Roll No. | | | | | | | |

TMA–101

B. Tech. (First Semester)

End Semester EXAMINATION, 2017

(All Branches)

ENGINEERING MATHEMATICS—I

Time : Three Hours] [*Maximum Marks : 100*

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) If $x = r\cos\theta, y = r\sin\theta$, then $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ is equal to ....

(b) If $\lambda_1, \lambda_2, \lambda_3$, are eigen value of an square matrix A, then the eigen value of A^{-1} is ....

(c) The minimum value of function $x^3 + y^3 = 3xy$ is ....

(d) If $\vec{F}$ is the velocity of a fluid particle, then $\int_C \vec{F} . dr$ represents ....

(e) Find the 10th derivative of $(2x+9)^{10}$.

2. Attempt any *five* parts : (3×5=15 Marks)

(a) Find the *n*th derivative of $\frac{1}{x^2+a^2}$.

(b) A fluid motion is given by $\vec{V}=(y+z)i+(z+x)j+(x+y)k$. Is this motion irrotational ? If so, find the velocity potential.

(c) If $f(x, y)=ax^2+2hxy+by^2$, then verify Euler theorem.

(d) Evaluate $\iint y\,dx\,dy$ over the area bounded by $x=0, y=x^2, x+y=2$ in the first quadrant.

(e) If $x=r\cos\theta, y=r\sin\theta, u=x^2-y^2$ and $v=2xy$, then find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

(f) Find the first six term of expansion of $e^x \log(1+y)$ in Taylor series in the near the point (0, 0).

(g) Evaluate $\int_0^1\int_0^y 2x\,dx\,dy$.

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) For what values of λ and μ, the following system of equation :

$$5x+3y+2z=9$$
$$-2x+3y+7z=8$$
$$\lambda x+3y+2z=\mu$$

will have :

(i) unique solution

(ii) infinite no. of solutions

(iii) no solution

(b) If $u=f(r)$ and $x=r\cos\theta, y=r\sin\theta$ or $r^2=x^2+y^2$, show that :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$$

(c) Find eigen values and eigen vectors of the matrix :

$$A=\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 3\\ 0 & 0 & 2\end{bmatrix}$$

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) If $x = r\sin\theta\cos\theta,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$, find $\dfrac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$.

(b) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

(c) A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Define the Gradient of a scalar function, Divergence and Curl of a vector function. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point (1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4).

(b) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)x\,y_{n+1} + (n^2+1)\,y_n = 0$.

(c) Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

6. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Find the point upon the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value and find this minimum value of *f*.

(b) Use Stokes' theorem to evaluate :

$$\int_C \vec{F}\cdot d\vec{r} \quad \text{for } \vec{F} = (x^2 + y^2)\hat{i} - 2xy\,\hat{j}$$

taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

(c) Apply Green's theorem to evaluate :

$$\int_C [(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy]$$

where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.