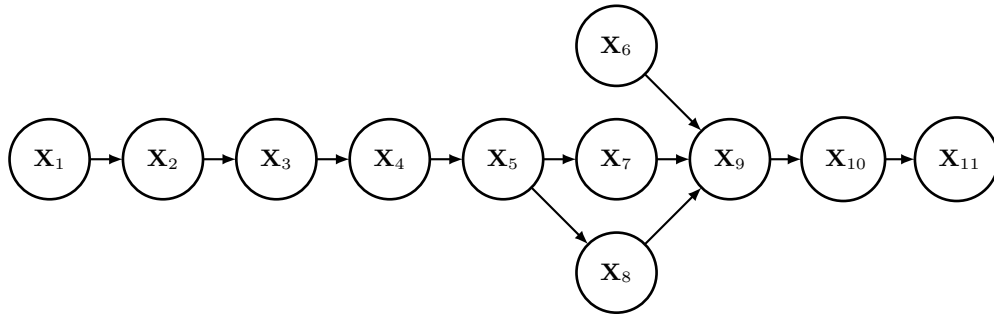SINGAPORE UNIVERSITY OF
TECHNOLOGY AND DESIGN

01.112 Machine Learning, Fall 2019
Homework 5

Due 13 Dec 2019, 5pm

This homework will be graded by Xiaobing Sun

In this homework, we would like to look at the Bayesian Networks. You are given a Bayesian network as below. All nodes can take 2 different values: $\{1, 2\}$.



**Question 1.** Without knowing the actual value of any node, are node $\mathbf{X}_2$ and $\mathbf{X}_6$ independent of each other? What if we know the value of node $\mathbf{X}_7$ and $\mathbf{X}_{11}$? *(5 points)*

**Question 2.** What is the number of *free* parameters needed to for this Bayesian network? What would be the number of *free* parameters for the same network if node $\mathbf{X}_3$ and $\mathbf{X}_9$ can take 3 different values: $\{1, 2, 3\}$, and all other nodes can only take 5 different values: $\{1, 2, 3, 4, 5\}$? *(5 points)*

**Question 3.** If we have the following probability tables (next page) for the nodes. Compute the following probabilities. Clearly write down all the necessary steps.

 (a) Calculate the following conditional probability:

$$P(\mathbf{X}_3 = 1 | \mathbf{X}_4 = 1)$$

*(6 points)*

 (b) Calculate the following conditional probability:

$$P(\mathbf{X}_5 = 2 | \mathbf{X}_3 = 2, \mathbf{X}_{11} = 2, \mathbf{X}_1 = 2)$$

*(9 points)*

(*Hint: find a short answer. The values in some of the probability tables may reveal some useful information.*)

| | $\mathbf{X}_1$ | |
|---|---|---|
| | 1 | 2 |
| | 0.5 | 0.5 |

| $\mathbf{X}_1$ | $\mathbf{X}_2$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.2 | 0.8 |
| 2 | 0.3 | 0.7 |

| $\mathbf{X}_2$ | $\mathbf{X}_3$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.3 | 0.7 |
| 2 | 0.3 | 0.7 |

| $\mathbf{X}_3$ | $\mathbf{X}_4$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.1 | 0.9 |
| 2 | 0.5 | 0.5 |

| $\mathbf{X}_4$ | $\mathbf{X}_5$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.5 | 0.5 |
| 2 | 0.6 | 0.4 |

| | $\mathbf{X}_6$ | |
|---|---|---|
| | 1 | 2 |
| | 0.6 | 0.4 |

| $\mathbf{X}_6$ | $\mathbf{X}_7$ | $\mathbf{X}_8$ | $\mathbf{X}_9$ | |
|---|---|---|---|---|
| | | | 1 | 2 |
| 1 | 1 | 1 | 0.8 | 0.2 |
| 1 | 1 | 2 | 0.1 | 0.9 |
| 1 | 2 | 1 | 0.9 | 0.1 |
| 1 | 2 | 2 | 0.7 | 0.3 |
| 2 | 1 | 1 | 0.3 | 0.7 |
| 2 | 1 | 2 | 0.2 | 0.8 |
| 2 | 2 | 1 | 0.2 | 0.8 |
| 2 | 2 | 2 | 0.9 | 0.1 |

| $\mathbf{X}_5$ | $\mathbf{X}_7$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.2 | 0.8 |
| 2 | 0.3 | 0.7 |

| $\mathbf{X}_5$ | $\mathbf{X}_8$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.8 | 0.2 |
| 2 | 0.7 | 0.3 |

| $\mathbf{X}_9$ | $\mathbf{X}_{10}$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.8 | 0.2 |
| 2 | 0.8 | 0.2 |

| $\mathbf{X}_{10}$ | $\mathbf{X}_{11}$ | |
|---|---|---|
| | 1 | 2 |
| 1 | 0.7 | 0.3 |
| 2 | 0.8 | 0.2 |