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“Analysis of Strategies of Robo Advisors based on the Example of Betterment“

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Abstract

This paper will give an overview over the industry of robo advisors. It will be discussed how they work, what kind of people they are suited for and the possible impact on the general industry of financial advisory. In addition to that, the main part of this paper will focus on the analysis of the leading stand-alone robo advisor, Betterment. The cornerstones of their strategy will be described and finally implemented to figure out if they add any value to the overall performance. It will be found, that, indeed, all elements of the strategy improve the performance and manage to finally provide a before-tax Sharpe ratio that is only slightly lower than the one of a target date fund with a similar equity share. Because of their tax-avoidance tools, which cannot be provided by target-date funds, it is possible that robo advisors like, Betterment, could outperform them nonetheless.

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List of Symbols

r	Asset returns
w	Asset weight
ρ	Correlation coefficient
Σ	Covariance matrix
ε	Error terms
λ	Implied risk-aversion coefficient
Ω	Matrix that determines uncertainty in views
n or N	Number of assets
T	Number of observed returns
$\hat{\delta}^*$	Optimal shrinking intensity
σ_{PF}	Portfolio variance
r_f	Risk-free rate
S	Sample covariance matrix
τ	Scalar
$\hat{\Sigma}_{Shrink}$	Shrunken covariance matrix
F	Structured estimator of the covariance matrix
P	Vector that determines all assets involved in views
Π	Vector of implied equilibrium returns
m	Vector of mean returns
X	Vector of returns
Q	Vector of views

List of Abbreviations

ETF	Exchange-traded fund
LTCG	Long-term capital gain
NBIM	Norges Bank Investment Management
REIT	Real estate investment trust
STCG	Short-term capital gain
TCP	Tax-Coordinated Portfolio
QDI	Qualified dividend income

1. Introduction

The robo advising industry is fairly new as it has only existed for roughly 9 years. Thus, there is, as yet, not much literature available regarding the topic in comparison to other branches of the financial industry. This paper will explain what robo advisors are, what the industry looks like, and how it will probably develop. Furthermore, this paper will work out how they can be distinguished from traditional robo advisors. From this, a conclusion can be drawn regarding what the potential target group is. Namely, the younger generation and people who cannot afford traditional financial advisors but nonetheless want to invest in financial markets, in order to save for certain goals or maximize their wealth in retirement.

However, it should be considered whether those machines and websites that are about to manage your entire savings are trustworthy and whether can they compete with intuitive do it yourself portfolio strategies or if they are even able to beat established low-costs financial instruments like target-date funds? To give an answer to these questions I will analyze the strategy of the leading stand-alone robo advisor, Betterment. In addition to that, I will implement a version of their strategy with the result that they provide a similar performance to a target-date fund and provide some tax avoidance tools that might be the decisive factor to beat even those funds after costs and tax.

2. Basic Information on Robo Advisors

The first part explains what robo advisors are, what they do and who are they suited for. After that the industry will be described and a brief overview of the development is given, followed by a discussion of "robo advisors versus human advisors". The last sub section will be a comparison between two of the leading stand-alone robo advisors, Betterment and Wealthfront, and a German startup Scalable Capital.

2.1. Introduction to Robo Advisors

Robo advisors are a form of financial advisor that try to manage your wealth with the help of computer algorithms. Their approach is to provide asset allocations that are suited to the goals of their clients and to offer financial advice without any human contact for less than traditional human advisors would charge. They use a passive form of investing, which means that they do not try to time the market or to pick single stocks which would be associated with higher costs for their clients (Betterment[1]). Instead they build long only portfolios with ETFs from different asset classes. Additionally, they provide tools like automatic rebalancing, optimization and asset location. The latter is a strategy to reduce the tax drag, which will be described later in this paper.

After explaining what robo advisors are, the question arises, who actually uses them? In fact, only 45% of US investors have heard about them at all and only 5% would use them (Fish et al., 2018). The reason for the low participation could be that only about a third of US citizens are looking for financial advice, because it is perceived to be too expensive (Fish et al., 2018). Furthermore, it normally takes a while until new technology is accepted by the general public, as older people do not trust websites which try to manage their money as much as they would trust their local wealth manager. Consequently, the average client of, for example, Betterment is a 36-year-old who appreciates modern technology (Wang and Padley, 2017, cited in Fish et al., 2018). In addition to that, the fact that most robo advisors have no or only low minimum balances is attractive to those who cannot yet afford a traditional financial advisor, who would normally require a minimum balance of between \$100,000 and \$500,000. Those people who do not have the required asset base are typically the younger generation (Fish et al., 2018).

2.2. Describing the Industry

The first robo advisors were Betterment and Wealthfront which were both founded in 2008; Betterment in New York City and Wealthfront in California. Neither of them offered their advice to retail investors until 2010 when Betterment launched their platform followed by Wealthfront in 2011 (Fish et al., 2018). Wealthfront did not even start as a robo advisor but as a mutual fund which has provided its knowledge to the tech community of the nearby Silicon Valley (Eule, 2017). Eule (2017) stated that these companies had an enormous impact on the financial industry and compared them to Tesla, which even accomplished pushing the center of the automobile industry, Detroit, in the direction of producing electro cars. Even though these companies did not reinvent the business, they made traditional financial advisors like Vanguard and Charles Schwab incorporate more technology into their strategy. Then, in 2015, Vanguard and Charles Schwab launched their own robo advisors, namely Vanguard PAS and Schwab Intelligent Portfolios (Eule, 2017).

In 2017 robo advisors had about $200 billion in assets under management, which sounds quite a lot, but, when considering that the global market is worth more than $80 trillion, they only make up about 0.25% of the global market (Eule, 2018). However, estimates show that strong growth can be expected in the next few years from about $1.4 trillion (Statista, 2019, cited in Fish et al., 2018) to $2.2 trillion (Regan, 2015, cited in Fish et al., 2018) to $8.1 trillion (Kocianski, 2016, cited in Fish et al, 2018) by 2020. Unfortunately, none of the leading, stand-alone robo advisor firms is listed on any stock exchange.

In Table 1 the most relevant robo advisors can be seen, measured according to assets under management. Quite interesting to see is that Vanguard´s and Charles Schwabs´s robo advisor have significantly more assets under management than the actual pioneers Betterment ($15 billion) and Wealthfront ($13 billion) which are ranked third and fourth.

The fourth column shows the fees which range from 0% to 0.4% per year, whereby the 0.4% also include contact with human advisors. Also important to mention is that just because Schwab Intelligent Portfolios do not charge any fees, it does not mean that they are cheaper than, for example, Betterment. Their customers have to pay fees on the products that are sold to them (Fish et al., 2018).

Robo Advisor	**Assets Under Management (in billion $)**	**Founded**	**Fees**	**Minimum Balance**	**Headquarters**
Vanguard PAS	$83.7	2015	Starting at 0.3%	$50,000	Malvern, PA
Schwab Intelligent Portfolios	$27	2015	0 (but fees on ETF)	$5,000	San Francisco, CA
Betterment	$15	2008	0.25% / 0.4%	$0	New York, NY
Wealthfront	$13	2008	0.25%	$500	Palo Alto, CA
Scalable Capital	$1	2014	0.75%	10,000€	Munich/London

Table 1: Data: Assets under management are either taken from the official homepage of the respective robo-advisor or from Fish et al. (2018) in the case of Schwab Intelligent Portfolios; all other information can be found on their websites: Vanguard https://investor.vanguard.com/advice/personal-advisor, Betterment https://www.betterment.com, Wealthfront https://www.wealthfront.com, Scalable Capital https://scalable.capital (as of 03-29-2019).

The next question is, how are those robo advisors regulated to ensure integrity? Since the Investment Act of 1940 each financial advisor has to register as a Registered Investment Advisor which is mandatory for both human and robo advisors (Fish et al., 2018).

Some robo advisors require their clients to transfer their assets to the company´s account, while others require them to transfer their assets to external brokerage houses. Those companies who hold their customers money themselves, have to register with the United States Securities and Exchange Commission (SEC) and the Financial Industry Regulatory Authority (FINRA) (Fish et al., 2018).

2.3. Robo versus Human Advisors

In the previous section, robo advisors were compared several times with traditional human advisors and mainly arguments in favor of robos were given. However, is there a chance that machines are able to replace humans completely? In the following subsection I want to discuss this question and give a few ideas regarding in which direction the industry could potentially develop.

Maybe the biggest plus for robo advisors is that they are cheaper and require a lower asset base than traditional wealth managers. They only charge between 0 and 40 bp while human advisors typically charge more than three times this fee; 1%-2% (Fish et. al, 2018). Furthermore, robo advisors require either no or only a small asset base between $0 (Betterment) and $50,000 (Vanguard), which makes their service accessible to a broader range of people which is a great way to democratize finance (Fish et al., 2018). In contrast to that, human advisors start to provide their services from $100,000 onwards. That is due to the fact that they have higher costs than robo advisor. While a human advisor can handle up to 100 clients with the help of an assistant, a robo advisor, like Betterment, has over 300,000 clients and only 200 employees (Fish et al., 2018).

Another reason for the lower costs is that one of the robo advisor´s main goal is to minimize the customer´s costs. They only use low price ETFs with a small bid-ask spread to decrease trading costs (Betterment[2]) and do not waste money on trying to predict or time the market while human advisors recommend primarily single stock or pricy products like mutual funds to earn money for their bank.

And this brings us to the next argument in favor of robos. As robo advisors only have a single algorithm for all of their customers, it is easier to monitor if they meet their regulations and valuate their strategy, while a human advisor that works for, let us say, a bank is only one person out of thousands (Fish et al., 2018). They need to follow a certain prescribed strategy, but each advisor will give a slightly different advice. Thus, it is nearly impossible to control each of them in detail, which makes it more likely that they will sell their customers higher priced products or follow a strategy which gains money for the bank in first place.

The last major advantage I want to mention here is the convenience of robo advisors. While human advisors have limited working hours a week, robo advisors are available 24/7 (Fish et al., 2018). If customers want to adjust their level of risk on a Sunday afternoon, they simply log into their account on the advisor´s website. Or, if they need to know some tax information, like their current tax impact, Betterment will provide this information on demand, to ensure that no customer is surprised by rising taxes (Betterment[15]).

After dealing with the positive aspects of robo advisors, it is only fair to mention their negative sides as well, to show that in some cases traditional advisors are still ahead.

I have previously stated that there might be a conflict of interest for human advisors because they sell affiliate products (Fish et al., 2018). Also, robo advisors are not free of this accusation. For example, Schwab Intelligent Portfolios consist primarily of their own products (which is the reason why they do not charge any managing fees) and most stock ETFs Betterment uses were issued by Vanguard. However, as Vanguard is the second largest issuer of ETFs after Black Rock (Forbes, 2018) this might not be a case of "affiliated products".

Another con is that there is no human contact. While this might be great in terms of low fees, some will miss a friendly hand guiding them through potential market downturns. Especially as no robo advisor has ever proven themselves in market crashes. It remains completely unexplored as to how they will react, since Betterment, the first of its kind, launched their advisor after the last crash in 2008. In this regard human advisors, in particular the more experienced, might have an advantage, because they have already experienced such events and have developed a sense of when it is time to switch to less risky assets like US Treasury Bonds.

In addition to that, human advisors might have an advantage in terms of determining a client´s level of risk and needs better than a standard questionnaire containing only four to twelve questions (FINRA, 2016).

It can be seen that both kinds of financial advisors have their advantages and disadvantages. Would it not be smart to combine human skills and technology to emphasize only the positive aspects? This is exactly what the next subsection will be all about.

2.4. The Future of Robo Advisors

In 2017, Betterment opened up a call center to add another level to their service. Customers who are willing to pay additional 0.15% and have a minimum balance of $100.000 are given unlimited access to a “Certified Financial Planner” via phone (Fish et al., 2018). Furthermore, they implemented a messaging service the same year, where customers get their requests answered within a business day (Eule, 2017).

All in all, the former stand-alone robo advisors have started to adopt more and more of the aspects of traditional financial advisors, while the latter have started to integrate more technology. They are both approaching each other and will, in Bo Lu´s opinion (co-founder of FutureAdvisor), end up in the middle, which is good news especially for the customers: high-tech funds with the benefits of traditional advisors at a lower price (Eule, 2017). Fish et al. (2018) call this development a human-robo-hybrid.

Other possible developments could obviously be stronger product diversification. Examples of that are companies like Ellevest, which is particularly focused on women, or Earthfolio which stands for socially responsible investing (Fish et al., 2018).

