Section Exercise 2 Solutions

1) Person A and person B run a website that produces local news stories and videos about the East Bay. In one week, A can produce 5 stories or 10 videos or any linear combination of the two, while person B can produce 7 stories or 8 videos or any linear combination of the two.

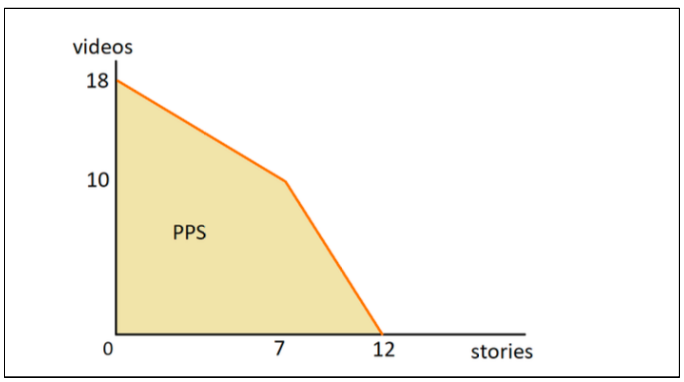
a) Who has the absolute advantage in producing each of the two goods? Explain how you know, and what that means.

A has the absolute advantage in producing videos, and B has the absolute advantage in producing stories. This is because with the given time A can produce more videos than B (10 versus 8) and B can produce more stories than A (7 versus 5).

b) Who has the comparative advantage in producing each of the two goods? Explain how you know, and what that means.

A has the comparative advantage in producing videos, and B has the comparative advantage in producing stories. This is because the opportunity cost of producing an extra video is $\frac{1}{2}$ of a story for A and $\frac{7}{8}$ of a story for B: the opportunity cost of producing an extra video is lower for A. Similarly, the opportunity cost of producing an extra story is 2 videos for A but only $\frac{8}{7}$ of a story for B. The opportunity cost of producing an extra video is lower for B.

c) Sketch the production possibility set for one week the two person team.



d) Compared to a situation in which each person worked equally on the two tasks, how much extra could they produce if they each specialized in one task?

If A split their time equally on the two tasks they could produce 2.5 stories or 5 videos; if B split their time equally they could produce 3.5 stories or 4 videos. In sum: 6 stories and 9 videos.
If each person specialized in the task in which they have the comparative advantage, they could produce 7 stories and 10 videos. That is: one more of each!

(This is the power of comparative advantage. Think about what it would be like if everyone in the world had to produce their own stuff—we'd have way less stuff than we actually do! By the way, if each specialized in the thing that they *don't* have the comparative advantage in, they'd get only 8 videos and 5 stories—less than splitting their time equally!)

e) What factors or forces might the team consider in answering the following normative question: what point in the production possibility set should the team choose?

Since this is a normative question, the number one key thing to think about is: what's the goal? To say what they 'should' do, we'd have to know what they want. If they are a for-profit enterprise, maybe they'd care about what revenue they earn from each type of content. Maybe this website is just for fun, and so they think mainly about what type of thing they like to do. It all depends on the goal!

For example, maybe person A really likes writing stories, and so the production plan has A write stories because. Maybe their website relies on advertising and they can make more money selling ads on stories than on videos, so they decide to produce 12 stories and no videos each week. Maybe their visitors prefer a mix of stories and videos so they choose something in the middle. Maybe they want to produce as much content as possible and so they choose to produce 18 videos per week since this gives them the biggest number of posts.

2) Let's take a look at matrix representations for four different two player, one shot, simultaneous move games. For each one, find the Nash equilibrium or equilibria in pure strategies, if any. In each case, does either player have a dominant strategy?

Game 1 (Player 1 rows, Player 2 columns)

	L	R
U	$3,3$	$0,5$
D	$5,0$	$1,1$

Game 2 (Player 1 rows, Player 2 columns)

	L	R
U	$1,1$	$0,5$
D	$5,0$	$3,3$

Game 3 (Player 1 rows, Player 2 columns)

	L	R
U	$1,1$	$0,0$
D	$0,0$	$1,1$

Game 4 (Player 1 rows, Player 2 columns)

	L	R
U	$1,-1$	$-1,1$
D	$-1,1$	$1,-1$

		Player 2: L	Player 2: R
Player 1	U	$3,3$	$0,5$
	D	$5,0$	$1,1$

The unique Nash equilibrium in this game involves player 1 playing the pure strategy "D" and player 2 playing the pure strategy "R". This is Nash because it is a mutual best response: each player's strategy yields them the highest possible payoff *given* the strategy used by the other player. In this example, both players' pure strategy in this Nash equilibrium is indeed a dominant strategy: it is always a best response, no matter what the other player does. (A game with this structure is known as a *prisoner's dilemma*!)

