50.007

Machine Learning

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Bayesian Networks (I)

Soft EM for HMM

E-Step

Run forward-backward algorithm
to collect fractional counts
from each instance

M-Step

$$a_{u,v} = \frac{\mathrm{count}(u,v)}{\mathrm{count}(u)} \qquad b_u(o) = \frac{\mathrm{count}(u \to o)}{\mathrm{count}(u)}$$

Soft EM for HMM

Finding the fractional count

$$\text{count}(u,v) = \sum_{i=1}^{m} \text{count}^{(i)}(u,v)$$

$$= \sum_{i=1}^{m} \sum_{\mathbf{y}} p(\mathbf{y}|\mathbf{x}^{(i)})\text{count}(\mathbf{x}^{(i)}, \mathbf{y}, u \to v)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} p(y_j = u, y_{j+1} = v|\mathbf{x})$$

$$\text{count}(u) = \sum_{i=1}^{m} \text{count}^{(i)}(u)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} p(y_j = u|\mathbf{x})$$

Soft EM for HMM

Finding the fractional count

$$\text{count}(u, v) = \sum_{i=1}^{m} \text{count}^{(i)}(u, v)$$

$$= \sum_{i=1}^{m} \sum_{\mathbf{y}} p(\mathbf{y}|\mathbf{x}^{(i)}) \text{count}(\mathbf{x}^{(i)}, \mathbf{y}, u \to v)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} p(y_j = u, y_{j+1} = v|\mathbf{x})$$

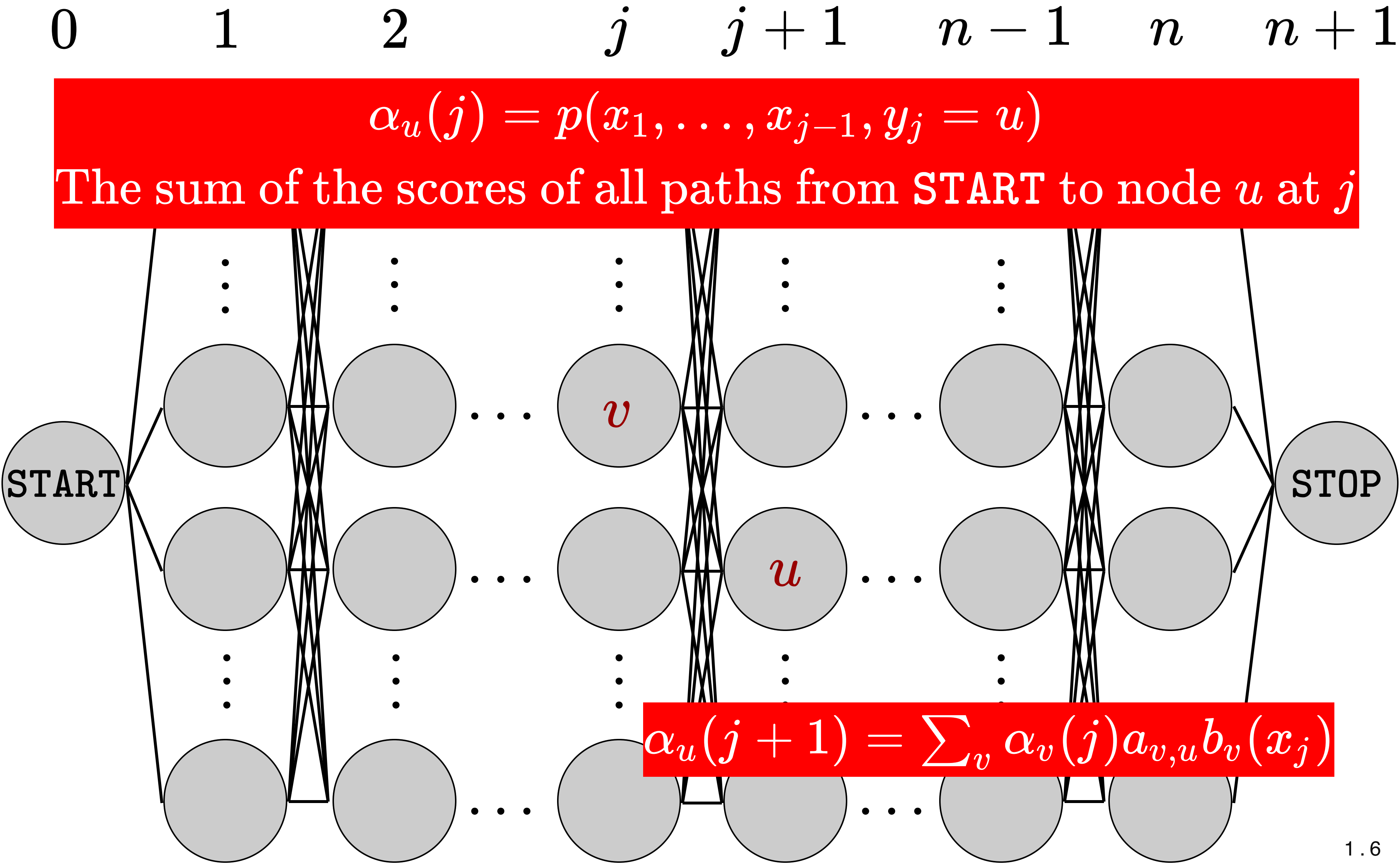
$$\sum_{j=0}^{n} \frac{\alpha_u(j) \cdot b_u(x_j) \cdot a_{u,v} \cdot \beta_v(j+1)}{\sum_v \alpha_v(k) \beta_v(k)}$$

$$\text{count}(u) = \sum_{i=1}^{m} \text{count}^{(i)}(u)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} p(y_j = u|\mathbf{x})$$

$$\sum_{j=0}^{n} \frac{\alpha_u(j) \beta_u(j)}{\sum_v \alpha_v(k) \beta_v(k)}$$

Forward-Backward Algorithm

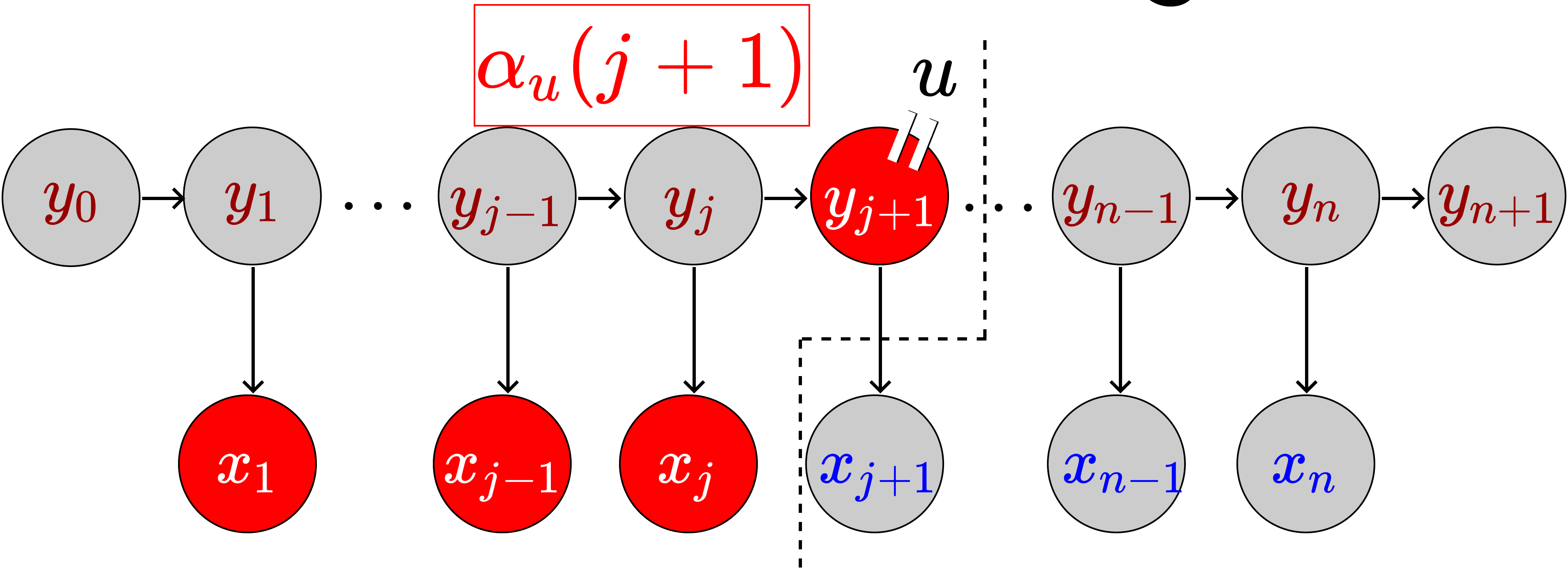


$$\alpha_u(j) = p(x_1, \ldots, x_{j-1}, y_j = u)$$

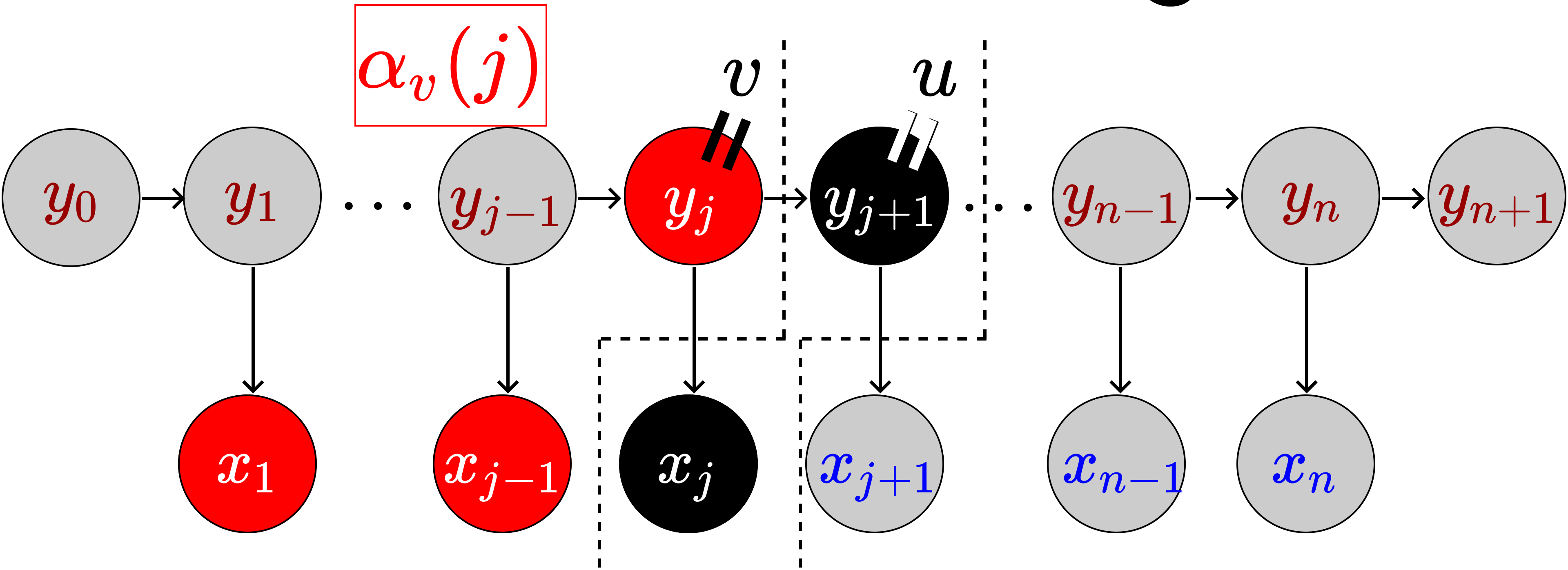
The sum of the scores of all paths from **START** to node u at j

