Roll No. ....

TMA–101

B. TECH. (FIRST SEMESTER)

END SEMESTER EXAMINATION, Jan., 2023

ENGINEERING MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Find the rank of matrix : (CO1/CO2)

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

(b) Find the value of λ for which the system of equations : (CO1/CO2)

$$3x - y + 4z = 3, x + 2y - 3z = -2, 6x + 5y + \lambda z = -3$$

has a trivial solution. Find the solution.

(c) State's Cayley-Hamilton Theorem and verify it for the matrix : (CO1/CO2)

$$A = \begin{bmatrix} 5 & -1 & 5 \\ 0 & 2 & 0 \\ -5 & 3 & -15 \end{bmatrix}.$$

2. (a) Find the expansion of $\sin xy$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ upto quadratic terms. (CO3)

(b) If $y = e^{\tan^{-1} x}$, prove that : (CO3)

$$(1+x^2)y_{n+2} + \left[2(n+1)x - 1\right] y_{n+1} + n(n+1)y_n = 0.$$

(c) State's Euler's Theorem and verify it for the function : (CO3)

$$f(x, y) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right).$$

3. (a) If u, v and w are the roots of the equation

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0,$$

in λ, then prove that : (CO3/CO4)

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}.$$

(b) If $u = xy + yz + zx, v = x^2 + y^2 + z^2, w = x + y + z$, determine whether there is a functional relationship between u, v and w and if so, find it. (CO3/CO4)

(c) Find the extreme values of the function $u(x, y) = x^3 + y^3 - 3axy$. (CO3/CO4)

4. (a) Evaluate : (CO5)

$$\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx .$$

(b) Change the order of integration and evaluate : (CO5)

$$\int_0^\infty \int_0^x x \exp\left(\frac{-x^2}{y}\right) dx dy .$$

(c) Define Beta an Gamma functions and evaluate the integral : (CO5)

$$\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} dx .$$

5. (a) Verify Green's Theorem for $\int_C (x^2 - 2xy)dx + (x^2 y + 3)dy$ around the boundary C of the region $y^2 = 8x$ and $x = 2$. (CO6)

(b) Verify Stokes' Theorem for $\vec{F} = x^2\hat{i} - xy\hat{j}$ for the surface integrated around the square in the plane $z = 0$, and bounded by the lines $x = 0$, $y = 0, x = a, y = a$. (CO6)

(c) Show that : (CO6)

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2} ;$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

1150