Date - 14·05·18
Time - 1:30 PM

Roll No. ....

TMA–201

B. TECH. (SECOND SEMESTER) END SEMESTER EXAMINATION, 2018

(All Branches)

ENGINEERING MATHEMATICS—II

Time : Three Hours

Maximum Marks : 100

Note : (i) This question paper contains two Sections.

(ii) Both Sections are compulsory.

Section—A

1. Fill in the blanks/True-False : (1×5=5 Marks)

(a) The solution of $\frac{dy}{dx} = \frac{-y}{x}$ is ....

(b) The Laplace transform of $e^{-ax} \cos bx$ is $\frac{s}{(s-a)^2 + b^2}$. (True/False)

(c) The periodic function is defined as ....

(d) The differential equation of heat equation is $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$. (True/False)

(e) The solution of Partial differential equation $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} - \dfrac{\partial^2 z}{\partial y^2} = 0$ is ....

2. Attempt any *five* parts : (3×5=15 Marks)

(a) Obtain the general solution of $\dfrac{d^2y}{dt^2} - 7\dfrac{dy}{dt} + 12y = 0$.

(b) Solve :

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

(c) Find the Laplace transform of $f(t) = t\,e^{-2t}\sin t$.

(d) Solve :

$$\frac{\partial^2 u}{\partial x^2} - 14\frac{\partial^2 u}{\partial x \partial y} + 48\frac{\partial^2 u}{\partial y^2} = 0$$

(e) Find the Fourier cosine series of $f(x) = \cos x$ in the interval $0 \le x \le \pi$.

(f) Find the inverse Laplace transform of $\dfrac{s+4}{(s-1)(s^2+4)}$

(g) Express $f(x) = x^2$, as a Fourier series in $-\pi < x < \pi$.

Section—B

3. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Solve :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}\sin x$$

(b) Solve :

$$\frac{d^2y}{dt^2} + 4y = \sin 2x \sin 3x$$

(c) Solve by method of variation of parameters $\dfrac{d^2y}{dx^2} + a^2y = \cot ax$.

4. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Find the Laplace transform of :

$$f(x) = \frac{1-\cos x}{x^2}.$$

(b) Find the inverse Laplace transform of $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$, using Convolution theorem.

(c) Using Laplace transform, solve the following differential equation :

$$\frac{d^2y}{dt^2}+9y=6\cos 3t, \quad \text{given}$$

$$y(0)=2, y'(0)=0.$$

5. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) Represent the following function by a Fourier sine series :

$$F(x)=\begin{cases} x, & 0<x\le \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2}<x\le \pi \end{cases}$$

(b) Given that $f(x)=x+x^2$ for $-\pi<x<\pi$, find the Fourier expansion of $f(x)$. Deduce that :

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+.....$$

(c) Solve the Partial differential equation :

$$\frac{\partial^2 u}{\partial x^2}-7\frac{\partial^2 u}{\partial x\partial y}+18\frac{\partial^2 u}{\partial y^2}=e^{3x}\sin(2x+5y)$$

6. Attempt any *two* parts of choice from (a), (b) and (c). (10×2=20 Marks)

(a) A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form $y=k(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time 't'.

(b) Prove that the following Recurrence relation :

(i) $xJ_n'=nJ_n-xJ_{n+1}$

(ii) $xJ_n'=-nJ_n+xJ_{n-1}$

(c) State and prove Rodrigue's formula.