Roll No. ....

TMA–101

B. Tech. (FIRST SEMESTER)
MID SEMESTER EXAMINATION, 2019
(ALL BRANCHES)
ENGINEERING MATHEMATICS–I

Time : 1 : 30 Hours

Maximum Marks : 50

Note :(i) AU questions are compulsory.

(ii) Answer any *two* **subquestions** among (a), (b) and (c) in each main question.

(iii) Total marks for each main question are **ten.**

1. Attempt any *two* parts of choice from (a), **(b)** and (c). **(2×5=10** Marks)

(a) Define Rank of Matrix. Find the rank of the matrix A by reducing it to the normal **form :**

$$\begin{matrix} 0 & I & 3 & \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & Q \end{matrix}$$

P. T. O.

(b) Find the value of A, for which the system of equations :

$$x+y+4z=1$$

$$x+2y-2z=1$$

$$\lambda x+y+z=1$$

will have a unique solution.

(c) Examine the following system of vectors for linear dependence :

$X_i = (1, -1, 1)$

$X2 = (2, 1, 1)$

$X3 = (3, 0, 2)$

2. Attempt any *two* parts of choice from (a), (b) and (c). (2;5=10 Marks)

(a) Define **eigen** values of a matrix. If the **eigen** values of a matrix are 1, -3, 2, then find the eigen values of A^{-1} and A^2. Also find the trace of the matrix.

(b) Find the eigen vector corresponding to the **eigen** value A, = – 6 for the matrix

$$A : \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

(c) Define **Cayley-Hamilton** Theorem. Verify **Cayley-Hamilton** theorem for the matrix

$$\begin{bmatrix} 15 & 4 \\ 1 & 2 \end{bmatrix}$$

3. Attempt any *two* parts of choice from (a), (b) and (c). **(2×5=10** Marks)

(a) Define partial differentiation. If Y = **sin x.cos** x, find y_n.

(b) If y = cos (m $\sin^{-1}$ x), prove that

$$(1-x^2)\, y_{n+2} = (2n + \quad x y_{n+1}$$

$$+ (m^2 \quad n^2)\, y_n = 0$$

(c) Find the value of $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2 y^3}{x^2+y^2}$, x O, y O.

4. Attempt any *two* parts of choice from (a), (b) and (c): (2x5=10 Marks)

(a) If $u = \log(x^2+y^2) + \tan\left(\frac{v}{x}\right)$, then find the value of $\frac{82_u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

(b) Define **Euler's** theorem. If $u = \log\left(\frac{x^4 - y^4}{x - y}\right)$, prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} \quad 3.$$

(c) Find the nth derivative of $\frac{1}{z \quad 4}$.

5. Attempt any *two* parts of choice from (a), (b) and (c). (2x5=10 Marks)

(a) Expand *ex* cos y upto second degree terms by Taylor's series.

(b) Test the convergence of the following series :

$$\sum \frac{n + a}{2^n + a}$$

(c) Show that the matrix :

$$A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & a - i\gamma \end{bmatrix}$$

is **unitary** matrix if $a^2 + D2 + {}_y2 + 8^2 = 1$.