$

$= ABC + AB\bar{C} + AC + B\bar{C}$

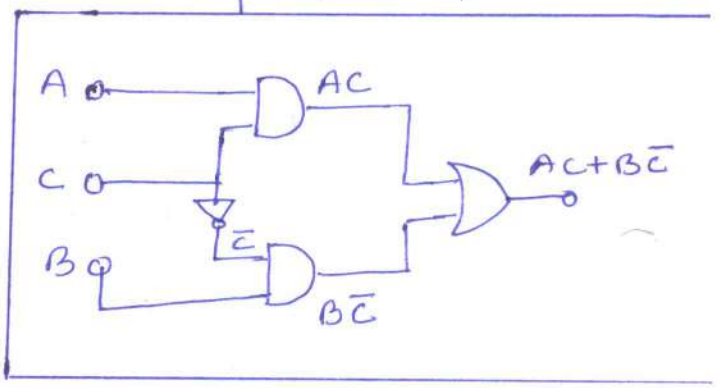
$= ABC + AC + AB\bar{C} + B\bar{C}$

$= AC(B+1) + B\bar{C}(A+1)$

$\boxed{Z = AC + B\bar{C}}$ ANS

$C+\bar{C} = 1$

$A+1 = 1$
$B+1 = 1$



(5) $Z = (A+B)(\bar{A}+C)$

$= A\bar{A} + AC + AB + BC$

$= AC + \bar{A}B + BC$

$= AC + \bar{A}B + (A+\bar{A})BC$

$= AC + \bar{A}B + ABC + \bar{A}BC$

$= AC + ABC + \bar{A}B + \bar{A}BC$

$= AC(1+B) + \bar{A}B(1+C)$

$\boxed{Z = AC + \bar{A}B}$ ANS

$A+\bar{A} = 1$

$1+B = 1$
$1+C = 1$

Realize the following using NAND gates.

(6) $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B + \bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B(C+\bar{C}) + (A+\bar{A})\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$

$= \bar{A}\bar{B}(C+\bar{C}) + \bar{A}B(\bar{C}+C) + A\bar{B}\bar{C}$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B}\bar{C}$

$= \bar{A}(\bar{B}+B) + A\bar{B}\bar{C}$

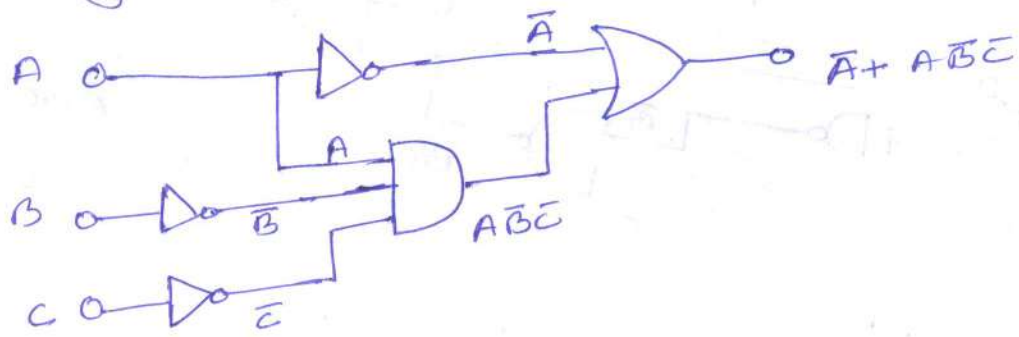
$\boxed{Z = \bar{A} + A\bar{B}\bar{C}}$ Ans