$$\alpha_u(j+1) = \sum_v \alpha_v(j) a_{v,u} b_v(x_j)$$

Forward-Backward Algorithm

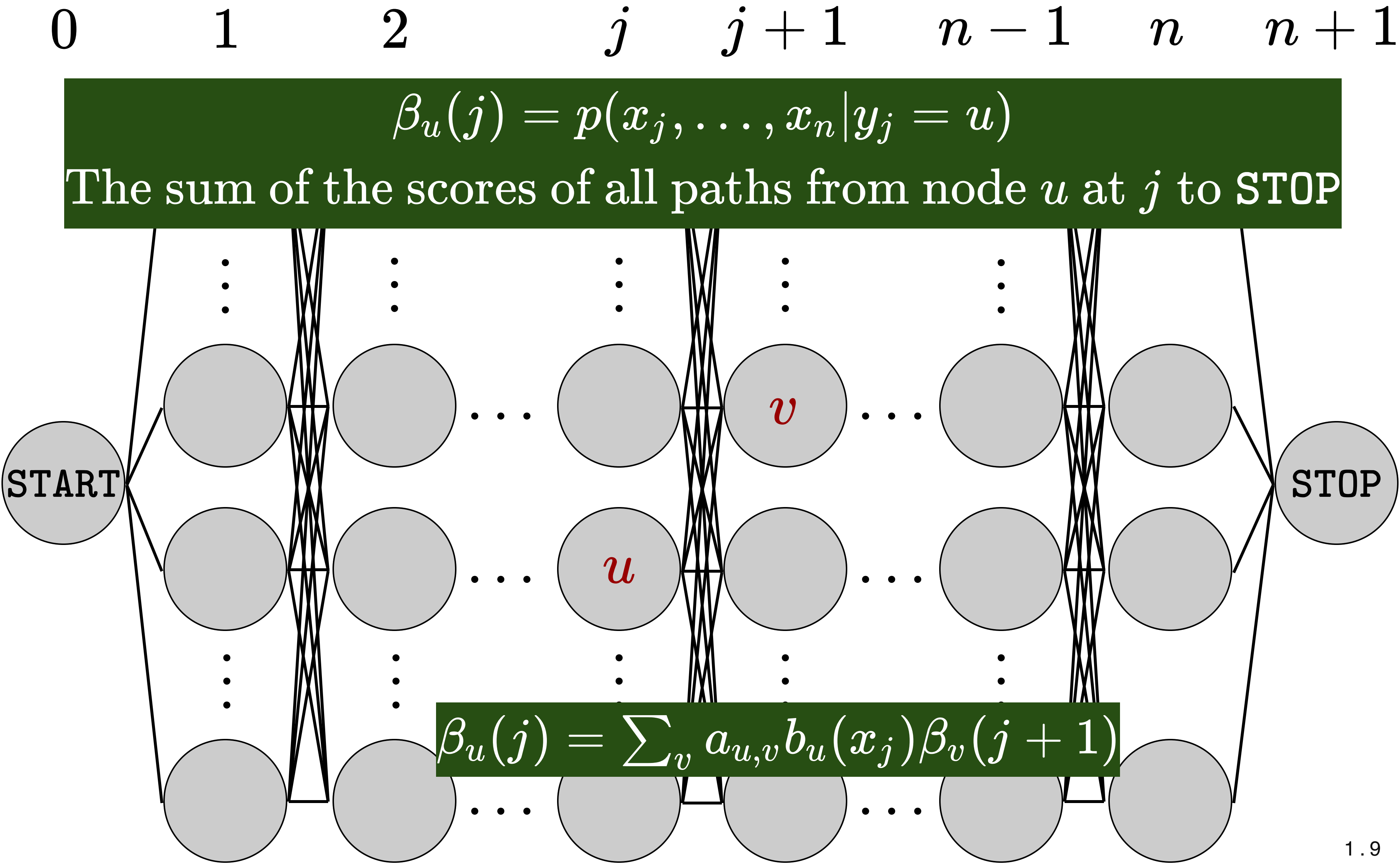


Forward-Backward Algorithm

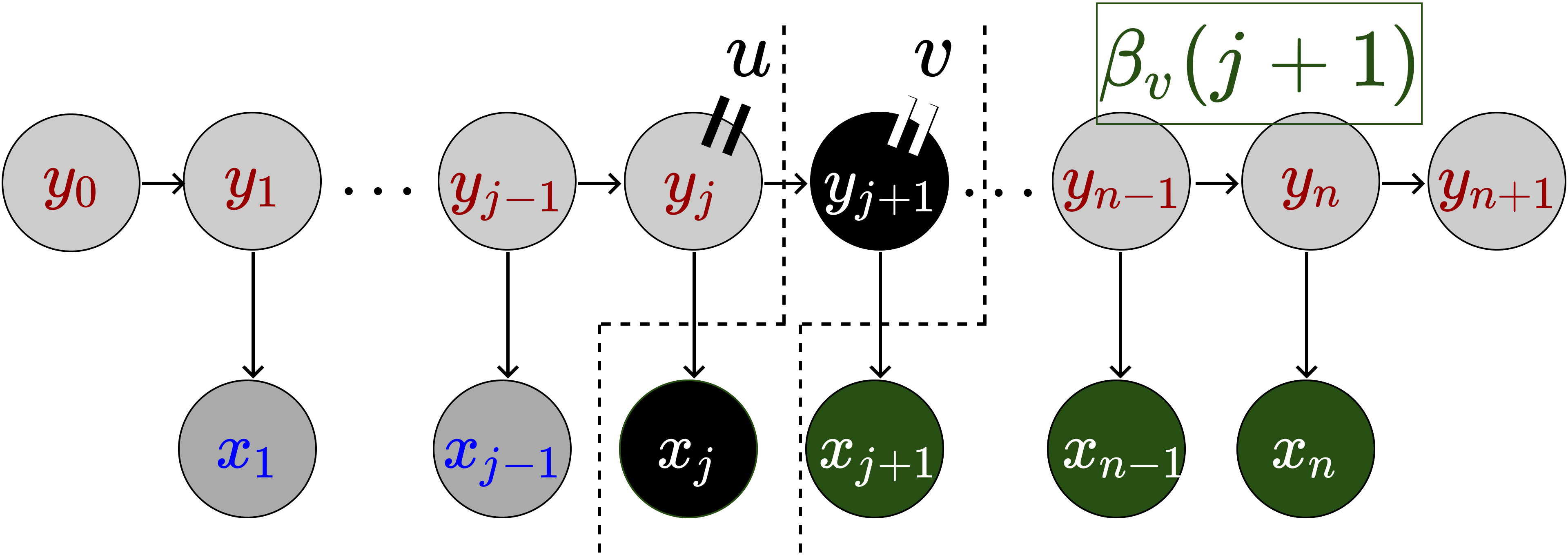


$$\boxed{\alpha_u(j+1) = \sum_v \alpha_v(j) a_{v,u} b_v(x_j)}$$

Forward-Backward Algorithm



Forward-Backward Algorithm



$$\boxed{\beta_u(j) = \sum_v a_{u,v} b_u(x_j) \beta_v(j+1)}$$

Soft EM for HMM

$$\boxed{\text{Finding the fractional count}}$$

$$\text{count}(u, v) = \sum_{i=1}^{m} \text{count}^{(i)}(u, v)$$

$$= \sum_{i=1}^{m} \sum_{\mathbf{y}} p(\mathbf{y}|\mathbf{x}^{(i)})\text{count}(\mathbf{x}^{(i)}, \mathbf{y}, u \to v)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} \frac{\alpha_u(j) \cdot b_u(x_j) \cdot a_{u,v} \cdot \beta_v(j+1)}{\sum_v \alpha_v(k)\beta_v(k)}$$

$$\text{count}(u) = \sum_{i=1}^{m} \text{count}^{(i)}(u)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} \frac{\alpha_u(j)\beta_u(j)}{\sum_v \alpha_v(k)\beta_v(k)}$$

In the M-Step:

$$a_{u,v} = \frac{\text{count}(u,v)}{\text{count}(u)}$$

Soft EM for HMM

Finding the fractional count

$$\text{count}(u \to o) = \sum_{i=1}^{m} \text{count}^{(i)}(u \to o)$$

$$= \sum_{i=1}^{m} \sum_{\mathbf{y}} p(\mathbf{y}|\mathbf{x}^{(i)})\text{count}(\mathbf{x}^{(i)}, \mathbf{y}, u \to o)$$

$$= \sum_{i=1}^{m} \sum_{j \text{ s.t. } x_j = o} \frac{\alpha_u(j)\beta_u(j)}{\sum_v \alpha_v(k)\beta_v(k)}$$

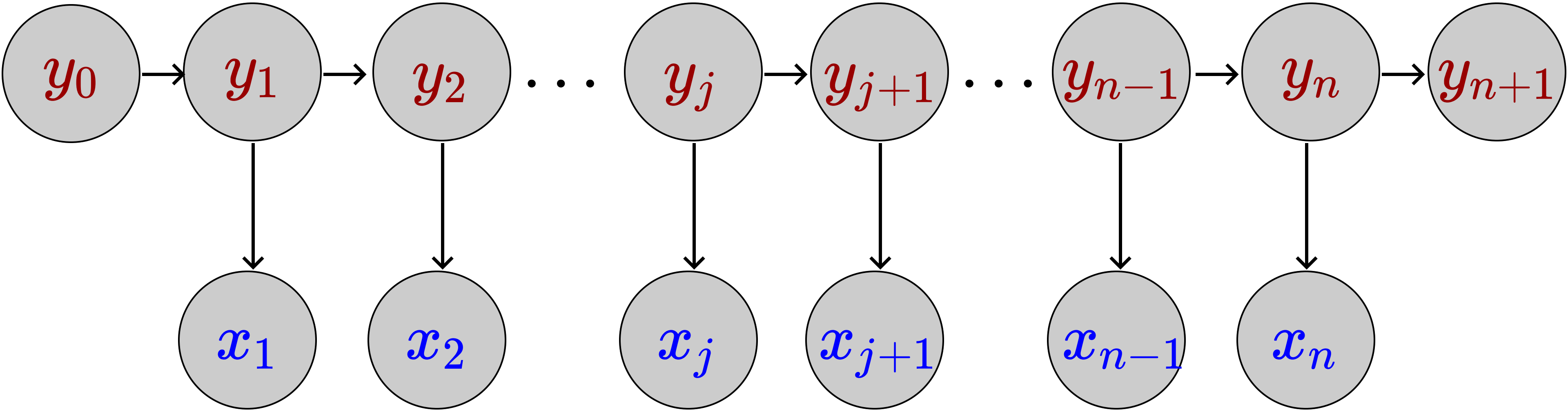
$$\text{count}(u) = \sum_{i=1}^{m} \text{count}^{(i)}(u)$$

$$= \sum_{i=1}^{m} \sum_{j=0}^{n} \frac{\alpha_u(j)\beta_u(j)}{\sum_v \alpha_v(k)\beta_v(k)}$$

In the M-Step:

$$b_u(o) = \frac{\text{count}(u \to o)}{\text{count}(u)}$$

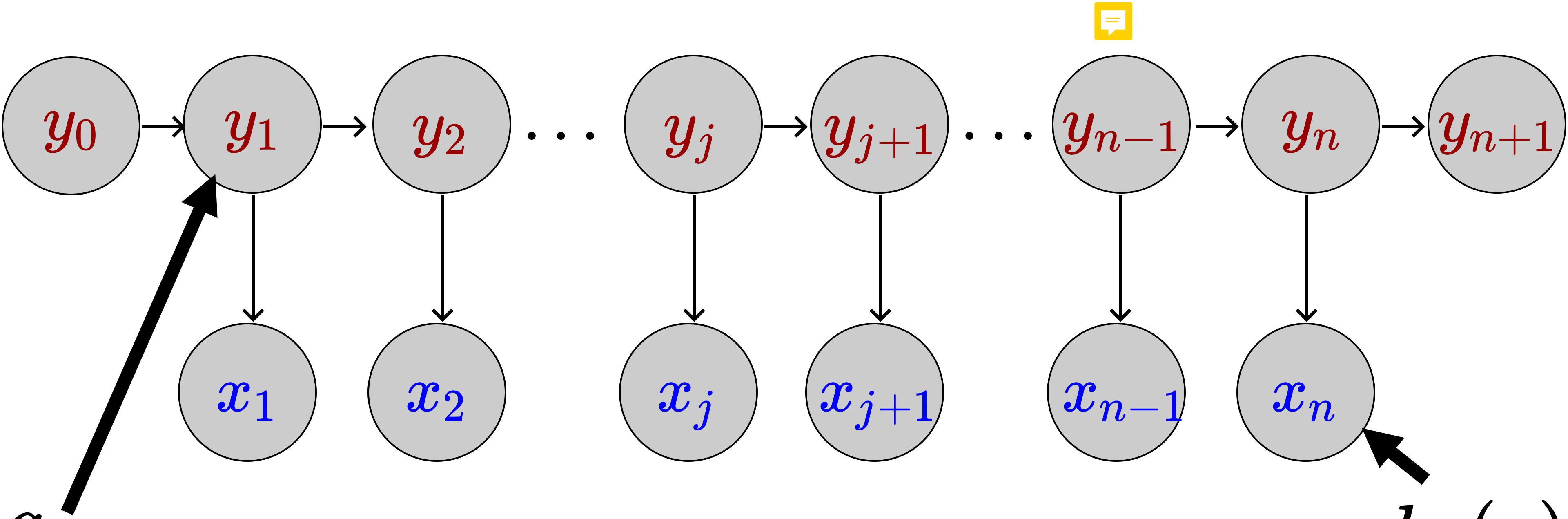
Hidden Markov Model



$$p(x_1, x_2, \ldots, x_{n-1}, x_n, y_0, y_1, y_2, \ldots, y_{n-1}, y_n, y_{n+1})$$

$$\underbrace{\prod_{j=0}^{n} a_{y_j, y_{j+1}}}_{\text{Transition probabilities}} \times \underbrace{\prod_{j=1}^{n} b_{y_j}(x_j)}_{\text{Emission probabilities}}$$

