Exponential distribution family

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Introduction

The exponential family is a series of probability distributions. These are all based on the

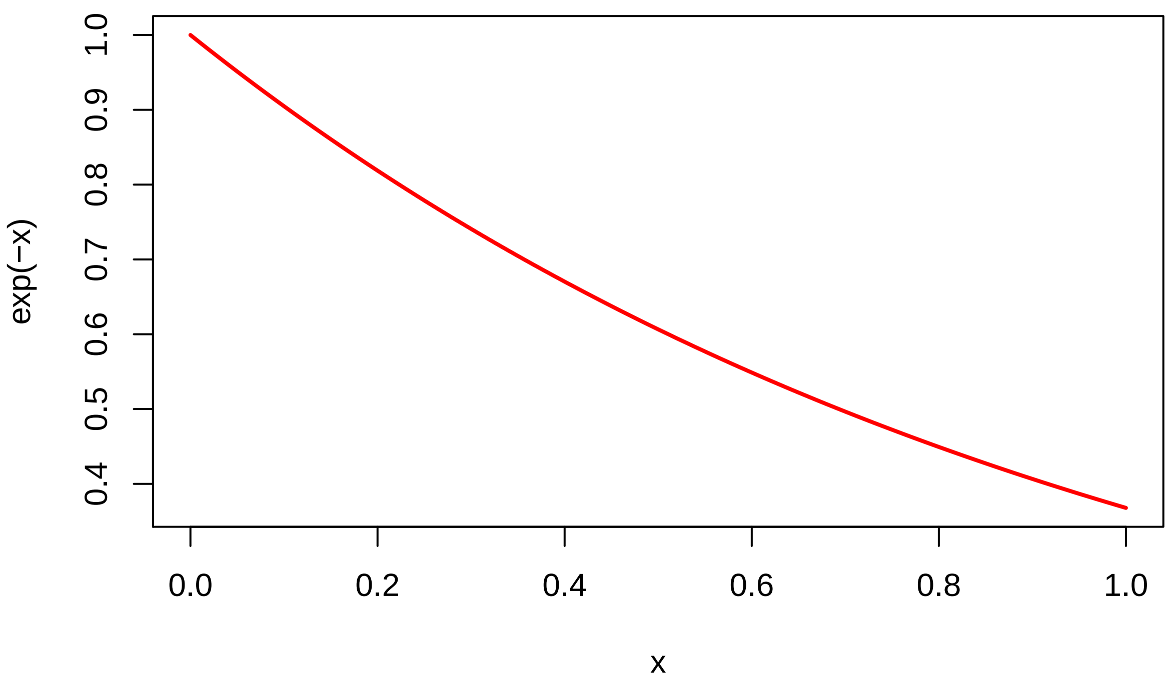
$$f(x) = e^x$$

function and are used in many different fields of application.

Exponential distribution

The most basic distribution is

$$e^{-x}$$



The sum under the curve has to be 1 to be a probability distribution

```
integrate(function(x) exp(-x), 0, Inf)
```

```
## 1 with absolute error < 5.7e-05
```