$\bar{A}B\bar{C} + \bar{A}B\bar{C} = \bar{A}B\bar{C}$

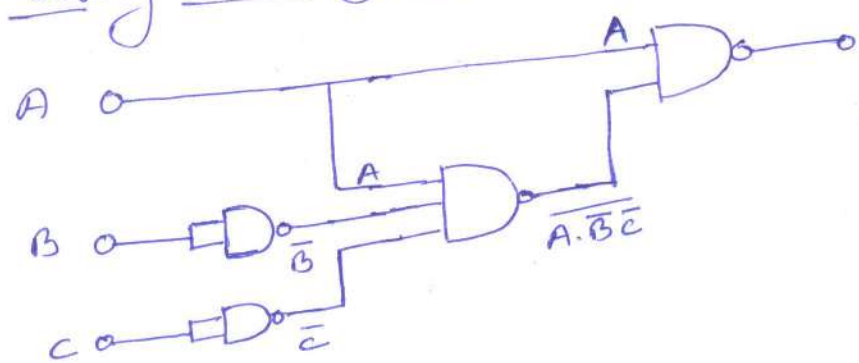
$B + \bar{B} = 1$

$C + \bar{C} = 1$

Using gates



Using NAND gate



$= \overline{A \cdot \overline{A\bar{B}\bar{C}}}$

$= \overline{\bar{A} \cdot \overline{A\bar{B}\bar{C}}}$

$= \overline{\bar{A} + A\bar{B}\bar{C}}$

$= \bar{A} + A\bar{B}\bar{C}$

(7) $Y = \bar{A}BC + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

$= \bar{A}BC + \bar{B}\bar{C}(\bar{A} + A)$ | $\bar{A} + A = 1$

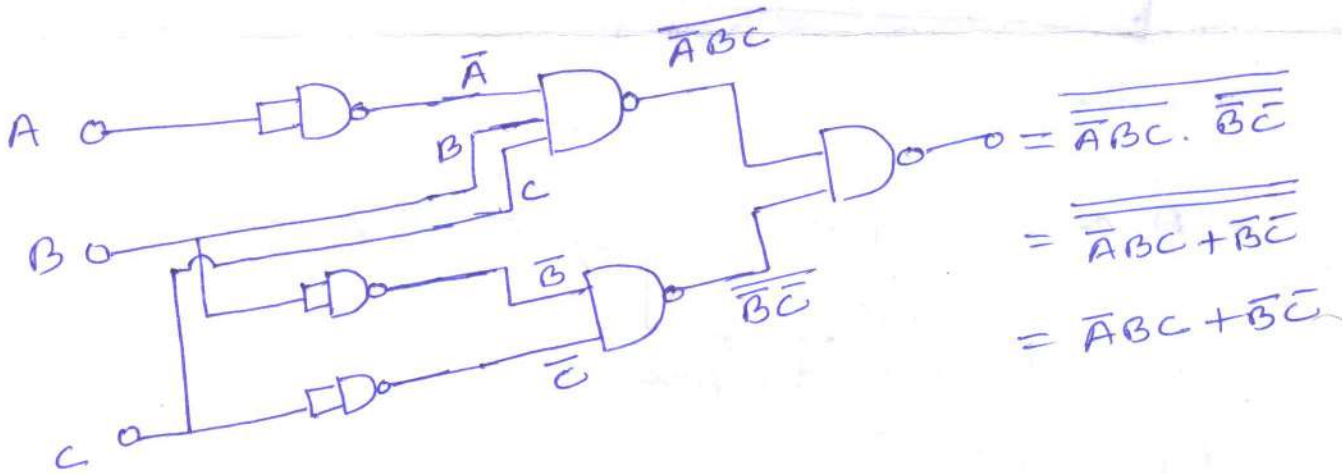
$\boxed{Y = \bar{A}BC + \bar{B}\bar{C}}$

for realization using NAND gates

$Y = \overline{\overline{\bar{A}BC + \bar{B}\bar{C}}}$

$= \overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}\bar{C}}}$

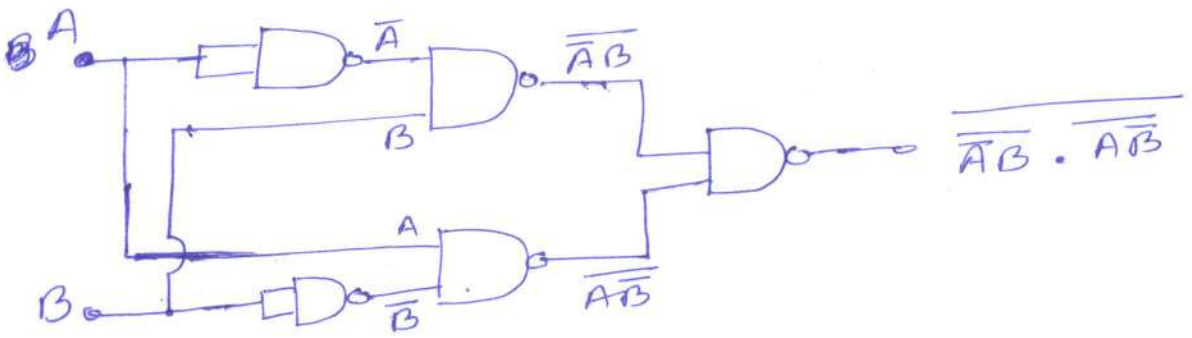
$= \overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}\bar{C}}}$