Hidden Markov Model



$p(x_1, x_2, \ldots, x_{n-1}, x_n, y_0, y_1, y_2, \cdots, y_{n-1}, y_n, y_{n+1})$

$u \backslash v$	A	B	STOP
START	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

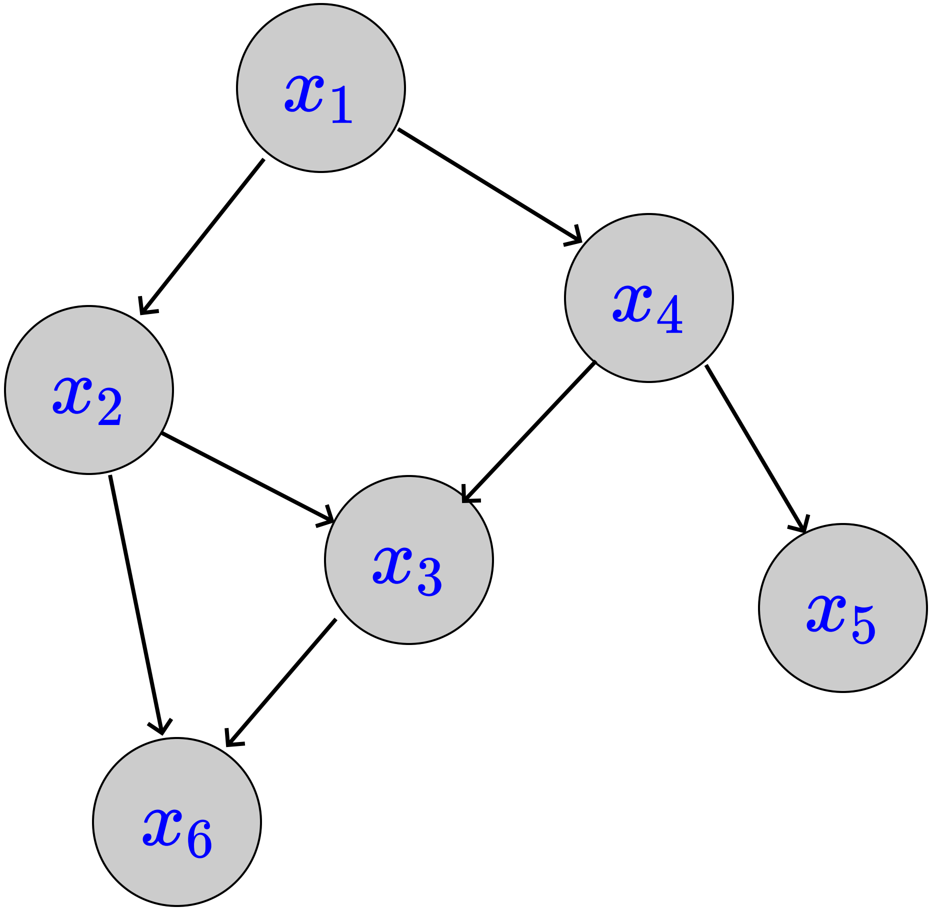
Transition probabilities

$\times$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9

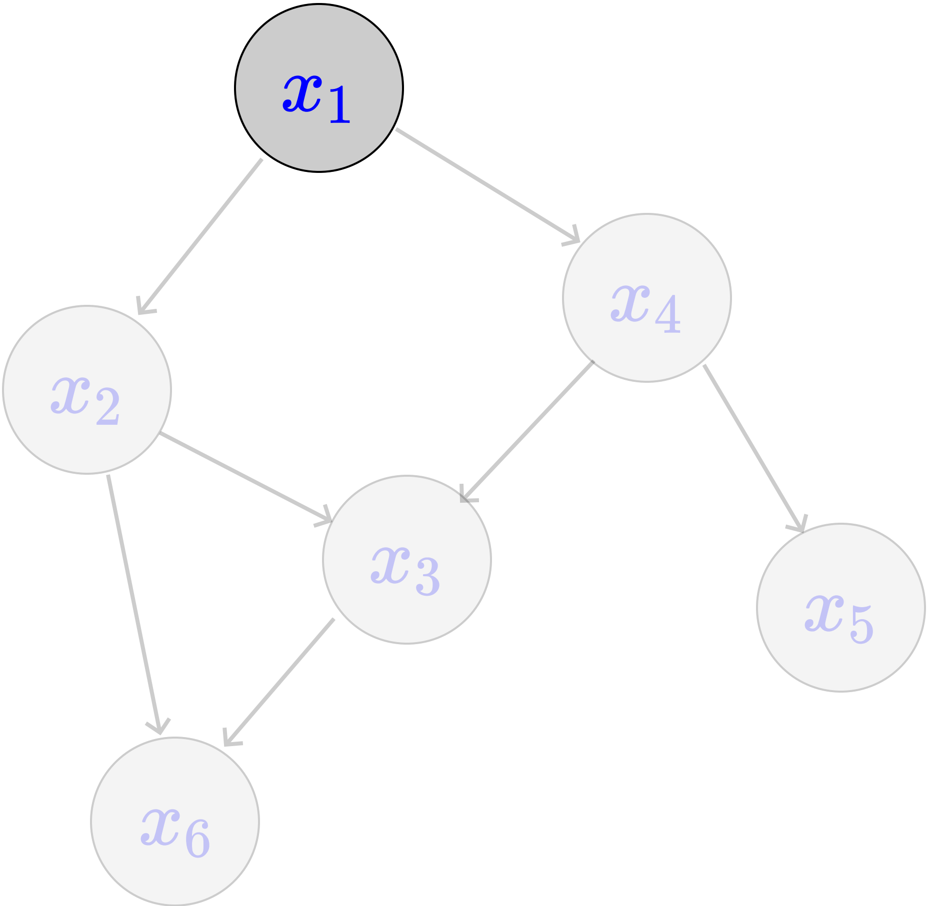
Emission probabilities

Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6) = ?$$

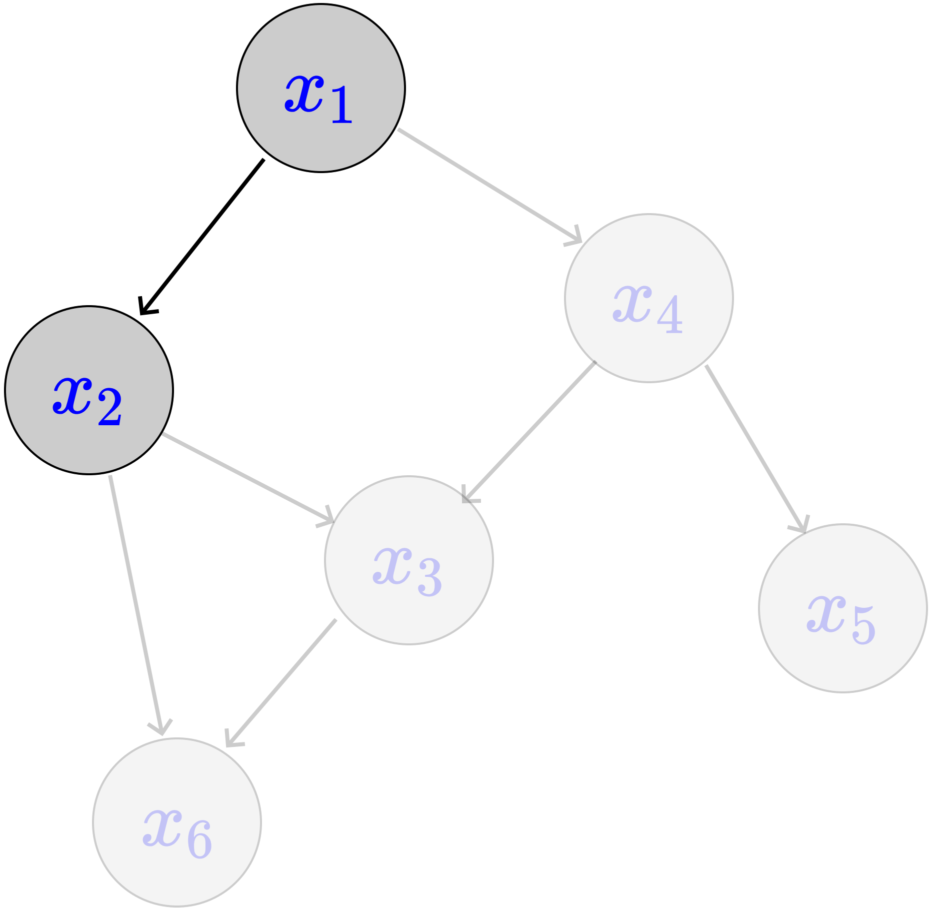
Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$

$$= p(x_1)$$

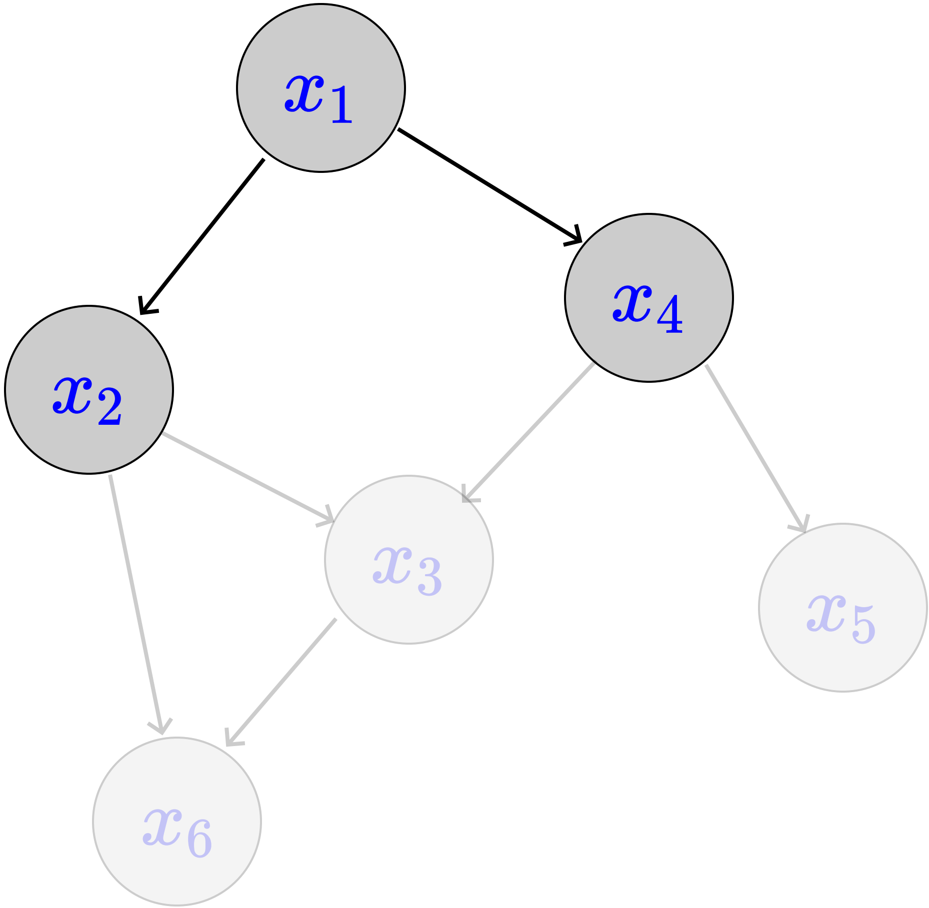
Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$

