Contents

Algorithm and parallelization of the software for

Bayesian analysis of hybrid EoS constraints with mass-radius data for compact stars

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Abstract

The algorithm, design and the schema of parallelization of the code for the newly suggested Bayesian analysis using disjunct mass and radius constraints for extracting probability measures for cold, dense nuclear matter equations of state is discussed.

1 Introduction

One of the question of the compact stars phsics is the discussions of deconfinement transition in the cores of stars and whether it proceeds as a crossover or rather as a first order transition. This question is relevant for the problem

of existence of a critical endpoint in the QCD phase diagram, which is actual in present and upcoming heavy-ion collision experiments.

The most basic features of a neutron star (NS) are the radius R and the mass M which so far have not been well determined simultaneously for a single object. In some cases masses are precisely measured like in the case of binary systems, but radii are quite uncertain [1]. In the other hand, for isolated neutron stars some radius and mass measurements exist but lack the necessary precision to allow conclusions about their interiors. In fact, it has been conjectured that there exists a unique relation between M and R for all neutron stars and their equation of state (EoS), thus constraining the properties of their interior [2]. For this reason, accurate observations of masses and radii are crucial to study cold dense nuclear matter expected to exist in neutron stars.

However, the presently observational data allow to make only a probabilistic estimations of the internal structure of the star, via Bayesian analysis (BA). This technique has been applied for the first time to this problem by Steiner et al. [3] who exemplified very well the power of this method. In their analysis, however, only a particular type of objects (X-ray bursters) has been considered under strongly model dependent assumptions. In this work is a continuation of our preliminary probabilistic studies of the super-dense stellar matter equation of state using Bayesian Analysis and modeling of relativistic configurations of neutron stars [4, 5] We put special emphasis on the choice of observational constraints and focus on investigations of the possible existence of deconfined quark matter in massive neutron stars such as the recently observed 2 $M_\odot$ pulsars [6, 7].

The steps to archive the goals of modeling and implementing of the BA are the following

1.1 Making a the Star configuration from given EoS

The microscopical properties of compact stars are modeled in the framework of general relativity, where the Einstein equations are solved for a static (non-rotating), spherical star resulting in the Tolman–Oppenheimer–Volkoff (TOV) equations [8, 9, 10]

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r) \tag{1}$$

$$\frac{dp(r)}{dr} = -G\frac{(\varepsilon(r) + p(r))(m(r) + 4\pi p(r)r^3)}{r(r - 2Gm(r))} \tag{2}$$

as well as the equation for the baryon number profile

$$\frac{dn_B(r)}{dr} = 4\pi r^2 m_N \frac{n_B(r)}{\sqrt{1 - 2Gm(r)/r}} \ . \tag{3}$$

These equations are integrated from the center of the star towards its surface, with the radius of the star R being defined by the condition $p(R) = 0$ and the gravitational mass by $M = m(R)$. In a similar manner, the baryon mass of the star is given by $M_B = m_N n_B(R)$, where m_N is the nucleon mass.

To complete the solution to the TOV equations the EoS is required. It is given by the relation $p = p(\varepsilon)$ which carries information about the microscopic ingredients of the dense nuclear matter, as mentioned before. Thus, the above equations have to be solved simultaneously using the equation of state under boundary conditions at the star centre ($r = 0$), taken as an input. In this way, for a given value of $\varepsilon(r = 0)$ the solution of the TOV equations are the $p(r)$ and $m(r)$ profiles and with them the parametric relationship $M(R)$ can be obtained.

For the present study we use the scheme suggested by Alford, Han and Prakash [11] for defining the hybrid EoS (shorthand: AHP scheme),

$$p(\varepsilon) = p_h(\varepsilon)\Theta(\varepsilon_H - \varepsilon) + p_h(\varepsilon_H)\Theta(\varepsilon - \varepsilon_H)\Theta(\varepsilon_H + \Delta\varepsilon - \varepsilon) + p_q(\varepsilon)\Theta(\epsilon - \epsilon_H - \Delta\epsilon), \tag{4}$$

where $p_h(\varepsilon)$ is a hadronic matter EoS and $p_q(\varepsilon)$ represents the high density matter phase, here considered as deconfined quark matter with the bag model type EoS