(8) $Y = A\bar{B} + \bar{A}B$ = XOR Gate using NAND

$$= \overline{\overline{A\bar{B} + \bar{A}B}}$$

$$= \overline{\overline{A\bar{B}} \cdot \overline{\bar{A}B}}$$

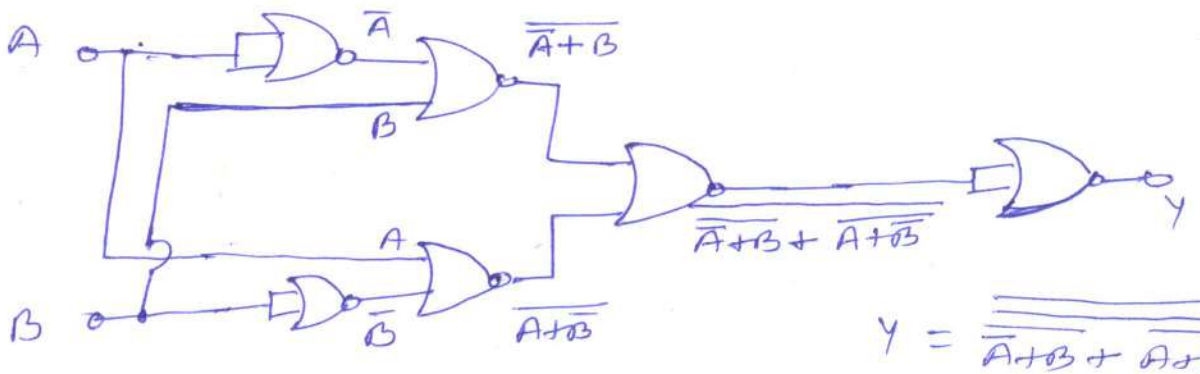


(9) ~~Using~~ XOR using NOR gates

$$Y = A\bar{B} + \bar{A}B = \overline{\overline{A\bar{B}}} + \overline{\overline{\bar{A}B}}$$

$$= \overline{\overline{A} + \overline{\bar{B}}} + \overline{\overline{\bar{A}} + \overline{B}}$$

$$= \overline{\overline{A} + B} + \overline{A + \overline{B}}$$



$$Y = \overline{\overline{\bar{A}+B} + \overline{A+\bar{B}}}$$

$$Y = \overline{\bar{A}+B} + \overline{A+\bar{B}}$$

Standard Representations of Logical Functions :

1. Sum of Products (SOP)

Ex. (i) $AB + \bar{A}B + A\bar{B}$

(ii) $AB + \bar{B}CD + AB\bar{C}$

2. Product of Sum (POS)

Ex. (i) $(A+\bar{B})(A+B+C)(B+C)$

(ii) $(\bar{A}+C)(\bar{B}+A)(A+\bar{B}+\bar{C})$

Canonical forms :-

When each term of a logical expression contains all literals, it is called a canonical form of the logical variables. If each term in SOP and POS forms contains all the literals then these are known as canonical or standard SOP and POS forms respectively.

Each individual term in standard SOP form is called minterm and in standard POS form as maxterm.

