

# Database and Big Data(2019)

## Week 4, S1: Functional dependencies

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# **DATABASE DESIGN**

How do we design a “good” database schema?

We want to ensure the integrity of the data.

We also want to get good performance.

# EXAMPLE DATABASE

**student(sid, course, room, grade, name, address)**

sid	course	room	grade	name	address
123	Philo	LT2	A	Agus	Labrador Park
456	Maths	LT5	B	Bron	Bukit Brown
789	Philo	LT2	A	Hannah	Sungei Buloh
012	Philo	LT2	C	Dewi	Bukit Puaka
789	Maths	LT5	A	Hannah	Sungei Buloh

# EXAMPLE DATABASE

**student**(sid, course, room, grade, name, address)

sid	course	room	grade	name	address
123	Philo	LT2	A	Agus	Labrador Park
456	Maths	LT5	B	Bron	Bukit Brown
789	Philo	LT2	A	Hannah	Sungei Buloh
012	Philo	LT2	C	Dewi	Bukit Puaka
789	Maths	LT5	A	Hannah	Sungei Buloh

# REDUNDANCY PROBLEMS

## Update Anomalies

- If the room number changes, we need to make sure that we **change all** students records.

## Insert Anomalies

- May not be possible to add a student unless they're enrolled in a course.

## Delete Anomalies

- If all the students enrolled in a course are deleted, then we lose the room number.

# EXAMPLE DATABASE

**student(sid,name,address)**

sid	name	address
123	Agus	Labrador Park
456	Bron	Bukit Brown
789	Hannah	Sungei Buloh
012	Dewi	Bukit Puaka

**course (sid,course,grade)**

Sid	course	grade
123	Philo	A
456	Maths	B
789	Philo	A
012	Maths	C
789	Philo	A

**room (course,room)**

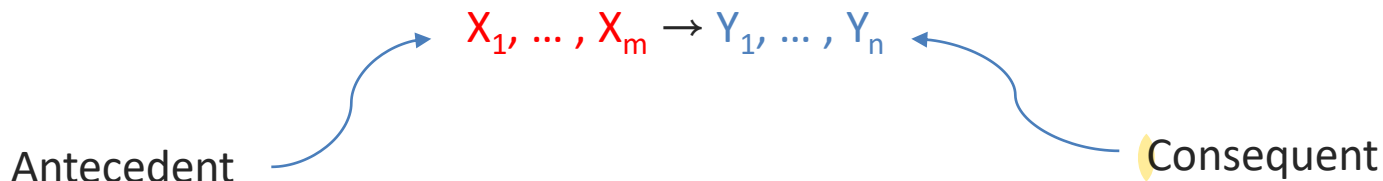
course	room
Philo	LT2
Maths	LT5

Why is this decomposition better?  
How to find it?

- If student graduates, we delete students and course table but we will still have room table.
- Decomposition is more robust.

# FUNCTIONAL DEPENDENCIES

A functional dependency is a **constraint** between two sets of attributes in a relation from a database.



The value of  $X_1, \dots, X_m$  functionally determines the value of  $Y_1, \dots, Y_n$

*Given  $X$ , we can determine  $Y$ .*

# FUNCTIONAL DEPENDENCIES

More formally,

A functional dependency  $X_1, \dots, X_m \rightarrow Y_1, \dots, Y_n$   
holds in a relation  $R$  if:

$$\forall t, t' \in R, t[X_1] = t'[X_1] \cap \dots \cap t[X_m] = t'[X_m] \rightarrow t[Y_1] = t'[Y_1] \cap \dots \cap t[Y_n] = t'[Y_n]$$

$R1(\underline{\text{sid}}, \text{name}, \text{address})$

sid	name	address
123	Agus	Labrador Park
456	Bron	Bukit Brown
789	Hannah	Sungei Buloh
012	Dewi	Bukit Puaka



# FUNCTIONAL DEPENDENCIES

e.g.,

$X_1$  : sid

$Y_1, Y_2$  : name, address

More formally,

A functional dependency  $X_1, \dots, X_m \rightarrow Y_1, \dots, Y_n$  holds in a relation  $R$  if:

$$\forall t, t' \in R, t[X_1] = t'[X_1] \wedge \dots \wedge t[X_m] = t'[X_m] \rightarrow t[Y_1] = t'[Y_1] \wedge \dots \wedge t[Y_n] = t'[Y_n]$$

