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Polynomial fitting

Solve a least square problem to find an optimal polynomial curve for a given set of two dimensional points.

Demonstrate the effect of the degree of polynomial in fitting a given set of points.

In [342]:

```
import matplotlib.pyplot as plt
import numpy as np
import math, random

#f(x) = 2x^3 + 10x^2 + x + 1

def polynomial_fitting(degree):
    x = np.linspace(0, 10, 20)
    y = 2*np.power(x, 3) + 10*np.power(x, 2) + x + 1
    noise_y = 2*np.power(x, 3) + 10*np.power(x, 2) + x + 1


    for i in range(-10, 10):
        noise_y[i] += (random.gauss(0.0, 70.0))

    plt.plot(x, y)
    plt.plot(x, noise_y, '.')
    plt.title('Original Polynomial and Noisy Points')
    plt.show()


    plt.xlabel('X Data')
    plt.ylabel('Y Data')

    energy = []
    for i in range(1, degree+1):
        p = np.polyfit(x, noise_y, i)
        new_y = np.polyval(p, x)

        label = '$f(x)='
        for j in range(len(p)):
            label += str("%0.3f" %  (p[j])) + 'x^' + str(i - j)
            if(j < len(p)-1 and p[j+1] >= 0):
                label += '+'
        label = label[:-3]
        label += '$'

        plt.plot(x, new_y, label = label)
        plt.legend(loc='center left', bbox_to_anchor = (1,0.5))
        energy.append(L2_distance(noise_y, new_y))

        plt.title('Approximating Polynomial Curve in Degree ' + str(i))
        plt.plot(x, noise_y, '.')
        plt.show()



    x = np.arange(1, degree+1, 1)
    plt.title('energy')
    plt.plot(x, energy)
    plt.show()


def L2_distance(source, target):
    sum =  0
    for i in range(len(source)):
        sum += (source[i] - target[i]) ** 2
    sum = math.sqrt(sum) / len(source)
    return sum
```

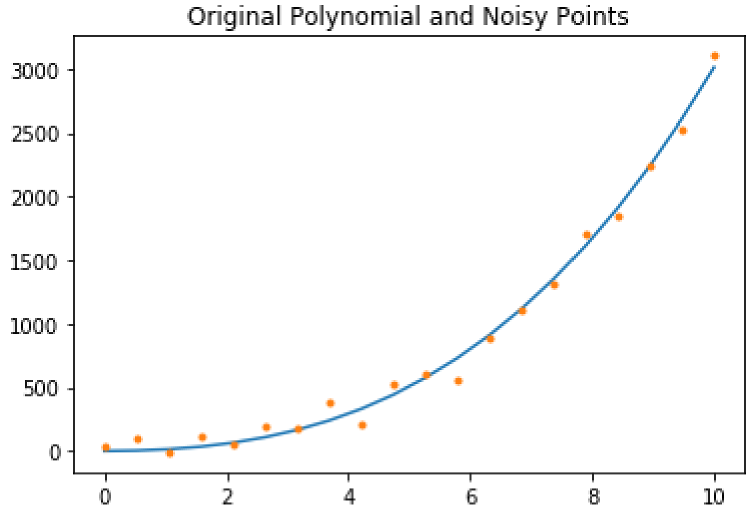
1. choose a polynomial curve and generate points along the curve with random noise