Exp (1) Convert $A + A\bar{B}$ into standard SOP form.

$$A + A\bar{B} = A(B+\bar{B}) + A\bar{B}$$

$$= AB + A\bar{B} + A\bar{B}$$

$$= AB + A\bar{B}$$

↓ minterm

(ii) Convert $(A+B)(B+C)$ into standard POS form.

$$(A+B)(B+C) = (A+B+C\bar{C})(B+C+A\bar{A})$$

$$= (A+B+C)(A+B+\bar{C})(B+C+A)(B+C+\bar{A})$$

same

$$= (A+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C)$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)$$

$[\because A \cdot A = A]$

↓ maxterm

∴ $A + BC = (A+B)(A+C)$

Minterm and Maxterm :-

$$Y = ABCD + A\bar{B}CD + A\bar{B}\bar{C}D + AB\bar{C}D$$

above expression can be expressed in the form of minterm.

$Y = ABCD + A\bar{B}CD + A\bar{B}\bar{C}D + AB\bar{C}D$

1111 → 15 → m_{15}

1011 → 11 → m_{11}

1001 → 9 → m_9

1101 → 13 → m_{13}

~~$Y = \Sigma m$~~

$$Y(A,B,C,D) = \Sigma m(9,11,13,15)$$

Then the corresponding maxterm form is

$$Y(A,B,C,D) = \Pi M(0,1,2,3,4,5,6,7,8,10,12,14)$$

Maxterm conversion can be done by given method.

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$$

$$\underbrace{0\ 0\ 1}_{\downarrow}\quad \underbrace{0\ 1\ \Phi}_{\downarrow}\quad \underbrace{1\ 0\ 0}_{\downarrow}\quad \underbrace{0\ 1\ 0}_{\downarrow}$$

$$1 \quad 3 \quad 4 \quad 2$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$M_1 \quad M_3 \quad M_4 \quad M_2$$