$R1(\underline{\text{sid}}, \text{name}, \text{address})$

sid	name	address
123	Agus	Labrador Park
456	Bron	Bukit Brown
789	Hannah	Sungei Buloh
012	Dewi	Bukit Puaka
123	Agus	Labrador Park

# FUNCTIONAL DEPENDENCIES

More formally,

A functional dependency  $X_1, \dots, X_m \rightarrow Y_1, \dots, Y_n$  holds in a relation  $R$  if:

$$\forall t, t' \in R, t[X_1] = t'[X_1] \cap \dots \cap t[X_m] = t'[X_m] \rightarrow t[Y_1] = t'[Y_1] \cap \dots \cap t[Y_n] = t'[Y_n]$$

$R1(\underline{\text{sid}}, \text{name}, \text{address})$

sid	name	address
123	Agus	Labrador Park
456	Bron	Bukit Brown
789	Hannah	Sungei Buloh
012	Dewi	Bukit Puaka



$\text{sid} \rightarrow \text{name}$

# FUNCTIONAL DEPENDENCIES

FD is a constraint that allows instances for which the FD holds

You can check (1) if a FD is violated by an instance, (2) but you cannot prove that a FD is part of the schema using an instance.

e.g.:

- (1) if  $\text{sid} \rightarrow \text{name}$  is given to us, we can identify a violation on the right table.
- (2) From the initial table we could not deduce a FD:  $\text{name} \rightarrow \text{address}$

R1(sid,name,address)

sid	name	address
123	Agus	Labrador Park
456	Bron	Bukit Brown
789	Hannah	Sungei Buloh
012	Dewi	Bukit Puaka
555	Agus	East Coast Park
456	Briac	Bukit Brown



# FUNCTIONAL DEPENDENCIES

Two FDs  $X \rightarrow Y$  and  $X \rightarrow Z$  can be written in shorthand as  $X \rightarrow YZ$ .

But  $XY \rightarrow Z$  is **not the same** as the two FDs  $X \rightarrow Z$  and  $Y \rightarrow Z$ .

# WHY SHOULD I CARE?

FDs seem important, but what can we actually do with them?

They allow us to decide whether a database design is correct.

→ Note that this is different than the question of whether it's a good idea for performance...

# IMPLIED DEPENDENCIES

**student(sid, course, room, grade, name, address)**

sid	course	room	grade	name	address
123	Philo	LT2	A	Agus	Labrador Park
456	Maths	LT5	B	Bron	Bukit Brown
789	Philo	LT2	A	Hannah	Sungei Buloh
012	Philo	LT2	C	Dewi	Bukit Puaka

Provided FDs

**sid** → **name, address**

**sid, course** → **grade**

Implied FDs

**name, address, course** → **grade**

**sid, course** → **sid**

**sid, course** → **course**

# IMPLIED DEPENDENCIES

Given a set of FDs  $\{f_1, \dots, f_n\}$ , how do we decide whether a FD  $g$  holds?

Compute the closure using **Armstrong's axioms** which is the set of all implied FDs.

# ARMSTRONG'S AXIOMS (Primary rules)

Reflexivity:

$$Y \subseteq X \Rightarrow X \rightarrow Y$$

e.g.,  $\{\text{sid}\} \subseteq \{\text{sid}, \text{course}\}$ , so,  $\{\text{sid}, \text{course}\} \rightarrow \{\text{sid}\}$

Augmentation:

$$X \rightarrow Y \Rightarrow XZ \rightarrow YZ$$

e.g.,  $\text{sid} \rightarrow \text{name}$ , so,  $\text{sid}, \text{course} \rightarrow \text{name}, \text{course}$

Transitivity:

$$X \rightarrow Y \wedge Y \rightarrow Z \Rightarrow X \rightarrow Z$$

e.g.,  $\text{sid}, \text{course} \rightarrow \text{sid}$  and  $\text{sid} \rightarrow \text{name}$ , so  $\text{sid}, \text{course} \rightarrow \text{name}$



