

13/12/2021

Morning

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S.S. 2028

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Roll No.

TMA-101

B. TECH. (FIRST SEMESTER)

MID SEMESTER EXAMINATION, NOV., 2021

(All Branches)

ENGINEERING MATHEMATICS—I

Time : 1½ Hours

Maximum Marks : 50

Note : (i) Answer all the questions by choosing any *one* of the sub-questions.

(ii) Each question carries 10 marks.

1. (a) Find the rank of the matrix :

10 Marks (CO1)

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

OR

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix}$.

10 Marks (CO1)

P. T. O.

2. (a) Investigate the values of a and b so that the equations :

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + az = b$$

have (i) Unique solution, (ii) Infinite solutions (iii) No solution.

10 Marks (CO2)

OR

- (b) Show that the set of vectors $X_1 = [2, 3, 1, -1]$, $X_2 = [2, 3, 1, -2]$, $X_3 = [4, 6, 2, 1]$ is linearly dependent.

10 Marks (CO2)

3. (a) State Cayley-Hamilton theorem and verify it for the matrix :

10 Marks (CO1)

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$$

OR

- (b) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then prove that :

10 Marks (CO3)

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u.$$

4. (a) If $y = e^{a \sin^{-1} x}$, prove that :

$$(1 - x^2) y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

10 Marks (CO3)

OR

- (b) If $y = \frac{x^3}{(x^2 - 1)}$, then find y_n at $x = 0$.

10 Marks (CO3)

(3)

5. (a) If $u = x^y$, then prove that :

10 Marks (CO3)

$$\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}.$$

OR

(b) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighborhood of (1, 1) and inclusive of second-degree terms.

10 Marks (CO3)