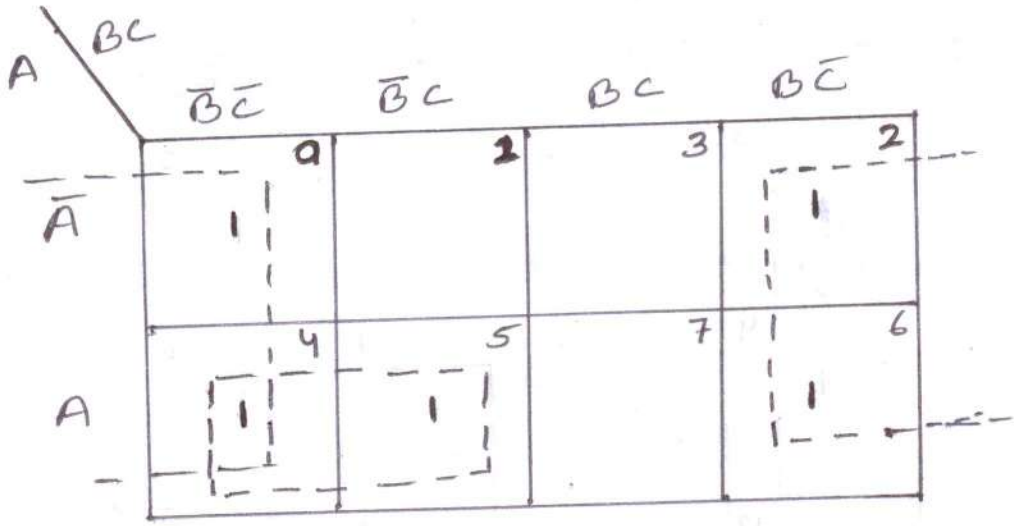
$$Y(A,B,C) = \Pi M(1,2,3,4)$$

then minterm form is

$$Y(A,B,C) = \Sigma m(0,5,6,7)$$

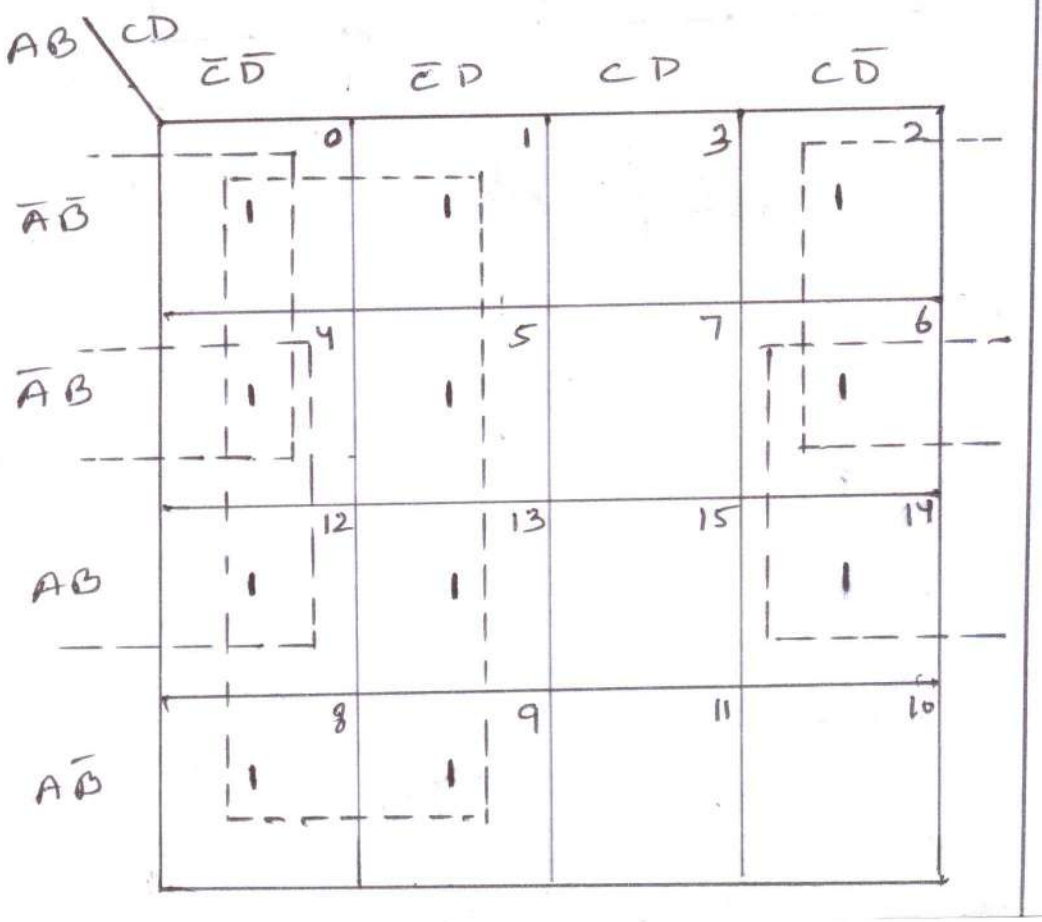
Karnaugh Maps

(1) $Y(A,B,C) = \Sigma m(0,2,4,5,6)$



$$Y = \bar{C} + A\bar{B}$$ Ans

