Roll No. .....

TMA–101

B. TECH. (FIRST SEMESTER)
MID SEMESTER EXAMINATION, 2018

(All Branches)

ENGINEERING MATHEMATICS—I

Time : 1:30 Hours

Maximum Marks : 50

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

   (a) A matrix is said to be involuntary matrix if ..... .

   (b) The characteristic values of the matrix $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ is ..... .

   (c) If $|A| = 0$, then at least one eigen value is zero. (True/False)

   (d) The fourth derivative of $y = \log 3x$ is ..... .

   (e) If $u = x^y$, then $\frac{\partial u}{\partial y} = 0$. (True/False)

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2. Attempt any *five* parts : (3×5=15 Marks)

(a) Show that the system of equation solve with the help of matrices, the simultaneous equations :

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

(b) Test the convergence of the series $\sum \frac{(n)^3}{3^n}$.

(c) Find whether the following set of vectors are linearly dependent or independent : [1, 2, 3], [3, – 2, 1], [1, – 6, – 5].

(d) Explain Leibnitz test for convergence of a series.

(e) State Cayley-Hamilton theorem and verify for $A = \begin{bmatrix} 5 & 7 \\ 3 & -1 \end{bmatrix}$.

(f) If $u = \sin^{-1} \frac{x + y}{\sqrt{x} - \sqrt{y}}$, prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$$

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Find rank by reducing it to normal form $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

(b) Find the eigen values for the matrix :

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(c) Find *n*th derivative of $y = \log (ax + b)$.

4. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) If $y = (\sin^{-1} x)^2$, then prove that :

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$$

(b) Expand $x^2 + xy + y^2$ in powers of $(x - 2)$ and $(y - 3)$ as far the terms of second degree using Taylor's theorem.

(c) If $u = \sin^{-1} \frac{x}{y}$, find the value of $\frac{\partial^2 u}{\partial x \partial y}$.

5. Attempt any *two* parts of choice from (a), (b) and (c). (5×2=10 Marks)

(a) Find the eigen vector corresponding to the eigen value $\lambda = 15$ for the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(b) Find *n*th derivative of $\frac{1}{8 - 6x + x^2}$.

(c) If $u = \log \frac{x^5 + y^5 + z^5}{x^2 + y^2 + z^2}$, prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3$$

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