$$p_q(\varepsilon) = c_q^2\varepsilon - B. \tag{5}$$

The stiffness of this EoS is given by c_q^2, the squared speed of sound. The positive bag constant B assures confinement, i.e. the dominance of the hadronic EoS at low densities. Note that a parametrization in this form by Haensel et al. [12] describes pretty well the superconducting NJL model derived in [13] and systematically explored in [14]. In the AHP scheme, the hadronic EoS is fixed and not subject to parametric variations. However, we shall use different well-known model EoS in our study the nonrelativistic variational EoS of Akmal et al. [15] (APR) and the density-dependent relativistic meanfield EoS of Typel and Wolter [16] (DD2) in its parametrization from Ref. [17]. Both EoS come eventually with extensions due to an excluded volume for baryons, as described for DD2 in [18].

The free parameters of the model are the transition density ε_H, the energy density jump $\Delta\varepsilon \equiv \gamma\varepsilon_H$ and c_q^2. The set of hybrid EoS in the plane pressure versus energy density is shown in Fig. **??** for APR as the hadronic EoS.

1.2 BA Formulation and Formalization

We define the vector of free parameters $\overrightarrow{\pi}\left(\varepsilon_c, \gamma, c_q^2\right)$ defining the hybrid EoS with a first order phase transition from nuclear to quark matter.

These parameters are sampled

$$\pi_i = \overrightarrow{\pi}\left(\varepsilon_k, \gamma_l, c_{q_m}^2\right), \tag{6}$$

where $i = 0 \ldots N-1$ (here $N = N_1 \times N_2 \times N_3$) as $i = N_1 \times N_2 \times k + N_2 \times l + m$ and $k = 0 \ldots N_1 - 1$, $l = 0 \ldots N_2 - 1$, $m = 0 \ldots N_3 - 1$, here N_1, N_2 and N_3 denote the number of parameters for ε_k, γ_l and $c_{q_m}^2$, respectively. Solving the TOV equations with varying boundary conditions for $\varepsilon(r = 0)$ generates a sequence of $M(R)$ curves characteristic for each of these N EoS. Subsequently, different neutron star observations with their error margins can be used to assign a probability to each choice in from the set of EoS parameters. We use three constraints: (i) the mass constraint for PSR J0348+0432 [7], (ii) the radius constraint for PSR J0437-4715 [19] and (iii) the constraint on the baryon mass at the well measured gravitational mass for the star B in the double pulsar system PSR J0737-3039 [20], which improved the earlier suggestion by Podsiadlowski et al. [21].

The goal is to find the set of most probable π_i based on given constraints using Bayesian Analysis (BA). For initializing BA we propose that a priori each vector of parameter π_i has probability equal one: $P\left(\pi_i\right) = 1$ for $\forall i$.

1.3 Inputs from Observations

1.3.1 Mass constraint for PSR J0348+0432

We propose that the error of this measurement is normal distributed $\mathcal{N}(\mu_A, \sigma_A^2)$, where $\mu_A = 2.01\,\mathrm{M_\odot}$ and $\sigma_A = 0.04\,\mathrm{M_\odot}$ are measured for PSR J0348+0432 [7]. Using this assumption we can calculate conditional probability of the event E_A that the mass of the neutron star corresponds to this measurement

$$P\left(E_A \left| \pi_i\right.\right) = \Phi(M_i, \mu_A, \sigma_A), \tag{7}$$

where M_i is the maximum mass obtained for π_i and $\Phi(x, \mu, \sigma)$ is the cumulative distribution function for the normal distribution.

1.3.2 Radius constraint for PSR J0437-4715

Recently, a radius constraint for the nearest millisecond pulsar PSR J0437-4715 have been obtained [19] giving $\mu_B = 15.5$ km and $\sigma_B = 1.5$ km. With

these data one calculates the conditional probability of the event E_B that the radius of a neutron star corresponds to this measurement

$$P\left(E_B \,|\, \pi_i\right) = \Phi(R_i, \mu_B, \sigma_B). \tag{8}$$

1.3.3 $M{-}M_B$ Relation Constraint for PSR J0737-3039(B)