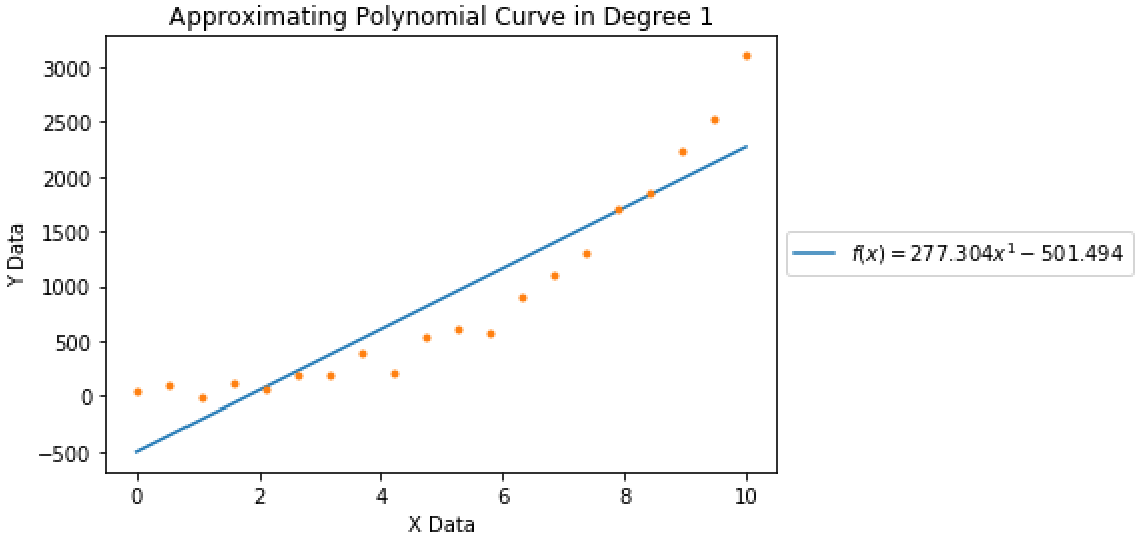
$$f(x) = 2x^3 + 10x^2 + x + 1$$

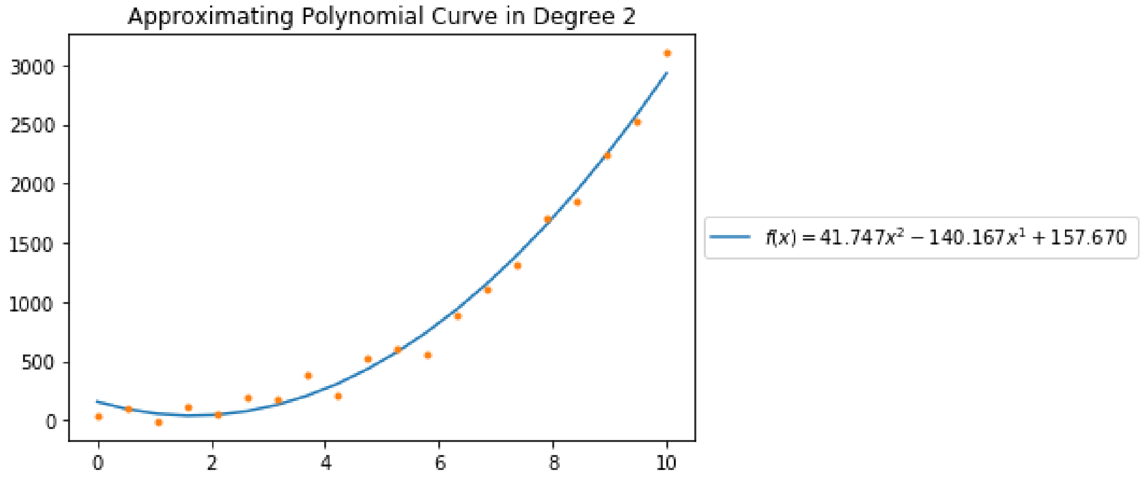
1. plot the generated noisy points along with its original polynomial without noise
2. plot the approximating polynomial curve obtained by solving a least square problem
3. plot the approximating polynomial curve with varying polynomial degree

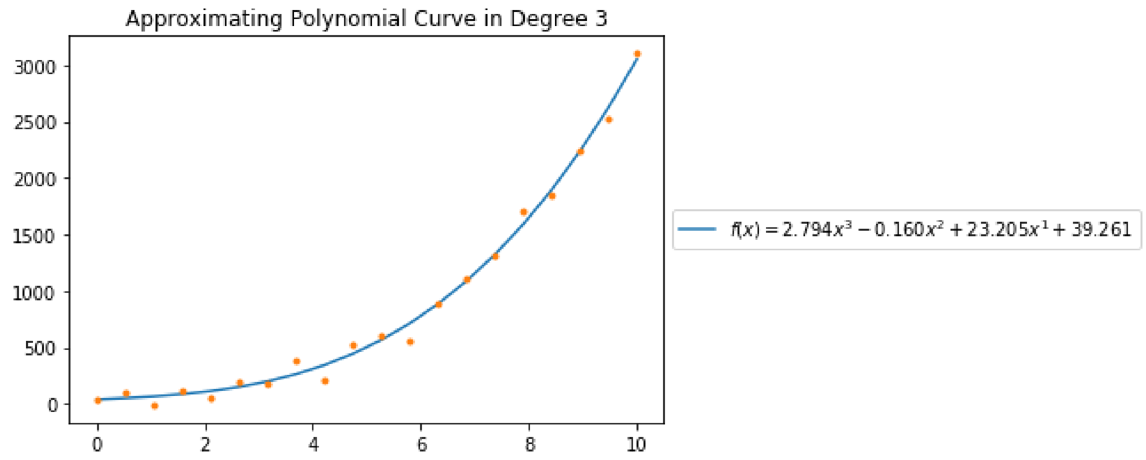
In [343]:

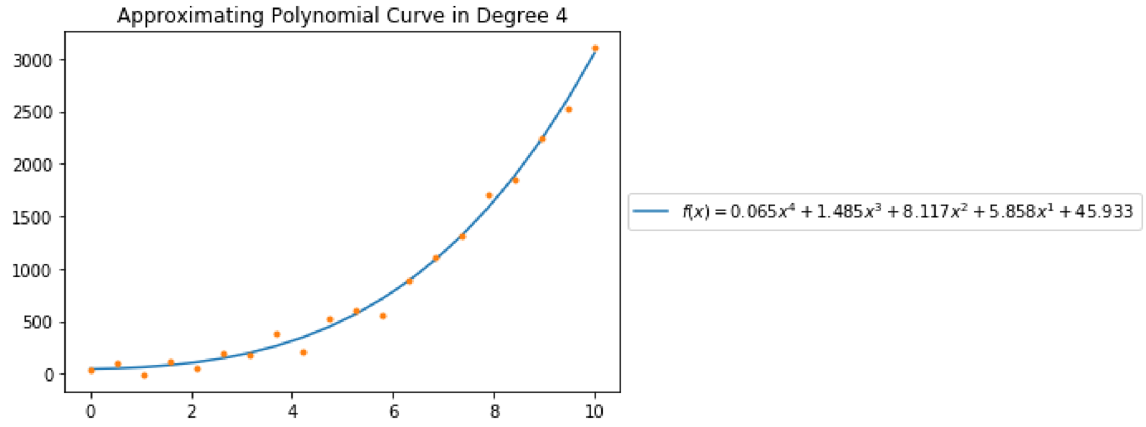
```
polynomial_fitting(10)
```

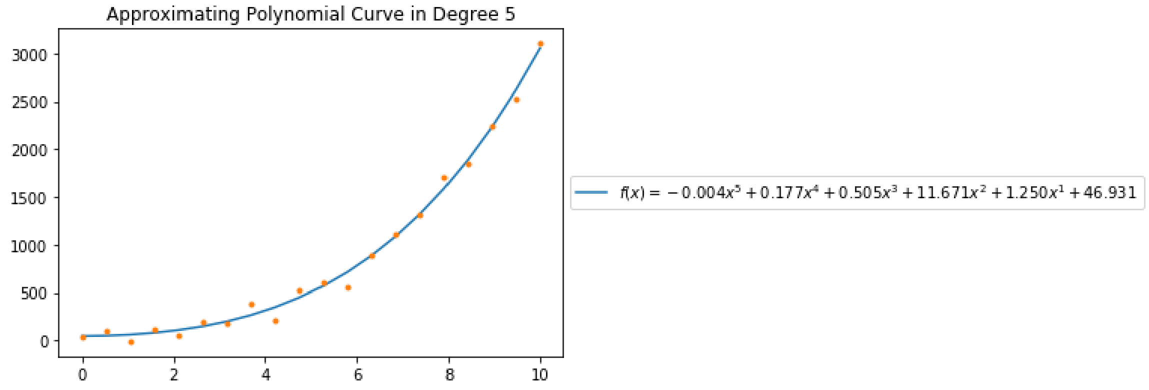

Original Polynomial and Noisy Points
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10