The last trend I want to touch on is that some traditional advisor firms acquire robo advisors to expand their distribution channels. For example, BlackRock has bought the above mentioned robo company FutureAdvisor in 2015 (Reuters, 2015) to sell their ETFs (Fish et al., 2018).

2.5. Portfolio Strategies of Betterment, Wealthfront and Scalable Capital in Comparison

In this subsection I want to compare the portfolio strategies of two of the leading stand-alone robo advisors, Betterment and Wealthfront, with a European one, Scalable Capital, that have a slightly different approach. Additionally, I will explain, at the end, why exactly I have chosen Betterment as subject of this paper.

Betterment and Wealthfront have a pretty similar strategy with only a few differences (Betterment[3] and Wealthfront[1]). Both use a long only portfolio. As short selling is indeed important for an efficient market (Saffi and Sigurdsson, 2011), but bears some serious risks I do not want to discuss in this paper (Kumar, 2011, pp. 209, 2015). Furthermore, all three companies use nothing but ETFs as an investment vehicle to reach sufficient diversification. They are invested in between 12 to 15 ETFs to cover all relevant markets around the world, split into two main asset classes - bonds/fixed income and stocks/equities. The funds Betterment invests in will be described later in this paper. An important point to be aware of at this stage is that Betterment is the only one of the three that strictly precludes commodities and real estate, and even dedicated a whole article to explain why they do so (Betterment[4]).

They use a variation of the Modern Portfolio Theory (MPT) by Harry Markowitz (1952) where they try to maximize the aggregated return of the portfolio, given a certain level of risk, by altering the weights of the certain assets. MPT requires two inputs: risk and expected return; and this is where the companies do something different. Scalable Capital claims that the assumption Markowitz made does not reflect the true nature of the asset´s behavior. MPT assumes that returns are normally distributed and that the covariances are the same for down- and upturns in the market, which is not the case (Scalable Capital,2016). Instead of variance as a proxy for risk, they use key figures like value at risk, expected shortfall and maximum drawdown. As input for their optimizing algorithm they use the 95% value at risk. By comparison, Betterment and Wealthfront do assume the normal distribution of returns, but see a different and more serious problem in this model. The mean variance optimizer is quite sensitive to its input, which means that one will get unbalanced weights which differ drastically when inputs are altered only slightly (Black and Litterman, 1991). What I mean by that, I will explain in more detail in chapter four.

To make the inputs more robust Betterment and Wealthfront use the so called Black-Litterman Model (Black and Litterman, 1991/1992). They do not use the average return as proxy for future expected returns since the past is not a good estimation of the future, in most cases. Instead they assumed the global market to be in an equilibrium in which the global market portfolio, composed according to its market capitalization, happens to be the optimal portfolio (Litterman, 2003, cited in Idzorek, 2007). Taking the weights as given you see the returns which would be optimal in this situation can be recovered. With the help of the Bayesian approach the weights of the portfolio can be tilted towards certain expectations. Betterment calculates their expectations with the help of the Fama-French three-factor model (Fama and French, 1992). Wealthfront, on the contrary, uses a self-developed model called Wealthfront Capital Market Model which is based on Fama and French (Wealthfront[1]).

The second input, the risk, is typically passed in form of the sample covariance matrix. The problem here is that this matrix bears lots of estimation error, which could lead to skewed results, especially when estimating for a large number of assets. Luckily, Ledoit and Wolf (2003b) developed a procedure to offset this error by “shrinking” the matrix.

To receive the desired level of risk they simply adjust the mixture of stocks and bonds in a portfolio. The percentage of stocks declines over the time, the closer you come to your target date, to ensure reaching the goal with a high probability (Betterment[5]). This is, by the way, a popular strategy of target-date funds, as well.

The robo advisors get the information they need to determine an adequate level of risk from a standardized questionnaire, since it is not planned that clients will be interviewed about their goals by a human advisor.

In addition to a well-diversified and optimized portfolio suited to the client´s needs, Betterment and Wealthfront also provide automatic, threshold-based rebalancing (Betterment[6]) and asset location, which is a strategy to reduce tax drag by locating assets in certain bank accounts (Betterment[1]).

To differentiate themselves more from target-date funds, Betterment provides four more investment goals in addition to saving for retirement. The other four goals are: Retirement Income, Safety Net, Major Purchase and General Investing (Betterment[3]). The differences between those goals are the structure of cashflows and the time horizon. When a customer set the goal “General Investing”, for example, she does not save for something particular, so the money will not be needed at a specific date. Thus, your asset allocation will be more aggressive. Furthermore, it will be expected, that withdrawals are taken, from time to time and deposits are made on a regular basis (Betterment[7]).

If a client wishes to fight the inflation of her extra cash, but doesn´t want to take extra risk, the Two-way Sweep service might be perfect (Betterment[8]). Betterment cooperation partner Quovo (Betterment[9]) estimates the money that will be needed for the next few weeks and transfers the surplus from the checking account to the Betterment Smart Saver account. This account consists of nothing but U.S. Short-Term Treasury Bonds and U.S. Short-Term Investment-Grade Bonds, which are ultra-low risk and used, as mentioned above, to tackle the annual inflation of approximately 2% (Betterment[10]).

Some other services include socially responsible investing (Betterment[11]), which only invests in companies or ETFs with a good ESG (Environmental, Social and Governance) score, and Charitable Giving. The latter donates unrealized capital to charity, without selling the asset first. This means you pay less tax and more money is able to go to charity (Betterment[12]).

Another service to reduce tax drag is Tax Loss Harvesting, which I only want to mention for the sake of completeness. The idea is to realize losses by accident to reduce or defer taxes. For more detail, Betterment has written a whitepaper regarding this topic (Betterment[13]).

Lastly, I have to answer the question, why have I chosen Betterment for the following analysis of their strategy? First of all, it was important to me to choose a company that's main business is to build those robo advisors. I did not want a company like Vanguard or Charles Schwab where this is only a "side business", even though I am aware that they have much more assets under management than stand-alone robo advisors. The second reason for choosing Betterment was that they were the first; the original if you want. They had the initial idea back in 2008 and developed their business steadily. They even have an entire archive of numerous articles and papers regarding their portfolio strategy and overall business, which makes them by far the most transparent leading robo advisor.

All in all, Betterment was the pioneer, has the most round down portfolio strategy, in my opinion, and attaches a lot of importance to research, which makes this company my subject of choice.

3. Asset Allocation: Selecting the Assets

In the following two chapters I will explain how Betterment composes their portfolio, starting with the selection of assets. This chapter will be followed by the process of optimization and, at the end, I will discuss the methodology of choosing the appropriate level of risk for a client. This whole procedure is called asset allocation, which, in short, describes the sensible distribution of capital to different assets so as to reduce risk and maximize returns (Garz et al., 2012, p. 15).

3.1. Importance of Diversification

I would like to start this subsection with a question: Why do risk-averse investors invest in more than just one asset? Bernoulli wrote in 1738 that "it is advisable to divide goods which are exposed to some small danger into several portions rather than risk them all together" (Bernoulli, 1738, cited in Rubinstein, 2002). If an investor´s capital is spread over several different assets, this will reduce the risk of a portfolio. This is called diversification because if she only invests in one company and that company´s stock price goes to zero, the portfolio would lose 100% of value. However, if there are, let us say, 30 assets in the portfolio and again only one company goes bankrupt, the portfolio only will lose a small fraction of worth, as the companies are, in most cases, not perfectly correlated (Garz et al., 2012, p. 42). In other words, the success of the other companies in this portfolio does not only depend on this one company.

Let us say, there are n assets in an equally weighted portfolio where all assets have the same pairwise correlation c and also the same volatility s. After some transformations we will get the following equation, describing the portfolio´s volatility (Garz et al., 2012, p. 43):

$$\sigma_{PF} = \sqrt{c + (s^2 - c) * \frac{1}{n}} \tag{3.1}$$

It can be seen that the volatility decreases in the number of assets, therefore, the more assets, the better. However, for $n \to \infty$ the risk will not go down to zero. Instead, it approaches a certain level of risk which is called systematic risk and is symbolized by β (Garz et al., 2012, p. 43). The risk, which can be reduced by diversification, is called unsystematic risk. To ensure sufficient diversification, the portfolio should include at least 30 assets with a correlation of below one. To do so, an investor should include several asset classes across countries and hold bonds as well as stocks. However, since the marginal diversification effects are declining, it might be not optimal to add unlimited assets to the portfolio as it will become more difficult to handle.

3.2. The Role of ETFs

Exchange Traded Funds can be seen as portfolios itself, as they passively replicate indexes that contain a great number of individual securities, weighted by their market capitalization. This means that you can obtain sufficient diversification by only buying a small number of ETFs from different sectors. Thus, Betterment is able to hold, more or less, the entire market portfolio by currently only being invested in eleven ETFs plus optionally municipal bonds (Betterment[2]). This makes the optimization and rebalancing process, which I will describe in the following chapters, way easier. Additionally, the fact that they are passively managed means that they are available for lower fees, as there are no extra costs for trying to beat the benchmark.

Another advantage of ETFs is that they are traded just like stocks. No matter if they replicate equities, fixed income, commodities, natural resources or real estate. They are all treated the same. The two sources of returns are capital gains and dividends, also for bonds, where those dividends replace the interest (Betterment[1]).

The fact that they are traded like stocks also means that an investor is able to buy and sell them during open market hours for a steadily adjusting price. In comparison, a mutual fund only determines their net asset value, the price for which the fund is offered, once a day (Betterment[2]).

ETFs have also an advantage in terms of tax. Some qualified dividends that are distributed to the investors are considered to be long-term capital and taxed at a lower, preferential rate (Betterment[1]). To be classified as qualified dividend the Jobs and Tax Relief Reconciliation Act of 2003 requires that those dividends have to be paid by a domestic company and that the holding period must be a minimum of 61 days of the 120 days surrounding the payout date. Since actively managed funds have a higher turnover, it is more likely that they do not meet the required holding period, which increases the tax liabilities of their investors (Betterment[2]).

Let us see which criteria Betterment uses to select their ETFs. They developed a scoring method called TACO, which is short for "total annual cost of ownership". TACO is the sum of the cost to trade and of the cost to hold (Betterment[2]).

The cost to trade includes the bid-ask spread and the volume of the fund. To reduce the costs for their clients, Betterment only selects ETFs that are heavily traded and, thus, have a small bid-ask spread or lower market premiums. The other part, the volume, is a measure of how many shares are traded each day. The volume should be high enough to ensure that Betterment´s trades do not affect the prices of the fund. To minimize the market impact, Betterment calculates two key figures (Betterment[2]):

$$RS_{AUM} = \frac{AUM_{Betterment}}{AUM_{ETF}} \qquad\qquad RS_{Vol} = \frac{Vol_{Betterment}}{Vol_{ETF}} \tag{3.2}$$

It is important to mention that even if only one criteria does not meet the standards of Betterment, the ETF is disqualified (Betterment[2]).

The cost to trade contains the expense ratio, so the fees an investor has to pay for holding an ETF one year and the tracking error. The problem is that there is a tradeoff between those two criteria. The better the fund replicates its benchmark, the more of the benchmark assets it needs to hold, which is related to higher transaction costs and leads to higher fees for the fund. The trick is to find a compromise between costs and tracking error (Betterment[2]). The ETFs Betterment chooses have an average portfolio expense ratio of between 0.07% and 0.15%. See Table 4 in Appendix for the fund´s expense ratios.

3.3. Betterment´s ETFs

In Table 4 in the Appendix you can see the ETFs Betterment currently uses. You may notice that the main issuer of ETFs is Vanguard (ETFs beginning with “V”).

Furthermore, it can be seen, that Betterment has invested in almost the total market, with the exception of assets for which costs or lack of data outweigh their benefits of being added to the portfolio. Those asset classes are natural resources, private equity, commodities and real estate investment trusts (REITs). The latter is only excluded as an own asset class, since companies that are in the real estate industry have already been included in other funds Betterment holds (Betterment[4]).

As I mentioned in the previous part, Betterment is the only one of the three robo advisors I have compared that does not use commodities. What, then, are the reasons for this decision? Betterment even wrote an article about it (Betterment[4]).