# ARMSTRONG'S AXIOMS (Secondary rules)

## Union:

$$(X \rightarrow Y) \wedge (X \rightarrow Z) \Rightarrow X \rightarrow YZ$$

e.g.,  $\text{sid} \rightarrow \text{name}$  and  $\text{sid} \rightarrow \text{address}$ , so  $\text{sid} \rightarrow \text{name, address}$

## Decomposition

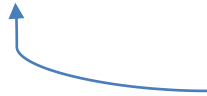
$$X \rightarrow YZ \Rightarrow (X \rightarrow Y) \wedge (X \rightarrow Z)$$

e.g.,  $\text{sid} \rightarrow \text{name, address}$ , so,  $\text{sid} \rightarrow \text{name}$  and  $\text{sid} \rightarrow \text{address}$

## Pseudo-transitivity

$$X \rightarrow Y \wedge YW \rightarrow Z \Rightarrow XW \rightarrow Z$$

e.g.,  $\text{sid} \rightarrow \text{sid}$  and  $\text{sid, course} \rightarrow \text{grade}$ , so  $\text{sid, course} \rightarrow \text{grade}$



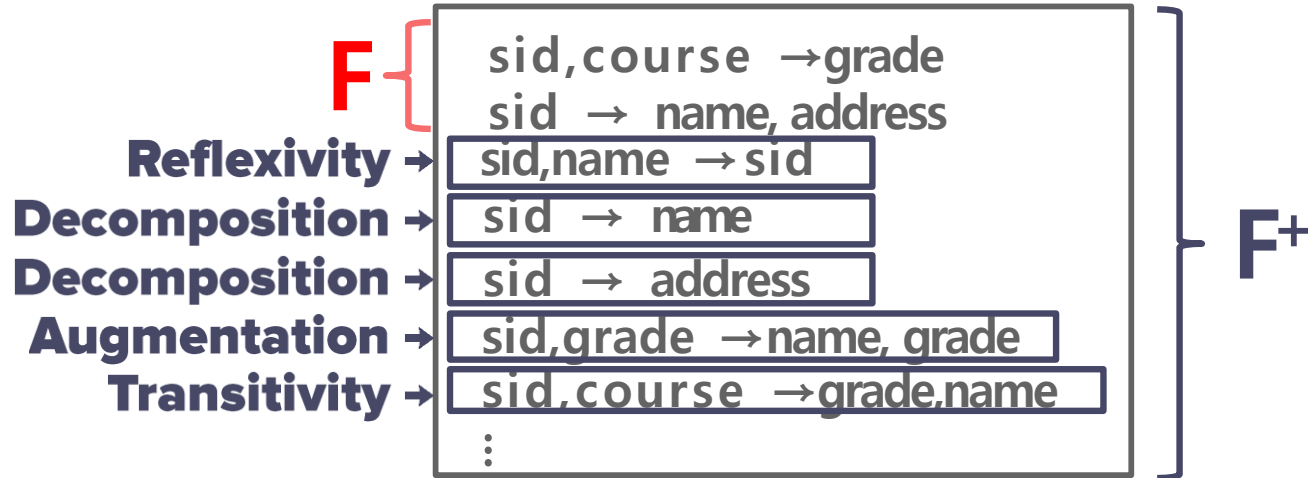
Why?

*sid -> sid  
reflexivity rule used.*

# CLOSURES

Given a set **F** of FDs  $\{f_1, \dots, f_n\}$ , we define the closure **F<sup>+</sup>** is the set of all implied FDs.

**student**(sid, course, room, grade, name, address)



# WHY DO WE NEED THE CLOSURE?

With the closure we can find all FD's easily and then compute the attribute closure:

→ For a given attribute  $X$ , the closure  $X^+$  is the set of all attributes such that  $X \rightarrow A$  can be inferred using the Armstrong's Axioms.