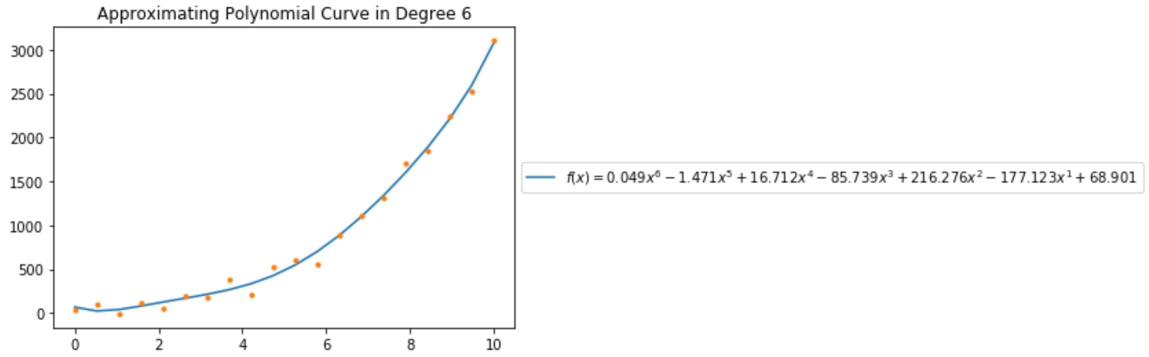

Approximating Polynomial Curve in Degree 1
Y Data
3000
2500
2000
1500
1000
500
0
−500
0
2
4
6
8
10
X Data
$f(x) = 277.304x^1 - 501.494$

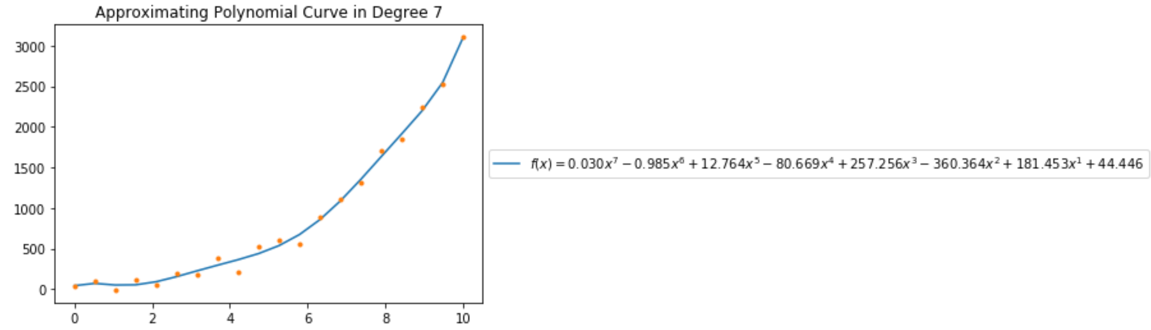

Approximating Polynomial Curve in Degree 2
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 41.747x^2 - 140.167x^1 + 157.670$

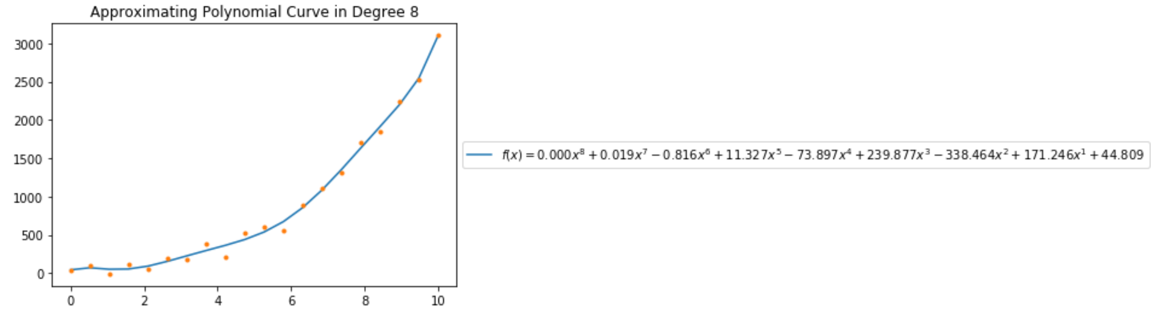

Approximating Polynomial Curve in Degree 3
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 2.794x^3 - 0.160x^2 + 23.205x^1 + 39.261$

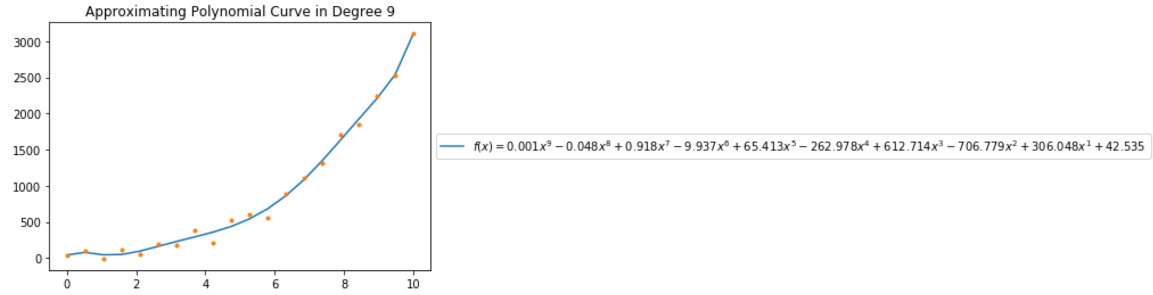

Approximating Polynomial Curve in Degree 4
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 0.065x^4 + 1.485x^3 + 8.117x^2 + 5.858x^1 + 45.933$