		Player 2: L	Player 2: R
Player 1	U	$1,1$	$0,5$
	D	$5,0$	$3,3$

Again here the unique Nash equilibrium in this game involves player 1 playing the pure strategy "D" and player 2 playing the pure strategy "R", and again both players' pure strategy in this Nash equilibrium is indeed a dominant strategy: it is always a best response, no matter what the other player does. By comparing these two games we see that this 'selfish' strategic reasoning sometimes leads to mutually destructive outcomes, and sometimes leads to mutually desirable outcomes, in the utilitarian sense!

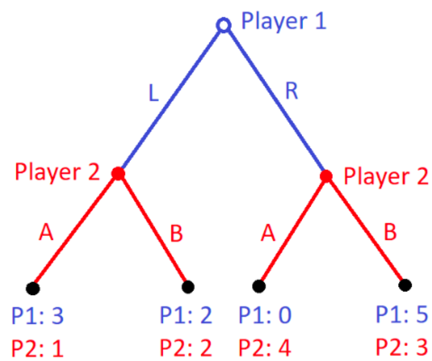
		Player 2: L	Player 2: R
Player 1	U	$1,1$	$0,0$
	D	$0,0$	$1,1$

In this game there are two pure strategy Nash equilibria. In one, player 1 plays U and 2 plays L. In the other, player 1 plays D and 2 plays R. In both cases each player's pure strategy is a best response *given* what the other is doing. In this game, there is no dominant strategy for either player: there is no strategy that is always a best response regardless of what the other is doing. (This type of game is known as a *coordination game*, for obvious reasons!)

		Player 2: L	Player 2: R
Player 1	U	$1,-1$	$-1,1$
	D	$-1,1$	$1,-1$

This game has no pure strategy Nash equilibria. There is no pair of pure strategies such that each player is best responding—in each of the four possibilities one player could have done better given what the other did. Again there is no dominant strategy for either player, for the same reason as in the previous game. (This type of game is known as *matching pennies*, for obscure reasons. Notice that it's like rock-paper-scissors but with 2 actions instead of 3! It turns out that there *is* a Nash equilibrium in this game, but it involves randomization in a way known as a *minimax* strategy. Intuitively: it pays in rock-paper-scissors to be as unpredictable as possible, to make life as difficult for your opponent as you can. This turns out to be a general concept for so-called *zero-sum games* in which each outcome has a winner and an equal and opposite loser. For much more on all of this stuff, take a course that involves some game theory—it's fun!)

3) The following game tree illustrates the order of moves and the payoffs in a two player, sequential move game.



What is the unique outcome of this game that survives the process of backward induction? Explain your answer.

The unique outcome that survives backward induction sees each player earn a payoff of 2. To apply backward induction, first we would look at the last moves before the end of the game and then work back up. In the last stage of this game, player 2 must decide whether to play A or B. If player 2 finds herself at the left-hand decision node (that is, if player 1 played L) then B would earn a higher payoff than A. If however player 2 finds herself at the right-hand decision node (that is, if player 1 played R) then A would earn a higher payoff than B.

Working back up, player 1 can therefore anticipate that if she plays L then player 2 will play B, and if she plays R then player 2 will play A. Therefore, player 1 would do better to play L, earing a payoff of 2, than to play R, earning a payoff of 0. The unique outcome surviving backward induction has player 1 play L and player 2 play B.

(In game theory, there are many more things one could think about in modeling a strategic situation. Do the players know each other's payoff structure? Is there randomness involved in the situation or are outcomes deterministic? Do players know how the other player thinks? Can the players communicate with each other? There is a lot to consider and explore!)

4) Consider the following matrix representation of a two player, one shot, simultaneous move game. Does either player have a dominant strategy? Why or why not? Find all the Nash equilibria in pure strategies, if any exist. Explain why your answers are Nash equilibria, if they exist, or why there are none, if they don't. (Please remember that if there are any Nash equilibria you should write them as what strategy each player uses, not what payoff each player gets.)