This constraint gives a region in the $M{-}M_B$ plane. We need to estimate the probability of a point $\mathcal{M}_i = (M_i, M_{Bi})$ to be close to the point $\mu = (\mu_G, \mu_B)$. The mean values $\mu_G = 1.249$, $\mu_B = 1.36$ and standard deviations $\sigma_M = 0.001$, $\sigma_{M_B} = 0.002$ are given in [20]. The probability can be calculated by following formula:

$$P\left(E_K \,|\, \pi_i\right) = \left[\Phi\left(\xi_G\right) - \Phi\left(-\xi_G\right)\right] \cdot \left[\Phi\left(\xi_B\right) - \Phi\left(-\xi_B\right)\right], \tag{9}$$

where $\Phi\left(x\right) = \Phi\left(x, 0, 1\right)$, $\xi_G = \dfrac{\sigma_M}{d_M}$ and $\xi_B = \dfrac{\sigma_{M_B}}{d_{M_B}}$, d_M and d_{M_B} are absolute values of components of vector $d = \mu - M_i$, here $\mu = \left(\mu_G, \mu_B\right)^T$ was given in [20] and $M_i = \left(M_i, M_{Bi}\right)^T$ stems for the solution of TOV equations for the ith vector of EoS parameters π_i. Note that formula (9) does not correspond to a multivariate normal distribution.

1.4 Calculation of a posteriori Probabilities

Note, that these measurements are independent on each other. Therefore, the complete conditional probability of the event that a compact object constructed with an EoS characterized by π_i fulfils all constraints is

$$P\left(E \,|\, \pi_i\right) = P\left(E_A \,|\, \pi_i\right) \times P\left(E_B \,|\, \pi_i\right) \times P\left(E_K \,|\, \pi_i\right). \tag{10}$$

Now, we can calculate probability of π_i using Bayes' theorem:

$$P\left(\pi_i \,|\, E\right) = \frac{P\left(E \,|\, \pi_i\right) P\left(\pi_i\right)}{\displaystyle\sum_{j=0}^{N-1} P\left(E \,|\, \pi_j\right) P\left(\pi_j\right)}. \tag{11}$$

2 Algorithm of the BA

We apply the scheme of BA for probabilistic estimation of the EoS given by vector parameter $\vec{\pi}$. Varying the three parameters in the intervals $400 <$

$\varepsilon[\text{MeV/fm}^3] < 1200$, $0 < \gamma < 1.5$ and $0.3 < c_s^2 < 1.0$ with $N_1 = N_2 = N_3 = 13$ we explore a set of $N = 2197$ hybrid EoS for each choice of hadronic EoS. [22, 23, 18]).

Note that the occurrence of high-mass twins is testable by observations. It requires precise radius measurements of $2\text{M}_\odot$ pulsars in order to verify the existence of an almost horizontal branch of high-mass pulsars with almost the same mass but radii differing by up to a few kilometers. As a necessary condition for such sequences the hadronic star sequence should have radii exceeding 12 km. The observation of this striking high-mass twin phenomenon would indicate a strong first-order phase transition in neutron star matter which gives important constraints for searches of a critical endpoint in the QCD phase diagram.

Fig. 1...

(see the Algorithm 1)

3   Parallelisation

-speedup whenn parallel

-scalability: up to which parallelitt is code working

-load balance: small computations(e.g. small nc) in compare to big computations

Flow chat of regions to be parallelized. Adding automated part to find extemum instead of visual manual analysis

4   Conclusion and Outlook

-full size problem solution for pre ToV at 0

-estimation for cooling

-estimation for rotation

-MonteCarlo: a way to reduce computation amount, but not applicable for correct bayes as of sum will not be full and for all region