To check if  $X \rightarrow A$  :

→ Compute  $X^+$

→ Check if  $A \in X^+$

## BUT AGAIN, WHY SHOULD I CARE?

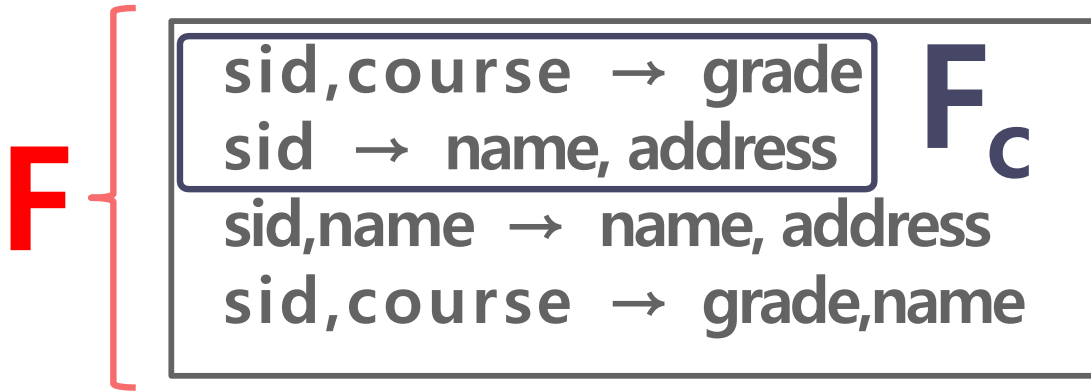
Maintaining the closure at runtime is expensive:

→ The DBMS has to check all the constraints for every **INSERT**, **UPDATE**, and **DELETE** operation.

We want a **minimal set** of FDs that was enough to ensure correctness.

# CANONICAL COVER

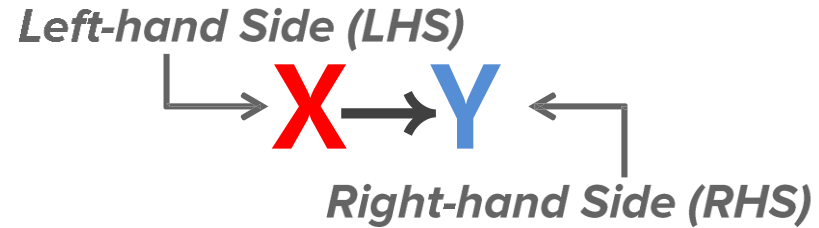
Given a set **F** of FDs  $\{f_1, \dots, f_n\}$ , we define the canonical cover **F<sub>c</sub>** as the minimal set of all FDs.



# CANONICAL COVER DEFINITION

A canonical cover  $F_c$  must have the following properties:

1. The RHS of every FD is a single attribute.
2. The closure of  $F_c$  is identical to the closure of  $F$  (i.e.,  $F_c^+ = F^+$  are equivalent).
3. The  $F_c$  is minimal (i.e., if we eliminate any attribute from the LHS or RHS of a FD, property #2 is violated).

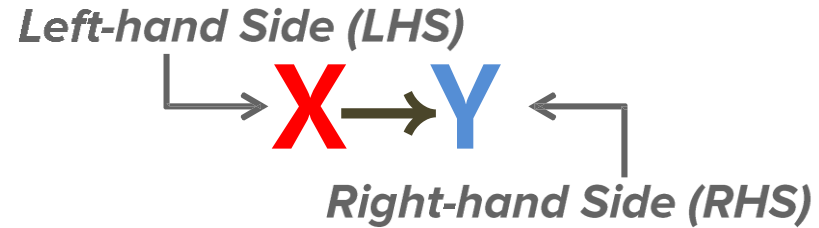


# COMPUTING THE CANONICAL COVER

Given a set **F** of FDs, examine each FD:

- Drop extraneous LHS or RHS attributes; or redundant FDs.
- Make sure that FDs have a single attribute in their RHS.

Repeat until no change.