The first and most important reason is that commodities, like copper and oil, have no intrinsic value. They are just inputs to generate output. So, their price only depends on what people are willing to pay (Betterment[4]). In other words: supply and demand. In contrast, financial assets grow wealth by developing technology and their efficiency. Indeed, supply and demand can be responsible for fluctuation in returns, as well, but in the long-term they revert to their intrinsic value.

Thus, financial assets are rather a long-term bet, while significant commodity returns only can be made by predicting the short-term demand (Betterment[4]).

If you compare the long-term performance of commodities (01.01.1971 – 05.01.2019) with the other asset classes in Table 2 and, especially, the Wilshire 5000 Total Market Full Cap Index over the same duration as the commodities, it can be seen that the latter’s returns are rather comparable to bond returns. Additionally, the long-term volatility is also higher than those of the bonds with the exception of emerging market bonds.

Furthermore, if substitutes for commodities are found, for example the development of a new efficient source of energy generation that makes oil redundant, the prices for those

ETFs will decrease promptly. Especially because commodity ETFs consist, to a large extent, of oil (Betterment[4]).

Asset	Annualized Mean	Volatility	Sharpe Ratio
VTI	12.42 %	13.05 %	0.95
VTV	11.44 %	12.55 %	0.91
VOE	10.52 %	13.03 %	0.81
VBR	10.16 %	14.14 %	0.72
VEA	4.94 %	13.86 %	0.36
VWO	5.19 %	18.15 %	0.29
VTIP	0.77 %	1.64 %	0.47
SHV	0.65 %	0.21 %	3.08
NEAR	1.33 %	0.59 %	2.26
BNDX	4.06 %	2.66 %	1.53
EMB	5.08 %	6.02 %	0.84
Wilshire 5000 Total Market Full Cap Index	11.23 %	15.46 %	0.73
PPIACO (Commodities)	3.55 %	3.27 %	1.06

Table 2: Overview of the Betterment ETFs and their particular performance characteristics. In addition to that, the Wilshire 5000 Total Market Full Cap Index which depicts all companies with headquarters in the US and a commodities ETF is added to give a reference point of how well the ETFs that Betterment has chosen perform in comparison. The key figures for the last two rows were calculated over the period from 01-01-1971 to 05-01-2019 and the rest from 07-31-2014 to 04-29-2019. Data: Yahoo! Finance https://finance.yahoo.com and FRED, Federal Reserve Bank of St. Louis https://fred.stlouisfed.org (as of 05-01-2019).

All in all, the lower returns, higher level of volatility and additional risk factors, namely the uncertainty in future demand, makes commodities less attractive which leads to Betterment´s decision to not include them in their portfolio.

4. Asset Allocation: Finding the optimal Asset Weights

After choosing the assets, in the form of ETFs, that I want to have in my portfolio, I have to figure out how much weight I want to give to each asset. The most popular way to do this is the mean variance optimization Harry Markowitz described in his paper “Portfolio Selection” (1952) for which he was later awarded a Nobel Prize.

The idea is to identify optimal portfolios. A portfolio is considered to be optimal if there is no other portfolio with the same expected return but lower risk, or vice versa, if there is no portfolio with the same level of risk but higher expected return. Those optimal portfolios form the so-called “efficient frontier” (Markowitz, 1952).

To determine such an optimal portfolio the following optimization problem has to be solved:

$$\max_{w} \mu^* = w\mu \tag{3.3}$$

$$s.t. \qquad \sigma_{PF}^2 = w\Sigma w'$$

$$1'w = 1$$

The goal is to maximize the expected portfolio return μ^* for a given level of risk σ_{PF}^2. Furthermore, the weights of the particular assets have to add up to 1. In the event that an investor desires a long only portfolio, another restriction can be added, which prevents negative asset weights:

$$w \geq 0 \tag{3.4}$$

Apart from the desired level of risk, which is given by the investor, there are only two required inputs, the $n \times 1$ vector of expected returns μ and the $n \times n$ variance-covariance matrix Σ (Markowitz, 1952).

Nonetheless, the model in this form is not widely used in practice, as it bears some significant weaknesses (Idzorek, 2007). One problem is that the optimization problem is quite sensitive to its inputs, which makes it difficult for investors to incorporate their subjective expectations. As Best and Grauer (1991) show, even small changes in expected returns can lead to portfolio weights which are not reasonable anymore. The portfolio can be forced into single, highly concentrated positions, while the remaining assets are no longer included. Using historical returns as proxy for expected returns can lead to unreasonable portfolio weights as well, with large long and short positions. Apart from that, historical returns might not be appropriate to estimate expected returns (Betterment[3]).

To avoid the problem of sensitiveness and to ensure a reasonable, balanced portfolio that includes individual views, Black and Litterman provide a model which was introduced in 1991 and extended in a following paper (1992).

However, before dealing with the Black-Litterman model, another weakness has to be eliminated, the variance-covariance matrix. This matrix potentially bears some significant estimation error, especially if a large number of assets is included (Ledoit and Wolf, 2003b). In the following I will describe a method by Ledoit and Wolf (2003a) to estimate a variance-covariance matrix that minimizes estimation error and leads to more robust results.

4.1. Shrinking the Covariance Matrix

Beside expected returns, the covariance matrix is the second important input required for a mean variance optimization. The most intuitive way to estimate this matrix would be to calculate the so-called sample covariance matrix which is based on a sample of historical returns:

$$S = \frac{1}{T}(X - m)'(X - m) \tag{3.5}$$

T is the number of observed returns, X is a $T \times N$ matrix of returns where N symbolizes the number of assets and m is a $1 \times N$ vector of mean returns. One of the sample covariance´s benefits is that it provides maximum likelihood under normality. This means, S converges towards the true covariance matrix Σ when the number of observations goes to infinity (Ledoit and Wolf, 2003a). Unfortunately, according to Ledoit and Wolf (2003a), in most cases there is only a limited number (maybe about 10 years of monthly returns), while there are thousands of stocks from which an investor wants to pick the best. Thus, in most cases it is $N > T$ which means that the matrix is always singular and therefore not invertible. So, if N is not larger than $T - 1$ a rank deficit occurs leading to "small sample problems" due to lack of data (Ledoit and Wolf, 2003a). Those estimation errors lead to extreme values within the matrix which distort a mean variance optimizer that tends to "place its biggest bets on those coefficients which are the most extremely unreliable" (Ledoit and Wolf, 2003b). According to Michaud (1989) this "error maximization" happens because a mean variance optimizer overweighs assets with the smallest variances or covariances and highest returns, which happens to be those with the highest amount of estimation error.

A solution to this problem is to impose structure, because a structured estimator bears less estimation error than the sample covariance matrix. On the other hand, this estimator is usually biased due to its misspecification, as the true structure is unknown a priori (Ledoit and Wolf, 2003a).

Structure is imposed by only allowing a few free parameters. To do so, a K-factor model can be used with $1 < K < N$. The number of factors indicates the level of structure, while the latter falls in K. The problem is that nobody can tell in advance how many factors would perform best, without considering out of sample returns. Thus, the model selection is ad hoc and there is no solution that fits all situations (Connor and Korajczyk, 1995, p. 127).

To avoid this kind of “guessing”, Wolf and Ledoit (2003a) have a different approach. They use Sharpe´s (1963) single-factor model and combine it with the sample covariance matrix. As mentioned above, both have their pros and cons: The structured estimator, also referred to as shrinkage target, has little estimation error but a lot of bias and the sample covariance has little bias but a lot of estimation error. Thus, a tradeoff is apparent and the question is in which ratio both should be combined. This process is called shrinkage and was first introduced by Stein (1956). Shrinkage relativizes the extreme values in the sample covariance matrix and pulls them towards the middle to decrease estimation error (Ledoit and Wolf 2003b).

However, neither I nor Betterment use Sharpe´s single-factor model for shrinkage, since Ledoit and Wolf (2003b) developed another model that provides similar results, but is easier to apply (Betterment[3]). Their new approach is to assume constant correlation.

Let us, though, start from the beginning. As already indicated, the shrunken matrix is nothing other than a weighted average of the sample covariance matrix and a shrinkage target that has the following form:

$$\hat{\Sigma}_{Shrink} = \hat{\delta}^{*}F + \left(1 - \hat{\delta}^{*}\right)S \tag{3.6}$$

S is the sample covariance matrix, F is referred to as the highly structured estimator and $\hat{\delta}^{*}$ is the optimal weight put on the structured estimator, or, in short, the optimal estimated shrinking constant. This constant adopts a value between 0 and 1 (Ledoit and Wolf, 2003b).

In the following, I will briefly explain how to build the remaining two inputs, $\hat{\delta}^{*}$ and F.

4.1.1. The Shrinkage Target

Ledoit and Wolf (2003b) assume constant correlation, which means that the covariances have to be altered in a way that, in the end, all covariances lead to the same correlation. The first step is to calculate the correlation for each asset pair.

$$r_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}} \tag{3.7}$$

The correlation coefficient of asset i and j is denoted by r_{ij} and s_{ij} is the according covariance from the sample covariance matrix. The second step is to calculate the average correlation $\bar{r}$. This is done by summing all correlation coefficients that lie above the diagonal and dividing it by its amount.

$$\bar{r} = \frac{2}{(N-1)N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} r_{ij} \tag{3.8}$$

The final step is to create a matrix F that has the shape $N \times N$, like the sample covariance matrix, and contains covariances that lead to the same correlation $\bar{r}$. The particular values for F can be calculated in the following way:

$$f_{ij} = \bar{r}\sqrt{s_{ii}s_{jj}} \tag{3.9}$$

This equation is basically equation 3.7 rearranged and the average correlation substituted for r_{ij}.

4.1.2. The Shrinkage Intensity

The problem is that, so far, all shrinkage estimators have broken down whenever the number of assets has exceeded the number of observations. To determine those shrinkage estimators a loss function is used which involves the inverse of the covariance matrix. Unfortunately, a matrix is not invertible in those cases. Ledoit and Wolf (2003b) take a different approach, without involving an inverse. The idea is to minimize the difference between the combination of the shrinkage target and sample covariance matrix, namely the shrinkage estimator, and the true covariance matrix based on the Frobenius norm:

$$L(\delta) = \|\delta F + (1-\delta)S - \Sigma\|^2 \tag{3.10}$$

To do so, the optimal value for δ must be found. Ledoit and Wolf (2003a) found that this optimal value δ^* asymptotically behaves like a constant, when the number of observations goes to infinity. To determine this constant, the following equation needs to be satisfied:

$$\kappa = \frac{\pi - \rho}{\gamma} \tag{3.11}$$

π is the sum of asymptotic variances and ρ the sum of asymptotic covariances. The difference between both is divided by the sum of squared differences between the constant correlations covariances and the historic covariances derived in the last section.

The values on the diagonal will be zero, since the variances of the shrinkage target are equal to the variances of the sample covariances matrix, as only the covariances have been adjusted. To find more detail on how to derive consistent values to estimate κ, see Appendix B of Ledoit and Wolf (2003b).

As soon as $\hat{\kappa}$ is estimated, the optimal shrinkage intensity is $\hat{\kappa}$ divided by the number of observations, as long as this value is between zero and one:

$$\hat{\delta}^* = max\left\{0, min\left\{\frac{\hat{\kappa}}{T}, 1\right\}\right\} \tag{3.12}$$

Now that all required inputs have been derived, the shrunken covariance matrix can be calculated by simply inserting them into equation 3.6.

4.2. The Black Litterman Model

The Black and Litterman Model merges the ideas of mean variance optimization (Markowitz, 1952) with the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). The idea is to calculate a new set of expected returns which will basically be a sophisticated weighted average of equilibrium returns and an investor´s individual views.

4.2.1. Equilibrium Returns

The implication of CAPM is that prices adjust until all investors are comfortable holding assets at the particular price and until the relative share of an asset´s market capitalization on the overall global market capitalization approximates the optimal portfolio weights (Black and Litterman, 1991). Betterment´s global market portfolio consists of the eleven ETFs that were introduced in the previous section.

As the optimal portfolio weights are known, the implied excess equilibrium returns Π can be recovered with the help of reverse optimization (Sharpe, 1974) to receive a neutral starting point that can later be combined with the investor´s unique views.

