50.002 COMPUTATIONAL STRUCTURES

INFORMATION SYSTEMS TECHNOLOGY AND DESIGN

Problem Set 7

1 FSM Possibility

We saw that certain functions, such as parentheses checking, cannot be performed by any finite state machine. Which of the following can be performed by an FSM? Assume, in each case, that the device is to take a series of 0s and 1s that represent the digits of a binary number entered left-to-right. The device is to have a single output, which is 1 only under the specified conditions:

1. When the last 277 digits entered have been alternate 1s and 0s.
   Solution:
   For the rest of this questions, always note that you can use an FSM as long as you have FINITE states.

   So, yes. Its a bit tedious for 277 digits, but you should be able to sketch one for 3 or 4 digits.

2. When more 0s than 1s have been entered.
   Solution:
   No. Requires unbounded counting.

3. When the number entered thus far is divisible by 3.
   Solution:
   Yes, can be done by a 3-state machine.

4. When an odd number of 1s and and even number of 0s have been entered.
   Solution:
   Yes, simple exercise.

5. When the number entered corresponds to a year in which the Red Sox win the world series
   Solution:
   (Easy answer: always 0!). Assuming there are a bounded number of such years (otherwise, dream on...) its easy to design such an FSM from a list of the years.

2 Turing Machine

1. Ben Bitdiddle's proposed Ph.D. thesis involves writing a program to compute a function f(x) on a Cray supercomputer. Ben's advisor points out that f cannot be computed on any Turing machine. Should Ben care? Why?

   Solution:
   If the function can't be computed on any Turing machine, then it can't be computed on any physically realizable machine that we know of. So Ben is out of luck... a Cray supercomputer isn't "super" in that sense.

2. Discouraged by your answer to the last question, Ben has turned his attention to an alternative thesis topic. He now proposes to invent the universal FSM, which will be to FSMs what a universal Turing machine is to Turing machines. Ben's idea is to build an FSM that can fed a sequence of inputs describing any other FSM and the inputs to that FSM. The universal FSM would then emulate the behavior of the described FSM on the specified inputs. Is Ben's idea workable? Why or why not?
   Solution:
   Unfortunately, the Universal FSM will have some fixed number (N) of states built into its design. So it won't have enough states to emulate machines with more than N states. Ben's idea isn't workable.

3 CPU

1. What are the four most important components of the von Neumann machine? Draw their correlations.
   Solution:
   CPU, Memory, Device, and Bus. The connection is that the CPU, Memory, and Device are interconnected by the bus.

2. What are the four most important components of a CPU?
   Solution:
   This is the β architecture: PC, Registers, Control Unit, and ALU

3. How much memory can a 32-bit von Neumann machine have? Why?
   Solution:
   2^{32} because each address is 32 bits long (A word).

4. Can a CPU have as many registers as possible, in theory?
   Solution:
   No. The registers must be encoded in instructions, i.e: 5 bits for 32 registers. An instruction is 32 bits long for β architecture, so having too many registers will make encoding infeasible.