# COMPUTING THE CANONICAL COVER (1)

**F:**

$AB \rightarrow C$  (1)

$A \rightarrow BC$  (2)

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)



# COMPUTING THE CANONICAL COVER (1)

**F:**

$AB \rightarrow C$  (1)

$A \rightarrow BC$  (2)

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

*Split (2)*



**F<sub>1</sub>:**

$AB \rightarrow C$  (1)

$A \rightarrow B$  (2')

$A \rightarrow C$  (2'')

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

## COMPUTING THE CANONICAL COVER (2)

**F<sub>1</sub>:**

$AB \rightarrow C$  (1)

$A \rightarrow B$  (2')

$A \rightarrow C$  (2'')

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

*Eliminate (2')*



**F<sub>2</sub>:**

$AB \rightarrow C$  (1)

$A \rightarrow C$  (2'')

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

## COMPUTING THE CANONICAL COVER (3)

**F<sub>2</sub>:**

$AB \rightarrow C$  (1)

$A \rightarrow C$  (2'')

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

*Eliminate (2'')*



**F<sub>3</sub>:**

$AB \rightarrow C$  (1)

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

# COMPUTING THE CANONICAL COVER (4)

**F<sub>3</sub>:**

**A**B → C    (1)  
B → C    (3)  
A → B    (4)

*Eliminate A  
from (1)*



**F<sub>4</sub>:**

B → C    (1')  
B → C    (3)  
A → B    (4)



# COMPUTING THE CANONICAL COVER (5)

**F<sub>4</sub>:**

**B → C** (1')

**B → C** (3)

**A → B** (4)

*Eliminate (1')*



**F<sub>5</sub>:**

**B → C** (3)

**A → B** (4)

# COMPUTING THE CANONICAL COVER (5)

$F_c$

$F_4:$

$B \rightarrow C$  (1')

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

*Eliminate (1')*



$F_5:$

$B \rightarrow C$  (3)

$A \rightarrow B$  (4)

- ✓ Nothing is extraneous
- ✓ All RHS are single attributes
- ✓ Final & original set of FDs are equivalent (same closure)

# NO REALLY, WHY SHOULD I CARE?

The canonical cover is the minimum number of assertions that we need to implement to make sure that our database integrity is correct.

It allows us to find the super key for a relation.

# RELATIONAL MODEL: KEYS (1)

## **Super Key:**

- Any set of attributes in a relation that functionally determines all attributes in the relation.

## **Candidate Key:**

- Any super key such that the removal of any attribute leaves a set that does not functionally determine all attributes.



# RELATIONAL MODEL: KEYS (2)

## Super Key:

- Set of attributes for which there are **no two distinct tuples** with the same values for the attributes in this set.

## Candidate Key:

- Set of attributes that **uniquely** identifies a tuple according to a key constraint.

# RELATIONAL MODEL: KEYS (3)

## **Super Key:**

→ A set of attributes that uniquely identifies a tuple.

## **Candidate Key:**

→ A minimal set of attributes that uniquely identifies a tuple.

## **Primary Key:**

→ Usually just the candidate key.

# RELATIONAL MODEL: KEYS

student(sid, course, room, grade, name, address)

sid	course	room	grade	name	address
123	Philo	LT2	A	Agus	Labrador Park
456	Maths	LT5	B	Bron	Bukit Brown
789	Philo	LT2	A	Hannah	Sungei Buloh
012	Philo	LT2	C	Dewi	Bukit Puaka

Provided FDs

**sid** → name, address

**sid, course** → grade

**course** → room

**sid, course, room** is a **superkey**.

**sid, course** is a **candidate key**, that we can choose as our **primary key**. When a key has several attributes, we say that it is a **composite key**.

# BUT WHY CARE ABOUT SUPER KEYS?

They help us determine whether it is okay to decompose a table into multiple sub-tables.

Super keys ensure that we are able to recreate the original relation through joins.

# SCHEMA DECOMPOSITIONS

Split a single relation **R** into a set of relations  **$\{R_1, \dots, R_n\}$** .

Not all decompositions make the database schema better:

- Update Anomalies
- Insert Anomalies
- Delete Anomalies
- Wasted Space

# DECOMPOSITION GOALS

## Loseless Joins

- Want to be able to reconstruct original relation by joining smaller ones using a natural join.

## Dependency Preservation

- Want to minimize the cost of global integrity constraints based on FD's.

## Redundancy Avoidance

- Avoid unnecessary data duplication.

# DECOMPOSITION GOALS

## Loseless Joins

→ Want to be able to reconstruct original relation by joining smaller ones using a natural join.

← **Mandatory!**

## Dependency Preservation

→ Want to minimize the cost of global integrity constraints based on FD's.