$$= p(x_1)p(x_2|x_1)$$

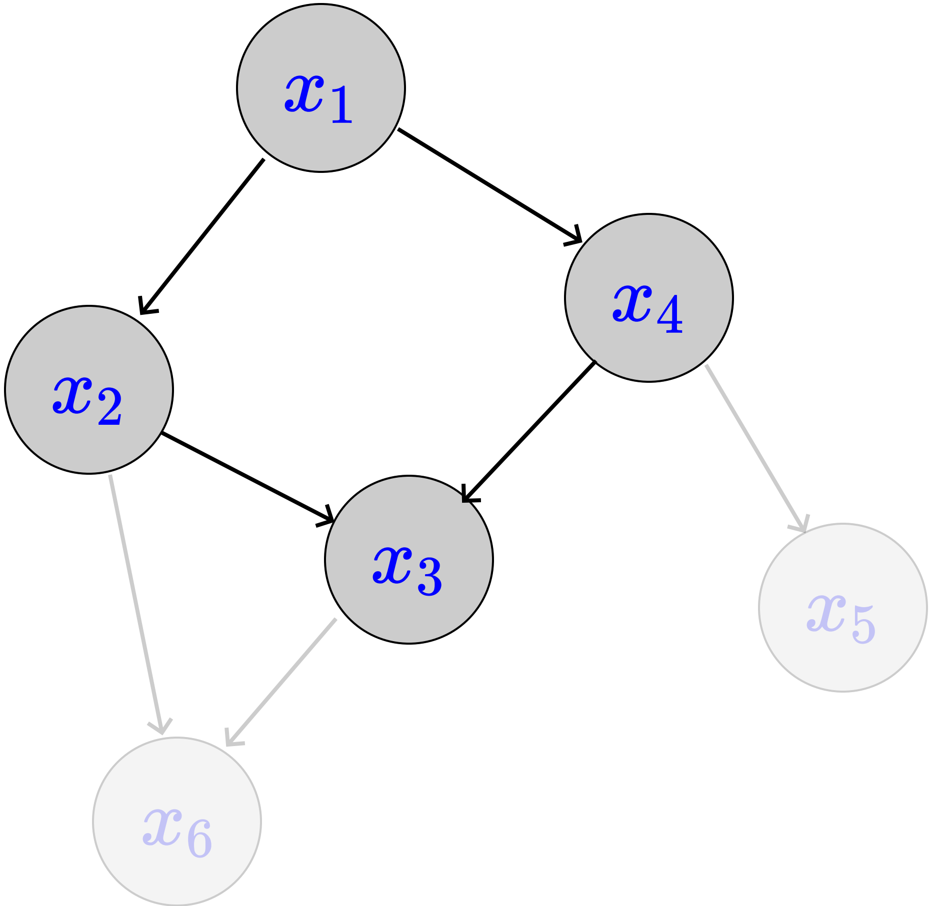
Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$

$$= p(x_1)p(x_2|x_1)p(x_4|x_1)$$

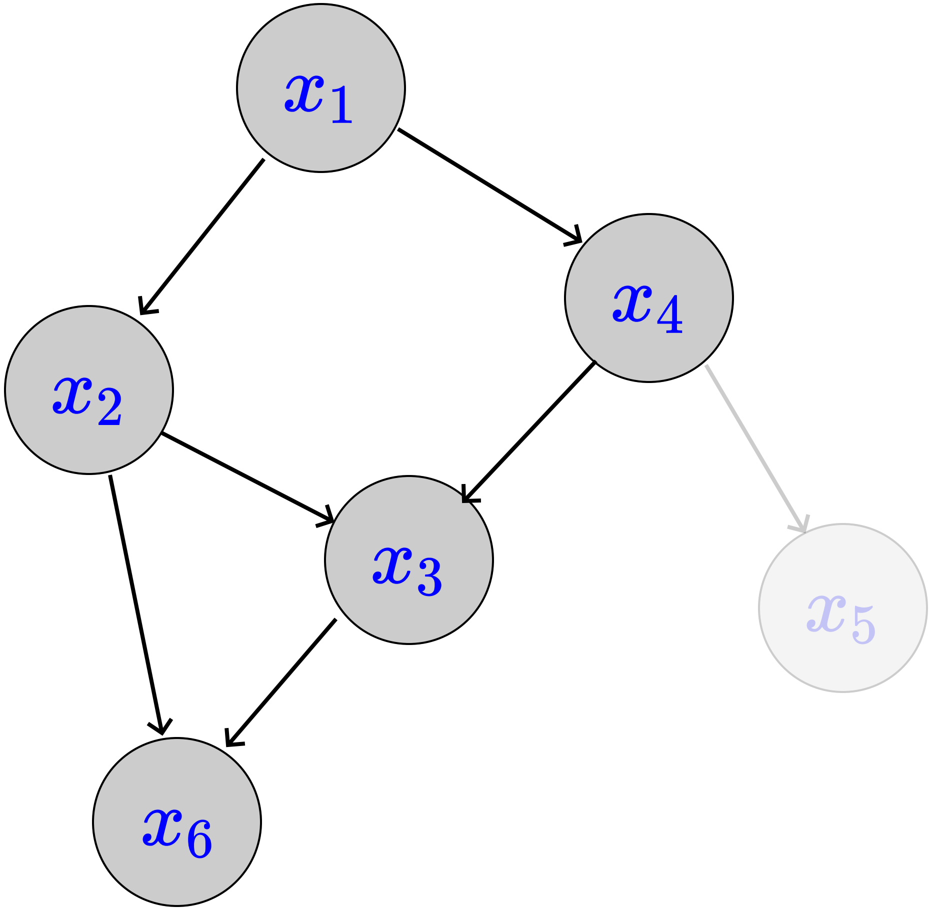
Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$

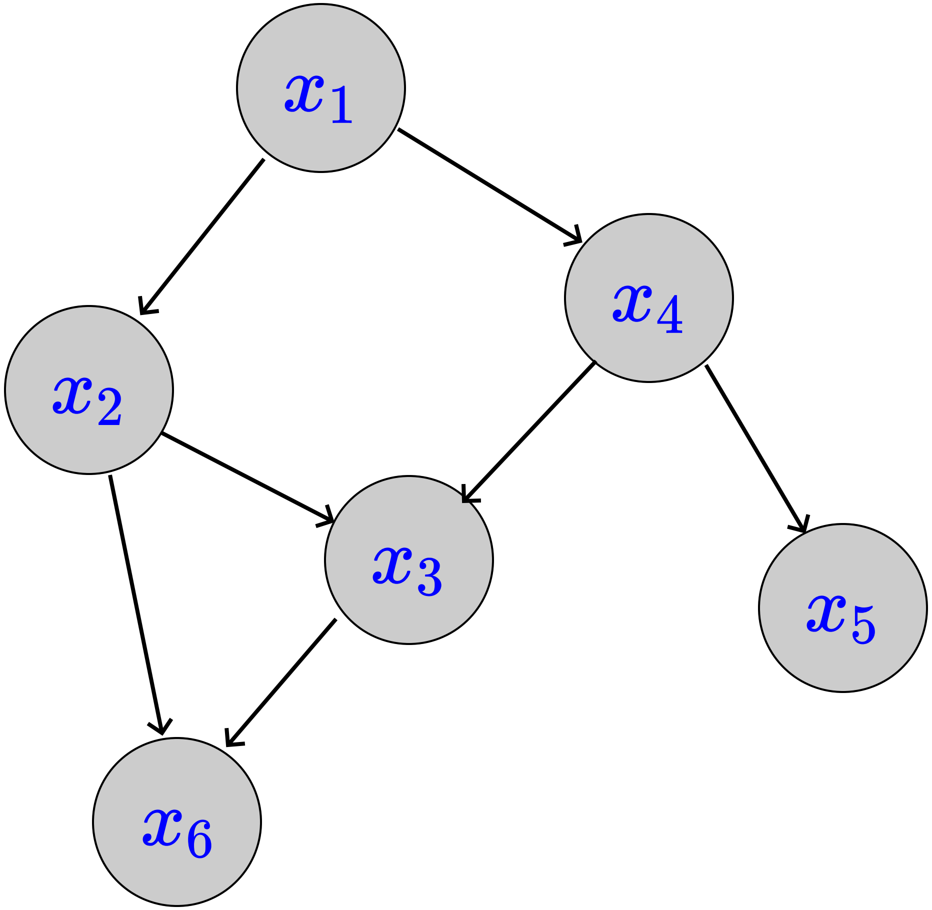
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)$$

Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)$$

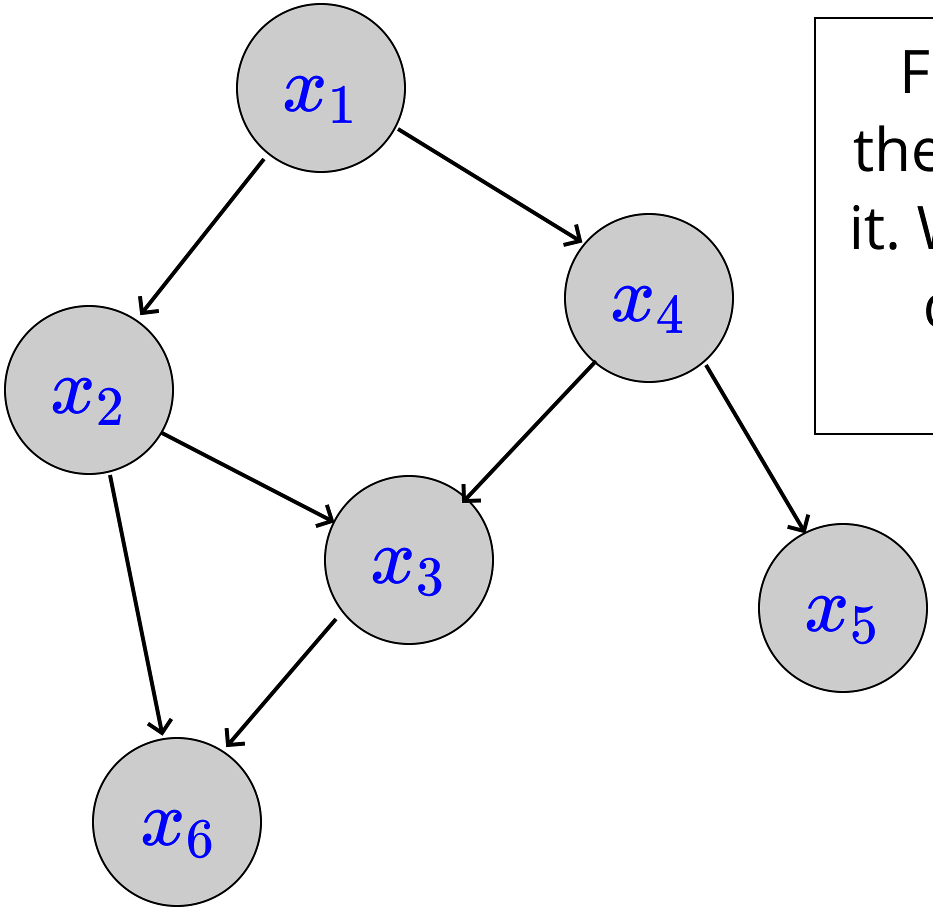
Generative Model



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$

$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model

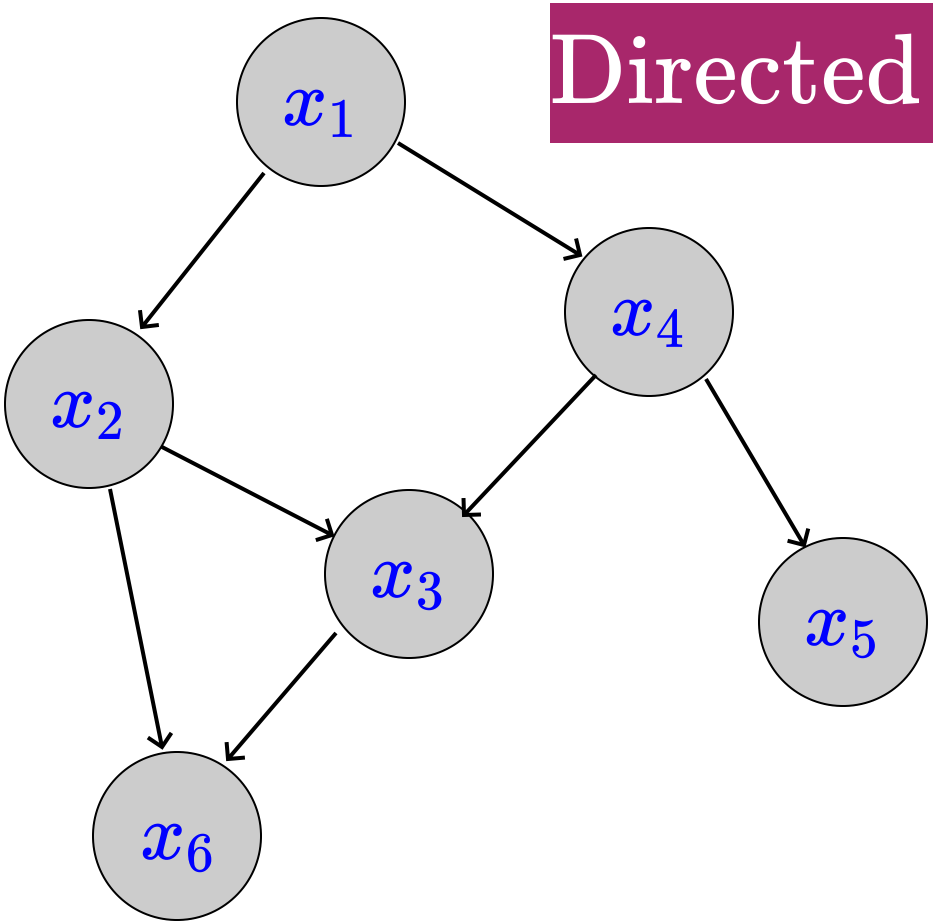


For a generative model, there is a graph underlying it. What types of properties do we have for such a graph?

$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

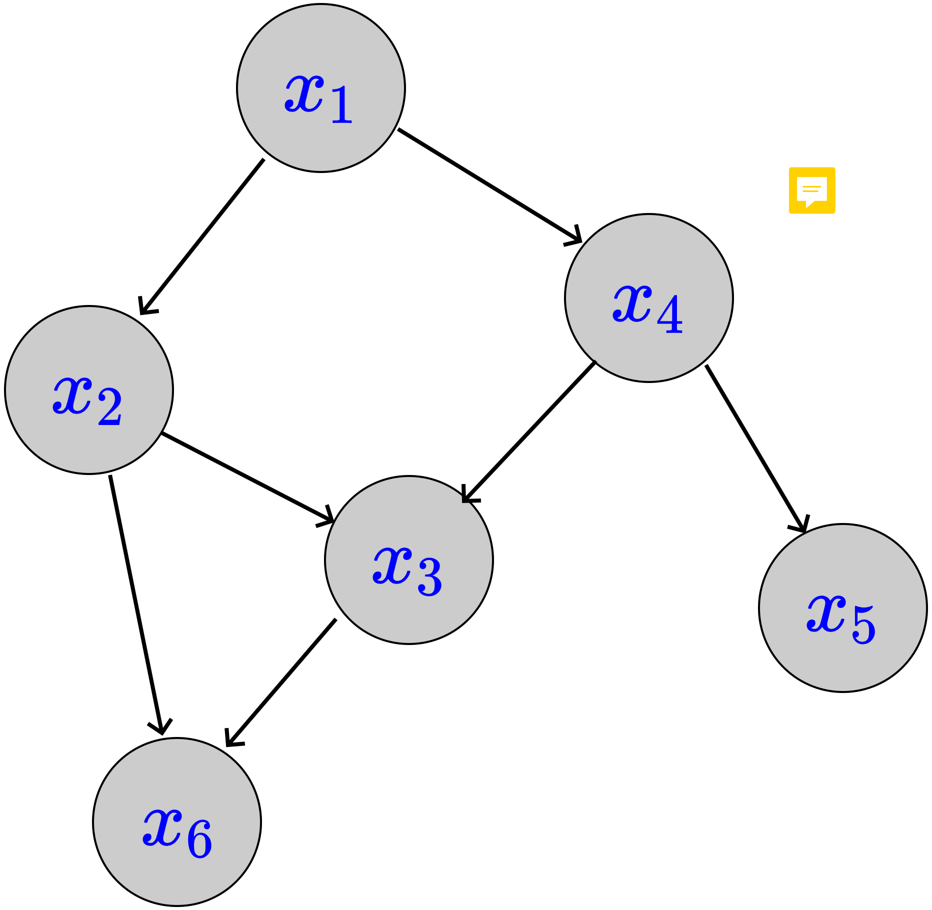
Generative Model

Directed Acyclic Graphs



$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model

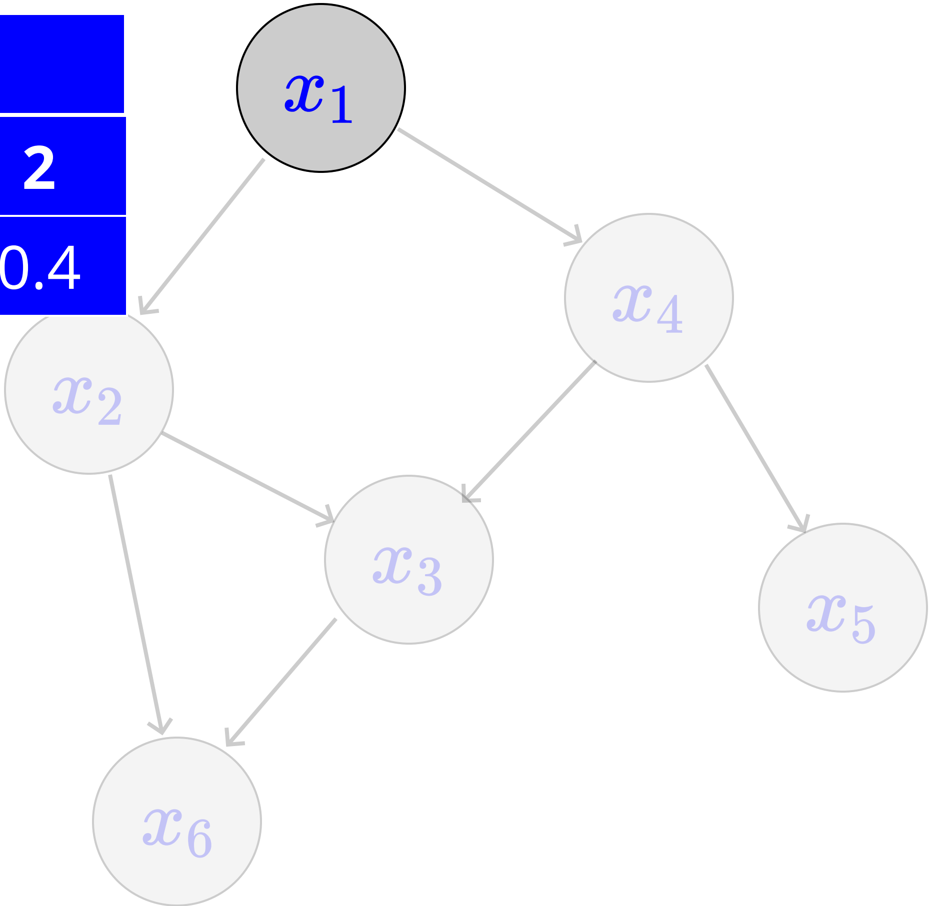


$$x_1 \in \{1, 2\}$$
$$x_2 \in \{1, 2\}$$
$$x_3 \in \{1, 2, 3\}$$
$$x_4 \in \{1, 2, 3\}$$
$$x_5 \in \{1, 2, 3, 4, 5\}$$
$$x_6 \in \{1, 2, 3, 4\}$$

$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model

x_1	
1	**2**
0.6	0.4



$x_1 \in \{1, 2\}$

$x_2 \in \{1, 2\}$

$x_3 \in \{1, 2, 3\}$

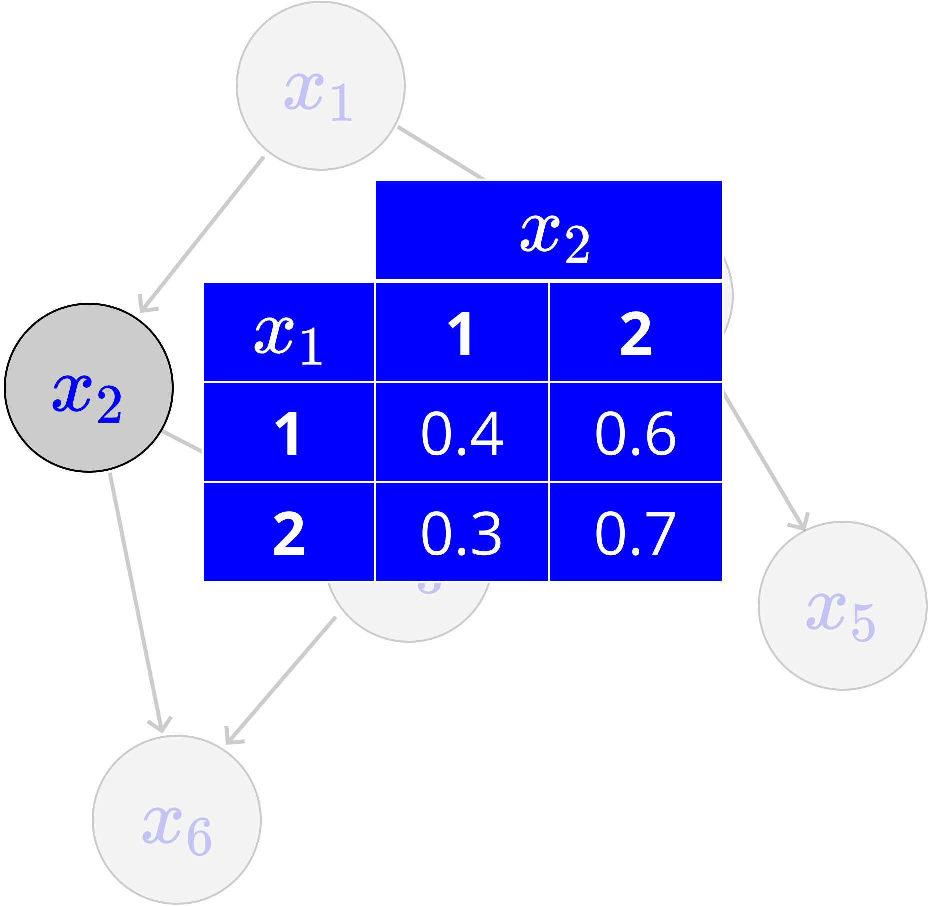
$x_4 \in \{1, 2, 3\}$

$x_5 \in \{1, 2, 3, 4, 5\}$

$x_6 \in \{1, 2, 3, 4\}$

$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model



	x_2	
x_1	**1**	**2**
1	0.4	0.6
2	0.3	0.7

$$x_1 \in \{1, 2\}$$
$$x_2 \in \{1, 2\}$$
$$x_3 \in \{1, 2, 3\}$$
$$x_4 \in \{1, 2, 3\}$$
$$x_5 \in \{1, 2, 3, 4, 5\}$$
$$x_6 \in \{1, 2, 3, 4\}$$