(2) $Y(A,B,C,D) = \Sigma m(0,1,2,4,5,6,8,9,12,13,14)$

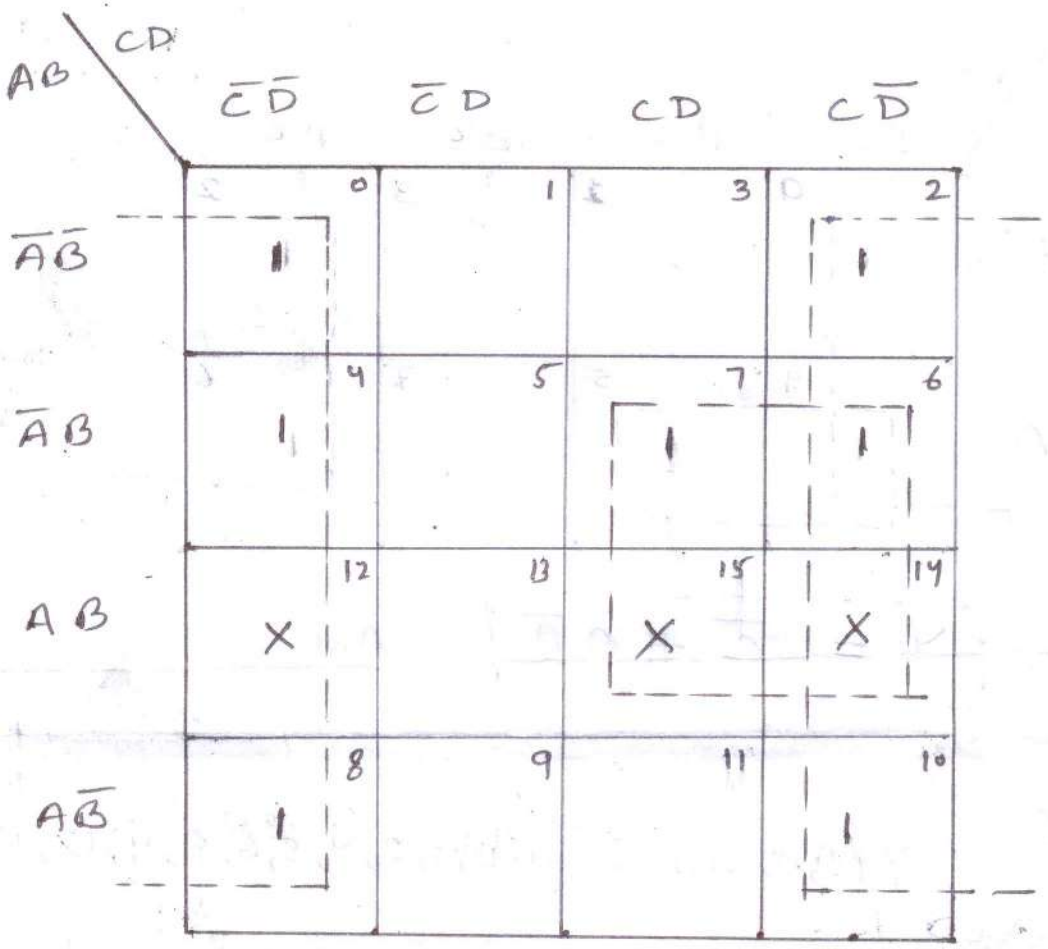


$Y = \bar{C} + \bar{A}\bar{D} + B\bar{D}$

Ans

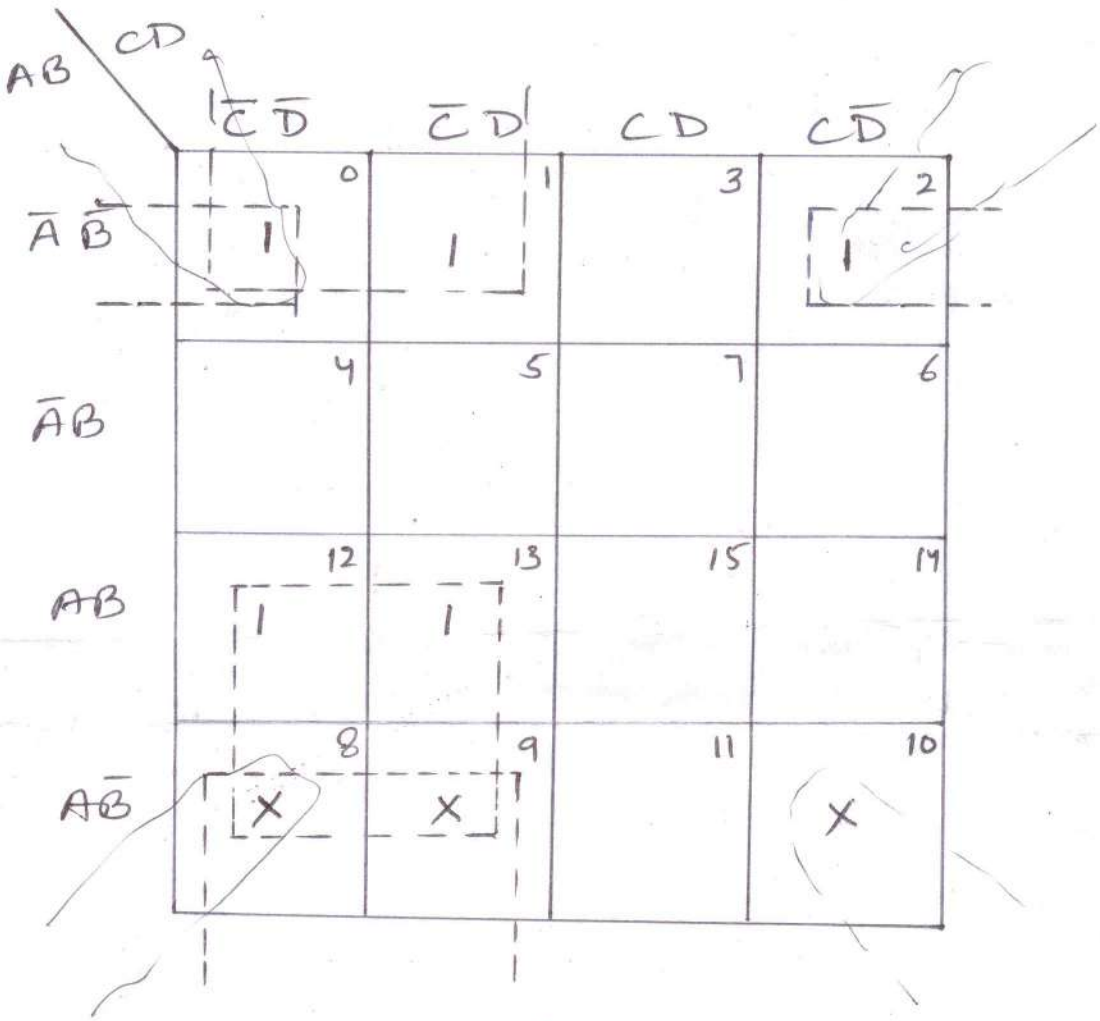
③ $Y(A,B,C,D) = \Sigma m(0,2,4,6,7,8,10)$