## Redundancy Avoidance

→ Avoid unnecessary data duplication.

← **Nice to have,  
but not required**

# LOSSLESS DECOMPOSITION (1)

Provided FDs  
**bname** → **bcity, assets**  
**loanId** → **amt, bname**

loans(bname, bcity, assets, cname, loanId, amt)

bname	bcity	assets	cname	loanId	amt
Downtown	Pittsburgh	\$9M	Andy	L-17	\$1000
Downtown	Pittsburgh	\$9M	Oswin	L-23	\$2000
Compton	Los Angeles	\$2M	Andy	L-93	\$500
Downtown	Pittsburgh	\$9M	Damian	L-17	\$1000



# LOSSLESS DECOMPOSITION (1)

Provided FDs  
**bname** → **bcity, assets**  
**loanId** → **amt, bname**

loans (bname, bcity, assets, **cname**, loanId, amt)

R1(bname, bcity, assets, cname)

bname	bcity	assets	cname
Downtown	Pittsburgh	\$9M	Andy
Downtown	Pittsburgh	\$9M	Oswin
Compton	Los Angeles	\$2M	Andy
Downtown	Pittsburgh	\$9M	Damian

R2(cname, loanId, amt)

cname	loanId	amt
Andy	L-17	\$1000
Oswin	L-23	\$2000
Andy	L-93	\$500
Damian	L-17	\$1000

# LOSSLESS DECOMPOSITION (1)

Provided FDs  
**bname** → **bcity**, **assets**  
**loanId** → **amt**, **bname**

R1(bname, bcity, assets, cname)

bname	bcity	assets	cname
Downtown	Pittsburgh	\$9M	Andy
Downtown	Pittsburgh	\$9M	Oswin
Compton	Los Angeles	\$2M	Andy
Downtown	Pittsburgh	\$9M	Damian

R2(cname, loanId, amt)

cname	loanId	amt
Andy	L-17	\$1000
Oswin	L-23	\$2000
Andy	L-93	\$500
Damian	L-17	\$1000



# LOSSLESS DECOMPOSITION (1)

Provided FDs  
**bname** → **bcity**, **assets**  
**loanId** → **amt**, **bname**

R1(bname, bcity, assets, cname)

bname	bcity	assets	cname
Downtown	Pittsburgh	\$9M	Andy
Downtown	Pittsburgh	\$9M	Oswin
Compton	Los Angeles	\$2M	Andy
Downtown	Pittsburgh	\$9M	Damian

R2(cname, loanId, amt)

cname	loanId	amt
Andy	L-17	\$1000
Oswin	L-23	\$2000
Andy	L-93	\$500
Damian	L-17	\$1000



bname	bcity	assets	cname	loanId	amt
Downtown	Pittsburgh	\$9M	Andy	L-17	\$1000
Downtown	Pittsburgh	\$9M	Andy	L-93	\$500
Downtown	Pittsburgh	\$9M	Oswin	L-23	\$2000
Compton	Los Angeles	\$2M	Andy	L-17	\$1000
Compton	Los Angeles	\$2M	Andy	L-93	\$500
Downtown	Pittsburgh	\$9M	Damian	L-17	\$1000

# LOSSLESS DECOMPOSITION (2)

R1(bname,bcity,assets,cname)

bname	bcity	assets	cname
Downtown	Pittsburgh	\$9M	Andy
Downtown	Pittsburgh	\$9M	Oswin
Compton	Los Angeles	\$2M	Andy
Downtown	Pittsburgh	\$9M	Damian



R2(bname,loanId,amt)

bname	loanId	amt
Downtown	L-17	\$1000
Downtown	L-23	\$2000
Compton	L-93	\$500

Provided FDs

bname → bcity, assets

loanId → amt, bname

# LOSSLESS DECOMPOSITION (2)

R1(bname,bcity,assets,cname)

bname	bcity	assets	cname
Downtown	Pittsburgh	\$9M	Andy
Downtown	Pittsburgh	\$9M	Obama
Compton	Los Angeles	\$2M	Andy
Downtown	Pittsburgh	\$9M	Damian