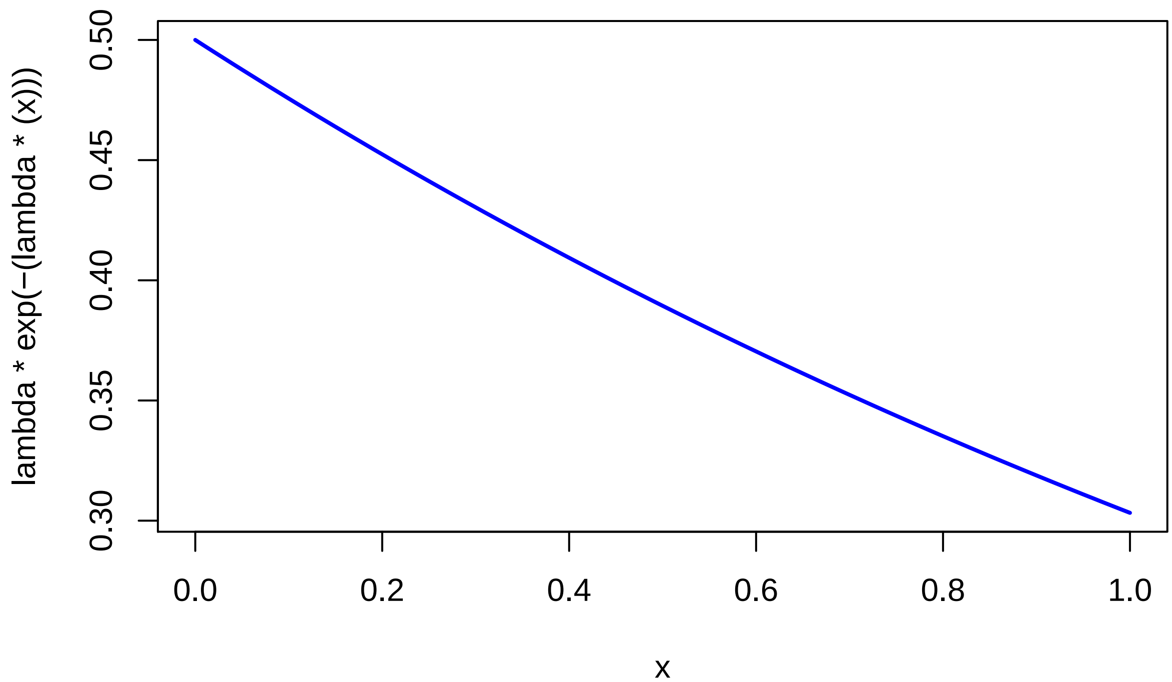
Other exponential distributions

Laplace distribution

$$\lambda \cdot e^{-\lambda \cdot x}$$

```r
x <- seq(0,1,0.01)

lambda <- 0.5
plot(x,lambda*exp(-(lambda*(x))),type = "l",col = "blue", lwd = "2")
```



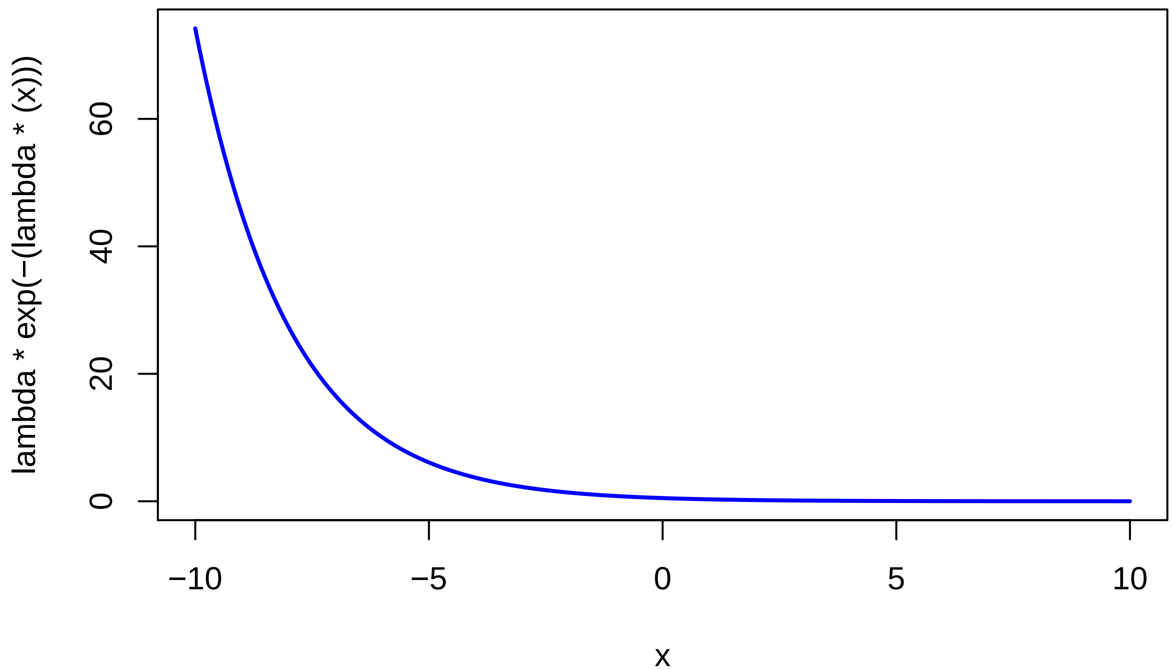

```r
integrate(function(x)
  lambda * exp(-lambda * (x)), 0, Inf)
```

```
## 1 with absolute error < 3.4e-05
```

$$\lambda \cdot e^{-\lambda \cdot (x-\mu)}$$

The additional parameter µ provides the possibility to use negative values and shift the curve along the x axis

```r
x <- seq(-10,10,0.01)
mu <- 20
lambda <- 0.5
plot(x,lambda*exp(-(lambda*(x))),type = "l",col = "blue", lwd = "2")
```




```r
integrate(function(x)
  lambda * exp(-lambda * (x - mu)), mu,Inf )
```

```
## 1 with absolute error < 3.4e-05
```