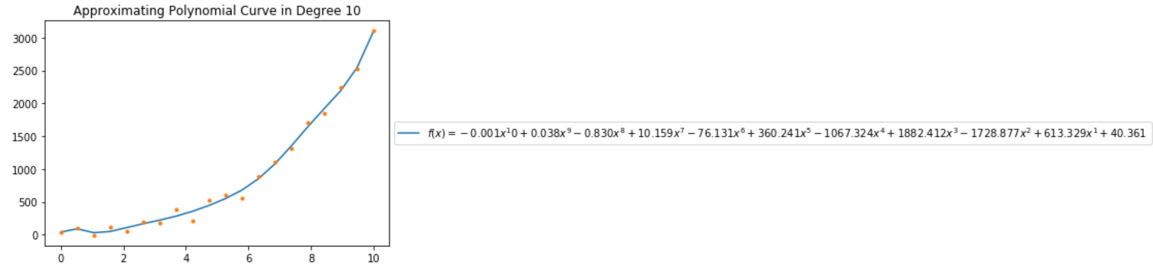

Approximating Polynomial Curve in Degree 5
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = -0.004x^5 + 0.177x^4 + 0.505x^3 + 11.671x^2 + 1.250x^1 + 46.931$

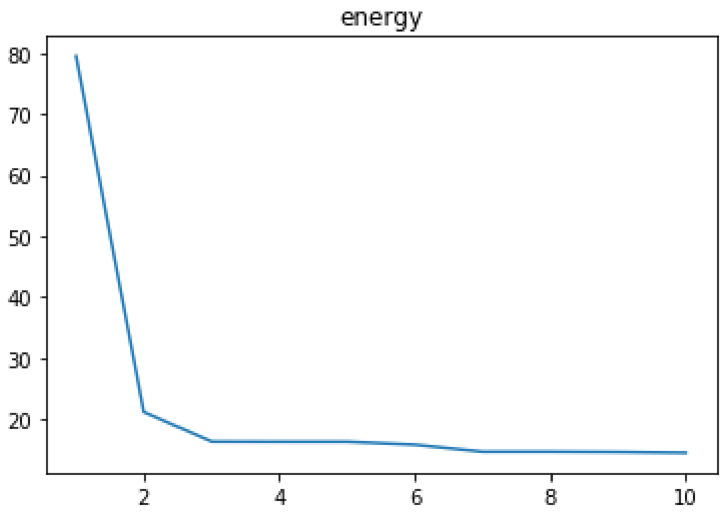

Approximating Polynomial Curve in Degree 6
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 0.049x^6 - 1.471x^5 + 16.712x^4 - 85.739x^3 + 216.276x^2 - 177.123x^1 + 68.901$


Approximating Polynomial Curve in Degree 7
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 0.030x^7 - 0.985x^6 + 12.764x^5 - 80.669x^4 + 257.256x^3 - 360.364x^2 + 181.453x^1 + 44.446$


Approximating Polynomial Curve in Degree 8
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 0.000x^8 + 0.019x^7 - 0.816x^6 + 11.327x^5 - 73.897x^4 + 239.877x^3 - 338.464x^2 + 171.246x^1 + 44.809$


Approximating Polynomial Curve in Degree 9
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = 0.001x^9 - 0.048x^8 + 0.918x^7 - 9.937x^6 + 65.413x^5 - 262.978x^4 + 612.714x^3 - 706.779x^2 + 306.048x^1 + 42.535$


Approximating Polynomial Curve in Degree 10
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
$f(x) = -0.001x^10 + 0.038x^9 - 0.830x^8 + 10.159x^7 - 76.131x^6 + 360.241x^5 - 1067.324x^4 + 1882.412x^3 - 1728.877x^2 + 613.329x^1 + 40.361$


energy
80
70
60
50
40
30
20
2
4
6
8
10