Roll No. | | | | | | | |

TMA–201

B. Tech. (Second Semester)

End Semester EXAMINATION, 2017

(All Branches)

ENGINEERING MATHEMATICS

Time : Three Hours] *[Maximum Marks : 100*

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) The particular integral of $4\frac{d^2y}{dx^2} - y = 2\cos 3x$ is ....

(b) $L\{e^{at}t^n\} =$ ....

(c) If $f(x) = x^4$ in $(-1, 1)$, then the Fourier coefficient $a_0 =$ ....

(d) If $P\,dx + x\sin y\,dy = 0$ is exact, then P can be ....

(e) The PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is of the type ....

571

2. Attempt any *five* parts : (3×5=15 Marks)

(a) Solve of ODE $(D^2 - 4D + 4)\, y = 0$

(b) Solve :

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$

(c) Find the Laplace transform of the function $t \sin(at)$.

(d) Use the convolution theorem evaluate

$$L^{-1}\left(\frac{2}{(s-3)(s^2+4)}\right).$$

(e) if $f(x) = |x|, -\pi < x < \pi$, obtain the Fourier Series of $f(x)$.

(f) Solve the PDE $\frac{\partial^2 z}{\partial x^2} - 14\frac{\partial^2 z}{\partial x \partial y} + 40\frac{\partial^2 z}{\partial y^2} = 0$

(g) Prove that :

$$\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$$

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Solve :

$$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 4\cos(\log(x))$$

(b) Using method of variation of parameters, solve the ODE $\frac{d^2y}{dx^2} + 4y = \tan(2x)$.

(c) Find general solution of the PDE

$$\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x\, \partial x} + 12\frac{\partial^2 z}{\partial y^2} = e^{4x}\sin(3x + 2y)$$

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=10 Marks)

(a) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity :

$$\frac{\partial y}{\partial t}\Big|_{t=0} = b\sin^3\frac{\pi x}{l}$$

find the displacement $y(x, t)$

(b) Find the inverse Laplace transform :

$$L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$$

(c) Compute the integral $\int_0^\infty x e^{-3x} \sin 5x\, dx\ \ dy$ Laplace transform method.

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=10 Marks)

(a) Find the Fourier series to represent the function $f(x)$ given by :

$$f(x) = \begin{cases} -k & \text{for } -\pi < x < 0 \\ k & \text{for } 0 < x < \pi \end{cases}$$

Hence show that :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$$

572

(b) Find the half range cosine series for the function $f(x) = x^2$ in the interval.

(c) Using Laplace transform, find the solution of the initial value problem $y'' - 4y' + 4y = 64 \sin 2t$ given that $y(0) = 0,\ y'(0) = 1$.

6. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=10 Marks)

(a) Prove that :

$$J_{3/2}(x)\sqrt{\frac{2}{\pi x}}\left\{\frac{\sin x}{x} - \cos x\right\}$$

(b) For the Legendre functions, establish following recurrence relation :

$$(n+1)\,P_{n+1} - (2n+1)\,xP_n + nP_{n-1} = 0$$

(c) Evaluate :

$$L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$$