The unconstrained optimization problem is: $\max_{w} w'\mu - \lambda w'\Sigma w/2$. After differentiating, substituting Π for μ and transforming the equation according to the latter, one will receive the following solution (Idzorek, 2007):

$$\Pi = \lambda \Sigma w_{mkt} \tag{3.13}$$

Π is a $N \times 1$ vector of implied excess equilibrium returns, Σ again a $N \times N$ variance-covariance matrix and w_{mkt} is a $N \times 1$ column vector of market capitalization weights. The last, and to this point unknown, parameter λ is the so-called risk-aversion coefficient. It indicates how much extra risk an investor is willing to take in exchange for an additional unit of return and, furthermore, it is used to scale the returns, which are to be calculated. Grinold and Kahn (1999, cited in Idzorek, 2007) estimate this implied risk-aversion coefficient by dividing the expected excess return of the market by the market´s variance:

$$\lambda = \frac{E(r) - r_f}{\sigma^2} \tag{3.14}$$

The implied excess equilibrium returns are basically those returns that the assets need to provide to make the market capitalization portfolio the optimal portfolio (Idzorek, 2007). As Sharpe (1964) has shown, the optimal portfolio, among all portfolios that lie on the efficient frontier, is the one that provides the most expected excess return per taken risk. In other words, it is the combination of assets that has the maximal Sharpe Ratio. This particular portfolio is often referred to as a tangency or market portfolio. It is exactly this portfoliothat equation 3.13 is about to calculate.

In literature, the implied equilibrium return vector, which is determined by reverse optimization, is often referred to as CAPM returns that can be easily confused with the CAPM returns which are calculated with betas that are based on a regression of asset returns on market returns (Sharpe, 1964). Those returns are only equal, if the market capitalization weights were used to calculate market returns, which are required for the regression (Idzorek, 2007).

4.2.2. Combining Equilibrium Returns with an Investor´s views

After deriving the neutral starting point, it is time to describe how an investor can express her personal views on how single assets will perform in the future and how to combine them with equilibrium returns to calculate a new set of combined returns $E[R]$, which still leads to a perfectly balanced portfolio.

The Black-Litterman formula (Black and Litterman, 1992) that uses a Bayesian approach is

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \tag{3.15}$$

where τ is a scalar, Σ is the variance-covariance matrix of excess returns ($N \times N$), P a matrix that determines all assets that are involved in views ($K \times N$ matrix, where K is the number of views), Ω is a diagonal covariance matrix ($K \times K$) that identifies the uncertainty in views, Π the $N \times N$ vector of implied equilibrium returns and Q the $K \times 1$ vector of views (Idzorek, 2007).

In the following, I will briefly describe how to derive those inputs and what kinds of views an investor can express.

4.2.3. Expressing Views

In their paper, Black and Litterman (1991) present three different ways to create a view. To give an idea of what those views can potentially look like, I will give an example for each type.

- View 1: U.S. Total Stock Market will yield an absolute excess return of 7% the following year.
- View 2: U.S. Value Stocks – Large Cap will outperform International Developed Market Stocks by 5.7%.
- View 3: U.S. Value Stocks – Small Cap and International Emerging Market Stocks will outperform U.S. Value Stocks – Large Cap and U.S. Value Stocks – Mid Cap by 3.5%.

The first view is an absolute view. It says that the U.S. Total Stock Market will yield an absolute excess return of 7% and, thus, the investor expects it to outperform the asset´s equilibrium returns by 0.35 basis points (see Table 5 in Appendix). As a result, the Black and Litterman formula will increase the weight in U.S. Total Stock Market. Vice versa, if an investor has the view that the asset will underperform its own equilibrium return, the formula will decrease the weight (Idzorek, 2007).

The second view is an example of relative views. This type of view is easier to use in practice, as investors have feelings about how much an asset will outperform another more often than determining specific expected returns (Idzorek, 2007). When I implement Betterment´s portfolio strategy, I will nonetheless use the absolute view and express views on each asset.

If the difference between the implied equilibrium returns (see Table 5 in Appendix) of the assets in View 2 (5.98%) is larger than the view expects it to be (5.71%) this means that either the International Developed Market Stocks ETF performs better than equilibrium indicates, or the U.S. Value Stocks – Large CapETF performs worse or one asset performs less badly than another relative to their equilibrium. Either way, the model tilts the weights towards the better performing asset in relation to equilibrium. In this case, the weight of International Developed Market Stocks will increase (Idzorek, 2007).

The third view is again a relative view, but with the difference that there are multiple assets involved. The number of outperforming assets does not have to be equal to the number of underperforming assets, as the example shows. Another example could be that there are three assets that outperform two other assets by a certain number of basis points. To see which asset weights will increase, and which will decrease, two portfolios are formed. The first portfolio contains the assets that outperform and the second portfolio those that underperform (Idzorek, 2007). In each portfolio, the relative weights were determined by dividing the market capitalization of a particular asset by the total sum of the mini-portfolio´s market capitalization. Afterwards, the weighted average returns for each mini-portfolio can be calculated. In the case that the difference between both portfolio-returns is larger than the view indicates, the investor expects the nominal outperforming assets to perform worse and thus reduces their weights (Idzorek, 2007).

Furthermore, there does not have to be a view on each asset. If an investor only has an opinion on half of the assets´ future returns, that is perfectly fine. This is another advantage of the Black-Litterman model; assets without views maintain their market capitalization weights (Idzorek, 2007).

4.2.4. Preparing the Inputs

After explaining how views can be created, it is time to show how to transform them into matrixes that can be passed into the Black-Litterman formula and how to build the remaining required inputs.

The first input is the $K \times 1$ vector Q (K is the number of views), or in our case the 3×1 vector that includes the views, e.g. how much an asset will outperform another. As a view has some kind of uncertainty, a vector of independent, normally-distributed error terms with a mean of zero ($\varepsilon \sim n.i.d.(0, \sigma^2)$, often referred to as "white noise") will be added:

$$\text{General case: } Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \qquad \text{Our case: } Q + \varepsilon = \begin{bmatrix} 7 \\ 5.7 \\ 3.5 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \tag{3.16}$$

However, the error terms (ε) will not enter the equation directly (Idzorek, 2007). Instead, the uncertainty of views will be expressed by the matrix Ω ($K \times K$) which has the error term´s variance (ω) on its diagonal. The covariances, i.e. the off-diagonal positions, are set to zero, as those error term variances are independently among each other :

General case:
$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix} \tag{3.17}$$

Finding values for ω and the scalar (τ) are the most abstract and difficult part of this model (Idzorek, 2007). Unfortunately, they are quite important inputs as the ratio of them determines how much an asset´s weight deviates from its market capitalization weight, in the case it is subject to a view. As already mentioned above, if an asset is not involved in a view, its weight does not change at all. All in all, ω should represent the confidence in a certain view. If an investor is 100% sure, the variance of error terms would be 0. Nonetheless, there is still a random error (ε) but without any deviation.

He and Litterman (1999) think that the magnitude of departure ($^{\omega}/_{\tau}$) should be equal to the variance of a respective view portfolio ($p_k \Sigma p_k'$), where p_k is the k-th row of P-vector. After rearranging the equation, one will get the following expression:

$$\omega = (p_k \Sigma p_k') * \tau \tag{3.18}$$

This estimation for ω is really convenient, because it makes τ irrelevant. Now, the latter is only responsible for the size of the values in Ω, but has no influence on the new combined vector of returns ($E[R]$) anymore. To receive somehow reasonable values for ω, even though they do not really matter for the final result, there are several assumptions for τ in literature. Lee (2000, p. 174) suggests that the scalar must be close to zero. Conversely, Satchell and Scowcroft (2000) choose 1 as appropriate value and Blamont and Firoozy (2003, cited in Idzorek, 2007) consider $\tau\Sigma$ to be the standard error of Π, which means they set τ equal to 1 deviated by the number of observations. All in all, all values for τ, mentioned in literature, are close to zero. For my calculations I will simply assume $\tau = 1$ since I am rather interested in the eventual results for $E[R]$. The matrix Ω is now the following:

$$\Omega = \begin{bmatrix} (p_1 \Sigma p_1') * \tau & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & (p_k \Sigma p_k') * \tau \end{bmatrix} \tag{3.19}$$

The last input that has to be built is the $K \times N$ matrix P that identifies the assets that are contained in a view. Each row represents one view, while nominal outperforming assets have a positive sign and underperforming assets a negative. The three views example from above is depicted again. The position of assets is chosen according to Table 4 in Appendix.

$$\text{Our case:} \qquad P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -0.85 & -0.15 & 0.28 & 0 & 0.72 \end{bmatrix} \qquad (3.20)$$

It can be seen that the rows of the relative views (View 2 and View 3) sum to zero, while the row of the first view does not. This leads to the fact that after calculating the new combined returns and calculating the optimal weights by rearranging equation 3.13, the summed weights do not result in 100%, anymore (Idzorek, 2007). If the absolute view implies that the asset outperforms, the summed weights would be >100% and vice versa. In the case that only relative views are involved, the portfolio will still sum to 100% since the increasing weight of the outperforming asset will be relativized by decreasing the underperforming asset´s weight (Idzorek, 2007). If the weight of U.S. Treasury Bonds increases by 20 bp, due to their relative outperformance, the weight of U.S. Value Stocks decreases by 20 bp. Thus, the net change of the view portfolio is zero.

Investors who want to have a 100% portfolio but also want to express absolute views, have to use a quadratic optimizer with corresponding restrictions (Idzorek, 2007) or simply divide each asset weight by the sum of all asset weights.

The depiction of matrix P above shows that in View 3 the assets are not represented by 1 with a sign, corresponding to out- or underperforming, anymore. In this case, there are two assets that nominally outperform. They can respectively be represented by one divided by the number of outperforming assets. The same can be said for the other mini portfolio. This method is based on Satchell and Scowcroft (2000).

But there is another way that uses a market capitalization scheme (Idzorek, 2007) and can be seen in 3.20. Each mini portfolio, the long and the short, has to result in 1. The idea is to weight the assets within each mini portfolio by measuring them according to their market capitalization. For example, if the two outperforming assets have a market capitalization of 28 and 72 they are represented in P by +0.28 and +0.72. If the market capitalizations of the assets in the short portfolio are 85 and 15, their corresponding values would be -0.85 and -0.15.

Now that all inputs have been constructed, they have to simply be inserted into equation 3.15. The result is a $N \times 1$ vector of new returns, which are a weighted average of implied equilibrium returns and views. To extract the new weights for the tilted portfolio, the equation can be rearranged according to w (Idzorek, 2007):

$$w = (\lambda\Sigma)^{-1}\Pi \tag{3.21}$$

Idzorek (2007) also allows that users can determine their level of certainty in views themselves, by intuitively choosing a confidence level between 0% and 100%. I do not want to discuss this possibility in this paper, because I want to keep the algorithm as objective as possible and exclude each parameter that contains any kind of subjective opinions. For this purpose, Betterment derives their views by using the Fama-French three factor model (Fama French, 1992). Unfortunately, this process is not sufficiently explained in their publications, which is why I do not go into more detail on this topic.

4.3. Adjusting the Portfolio Composition over Time

NBIM, short for Norges Bank Investment Management (2012), which is the Norwegian central bank, suggest that short-term investors should hold different portfolios than long-term investors. If there are many years left until the money is needed, an investor should hold a riskier portfolio because more volatility goes with higher growth in the long term. To adjust the risk, Betterment mixes the stock portfolio we derived in the previous sections with a fixed income or bond portfolio which typically bears a lower level of risk (Betterment[5]). In the beginning, the equity share is quite high and diminishes the more the target date approaches to reach the financial goal with a high probability. This strategy of managing the downside or "uncertainty optimization" is widely used by target-date funds (Fish et al., 2018).

To determine the optimal expected mixture of stocks and bonds Betterment simulates potential outcomes for several different portfolio combinations (Betterment[5]). After that, they take the 15th percentile of returns and compare them to each other. In the first few years portfolios with a high equity share only realize losses and are outperformed by bond heavy portfolios. However, over time, even the 15th worst outcome scenarios of stock portfolios begin to perform better than bond portfolios (Betterment[5]). When plotting the 15th percentile of out outcomes, with the equity share on the y-axis and the time until the target date on the x-axis, a line of optimal portfolio allocation becomes visible. This line connects the portfolios with the highest median outcome at each point in the investment horizon. Betterment found that after only twelve years the first portfolios that have an equity share of >50% begin to perform better than bond heavy portfolios and as late as after 20 years all equity heavy portfolios are the best option (Betterment[5]).

This process will be repeated for all percentiles between the 5^{th} and the 50^{th} percentile. The reason why no outcome above the 50^{th} percentile is considered is that in these cases the stock only portfolio will always provide the highest median of outcomes (Betterment[5]). Next, all of these optimal allocation lines can be plotted in one chart. Calculating the average equity share through all percentiles for each point of time derives another line that represents the best equity share in order to improve the investors chances of "making money and *not* losing it" (Betterment[5]).