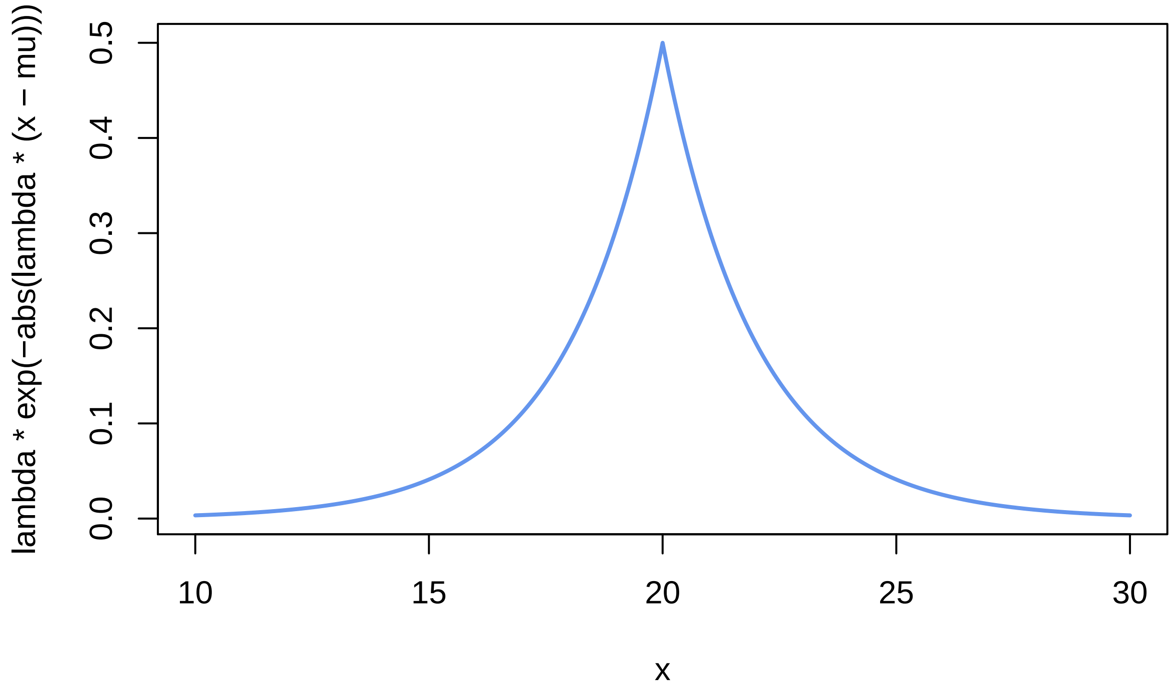
To ensure a result of 1 after integration, we need to integrate from *mu* to *Infinity*

symmetry and normalization

$$\frac{1}{2}\lambda e^{-|(\lambda(x-\mu)|}$$

```r
x <- seq(10,30,0.01)

lambda <- 0.5
mu <- 20
plot(x,lambda*exp(-abs(lambda*(x-mu))),type = "l",col = "cornflowerblue", lwd = "2")
```



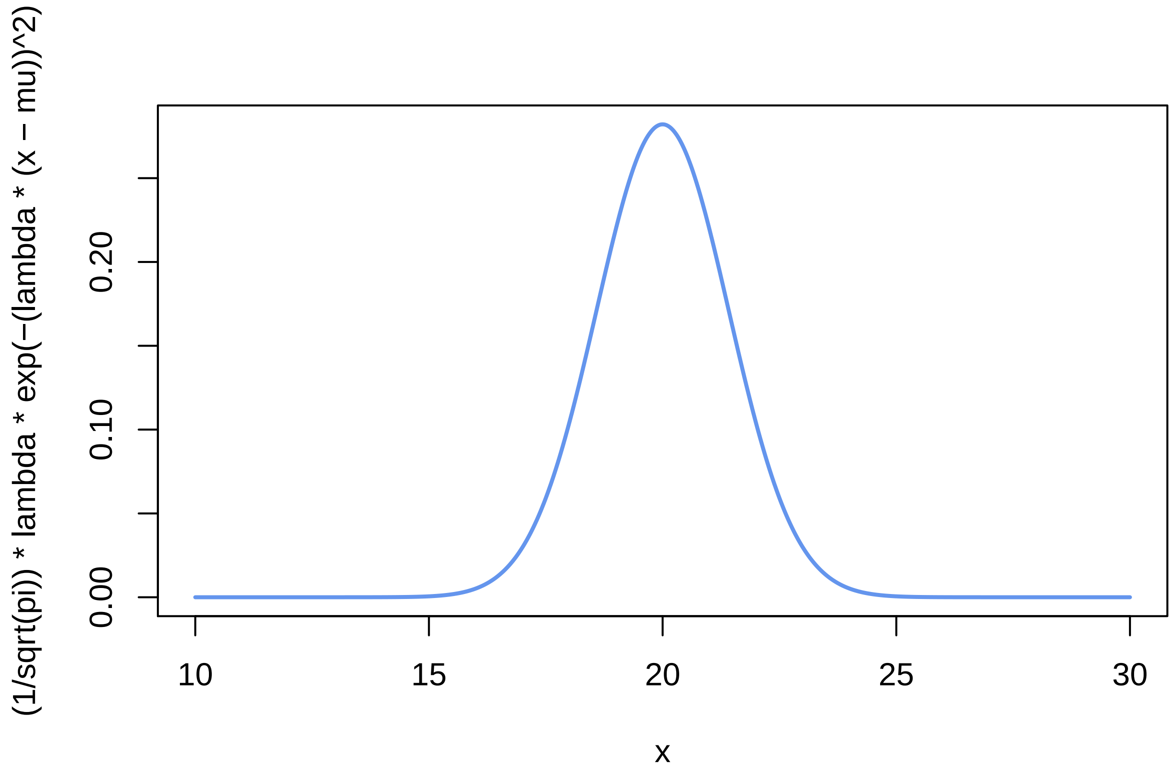

```
integrate(function(x) exp(-x), 0, Inf)
```

```
## 1 with absolute error < 5.7e-05
```