5   Acknowledgment

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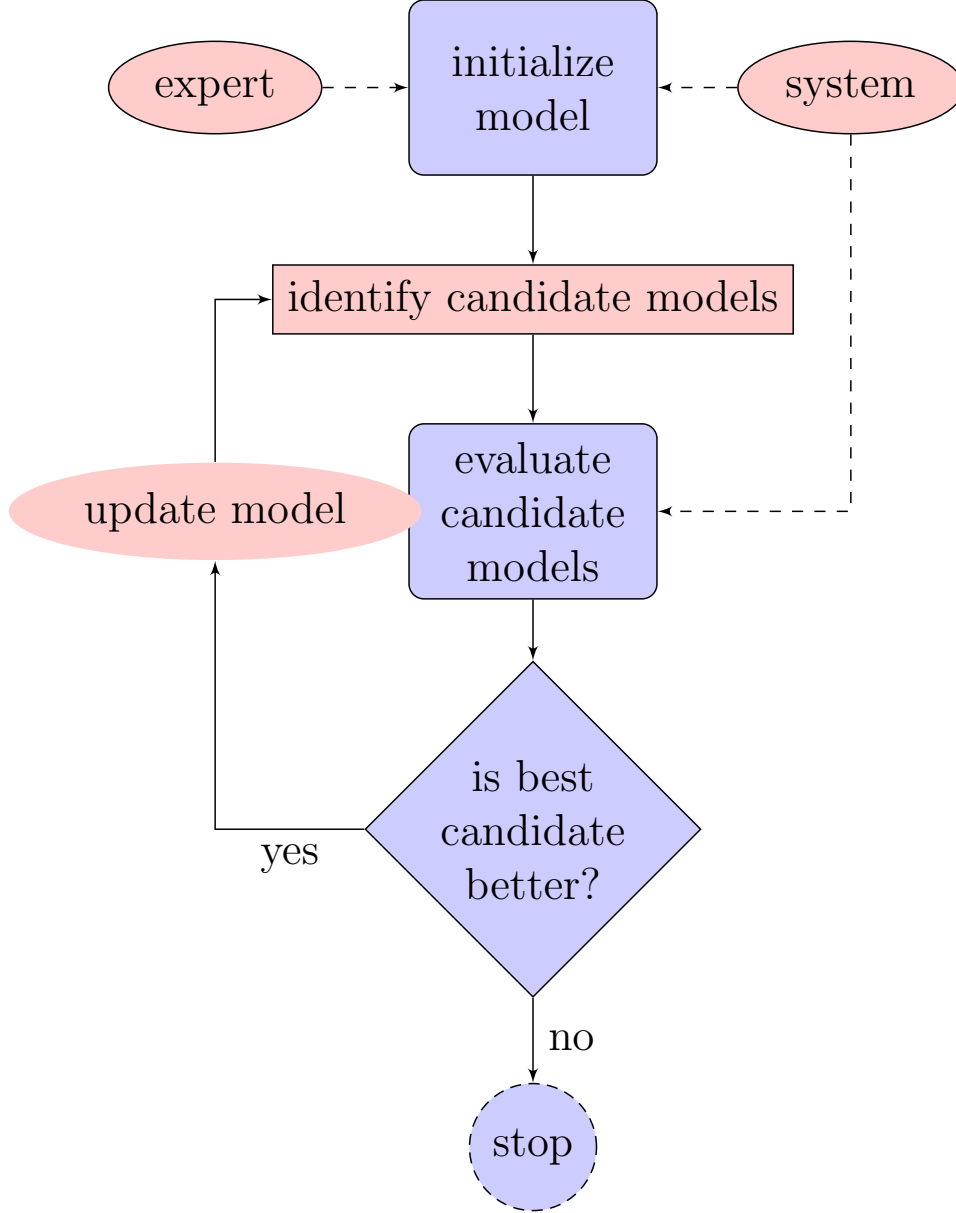
Figure 1: Flow diagram of the `parallel bayes` for running `TOV` at different frequency bands in separate MPI worlds/groups. Bricks represent usual algorithmic steps; diamonds are steps with decisions about future direction of run; ellipses are steps when message passing take place represented as dash-dotted paths; blocks with rounded corners represent steps in which `MASTER` or `SLAVE` ranks are waiting until a new MPI message is received.

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Algorithm 1: Algorithm for initialization, domain decomposition and bookkeeping

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Data: MASTER rank
Result: Preparation of parallel simulations
Read and sort the band table;
Initializing the bookkeeping;
Define the domains as groups;
while *infinitely* **do**
    Check and count free slaves;
    if *not enough free slaves* **then**
        | go into waiting state
    end
    Check bookkeeping for bands to run;
    if *all groups fit* **then**
        for *each group* **do**
            Send group size and band value to slaves;
            Create new communicators;
            Set the slaves as busy;
            Set the groups as submitted;
        end
    else
        if *biggest group does not fit* **then**
            | Stop simulation;
        else
            if SLAVE*s are busy* **then**
                Go into waiting state;
                Finished message received;
                Set slaves free;
            else
                | Finish simulation
            end
        end
    end
end