R2(cname,loanId,amt)

bname	loanId	amt
Downtown	L-17	\$1000
Downtown	L-23	\$2000
Compton	L-93	\$500



bname	bcity	assets	cname	loanId	amt
Downtown	Pittsburgh	\$9M	Andy	L-17	\$1000
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Compton	Los Angeles	\$2M	Andy	L-93	\$500
Downtown	Pittsburgh	\$9M	Damian	L-23	\$1000



# LOSSLESS DECOMPOSITION (3)

R1(bname,assets,cname,loanId)

bname	assets	cname	loanId
Downtown	\$9M	Andy	L-17
Downtown	\$9M	Oswin	L-23
Compton	\$2M	Andy	L-93
Downtown	\$9M	Damian	L-17



R2(loanId,bcity,amt)

loanId	bcity	amt
L-17	Pittsburgh	\$1000
L-23	Pittsburgh	\$2000
L-93	Los Angeles	\$500

Provided FDs  
**bname** → **bcity**, **assets**  
**loanId** → **amt**, **bname**

# LOSSLESS DECOMPOSITION (3)

R1(bname,assets,cname,loanId)

bname	assets	cname	loanId
Downtown	\$9M	Andy	L-17
Downtown	\$9M	Oswin	L-23
Compton	\$2M	Andy	L-93
Downtown	\$9M	Damian	L-17



R2(loanId,bcity,amt)

loanId	bcity	amt
L-17	Pittsburgh	\$1000
L-23	Pittsburgh	\$2000
L-93	Los Angeles	\$500

Provided FDs

bname → bcity, assets

loanId → amt, bname



bname	bcity	assets	cname	loanId	amt
Downtown	Pittsburgh	\$9M	Andy	L-17	\$1000
Downtown	Pittsburgh	\$9M	Oswin	L-23	\$2000
Compton	Los Angeles	\$2M	Andy	L-93	\$500
Downtown	Pittsburgh	\$9M	Damian	L-17	\$1000

# DEPENDENCY PRESERVATION

A schema preserves dependencies if its original FDs **do not span multiple tables**.

Why does this matter?

→ It would be expensive to check (assuming that our DBMS supports ASSERTIONS).



# DEPENDENCY PRESERVATION (1)

**R1(bname,assets,cname,loanId)**

bname	assets	cname	loanId
Downtown	\$9M	Andy	L-17
Downtown	\$9M	Oswin	L-23
Compton	\$2M	Andy	L-93
Downtown	\$9M	Damian	L-17

**R2(loanId,bcity,amt)**

loanId	bcity	amt
L-17	Pittsburgh	\$1000
L-23	Pittsburgh	\$2000
L-93	Los Angeles	\$500

Provided FDs  
**bname** → **bcity,assets**  
**loanId** → **amt,bname**

# DEPENDENCY PRESERVATION (1)

R1(bname,assets,cname,loanId)

bname	assets	cname	loanId
Downtown	\$9M	Andy	L-17
Downtown	\$9M	Oswin	L-23
Compton	\$2M	Andy	L-93
Downtown	\$9M	Damian	L-17

R2(loanId,bcity,amt)

loanId	bcity	amt
L-17	Pittsburgh	\$1000
L-23	Pittsburgh	\$2000
L-93	Los Angeles	\$500

canonical form

*loadid -> bname*

*bname -> bcity*

*bname -> assets*

*loadid -> amt*

Provided FDs  
**bname** → **bcity,assets**  
**loanId** → **amt,bname**



# DEPENDENCY PRESERVATION (1)

R1(bname,assets,cname,loanId)

bname	assets	cname	loanId
Downtown	\$9M	Andy	L-17
Downtown	\$9M	Obama	L-23
Compton	\$2M	Andy	L-93
Downtown	\$9M	DJ Snake	L-17

R2(loanId,bcity,amt)

loanId	bcity	amt
L-17	Pitt burgh	\$1000
L-23	Pitt burgh	\$2000
L-93	Los Angeles	\$500

Provided FDs  
**bname** → **bcity,assets**  
**loanId** → **amt,bname**

# DEPENDENCY PRESERVATION

To test whether the decomposition  $R = \{R_1, \dots, R_n\}$  preserves the FD set  $F$ :

