Roll No. Healthy Living & fitness

TMA–101

B. TECH. (FIRST SEMESTER)
END SEMESTER EXAMINATION, 2019
(ALL BRANCHES)

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

1. Attempt any *two* parts of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. (CO1)

(b) Find the Eigen values and Eigen vectors of matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (CO1)

(c) Determine the values of a and b for which the system : (CO2)

$$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$$

(i) has a unique solution

(ii) has no solution, and

(iii) has infinitely many solutions.

2. Attempt any *two* parts of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the nth derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$. (CO3)

(b) If $z(x+y)=x^2+y^2$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial x}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial x}\right)$. (CO3)

(c) If $u=\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, evaluate $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. (CO3)

3. Attempt any *two* parts of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Find the stationary points and extreme values of the function : (CO4)

$$f(x, y)=x^3y^2(1-x-y)$$

(b) If $x^2+y^2+u^2-v^2=0$ and $uv+xy=0$, find the value of $\frac{\partial(u, v)}{\partial(x, y)}$.

(CO3)

(c) Are $u=x+y+z$, $v=x-y+z$, $w=x^2+y^2-2yz$ functionally dependent ? If yes, find the relation also. (CO3)

4. Attempt any *two* parts of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Evaluate : (CO5)

$$\int_1^2\int_0^1\int_{-1}^1(x^2+y^2+z^2)\,dx\,dy\,dz$$

(b) Evaluate : (CO5)

(i) $\int_0^1 x^4(1-\sqrt{x})^5\,dx$

(ii) $\int_0^\infty \sqrt{x}\,e^{-x}\,dx$

(c) Change the order of integration in the following integral and evaluate :

(CO5)

$$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$$

5. Attempt any *two* parts of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define curl and divergence of a vector point function. If a vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$. Is this field irrotational ?

(CO6)

(b) Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^3 = 3$ at the point (1, 1, 1). (CO6)

(c) State Gauss's Divergence theorem $\iint_S \vec{F}\,\hat{n}\,dS$ where : (CO6)

$\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.