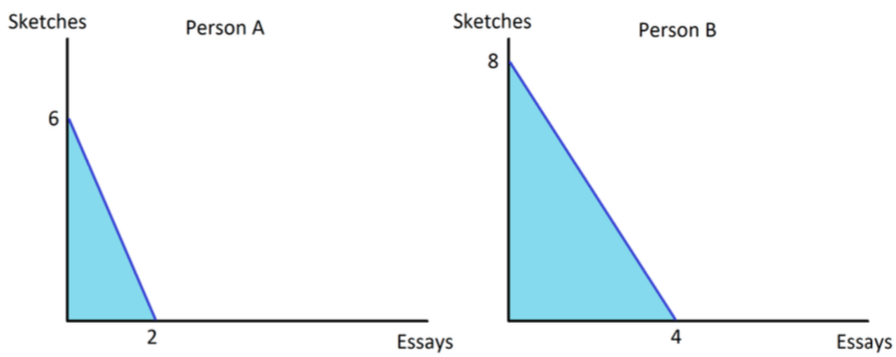
		Player 2	
		L	R
Player 1	U	$0,2$	$6,6$
	D	$2,2$	$2,0$

Neither player has a dominant strategy. It is not the case for either player that there is a particular strategy that always guarantees them a higher payoff than the other regardless of what the other player chose to do. For example: for player 1, U yields a higher payoff than D if player 2 chose R, but D yields a higher payoff than U if player 2 chose L. Therefore, they have no dominant strategy. The same is true for player 2.

There are two Nash equilibria in pure strategies. First: player 1 plays U, player 2 plays R. Second: player 1 plays D, player 2 plays L. In both cases, neither player could have gotten a higher payoff by choosing another strategy, given the strategy chosen by the other player. Each one is doing the best they can given what the other did. For example, in the first one I mentioned: if player 1 plays U then player 2's best response is R, since it gets them a payoff of 6 rather than 2 if they'd chosen L. If player 2 chooses R, player 1's best response is U, since it gets them a payoff of 6 rather than 2 if they'd chosen D. Therefore, each is best-responding given what the other did; it's a Nash equilibrium.

(By the way, in case you're interested, this game is a version of the so-called 'stag hunt' 2x2 game that is sometimes used as a building block in models or studies of social trust.)

5) Two people, A and B, can each produce essays or sketches, or some combination of both. The following two diagrams show the production possibility set for one day for each person.



a) Let's say that A and B, as a team, need to produce a total of 5 essays, but they also want to produce as many sketches as possible. How many sketches will they produce in total? How much of each good will each person produce? Explain your answers.

They will produce 3 sketches in total. Person A will produce 1 essay and 3 sketches; person B will produce 4 essays. The reason is that the opportunity cost of each essay is higher for person A than it is for person B. Each essay requires sacrificing three sketches for person A but only two sketches for person B. Therefore, the way to produce five essays while sacrificing as few sketches as possible is to have person B produce as many essays as they can (four) and only have person A produce essays when and if person B cannot produce any more. You might also have reasoned the other way around: if we're producing all essays, then we have six; if we want to reduce the number of essays by one, we'll get the most sketches if we have person A do them, since the opportunity cost of a sketch is lower for them. Finally, note also that you might have talked about comparative advantage to help explain your answer—person B has the comparative advantage in essays, so have them produce as much as possible—which is totally fine, just remember to try as much as possible to keep your explanations understandable even to someone who doesn't know the jargon!

b) Person C can produce up to 5 essays in one day, up to X sketches in one day, or, if they split their time, a linear combination of both. Person C has the comparative advantage over person A in the production of one of the two goods, but the comparative advantage over person B in the production of the other of the two goods. Suggest a value of X that would be consistent with this information. Explain.

X must be between 10 and 15. To have the comparative advantage in something means to have a lower opportunity cost, and so person C must have a lower opportunity cost of essays than one of the two other people and a lower opportunity cost for sketches than the remaining person. We also know, in case it helps, that if there are two outputs in question, if a person has the comparative advantage over you in one of them, you have the comparative advantage over them in the other.

Person A's opportunity cost for an essay is 3 sketches; person B's is 2 sketches. Person A's opportunity cost for a sketch is $\frac{1}{3}$ of an essay; person B's is $\frac{1}{2}$ of an essay. So we know that the opportunity cost of an essay for person C must be in between 2 and 3; the opportunity cost of a sketch for person C must be between $\frac{1}{3}$ and $\frac{1}{2}$. Any X between 10 and 12 satisfies this. For example: $X = 12$. In that case, person C would have an opportunity cost of 2.4 for an essay and roughly 0.42 for a sketch—they'd have the comparative advantage over person A for essays, but over person B for sketches!

Note that if $X < 10$ it won't work—in that case, both person A and person B would have the comparative advantage over C for sketches. Similarly, if $X > 15$ it won't work—in that case, both A and B would have the comparative advantage over C for essays.

Discussion prompts

1) "It's best if everyone focuses on what they're best at." Agree or disagree?