So, basically the riskiness of a portfolio depends on how much time is left until the investor needs the money. Although, the customers are of course able to incorporate their personnel risk preferences. If they tend to be more conservative, the lower percentiles are weighted heavier and if they are more aggressive, the upper percentiles are weighted heavier when taking the average for the glide-path (Betterment[5]).

5. Asset Location

Asset location has existed since the first qualified account was established in the 1970s (Betterment[1]). Since then, those qualified accounts have been used to save for retirement because they provide some advantages to reduce the tax burden if certain requirements are met. In general, there are three kinds of accounts: taxable, tax-deferred and tax-exempt accounts (Betterment[1]). The latter two have some benefits over the taxable account which will be described below.

Betterment provides a service that is called Tax-Coordinated Portfolio (TCP) which automatically locates assets in those three accounts to maximize the take-home value or after-tax return for their clients, in the cases where those accounts are held by the customer. This possibility of reducing the tax burden by placing the assets in the right account is a decisive advantage over target-date funds which cannot be split and placed in several accounts since they are to be treated like an ordinary stock (Betterment[1]).

A naïve approach of asset location could be to replicate the optimal portfolio we derived in the last few chapters in all three accounts. Of course, this can never be the optimal solution to this problem (Betterment[1]), but to understand why, the characteristics and advantages of each of the accounts need to be understood.

5.1. The three Account Types

Taxable Account

The first and most well-known account is the taxable account. Withdrawals, capital gains or distributions are conventionally taxed without any benefit (Betterment[1]). How each of the mentioned cash flows is taxed in particular, is the subject of the next section.

Tax-deferred Account

This account type belongs to the class of qualified accounts, as well as the tax-exempt account. Normally, if capital gains are realized within an account they are subject to tax, even if they were only realized to rebalance the portfolio. The advantage of a tax-deferred account is that all kinds of taxes will be deferred, until the owner of the account decides to make withdrawals, which should solely start in retirement (Betterment[1]). Thus, interest and dividends are also not taxed, which is quite important since there are assets which mainly generate return through periodic distributions.

Tax-deferral offers an advantage, because of the compounding effect (Betterment[1]). Since distributions and capital gains are not taxed in the year they accrue, but later, more money remains in the account which can be reinvested. The account is able to grow more as more capital means more return in absolute terms.

There are two accounts that provide the tax-deferral effect, a Traditional 401(k) and Traditional IRA account. To be able to contribute to the Traditional IRA the account holder must be under $70\frac{1}{2}$ (Vanguard, a) . The limit of the contribution depends on the age of the holder. If she is under 50, the restriction for the 2019 tax year is \$6,000 and for 50 years and older \$7,000. Besides this, there is no additional restriction that considers income (Vanguard, a).

If withdrawals are taken before the age of $59\frac{1}{2}$, a federal penalty tax of 10% needs to be paid. In addition to that, there is also a restriction that requires the account holder to take "required minimum distributions" (RMD). The first RMD needs to be taken by April 1 of the year after the holder turned $70\frac{1}{2}$. For each following year, the RMDs must be taken by December 31 (Vanguard, a).

All amounts that are regularly withdrawn are taxed at the ordinary tax rate, which can be a disadvantage if the initial tax rate of an asset was the preferential one. Hence, this account is only profitable, if there is enough time for the compounding effect to relativize the potentially higher tax rate.

Tax-Exempt Account

The second qualified account prevents, as the name indicates, all kinds of taxes within the account. But this does not mean that no taxes have to be paid, at all. Taxes are paid in advance, before the deposits get into the account. But from this point on, all capital gains and distributions are not taxed any longer (Vanguard, b).

Again, there are two accounts that have tax-exempt, Roth 401(k) and Roth IRA. The contribution limits of the Roth IRA account are similar to the Traditional IRA account, with the addition that the contribution limit is lower for those with income above a certain threshold (IRS, 2019).

The next difference is that there is no required minimum distribution. The only requirement regarding withdrawals, after the age of $59\frac{1}{2}$, is the 5-year-holding-period (Vanguard, a).

5.2. How are Capital Gains and Distributions taxed?

First of all, there are basically two federal tax rates at which returns are taxed, the ordinary and preferential rate. The ordinary rate goes from 25% to 39.6% and the preferential rate from 15% to 20%. High earners have to pay additional 3.8% on top of each rate. Therefore, every time taxes accrue, one of those two rates are applied (Betterment[1]).

One type of return is capital gains which can be subdivided into two categories, long-term capital gains (LTCG) and short-term capital gains (STCG). If an asset is held for more than a year before it is sold, the gains are considered as LTCG and taxed at the preferential rate (Betterment[1]). In contrast to that, STCG are returns that are realized with assets that are only held a year or less and taxed at the ordinary higher rate. Therefore, it is smart to only realize long-term capital gains in order to prevent the higher tax rate (Betterment[1]).

Betterment precludes STCG completely which could lead to portfolios not being rebalanced because there is no long-term capital or cashflow available. Usually this happens only to accounts that have been recently opened. In these cases, Betterment suggests their customers make deposits for rebalancing to avoid taxes (Betterment[6]).

The other returns that are taxed are dividends and interests. Dividends are paid by stocks and taxed at the preferential rate while interest is paid by bonds and taxed at the ordinary rate. The payments of ETFs are generally called dividends, even those of bond ETFs, as the interests are gathered and periodically distributed. Again, bond ETFs are taxed at the ordinary rate and stock ETFs at the preferential rate. There is, however, an exception to applying the preferential rate on stock ETFs. The lower rate applies only for the payments of stocks within a stock ETF that meets the requirements of a qualified dividend income (QDI) (Betterment[1]). To have an idea of how many assets of an ETF are qualified, and hence taxed at the preferential instead of the ordinary rate, ETFs typically release a QDI percentage (Vanguard, c). For example, the Vanguard Total Stock Market ETF (VTI) has a QDI percentage of 94.01% which means, that only 5.99% of distributions are taxed at the ordinary rate. Another exception of the upper tax rules are municipal bonds which are generally tax free (Betterment[1]).

Since bond funds mainly generate their return through dividends, which are annually taxed at the higher tax rate, instead of capital gains which are only taxed in the case of realization, bond funds are the classic example for a tax inefficient asset class (Betterment[1]). In contrast to that, stock ETFs are more tax efficient because their source of income consists mainly of capital gains. ETFs that depict emerging market stocks are most tax efficient in the Betterment portfolio, due to low distributions and high capital gains, while U.S. Big Cap ETFs are less tax efficient since they do not grow that fast anymore and instead pay larger dividends. All in all, the concept of tax efficiency is rather a relative concept (Betterment[1]).

5.3. Which Asset to locate in which Account?

Now, that the tax characteristics of each account and asset class are known, it is clear that replicating the whole portfolio in each account is not optimal. Rather, it seems more logical to locate assets according to their tax efficiency. Stocks that are tax efficient and thus do not cause much tax drag, go into the taxable account, while bonds would perform best in qualified accounts since they annually pay interests which are subject to tax (Betterment[1]).

Locating assets according to their tax efficiency, however, does not capture the whole picture. This strategy would only be optimal, if all asset classes have the same returns. Since bonds gain a lot less returns as stocks, the effect of sheltering bonds from taxes might be less effective in absolute terms than protecting the returns of stocks even if they are more tax efficient (Betterment[1]). Instead of solving according to tax efficiency, the after-tax return must be maximized which considers both tax efficiency and expected return. What really matters is the absolute amount of expected savings instead of just improving the tax efficiency of an asset which might contribute rather less return to the portfolio and wastes the valuable space of the qualified accounts (Betterment[1]).

5.4. How Betterment locates their Assets

Betterment solves the asset location problem by solving the so-called objective function with the help of linear optimization. For this purpose, the expected after-tax returns must be calculated. Those returns are derived by the Black-Litterman model which has been discussed in the previous chapter (Betterment[1]).

Since each asset can potentially be placed in each of the three accounts, there are three different after-tax returns for each asset (Betterment[1]). In a tax-exempt account, the after-tax return is equal to the expected return, as no taxes occur. In a tax-deferred account, the annual returns are compounded to project the expected overall return at the time of liquidation in retirement. This realized gain will be taxed at the ordinary rate and, from what is left after applying the tax, an annualized after-tax return can be calculated. In the taxable account, dividends and capital gains need to be taxed separately as they are subject to different tax rates, as discussed in section 5.2. For the taxation of capital gains, the preferential rate is solely assumed, as Betterment does not realize short-term capital (Betterment[1]).

The objective function is the sum of after-tax returns for each asset for each account multiplied by the unknown respective weight. This would mean that the objective function must handle up to 33 assets, if the customer holds all three accounts. Furthermore, the function is subject to constraints that depict the optimal asset allocation, that have been derived in the previous chapters (Betterment[1]).

Expected returns must be updated periodically due to new available information. This could potentially trigger changes in the asset´s locations but since reshuffling between the accounts does not realize any gains, no additional costs would arise beside paying the bid-ask spread (Betterment[1]).

5.5. Betterment´s Results

Betterment tested their asset location methodology with a Monte Carlo testing framework for which they required over 150,000 computer-hours and up to 3,000 Amazon Web Services servers running simultaneously (Betterment[1]).

The results show that TCP generates a tax alpha of between 0.10% and 0.82% per year. Interesting to note is that a higher bond share goes with higher tax alpha. An explanation for this could be the tax inefficiency of bonds. Another observation is that the presence of Roth accounts leads to higher alpha. This is quite intuitive as Roth accounts prevent any kind of tax within the account but this should be no reason for dumping the tax-deferring account. The results show that the highest tax alpha is generated if all three accounts (taxable, traditional, Roth) are involved and the fixed income share is high. On the other hand, the equity share should not be determined according to the tax alpha but according to the remaining number of years to liquidation as discussed in section 4.3. (Betterment[1]).

6. Rebalancing

The final important cornerstone of the Betterment Portfolio Strategy is rebalancing. Over time assets deviate from their target allocation, as some assets outperform others and the portfolio will end up concentrated in the outperforming assets. Rebalancing is basically the action of resetting the weights to the initial ones or, if new views are available, to the new optimal target allocation (Nardon and Kiskiras, 2013). This chapter will describe the benefits of rebalancing and show which kind of rebalancing performs best with and without the presence of transaction costs. One benefit is that rebalancing is responsible for maintaining diversification and reducing the risk. However, before dealing with this topic, I will talk about the second benefit, the claim that rebalancing is even able to outperform passive buy and hold portfolios in terms of returns.

6.1. Rebalancing Bonus

First of all, rebalancing can indeed gain some kind of rebalancing bonus, but only under certain conditions. The first to discover that a constant mix portfolio, i.e. a continuously rebalanced portfolio, could outperform a drifting portfolio were Perold and Sharpe

(1988). They found that a necessary condition is that assets behave in a mean reversing way, which means that they oscillate around a certain value. Perold and Sharpe (1988) have provided a good example: Let there be a portfolio consisting of 60% of stock with a fluctuating price and 40% of a bond with consistent price. The initial portfolio has an initial value of 100. Now the price of the stocks market drops from 100 to 90 and recovers back to 100 which means that there is no trend in this period for the price reversing. In Table 3 it can be seen that after the price drop, the percentage of stocks decreased and because a constant portfolio is desired, the position of the losing asset raised, nonetheless. When the stock´s price increases again the value of the portfolio exceeds its initial value because the position of stocks was previously increased to remain at 60%. To amplify this rebalancing bonus, a higher price drop, thus a higher volatility, would be beneficial.

Case	Stock Market	Value of Stock	Value of Bills	Value of Assets	Percantage in Stocks
Initial	100	60.00	40.00	100.00	60.0
After change	90	54.00	40.00	94.00	57.4
After rebalancing	90	56.40	37.60	94.00	60.0
After change	100	62.67	37.60	100.27	62.5
After rebalancing	100	60.16	40.11	100.27	60.0

Table 3: Taken from Perold and Sharpe (1995), p. 153. This table shows, how rebalancing bonus is generated if a stock´s price drops and recovers to its initial price.

But what would happen, if prices are not reversing? A constant portfolio means to sell winners or to buy losers. In a bull market this would mean, that stocks were sold to buy the underweighted bond and if stock prices rise further the portfolio will steadily decrease the number of stocks and thus always underperform the drifting portfolio (Perold and Sharpe, 1988).