$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model

$x_1 \in \{1, 2\}$

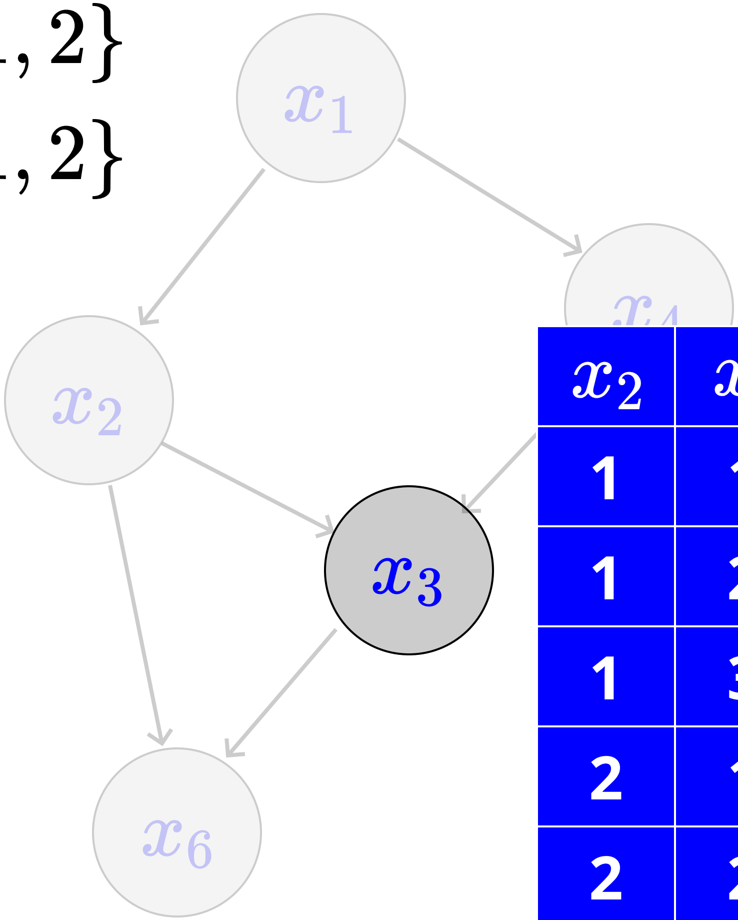
$x_2 \in \{1, 2\}$

$x_3 \in \{1, 2, 3\}$

$x_4 \in \{1, 2, 3\}$

$\{1, 2, 3, 4, 5\}$

$\{1, 2, 3, 4\}$

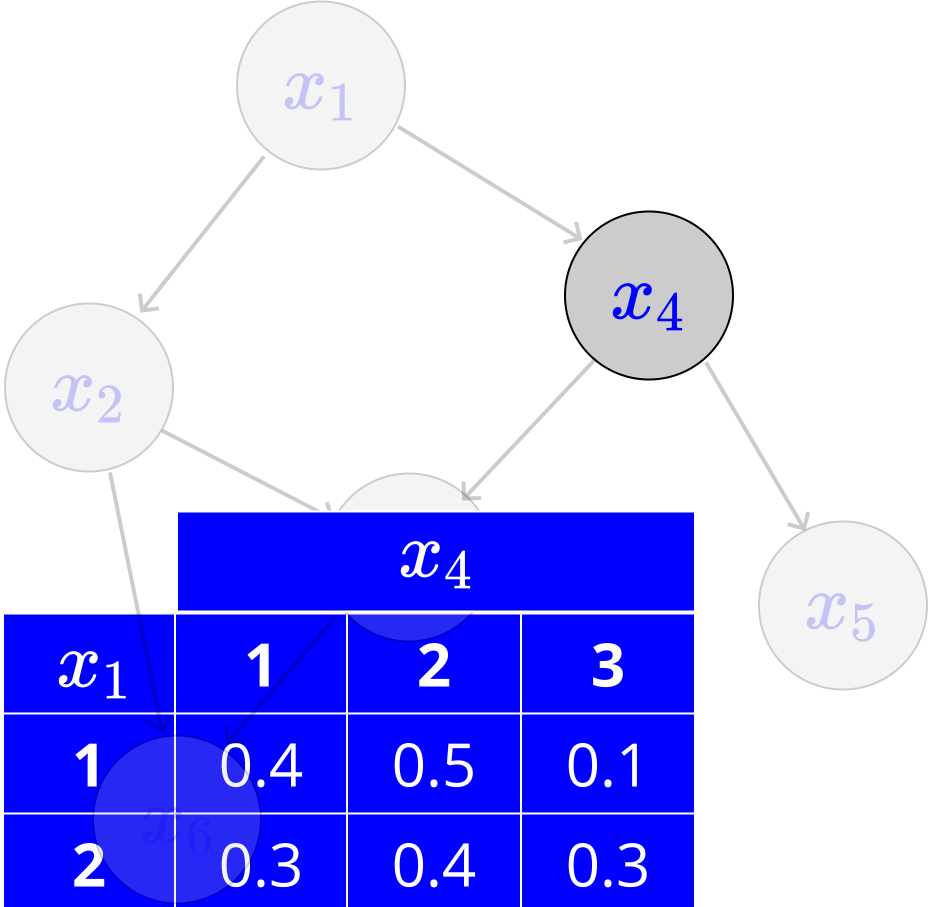


		x_3		
x_2	x_4	**1**	**2**	**3**
1	**1**	0.4	0.5	0.1
1	**2**	0.3	0.4	0.3
1	**3**	0.1	0.8	0.1
2	**1**	0.7	0.3	0.0
2	**2**	0.2	0.6	0.2
2	**3**	0.5	0.3	0.2

$p(x_1, x$

$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model

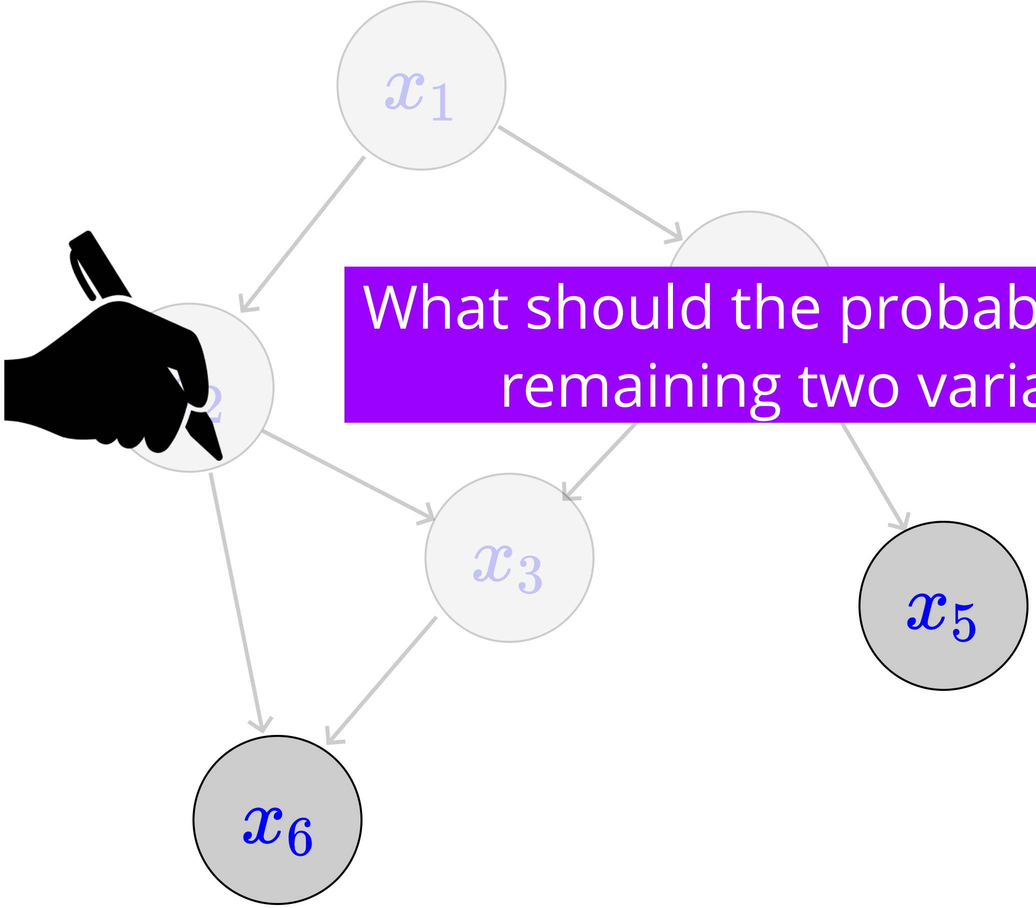


	x_4		
x_1	**1**	**2**	**3**
1	0.4	0.5	0.1
2	0.3	0.4	0.3

$$x_1 \in \{1, 2\}$$
$$x_2 \in \{1, 2\}$$
$$x_3 \in \{1, 2, 3\}$$
$$x_4 \in \{1, 2, 3\}$$
$$x_5 \in \{1, 2, 3, 4, 5\}$$
$$x_6 \in \{1, 2, 3, 4\}$$

$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Generative Model



What should the probability tables for the remaining two variables be like?

