TMA–101

B. TECH. (FIRST SEMESTER)

MID SEMESTER EXAMINATION, Nov., 2022

ENGINEERING MATHEMATICS—I

Time : 1½ Hours

Maximum Marks : 50

Note : (i) Answer all the questions by choosing any *one* of the sub-questions.

(ii) Each sub-question carries 10 marks.

1. (a) Find rank of the matrix : (CO1)

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

OR

(b) Determine whether the following set of vectors are linearly dependent or independent : $\{(0, 1, 1), (1, 0, 3), (1, 1, 4)\}$. (CO1)

P. T. O.

2. (a) For what value of λ and μ, the following system of equations   (CO2)

$$x + y + z = 6;$$

$$x + 2y + 3z = 10;$$

$$x + 2y + \lambda z = \mu$$

will have :

(i) Unique solution;

(ii) Infinite solutions;

(iii) No solution.

OR

(b) Diagonalize the matrix :   (CO2)

$$\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}.$$

3. (a) Find Eigen values and Eigen vectors of the matrix :   (CO1)

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

OR

(b) If $y = \sin (m \sin^{-1} x)$, prove that :   (CO3)

$$(1 - x^2)y_{n+2} - (2n + 1)x.y_{n+1} + (m^2 - n^2)y_n = 0.$$

4. (a) If $u = \log(x^3 + y^3 + z^3) - 3xyz$, then show that :   (CO3)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}.$$

OR

(b) If $u = \log_e\left(\dfrac{x^3 + y^3}{x + y}\right)$, show that :   (CO3)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2.$$

5. (a) Expand $e^x \cos y$ near the point $(0, 0)$ by Taylor's theorem upto third degree term.   (CO3)

OR

(b) Expand $x^2 y + 3y - 2$ in powers of $x - 1$ and $y + 2$ using Taylor's theorem.   (CO3)