The other way around, in a bear market this strategy will keep buying the losing stocks and again underperform the buy and hold strategy where the number of assets remain the same (Perold and Sharpe, 1988).

To quantify the rebalancing bonus in the two-asset case, Bernstein (1997, cited in Nardon and Kiskiras, 2013) came up with the following equation:

$$RB_{1,2} = w_1 w_2 \left(\frac{\sigma_1^2}{2} + \frac{\sigma_2^2}{2} - \sigma_{1,2} \right) \tag{5.1}$$

This equation shows that the rebalancing bonus is increased by rising variance (σ_1^2, σ_2^2) and decreases in $\sigma_{1,2}$, which denotes the covariance between both assets. Another important implication of this formula is that the maximum rebalancing bonus is generated when both assets are equally weighted ($w_i = 0.5$). Bernstein tested this equation with real data and compared both rebalancing bonuses. The R^2, which can also be interpreted as correlation, was 0.983, which is an indicator for Bernstein´s equation being a strong estimator for the rebalancing bonuses found in the data.

Fernholz and Shay (1982) came up with another version of an equation that describes the rebalancing bonus, which is quite similar to that by Bernstein and has the same implications:

$$RB = \frac{1}{2}\left(\sum_{i=1}^{n} w_i\sigma_i^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i,j}\right) \tag{5.2}$$

With the help of this description, Nardon and Kiskiras (2013) simulated rebalancing bonuses by generating 10,000 price series for two assets that had no trend, a volatility of $\sigma = 0.35$, a correlation of $\rho = -0.4$ and are equally weighted. The results showed that the rebalancing bonus can also be negative in this given setting which is depicted neither by Bernstein´s nor Fernholz and Shay´s formula. In fact, the rebalancing bonus is negatively correlated with the difference between the asset´s annualized returns and can be negative if those returns differ too much (Nardon and Kiskiras, 2013). This means that both assets should have similar returns or, at least, they should not differ substantially, in order to gain excess returns. Equation 5.2 does a good job of predicting the rebalancing bonus, in the case of similar returns (Nardon and Kiskiras, 2013).

In addition to that, Nardon and Kiskiras (2013) also changed the values for volatility and correlation. Their results are consistent with the implications of Bernstein´s formula. Although, in contrast to Perold and Sharpe, Nardon and Kiskiras (2013) found no evidence that the absence of a deterministic trend is required, as long as the trend is constant. This fluctuation around a deterministic trend relative to each other is called relative mean-reversion.

Furthermore, they found that a positive rebalancing bonus is associated with the two assets sharing the same initial and ending price and the same long-term trend. To carry it a little bit further, Nardon and Kiskiras (2013) state that an outperforming of the drifting portfolio takes place whenever the latter conditions hold, no matter the volatility or correlation. Moreover, if the frequency that both asset prices converge rises, the rebalancing bonus will also rise.

The last, as yet undiscussed, parameter that needs to be determined is the number of assets that are included in a portfolio. For this purpose, Kim and Mulvey (2009, cited in Nardon and Kiskiras, 2013) derived a formula which assumes that all assets are equally weighted (as proven above, the best weighting in order to increase the rebalancing bonus) and have the same variance and correlation:

$$RB = \frac{(n-1)\sigma^2(1-\rho)}{2n} \tag{5.3}$$

Again, RB is a decreasing function of the correlation ρ and an increasing function of the volatility σ. The number of assets (n), the actual part of interest, is also responsible for a rising RB. However, when building the second deviation, it can be noticed that the marginal effects are diminishing.

To sum it all up, the maximal rebalancing bonus can be obtained, if assets share the same price at the beginning and ending of a period, have the same trend and behave in a relatively mean-reversing fashion. To amplify this effect, the assets should be equally weighted, have a volatility and low correlation and, last but not least, the portfolio should include a larger number of assets.

6.2. Reducing Risk and gaining higher Sharpe Ratios

In the previous section I have shown that it is possible to outperform drifting portfolios by simply resetting the portfolio weight in a continuous manner, if certain market conditions are given. Up to this point we have disregarded transaction costs, theoretically making continuous rebalancing the optimal regime. It is true that Betterment does not charge any transaction costs for their customers but, nonetheless, the small, but still existent, bid-ask spread has to be paid (Betterment[14]). This section investigates which rebalancing regime is the best, in order to lower the portfolios´ risk and to gain the maximal Sharpe ratio in the presence of transaction costs.

First of all, the kinds of rebalancing need to be specified. In general, there are two classes, the calendar driven and threshold driven rebalancing (NBIM, 2012). Calendar driven rebalancing means that the portfolio is rebalanced with a certain frequency. Typical frequencies are monthly, quarterly or annually. In contrast, threshold driven rebalancing only resets weights when an asset deviates more from the target allocation than was defined in advance. For example, if there was an asset with a weight of 25% and a threshold of 3%, rebalancing would only be triggered if the asset´s portfolio share was either more than 28% or less than 22%. In their paper the Norges Bank Investment Management (NBIM, 2012) calls this area, in which no rebalancing takes place, a no-trade region.

The higher the threshold, or the longer the chosen periods, the less trades take place and the further the assets move away from their target allocation. Especially, if there are asset classes with significantly different expected returns and volatilities, for example, equities and fixed income. Equities have much higher volatility and in the long term they are likely to outperform bonds. Hence, the share of equities in a portfolio rises and so does the portfolio risk as the much higher volatility is more strongly weighted due to higher equity weights. All in all, appropriate thresholds and rebalancing frequencies are determined by the tradeoff between the deviation from target allocation and transaction costs (NBIM, 2012).

To figure out what differences those different strategies make, NBIM (2012) composed a portfolio consisting of 60% equities and 40% bonds of the American, European and Asian/Pacific market which were weighted according to their market capitalization. They used a set of historical monthly returns between 1970 and 2011.

At first, they considered the case where both the equity share and the regional allocation were rebalanced, i.e. the whole portfolio. For the threshold strategy, the weights were completely reset for all assets if they deviated more than the threshold allowed for two consecutive months.

In Table 6 in the Appendix, it can be seen that the volatility for all assets decreased by nearly 1% and the geometric average returns rose by about 0.65%. This obviously leads to higher Sharpe ratios. Since the values for all rebalancing strategies have clearly improved, the Sharpe ratios of the rebalanced portfolios were significantly higher than the one belonging to the drifting portfolio. It is noteworthy, that the values among the rebalancing strategies do not differ substantially. The fact that rebalancing was applied seems to be more important than choosing a specific strategy (NBIM, 2012).

In these numbers, transaction costs have not been included, yet. NBIM (2012) assume “fairly high estimates of transaction costs” for either 50 or 100 basis points. In the two rightmost columns of Table 7 in the Appendix you can see that even with transaction costs included, the Sharpe ratios are higher in all but one case.

Not only the values for returns and volatility improved, but also the 95% Value at Risk. In general, the effect of rebalancing is that it alters the distribution of possible outcomes (Ilmanen and Maloney, 2015). It cuts off the tails and makes the range of outcomes smaller. Furthermore, if all assets have the same expected returns this would lead to a higher median. All in all, the reshaping of distribution makes it more likely that the portfolio produces positive returns (Ilmanen and Maloney, 2015).

Another interesting question would be how much the performance would improve if only the share of equity was rebalanced NBIM (2012). This means, whenever rebalancing is triggered the only goal is to restore the 60% weight of equity. The assets within this assets class remain the same. The results can be seen in Table 7 in Appendix. Again, the values for returns, volatility, Sharpe ratio and Value at Risk improve. Although, they are not as good as those of the previous back testing with all asset weights being reset. This time, when including 100 basis points as transaction costs, not all rebalancing strategies´ Sharpe ratios are able to outperform the drifting portfolio. Though, NBIM (2012) also found that the turnover for most strategies is quite low and hence there are lower transaction costs in reality. In the end, each rebalanced portfolio would outperform drifting, even after costs.

However, even if only rebalancing the equity share does not yield the maximum performance, it is main driver of outperformance, while rebalancing within each asset classes further increases the effect (NBIM, 2012).

To sum this chapter up, rebalancing is an important part of a portfolio strategy. It ensures diversification by making sure that the portfolio weights are reset to target allocation on a regular basis to prevent riskier assets from overtaking the portfolio. This leads to lower volatility and higher returns, if certain market conditions are present, and thus to better Sharpe ratios. The fact that it is not important to decide on one strategy is, since the differences in performance are, if anything, marginal, quite notable.

However, Betterment defines the portfolio drift as the sum of the absolute deviation from target allocation divided by two (Betterment[6]). Rebalancing is triggered whenever a portfolio deviates 3% or more (checked once a day). Cash in- and outflows are used to buy underweighted asset classes and sell overrepresented ones. Typical cashflows are dividend payments, deposits or withdrawals. In the case that there are no cashflows the algorithm trades itself. As Betterment always tries to minimize tax drag, attention is only paid to selling assets with low tax drag (Betterment[6]). If a rebalancing trade would realize short term capital gains, which are taxed at a higher tax rate, rebalancing would not take place. Hence, it is possible that a Betterment portfolio drifts more than 3% from target allocation in some cases (Betterment[6]).

7. Implementing the Betterment Portfolio Strategy

In this part, I will implement the Betterment Portfolio Strategy and compare it to a number of different strategies. The question that I want to answer is whether Betterment is able to outperform intuitive "do it yourself" strategies and which part of the strategy adds what value.

7.1. Data Description

I have used adjusted close data from 11-01-2013 to 04-29-2019 for all eleven ETFs from Table 4 in the Appendix. These returns are from Yahoo! Finance which adjusts for dividends and stock splits. The observation period, for which all strategies are simulated, goes from 07-31-2014 to 04-29-2019 which means that the first ten months are used to calculate the required estimators. For some strategies market capitalization weights are required. These weights are collected from factsheets available on the respective website of the ETFs´ Benchmark. I used the current ones as a starting point and calculated them back on the basis of their returns.

7.2. Description of Strategies

All in all, there are seven strategies I want to compare to each other. At first, they are quite simple as they should mimic the possible strategies of potential Betterment customers. These customers are people who do not have much knowledge concerning the financial market, but nevertheless want to grow their capital for retirement or other investment goals. From strategy to strategy they become more sophisticated and more difficult to implement. Therefore, there is good reason to believe that those strategies become better, with the last performing the best.

Furthermore, all strategies are subject to different rebalancing regimes ranging from no rebalancing to continuous rebalancing. Rebalancing is triggered whenever the portfolio drift reaches the prescribed threshold. The portfolio drift is defined as in section 6.2., as the sum of absolute deviations from the target allocation divided by two.

- <u>Goldman Sachs + NEAR (60/40):</u> The easiest way to be invested is to randomly pick a Dow Jones 30 stock. To make all strategies comparable and enable investigation of the rebalancing effect, all portfolios consist of 40% bonds, as well. In this case U.S. Short-Term Investment-Grade Bonds are mixed with the Goldman Sachs stock.
- <u>MSCI World + SHV + NEAR (60/20/20):</u> A more advanced strategy would be to buy the MSCI World as it depicts more or less the worldwide economy and, therefore, it is well diversified. Furthermore, the portfolio consists of two popular bond ETFs which are often considered to be of high quality and bear a low level of risk, U.S. Short-Term Treasury Bonds and again U.S. Short-Term Investment-Grade Bonds.
- <u>Equally Weighted:</u> As the name possibly implies, the strategic asset allocation is to weigh all assets equally within the bond and stock portfolio. Again, the overall portfolio consists of 60% stocks. The assets that are used are those eleven that Betterment also uses. I will choose this strategy as benchmark to evaluate if optimizing the asset weights adds any value or if asset allocation is simply a waste of resources, as the paper written by DeMiguel, Garlappi and Uppal (2007) indicates.
- <u>Weights only set once:</u> This is more or less the Betterment Portfolio Strategy with historic averages as views, with the difference, that the optimal weights are only determined once, at the beginning.
- <u>Betterment w/o views:</u> This strategy is Betterment’s normal strategy, but without views. This means, that instead of tilted asset weights, the ten months average of the market capitalization weights is used. This should perform well, as this constellation is considered to be the optimal in regards to Sharpe (1964).
- <u>Betterment w/o shrunken cvm:</u> This strategy uses Betterment´s ETFs and calculates optimal asset weights with the Black-Litterman model on basis of the historic averages from the last ten months, which are updated four times a year. The difference to the actual Betterment strategy is that the sample covariance matrix is used instead of one that has been shrunk.
- <u>Betterment w historic average as view:</u> This strategy is the one that comes the closest to the actual Betterment Portfolio Strategy. There are some assumptions I

have to make, since this strategy was not explained in detail, which also applies to the strategies described above. First of all, I have used historic averages as views, while Betterment uses a Fama-French regression. The reason for this simplification is that they do not explain how they calculate estimators for future required input factors (excess returns, size factor, value factor). The most intuitive way would be an autoregressive model, but since the markets are assumed to be efficient this should lead to problems as stock prices follow a random walk and cannot be predicted (Fama, 1970). A reasonable alternative is to jump on a bandwagon and to buy winners and to sell losers. Jegadeesh and Titman (1993) released a paper where they found that it is possible to generate positive returns by making decisions based on the average of the past 12 months. In this case, I decide to use a ten-month average as it performs slightly better. Furthermore, the sample covariance matrix is not calculated with daily but, rather, monthly returns. As ten months is not enough for estimating a reasonable covariance matrix I need to cheat and use out of sample data, thus I assume that all variance and covariances are the same during the entire observation period. The reasons for this decision will be discussed in the next section. Another difference to Betterment is, that I assume that all Bond ETFs are weighted the same, as no information regarding market capitalization is available. In general, Betterment´s approach is to use Monte Carlo simulations to make the allocation weights more robust. This will not be possible for me, as well. In addition to that, I also disregard from cash flows, asset location and will only consider before-tax returns. To make it clear, this strategy is not the original Betterment Portfolio Strategy but a simplified version which incorporates the main cornerstones of their strategy. Betterment might perform even better in reality.