$x_1 \in \{1, 2\}$

$x_2 \in \{1, 2\}$

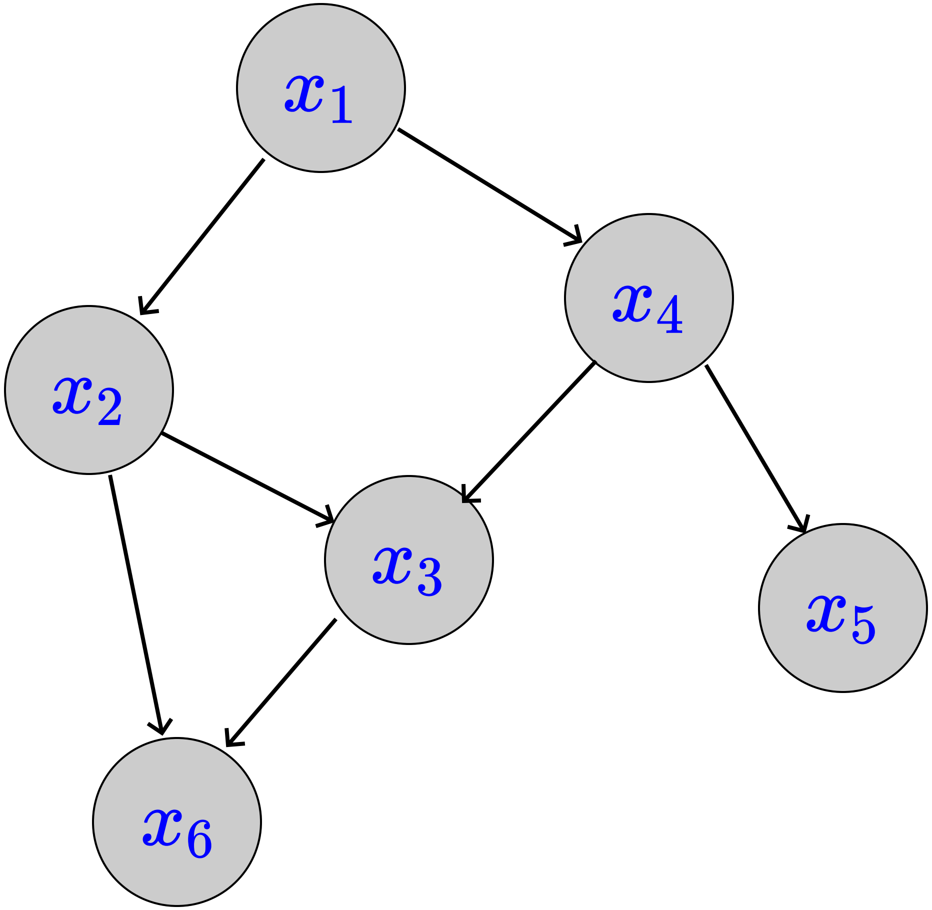
$x_4 \in \{1, 2, 3\}$

$x_5 \in \{1, 2, 3, 4, 5\}$

$x_6 \in \{1, 2, 3, 4\}$

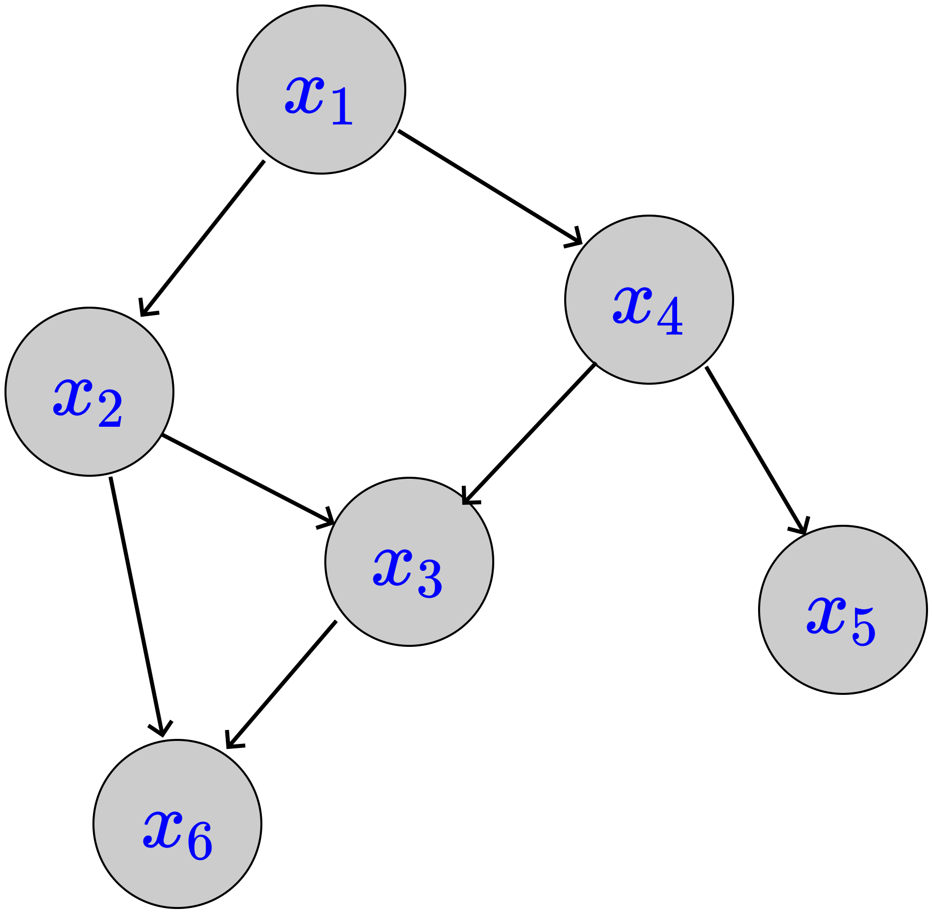
$$p(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$= p(x_1)p(x_2|x_1)p(x_4|x_1)p(x_3|x_2, x_4)p(x_6|x_2, x_3)p(x_5|x_4)$$

Bayesian Networks



$$p(x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 3, x_5 = 5, x_6 = 2)$$

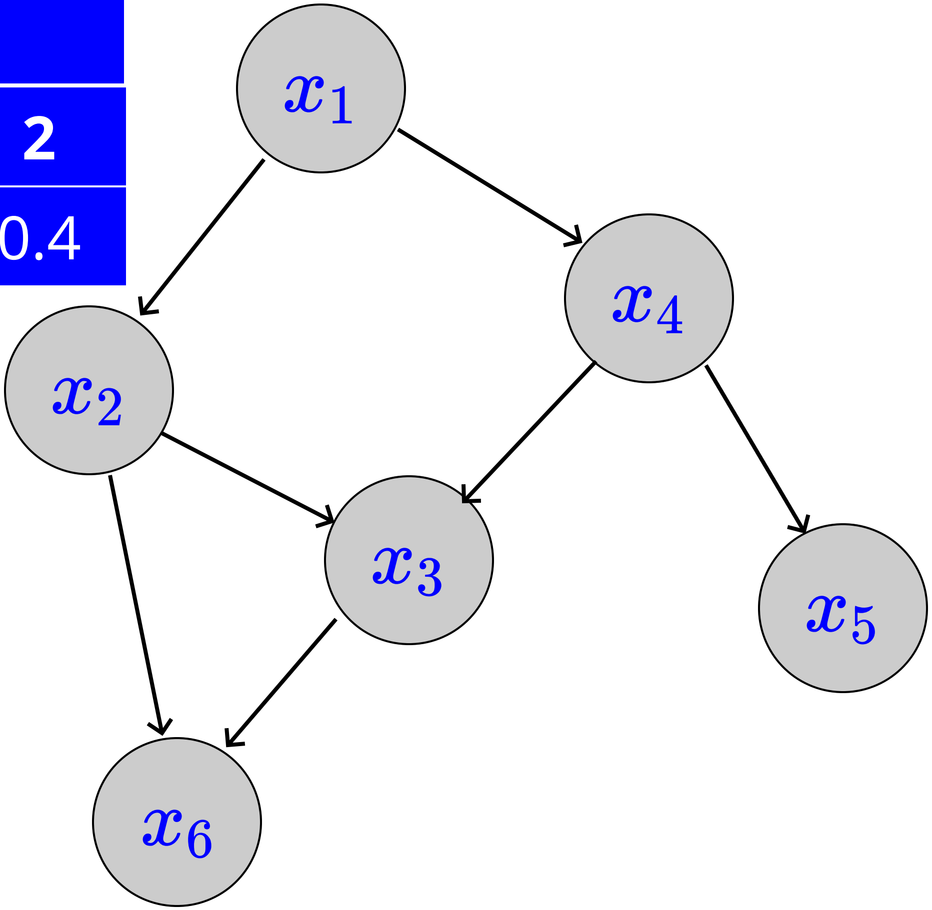
Bayesian Networks



$$
\begin{aligned}
&p(x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 3, x_5 = 5, x_6 = 2) \\
&= p(x_1 = 1) \times p(x_2 = 2 | x_1 = 1) \times p(x_4 = 3 | x_1 = 1) \\
&\times p(x_3 = 1 | x_2 = 2, x_4 = 3) \times p(x_6 = 2 | x_2 = 2, x_3 = 1) \times p(x_5 = 5 | x_4 = 3)
\end{aligned}
$$

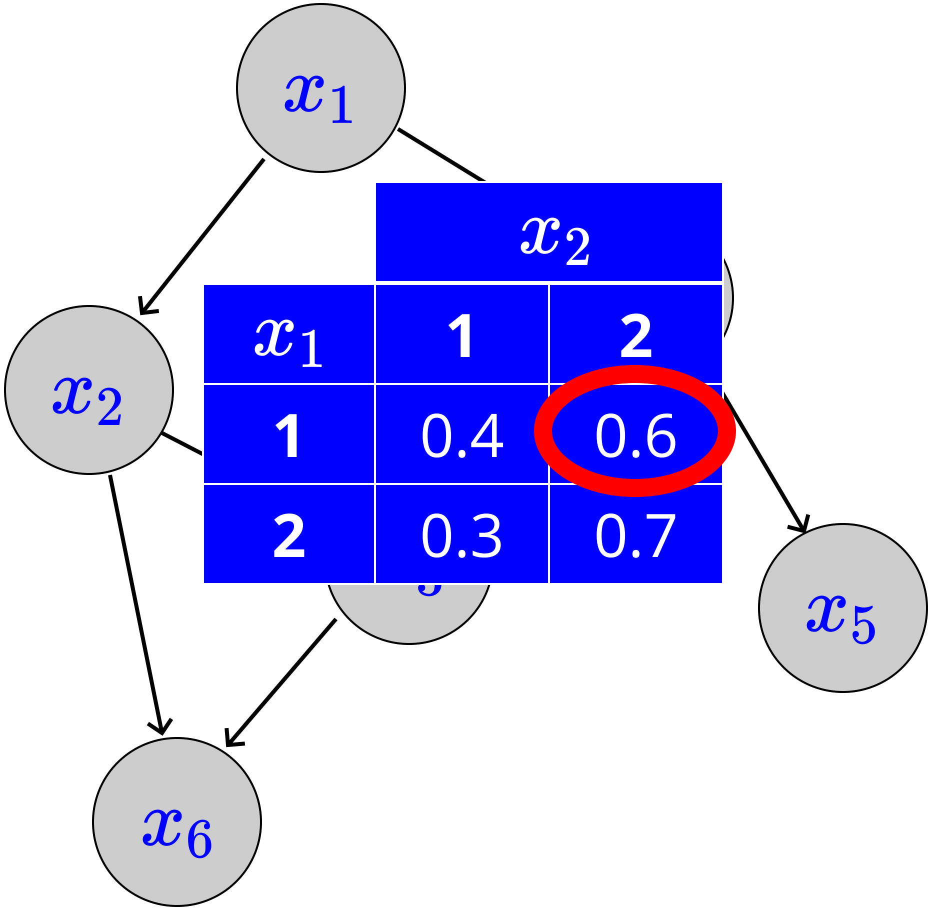
Bayesian Networks

x_1	
1	2
0.6	0.4



$$p(x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 3, x_5 = 5, x_6 = 2)$$
$$= p(x_1 = 1) \times p(x_2 = 2 | x_1 = 1) \times p(x_4 = 3 | x_1 = 1)$$
$$\times\, p(x_3 = 1 | x_2 = 2, x_4 = 3) \times p(x_6 = 2 | x_2 = 2, x_3 = 1) \times p(x_5 = 5 | x_4 = 3)$$