$$\frac{1}{\sqrt{\pi}}\lambda e^{-[(\lambda(x-\mu)]^2}$$

```
x <- seq(10,30,0.01)

lambda <- 0.5
mu <- 20
plot(x,(1/sqrt(pi))*lambda*exp(-(lambda*(x-mu))^2),type = "l",col = "cornflowerblue", lwd = "2")
```




```r
integrate(function(x) exp(-x), 0, Inf)
```

```
## 1 with absolute error < 5.7e-05
```

In these cases the factor infront of e is the important part regarding the normalization.

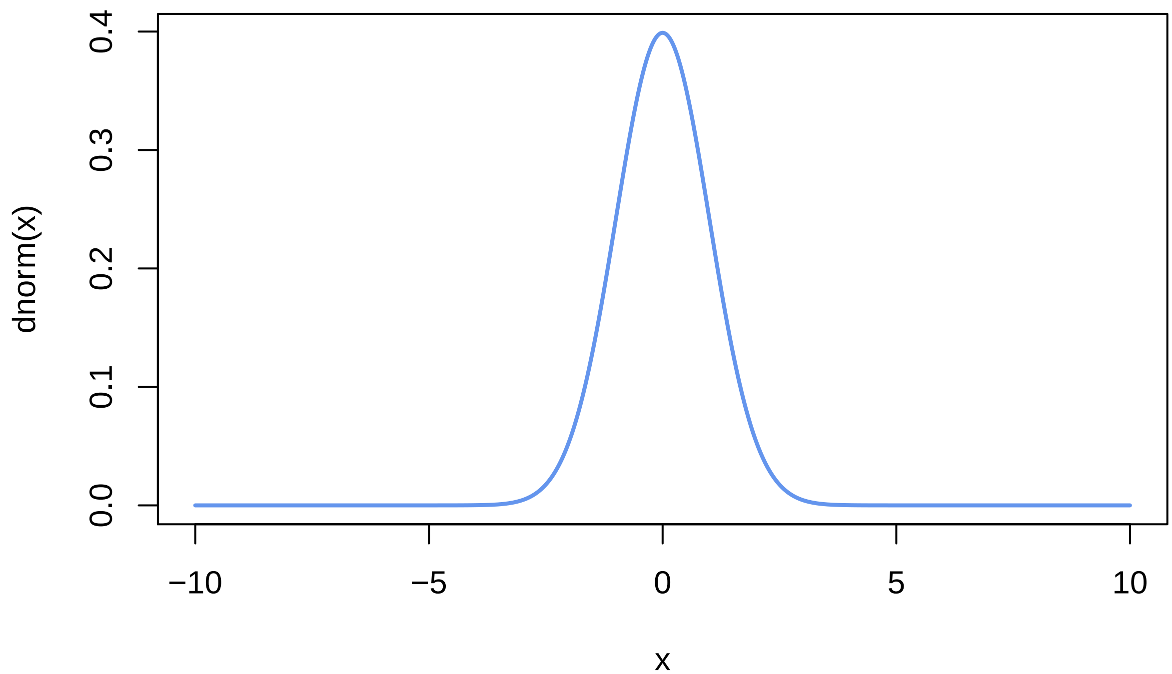
From here it is only a short way to the gaussian distribution

The Normal Distribution (Gaussian)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{(-\frac{(x-\mu)^2}{(2\sigma^2)})}$$

```r
x <- seq(-10,10,0.01)


plot(x,dnorm(x),type = "l",col = "cornflowerblue", lwd = "2")
```




```
integrate(function(x) exp(-x), 0, Inf)
```

```
## 1 with absolute error < 5.7e-05
```

Many natural phenomena occur approximately normal distributed, like bodysizes and windspeeds.