7.3. Occurring Problems and their Solutions

Using historic averages as views does not lead to those reasonable asset weights that Black-Litterman should actually generate. There are two possible reasons for this predicament. First, the inverse of the covariance matrix still bears a lot of error indicated by high conditional numbers. My thesis was to use monthly data instead of daily data to smooth out irrelevant short-term fluctuations which could be caused by the randomness of stock prices. Indeed, the conditional number becomes significantly better when using monthly data. Unfortunately, ten months is not enough to estimate a sample covariance matrix, so I need to use the entire sample. An alternative solution could be to use index data for this calculation, but as ETFs bear tracking errors in order to lower expense ratios I decided not to use them.

The second problem is that average historical returns differ much from implied equilibrium returns in most cases. Even the rankings of ETFs according to their returns differ significantly. This leads to asset weights well above 100% and below 0%. The basic idea of Black-Litterman is to tilt assets in the adequate direction. Obviously, historic averages tilt them too much. My idea is to decrease the deviation between average returns and implied equilibrium returns to only push each asset in the particular direction really gently:

$$Q = \Pi + \delta * (\bar{r} - \Pi) \tag{6.1}$$

Where Q is the vector of views, Π the vector of implied equilibrium returns, $(\bar{r} - \Pi)$ the difference between average historical returns and implied equilibrium returns, and δ the intensity for which I assume 0.1. The resulting asset weights lie mostly between 0 and 1 and are way more balanced. Furthermore, this solution avoids using linear optimization which, again, would lead to results that concentrate only in few assets while the rest would be set to zero. All in all, I have generated 209 asset weights with this set up and only three of them are slightly negative, while the rest is a long-position and is still tilted away from implied equilibrium returns clearly, just the way it should be. In those three cases I will enable short selling even if robo advisors solely hold long-only portfolios, as the signs for a market downturn seem to be really strong.

7.4. Empirical Results

My indicator for a well performing portfolio is a high Sharpe ratio disregarding risk-free rates, since risk adjusted returns are more informative as they exclude cheating by simply taking more risk.

As mentioned above, the strategies are ranked from intuitive to sophisticated. Thus, the Sharpe ratio can be expected to increase from strategy to strategy and should also increase the lower the rebalancing threshold is. All in all, those expectations are met with a few interesting exceptions. Figure 1 shows that Betterment w/o view performs worse than the strategy where an optimal allocation is derived only once. This is really unintuitive since the portfolio that is weighted according to market capitalization weights should represent the optimal market portfolio, which, again, should beat a portfolio where the portfolio weights are not adjusted over time.

Furthermore, it is interesting to see that rebalancing does not have that much of an impact. From left to right the boxes only become slightly darker. One explanation could be that for the strategies, where the asset allocation is updated every three months the weights are reset to the new allocation and thus a rebalancing takes place even if the threshold was not reached. Even a low threshold which lead to a high number of rebalances does not significantly increase the Sharpe ratio. In fact, the optimal regime for rebalancing is the 3% threshold which is, by the way, the official Betterment threshold, as well.

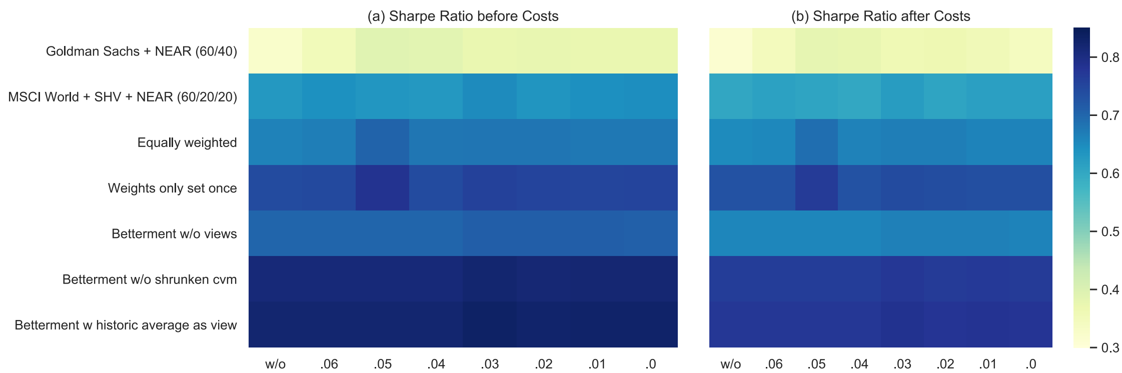

Figure 1: (a) Sharpe ratios for all strategies before costs and disregarding from the risk-free rate. (b) Sharpe ratios after costs (transaction costs, expense ratio, managing fees).

Even after costs (for Betterment: bid-ask spread, expense ratio of ETFs, managing fees), the Betterment Portfolio Strategy with a 3% rebalancing threshold is still the best strategy (see Figure 1 (b)). For every single strategy, the actual costs have been calculated and no assumptions were made. All in all, this strategy has one of the lowest volatilities and the highest after costs return. It beats the MSCI World strategy by about 1.23% and the equal weights strategy by about 0.77% (see Table 8 in the Appendix). This may not sound much, but all strategies are long-term strategies. The average Betterment customer is 36 years old (Fish et al., 2018), which means that there are 31 years left until retirement. If this customer invests $100,000 then an additional 0.77% per year would mean, that she has $134,855.27 more in her bank account in retirement.

Betterment´s Sharpe ratio is 0.8333 before costs and, hence, performs much better than the rest. To get a feeling how good this number is compared to real life funds, let us compare it to the Sharpe ratio of a target-date fund, which are robo advisor´s greatest competitors. A comparable target-date fund in terms of equity share would be the Black

Rock LifePath® Index 2025 Fund Institutional Shares (Fidelity Investments, 2019) which has a Sharpe ratio of 0.85 (time horizon: 05-31-2011 to 05-31-2019). Thus, target-date funds are slightly better but not out of range and taxes are not even included yet. However, this comparison should be treated with caution, as both Sharpe ratios were calculated over different time horizons and should only provide guidance.

A summary of my simulation can be seen in Table 8 in the Appendix. From this table, the effects of diversification, asset weights and covariance shrinking can be derived. To determine these effects, I subtracted the average Sharpe ratio from each other. Diversification has the largest impact. The Sharpe ratio of the equally weighted portfolio is 0.31 higher on average than those of the Goldman Sachs portfolio, where only two assets are involved. The weighting of assets has the second largest impact. The Sharpe ratio of the Betterment strategy without a shrunken covariance matrix is 0.14 higher than the Sharpe ratio of the equally weighted portfolio. The presence of a shrunken covariance matrix has the smallest effect with an, on average, 0.01 higher Sharpe ration in comparison to the strategy with the sample covariance. The effect, when rebalancing is triggered, is significant (p-value: 0.001) while the effect between the rebalancing regimes is not.

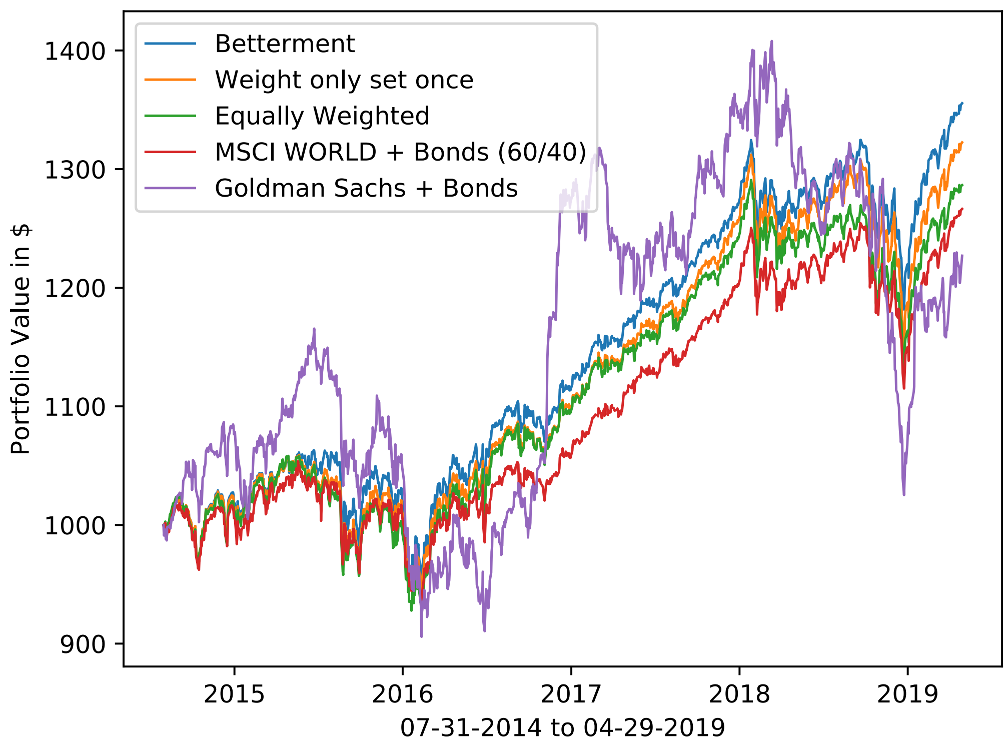

Figure 2: Price development for five of the seven strategies.

7.5. Outlook for future Research

As seen in the last section, the Betterment Portfolio Strategy clearly performs better than the rest. Admittedly, the period of observation of 4 years and 9 months is quite short. It would be interesting to see, how this strategy performs in the long run, especially if it has to face market downturns which are more persistent than the one at the end of 2018. The fact that short selling is not permitted might reduce the performance significantly.

Furthermore, future research could incorporate asset location and compare the after-tax performance with target-date funds. As already mentioned in chapter five, target-date funds cannot reduce tax drag by asset location. Thus, it would be interesting to see if Betterment could use this advantage to leave target-date funds behind once and for all.

8. Conclusion

Robo advisors are a good opportunity for younger people who do not have the required asset base in order to be managed by a traditional human advisor or cannot afford their fees. One option would be to try to manage their assets themselves but this would not lead to the same results as, Betterment. As shown, even small differences in returns can compound and have a huge impact on the investor´s future balance sheet. The other option would be to invest in target-date funds. Before tax they performed slightly better in contrast to my version of the Betterment Portfolio Strategy but after applying asset location and tax loss harvesting, robo advisors might be ahead. Furthermore, they provide portfolios which are suited to the customer´s goals and give advice on how to invest to reach a certain goal. In addition to that, robo advisors are incorporating humans more and more and thus are approaching traditional advisory at an affordable price.

All in all, robo advisor definitely do not yield poor results and are recommendable over do it yourself strategies and are at least on a par with target-date funds. I expect them to become a serious competitor in the financial industry as they provide high quality financial advice at lower price and thus are able to reach a broader range of the general public than traditional advisors.