- Compute  $F^+$
- Compute  $G$  as the union of the set of FDs in  $F^+$  that are covered by  $\{R_1, \dots, R_n\}$
- Compute  $G^+$
- If  $F^+ = G^+$ , then  $\{R_1, \dots, R_n\}$  is Dependency Preserving

# DEPENDENCY PRESERVATION (2)

Is  $R=\{R_1, R_2\}$  dependency preserving?

$$F^+ = \{A \rightarrow B, AB \rightarrow D, A \rightarrow D, C \rightarrow D\}$$

$R_1(A, B, C) \quad R_2(C, D)$
$F = \{A \rightarrow B, AB \rightarrow D, C \rightarrow D\}$

# DEPENDENCY PRESERVATION (2)

Is  $R=\{R_1, R_2\}$  dependency preserving?

$$F^+ = \{A \rightarrow B, AB \rightarrow D, A \rightarrow D, C \rightarrow D\}$$

$$G = \{A \rightarrow B\} \cup \{C \rightarrow D\}$$

FDs covered by  $R_1$       FDs covered by  $R_2$

$R_1(A, B, C) \quad R_2(C, D)$
$F = \{A \rightarrow B, AB \rightarrow D, C \rightarrow D\}$

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# DEPENDENCY PRESERVATION (2)

Is  $R=\{R_1, R_2\}$  dependency preserving?

$$(A \rightarrow D) \in F^+$$

$$F^+ = \{A \rightarrow B, AB \rightarrow D, \boxed{A \rightarrow D}, C \rightarrow D\}$$

$$G = \{A \rightarrow B\} \cup \{C \rightarrow D\}$$

$$G^+ = \{A \rightarrow B, C \rightarrow D\}$$

$F^+ \neq \boxed{G^+}$  because  $(A \rightarrow D) \in (F^+ - G^+)$   
 $(A \rightarrow D) \notin G^+$

$R1(A, B, C) \quad R2(C, D)$

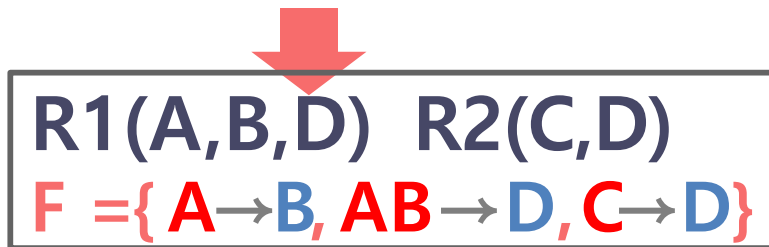
$$F = \{A \rightarrow B, AB \rightarrow D, C \rightarrow D\}$$

*Decomposition is not DP*



## DEPENDENCY PRESERVATION (3)

Is  $R=\{R_1, R_2\}$  dependency preserving?



## DEPENDENCY PRESERVATION (3)

Is  $R=\{R_1, R_2\}$  dependency preserving?

$$F^+ = \{A \rightarrow B, AB \rightarrow D, A \rightarrow D, C \rightarrow D\}$$

$$G = \{A \rightarrow B, A \rightarrow D, AB \rightarrow D\} \cup \{C \rightarrow D\}$$

$$G^+ = \{A \rightarrow B, AB \rightarrow D, A \rightarrow D, C \rightarrow D\}$$

$$F^+ = G^+$$



$R_1(A, B, D) \quad R_2(C, D)$   
 $F = \{A \rightarrow B, AB \rightarrow D, C \rightarrow D\}$

*Decomposition is DP*

# DECOMPOSITION SUMMARY

## Lossless Joins

- Motivation: Avoid information loss
- Goal: No noise introduced when reconstituting the main relation via joins
- Test: we will see that on Wednesday (chasing algorithm).

# DECOMPOSITION SUMMARY

## Dependency Preservation

- Motivation: asserting efficient FD
- Goal: guaranteeing that all the FDs specified in  $F$  are preserved at minimal cost in the decomposition.
- Test:  $R = (R_1 \cup \dots \cup R_n)$  is dependency preserving if the closure of FDs covered by each  $R_i$  = closure of the initial FDs covered in  $R$ .

# CONCLUSION

Functional dependencies are simple to understand.

They will allow us to reason about schema decompositions.