$+ \Sigma d(12,14,15)$



$\boxed{Y = \bar{D} + BC}$ ANS.

(4) $Y(A,B,C,D) = \Sigma m(0,1,2,12,13) + \Sigma d(8,9,10)$



$$Y = A\bar{C} + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{D}$$ ANS

Redundant group

Problems on K-map

1) $Y = \sum m(1,2,3,7) = B\bar{C} + \bar{A}B + A\bar{C}$

2) $Y = \sum m(0,3,5,6) = \bar{A}(\bar{B}\bar{C} + BC) + A(B\bar{C} + \bar{B}C)$

3) $Y = \sum m(0,1,3,5,6,7) =$ [redacted] $\bar{A}\bar{B} + AB + C$

4) $Y = \sum m(0,1,2,3,4,6) = \bar{B} + \bar{C}$

5) $Y = \sum m(0,1,2,4,6) = \bar{B} + \bar{A}\bar{C}$

6) $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C = \bar{A}B + \bar{A}\bar{C} + A\bar{B}C$

7) $Y = \sum m(1,5,7,9,11,13,15) = \bar{C}D + BD + AD$

8) $Y = \sum m(1,3,5,9,11,13) = D(\bar{B} + \bar{C})$

9) $Y = \sum m(1,3,4,5,7,9,11,13,15) = Y = \bar{A}B\bar{C} + D$

10) $Y = \sum m(1,2,9,10,11,14,15) = \bar{B}(\bar{C}D + C\bar{D}) + AC$

11) $Y = \sum m(4,5,8,9,11,12,13,15) = \bar{C}(A+B) + AD$

Redundant Group

12) $Y = \sum m(1,5,6,7,11,12,13,15) = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$

13) $Y = \sum m(0,1,2,5,13,15) = \bar{A}\bar{B}\bar{D} + ABD + \bar{A}\bar{C}D$

14) $Y = \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + A\bar{C} + \bar{B}$
$= \bar{B} + A\bar{C} + \bar{A}CD$

Don't Care Condn

15) $Y = \sum m(1,3,7,11,15) + d(0,2,5) = CD + \bar{A}\bar{B}$

16) $Y = \sum m(1,3,11,15) + d(0,2,5,8,14) = CD + \bar{A}\bar{B}$

17) $Y = \sum m(0,1,5,9,13,14,15) + d(3,4,7,10,11)$
$= D + (AC + \bar{A}\bar{C})$