9. Appendix

Asset Clas	Benchmark	ETF	Expense Ratio	Bid-Ask Spread	Avg. Market Cap. Weight	Holdings: ETF / (Benchmark)
Stocks						
U.S. Total Stock Market	CRSP US Total Market Index	VTI	0.03 %	0.01 %	40.0 %	3,607 / (3,557)
U.S. Value Stocks - Large Cap	CRSP US Large Cap Value Index	VTV	0.04 %	0.02 %	17.5 %	339 / (337)
U.S. Value Stocks - Mid Cap	CRSP US Mid Cap Value Index	VOE	0.07 %	0.02 %	3.1 %	203 / (202)
U.S. Value Stocks - Small Cap	CRSP US Small Cap Value Index	VBR	0.07 %	0.06 %	2.8 %	842 / (820)
International Developed Market Stocks	FTSE Developed All Cap ex US Index	VEA	0.05 %	0.02 %	29.3 %	3,954 / (3,884)
International Emerging Market Stocks	FTSE Emerging ACap CN A Inclus Idx	VWO	0.12 %	0.02 %	7.3 %	4,654 / (4,024)
Bonds						
U.S. Inflation-Protected Bonds	BloomBarc US 0-5 Year TIPS Index (Benchmark)	VTIP	0.06 %	0.02 %		16 / (15)
U.S. Short-Term Treasury Bonds	ICE U.S. Treasury Short Bond Index	SHV	0.15 %	0.01 %		50 / (84)
U.S. Short-Term Investment-Grade Bonds	-/-	NEAR	0.25 %	0.02 %		411
International Developed Market Bonds	BloomBarc GA ex-USD FlAdjRIC Cp Hgd	BNDX	0.09 %	0.02 %		5,796 / (10,518)
International Emerging Market Bonds	J.P. Morgan EMBI Global Core Index	EMB	0.39 %	0.01 %		479

Table 4: The eleven Betterment ETFs that are currently included in their portfolio strategy. Data: Vanguard https://investor.vanguard.com/advice/personal-advisor, Center for Research in Security Pirces http://www.crsp.com, Morning Star https://www.morningstar.com, iShares https://www.ishares.com/de and Bloomberg https://www.bloomberg.com/europe (as of 03-29.2019).

Asset Class	Expected Returns	Implied Equilibrium Return Vector Π
VTI	7.00 %	6.65 %
VTV	8.34 %	8.73 %
VOE	8.55 %	9.01 %
VBR	9.45 %	8.72 %
VEA	2.63 %	2.75 %
VWO	5.21 %	8.34 %

Table 5: Example of how the vectors of expected returns and implied equilibrium returns can potentially look like.

	Return	Volatility	Sharpe	VaR 95%	Sharpe 50 bp	Sharpe 100 bp
Panel A: Drifting Mix						
60/40 with regional factors	9.15 %	11.1 %	0.83	-4.4 %	0.83	0.83
Panel B: Calendar based						
Monthly	9.16 %	10.2 %	0.94	-4.0 %	0.85	0.80
Quarterly	9.72 %	10.2 %	0.95	-4.0 %	0.9	0.85
Annually	9.89 %	10.2 %	0.97	-3.9 %	0.92	0.87
Panel C: Threshold based						
1%	9.73 %	10.2 %	0.96	-4.0 %	0.90	0.86
2%	9.84 %	10.2 %	0.97	-4.0 %	0.92	0.87
3%	9.88 %	10.2 %	0.97	-4.0 %	0.92	0.87
4%	9.81 %	10.2 %	0.96	-4.0 %	0.91	0.86
5%	9.93 %	10.2 %	0.98	-3.9 %	0.92	0.88
6%	9.97 %	10.2 %	0.98	-4.0 %	0.93	0.88
7%	9.72 %	10.1 %	0.97	-4.1 %	0.91	0.86
8%	10.01 %	10.2 %	0.98	-3.8 %	0.93	0.88
9%	9.95 %	10.3 %	0.97	-3.8 %	0.92	0.87
10%	9.60 %	10.2 %	0.94	-3.9 %	0.89	0.84

Table 6: Borrowing from NBIM (2012), p. 8. This table compares different rebalancing regimes with each other in terms of return, volatility, Sharpe ratio (disregarding from risk-free rate) and the 95% value at risk. The portfolios consist of bonds and stocks and each asset class is subdivided into assets from various regions. Whenever rebalancing takes place, the equity share and regions are reset. Furthermore, two more Sharpe ratios are calculated that assume transaction costs of either 50 basis point or 100 basis points.

	Return	Volatility	Sharpe	VaR 95%	Sharpe 50 bp	Sharpe 100 bp
Panel A: Drifting Mix						
60/40 with regional factors	9.15 %	11.1 %	0.83	-4.4 %	0.83	0.83
Panel B: Calendar based						
Monthly	9.37 %	10.3 %	0.91	-4.1 %	0.86	0.81
Quarterly	9.49 %	10.3 %	0.92	-4.1 %	0.87	0.82
Annually	9.59 %	10.3 %	0.93	-4.1 %	0.88	0.83
Panel C: Threshold based						
1%	9.44 %	10.3 %	0.91	-4.1 %	0.87	0.82
2%	9.48 %	10.3 %	0.92	-4.1 %	0.87	0.82
3%	9.55 %	10.3 %	0.92	-4.1 %	0.88	0.83
4%	9.53 %	10.3 %	0.92	-4.1 %	0.88	0.83
5%	9.59 %	10.4 %	0.93	-4.1 %	0.87	0.83
6%	9.44 %	10.4 %	0.91	-4.1 %	0.86	0.81
7%	9.63 %	10.5 %	0.92	-4.2 %	0.87	0.82
8%	9.56 %	10.4 %	0.92	-4.2 %	0.87	0.82
9%	9.50 %	10.4 %	0.91	-4.1 %	0.87	0.82
10%	9.45 %	10.3 %	0.92	-4.1 %	0.87	0.82

Table 7: Borrowing from NBIM (2012), p. 13. This table compares different rebalancing regimes with each other in terms of return, volatility, Sharpe ratio (disregarding from risk-free rate) and the 95% value at risk. The portfolios consist of bonds and stocks and each asset class is subdivided into assets from various regions. Whenever rebalancing takes place, the only equity share is reset. Furthermore, two more Sharpe ratios are calculated that assume transaction costs of either 50 basis point or 100 basis points.

Strategy	no rb	6%	5%	4%	3%	2%	1%	0%
Panel A: Sharpe Ratio before Costs								
Goldman Sachs + NEAR (60/40)	0.3239	0.3557	0.3934	0.3897	0.3749	0.3790	0.3731	0.3750
MSCI World + SHV + NEAR (60/20/20)	0.6302	0.6426	0.6350	0.6302	0.6521	0.6370	0.6454	0.6480
Equally weighted	0.6643	0.6693	0.7050	0.6786	0.6820	0.6815	0.6763	0.6777
Weights only set once	0.7444	0.7458	0.7839	0.7445	0.7585	0.7543	0.7520	0.7542
Betterment w/o views	0.7036	0.7036	0.7036	0.7036	0.7114	0.7105	0.7105	0.7097
Betterment w/o shrunken cvm	0.8116	0.8116	0.8116	0.8116	0.8223	0.8162	0.8179	0.8186
Betterment w historic average as view	0.8226	0.8226	0.8226	0.8226	0.8333	0.8272	0.8293	0.8298
Panel B: Mean Return								
Goldman Sachs + NEAR (60/40)	0.0480	0.0505	0.0561	0.0549	0.0528	0.0534	0.0525	0.0528
MSCI World + SHV + NEAR (60/20/20)	0.0529	0.0523	0.0519	0.0517	0.0532	0.0520	0.0526	0.0528
Equally weighted	0.0561	0.0551	0.0578	0.0565	0.0566	0.0566	0.0561	0.0562
Weights only set once	0.0621	0.0604	0.0629	0.0603	0.0615	0.0610	0.0607	0.0610
Betterment w/o views	0.0568	0.0568	0.0568	0.0568	0.0575	0.0577	0.0578	0.0578
Betterment w/o shrunken cvm	0.0651	0.0651	0.0651	0.0651	0.0661	0.0659	0.0661	0.0662
Betterment w historic average as view	0.0659	0.0659	0.0659	0.0659	0.0670	0.0668	0.0670	0.0671
Panel C: Volatility								
Goldman Sachs + NEAR (60/40)	0.1483	0.1419	0.1425	0.141	0.1409	0.1410	0.1408	0.1408
MSCI World + SHV + NEAR (60/20/20)	0.0839	0.0813	0.0817	0.0821	0.0816	0.0816	0.0815	0.0815
Equally weighted	0.0845	0.0823	0.0821	0.0832	0.0830	0.0831	0.0830	0.0830
Weights only set once	0.0834	0.0810	0.0803	0.0810	0.0811	0.0809	0.0808	0.0808
Betterment w/o views	0.0807	0.0807	0.0807	0.0807	0.0809	0.0812	0.0814	0.0814
Betterment w/o shrunken cvm	0.0802	0.0802	0.0802	0.0802	0.0804	0.0807	0.0809	0.0809
Betterment w historic average as view	0.0801	0.0801	0.0801	0.0801	0.0804	0.0807	0.0808	0.0808
Panel D: Rebalancing Count								
Goldman Sachs + NEAR (60/40)	0	2	8	11	15	31	91	1193
MSCI World + SHV + NEAR (60/20/20)	0	1	1	1	6	7	35	1193
Equally weighted	0	1	3	3	6	14	44	1193
Weights only set once	0	1	3	2	5	12	39	1193
Betterment w/o views	18	18	18	18	19	24	57	1211
Betterment w/o shrunken cvm	18	18	18	18	20	23	58	1211
Betterment w historic average as view	18	18	18	18	20	23	59	1211
Panel E: Transaction Costs								
Goldman Sachs + NEAR (60/40)	0.0010	0.0011	0.0014	0.0015	0.0015	0.0017	0.0022	0.0042
MSCI World + SHV + NEAR (60/20/20)	0.0022	0.0023	0.0023	0.0022	0.0023	0.0023	0.0023	0.0026
Equally weighted	0.0011	0.0011	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013
Weights only set once	0.0011	0.0011	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013
Betterment w/o views	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0038
Betterment w/o shrunken cvm	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0039
Betterment w historic average as view	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.004
Panel F: Sharpe Ratio after Costs								
Goldman Sachs + NEAR (60/40)	0.3171	0.3478	0.3834	0.3792	0.3643	0.3666	0.3577	0.3449
MSCI World + SHV + NEAR (60/20/20)	0.6035	0.6149	0.6074	0.6029	0.6242	0.6092	0.6170	0.6163
Equally weighted	0.6509	0.6555	0.6910	0.6649	0.6682	0.6676	0.6622	0.6618
Weights only set once	0.7308	0.7318	0.7696	0.7305	0.7443	0.7400	0.7376	0.7381
Betterment w/o views	0.6584	0.6584	0.6584	0.6584	0.6662	0.6654	0.6653	0.6629
Betterment w/o shrunken cvm	0.7645	0.7645	0.7645	0.7645	0.7753	0.7694	0.7709	0.7698
Betterment w historic average as view	0.7754	0.7754	0.7754	0.7754	0.7861	0.7802	0.7821	0.7808
Panel G: Mean Return w Costs								
Goldman Sachs + NEAR (60/40)	0.0470	0.0494	0.0546	0.0534	0.0513	0.0517	0.0504	0.0485
MSCI World + SHV + NEAR (60/20/20)	0.0507	0.0500	0.0496	0.0495	0.0509	0.0497	0.0503	0.0502
Equally weighted	0.0550	0.0539	0.0567	0.0553	0.0555	0.0555	0.0550	0.0549
Weights only set once	0.0609	0.0593	0.0618	0.0592	0.0603	0.0599	0.0596	0.0596
Betterment w/o views	0.0531	0.0531	0.0531	0.0531	0.0539	0.0540	0.0542	0.0540
Betterment w/o shrunken cvm	0.0613	0.0613	0.0613	0.0613	0.0624	0.0621	0.0623	0.0622
Betterment w historic average as view	0.0621	0.0621	0.0621	0.0621	0.0632	0.0630	0.0632	0.0631

Table 8: Summary of the results of the empirical study in chapter 7. When calculating Sharpe ratios, the risk-free rate is not considered. Costs include transaction costs, expense ratios and Betterment´s managing fees.

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10.1. Research Object

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