Roll No. ....

TMA–101

B. TECH. (FIRST SEMESTER)

END SEMESTER EXAMINATION, 2018

(ALL BRANCHES)

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True/False : (1×5=5 Marks)

(a) If the eigen value of a square matrix (A) is λ, then the eigen value of its transpose (A^T) is $\frac{1}{\lambda}$. (True/False)

(b) The 3rd derivative of 2^{5x} is ....

(c) If the functions u, v of two independent variables x, y are not independent, then $\frac{\partial(u, v)}{\partial(x, y)} =$ ....

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(d) If $rt - s^2 > 0$, then the necessary condition for $f(x, y)$ to have a maximum value is that ....

(e) If the divergence of a vector is zero, then the vector is ....

2. Attempt any *five* parts out of seven : (3×5=15 Marks)

(a) Test the continuity of the function :

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 - y^2}, & x \neq 0, y \neq 0 \\ 0 & , x = 0, y = 0 \end{cases}.$$

(b) State Euler's theorem of differential calculus. If $u = \log \dfrac{x^2 + y^2}{xy}$, then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

(c) If $u = e^x \sin y, v = x \log \sin y$, then find the Jacobian :

$$\frac{\partial (u, v)}{\partial (x, y)}.$$

(d) Determine the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$.

(e) Using Beta and Gamma functions, evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^6 \theta \cos^4 \theta \, d\theta$.

(f) Define the Gradient of a scalar function, Divergence and curl of a vector function.

(g) Show that the matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) If $u = \log(x^2 + y^2 + z^2)^{\frac{1}{2}}$, find the value of $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}$.

(b) Find the eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

(c) Investigate the values of 'b' for which the system of homogeneous equations :

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

has a (i) trivial solution, (ii) non-trivial solution. Find the non-trivial solution using Matrix method.

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Determine the directional derivative of the function $f = 2x^2 - y^2 + z^2$ at the point P (3, 2, 1) in the direction of the line PQ, where Q is the point (4, 0, 5) and also calculate the magnitude of the maximum directional derivative.

(b) Expand $e^x \cos y$ in powers of x and $\left(y - \frac{\pi}{2}\right)$ upto the terms of degree 3.

(c) Prove that :

$$\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta \, d\theta = \frac{\left|\overline{\frac{p+1}{2}}\right. \left|\overline{\frac{q+1}{2}}\right.}{2\left|\overline{\frac{p+q+2}{2}}\right.}.$$

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Evaluate :

$$\iint_R (x^2 + y^2)\, dx, dy,$$

where R is the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Determine the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{a} + \frac{z}{a} = 1$ and the coordinate planes.

(c) By using the Stokes' theorem, evaluate

$$\int_C [(x+y)\,dx + (2x - z)\,dy + (y+z)\,dz]$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

6. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) The pressure P at any point (x, y, z) in space is $P = 400\,xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.

(b) If $\mu = \sin^{-1} x + \sin^{-1} y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$, then find $\dfrac{\partial (u, v)}{\partial (x, y)}$. Are they functionally related ? If yes, then determine the relationship between them.

(c) By using the Green's theorem, evaluate $\int_C \left[(2x^2 - y^2)\, dx + (x^2 + y^2)\, dy \right]$, where C is the boundary of the area enclosed by $y = 0$ and the upper half of the circle $x^2 + y^2 = a^2$.