Visualization of hyperbolic functions : $\tanh(\alpha) = \tan(\varphi)$ for φ corresponding to α.

Im

$\cosh(\alpha)$

P'

j

$\tan(\varphi) = \tanh(\alpha)$

C'

C

P

complex plane

$\dfrac{\alpha}{2}$

$\sinh(\alpha)$

$-\dfrac{\alpha}{2}$

O

1

A

Re

$\left|\overline{AC}\right| = \tanh(\alpha) = \tan(\varphi)$

$\left|\overline{OP}\right| = \sqrt{\cosh^2(\alpha) + \sinh^2(\alpha)}$

Area $d\alpha/2 = \dfrac{1}{2}\left|\overline{OP}\right|^2 d\varphi$ (to first order)

$\cosh^2(\alpha) - \sinh^2(\alpha) = 1$