Bayesian Networks

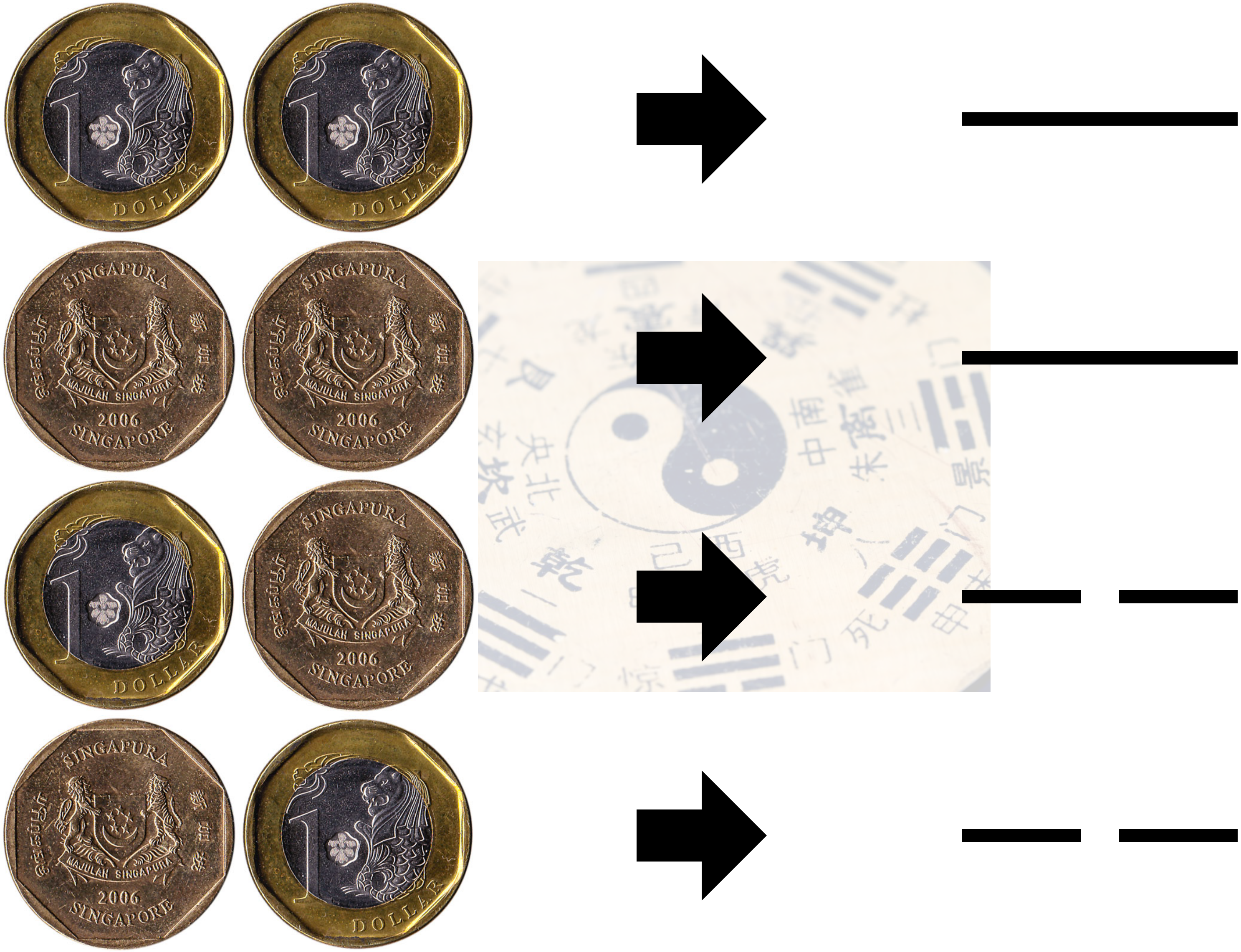


	x_2	
x_1	**1**	**2**
1	0.4	0.6
2	0.3	0.7

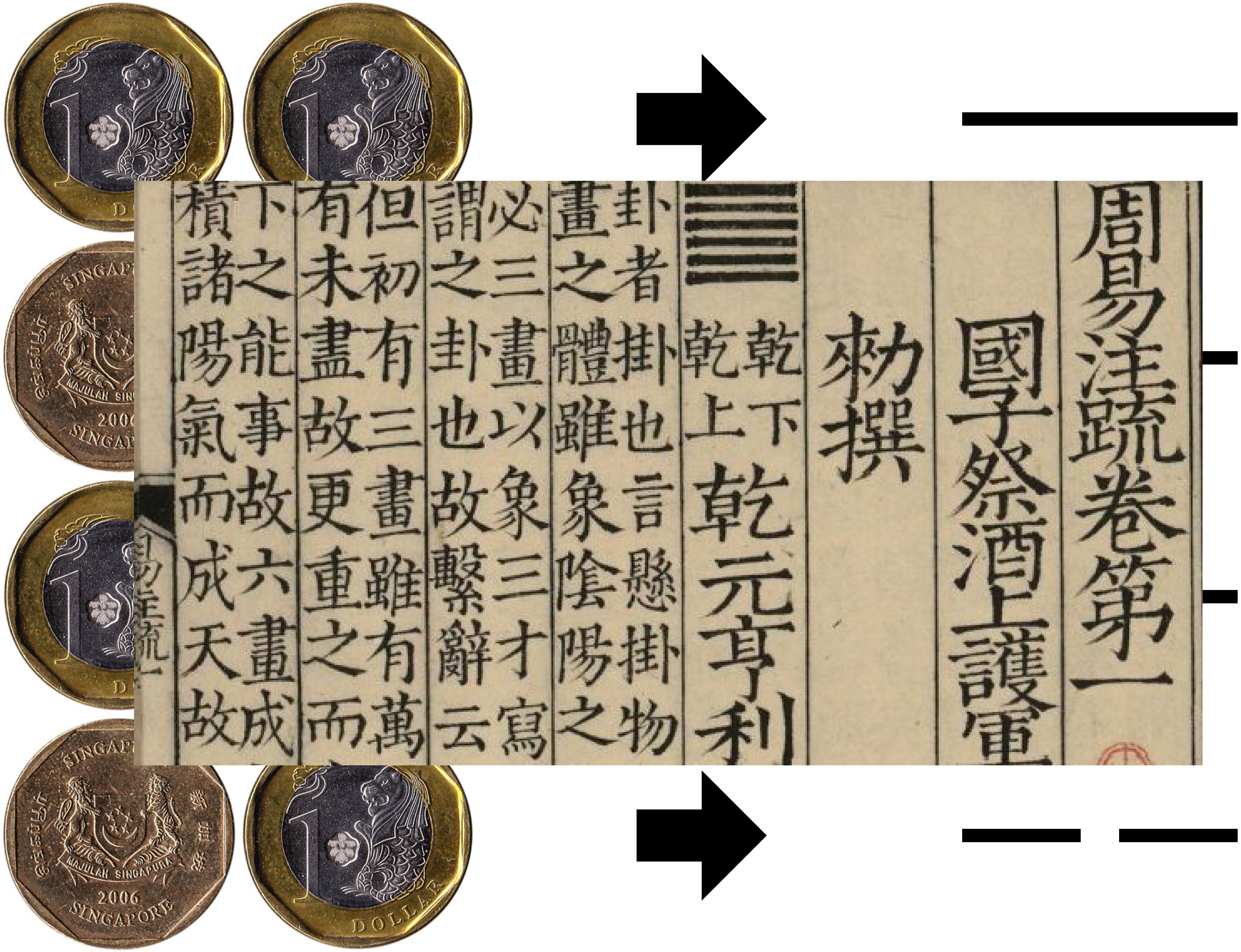
$$p(x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 3, x_5 = 5, x_6 = 2)$$
$$= p(x_1 = 1) \times p(x_2 = 2 | x_1 = 1) \times p(x_4 = 3 | x_1 = 1)$$
$$\times p(x_3 = 1 | x_2 = 2, x_4 = 3) \times p(x_6 = 2 | x_2 = 2, x_3 = 1) \times p(x_5 = 5 | x_4 = 3)$$

Bayesian Networks

Bayesian Networks



Bayesian Networks



周易注疏卷第一

國子祭酒上護軍

勑撰

乾下乾上乾元亨利

卦者掛也言懸掛物

畫之體雖象陰陽之

必三畫以象三才寫

謂之卦也故繫辭云

但初有三畫雖有萬

有未畫故更重之而

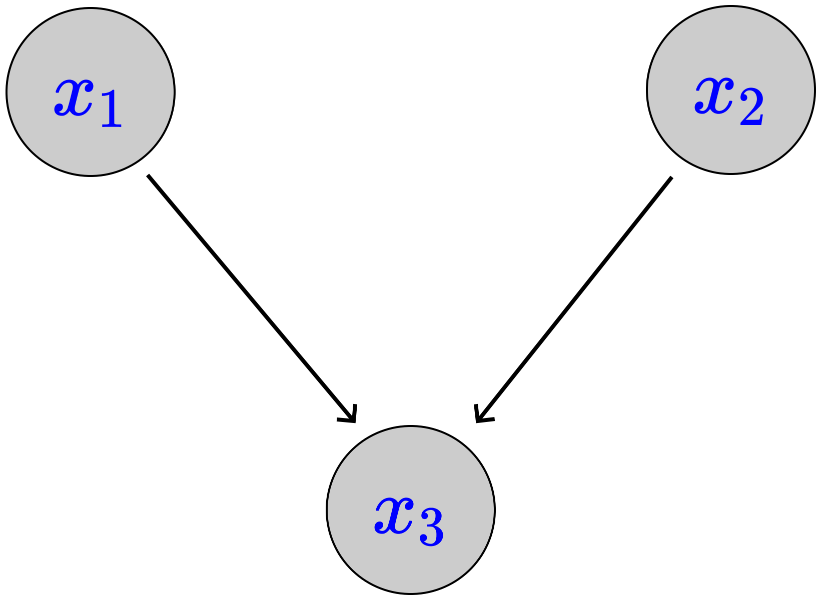
下之能事故六畫成

積諸陽氣而成天故

Bayesian Networks

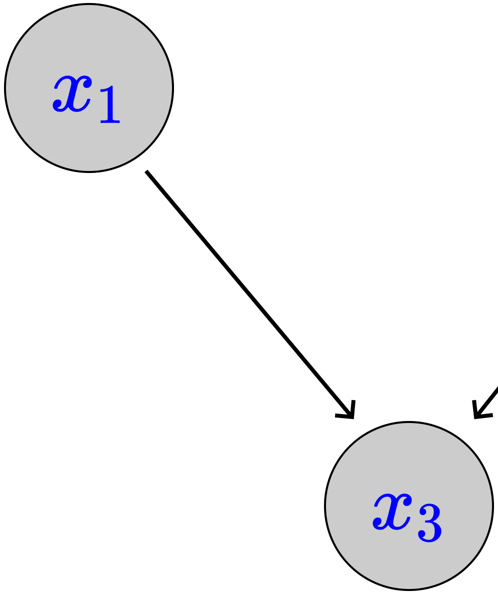
Can we represent this generative process with a Bayesian Network?

Bayesian Networks



Bayesian Networks

x_1	
1	2
0.5	0.5

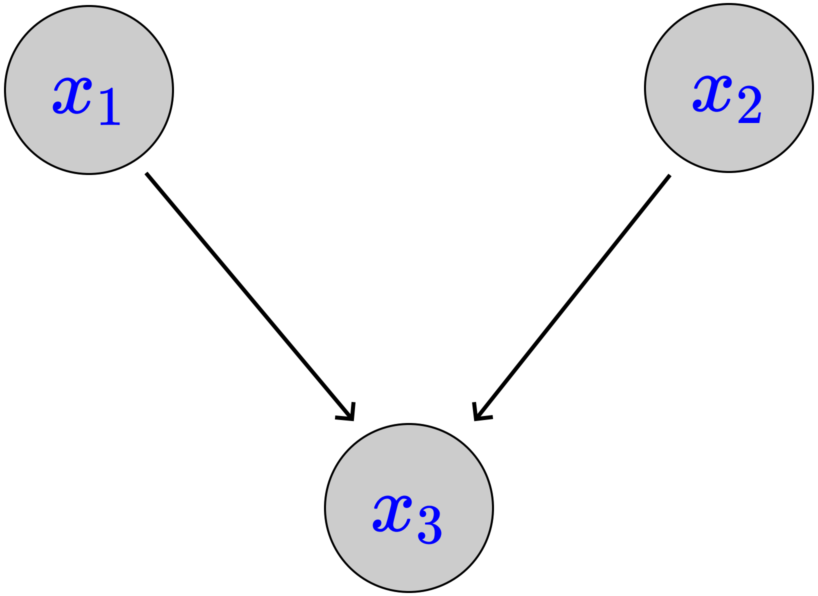


x_2	
1	2
0.5	0.5

		x_3	
x_1	x_2	—	– –
H	H	1	0
H	T	0	1
T	H	0	1
T	T	1	0

Bayesian Networks

x_1	
1	2
0.5	0.5



x_2	
1	2
0.5	0.5

		x_3	
x_1	x_2	—	- -
H	H	1	0
H	T	0	1
T	H	0	1
T	T	1	0

Are x_1 and x_2 independent?

What if x_3 is given?

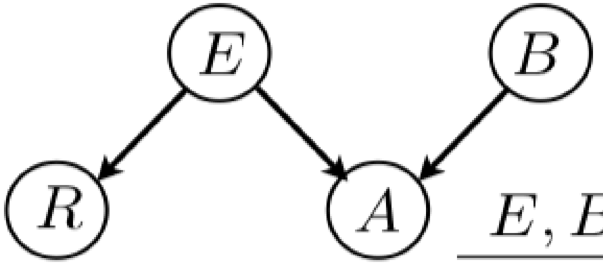
Bayesian Networks

E :

T	F
0.01	0.99

B :

T	F
0.01	0.99



R :

E	T	F
T	1	0
F	0	1

$A|E,B$:

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$P(E = e, B = b, A = a, R = r)$$

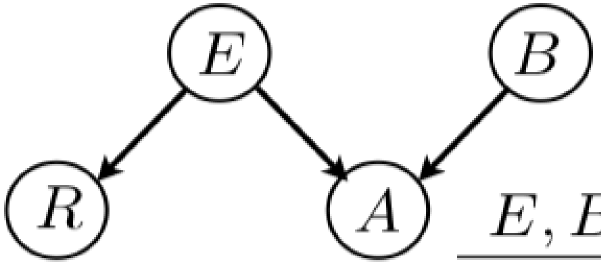
Bayesian Networks

E:

	T	F
	0.01	0.99

B:

	T	F
	0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$P(E=e,B=b,A=a,R=r)$$
$$=P(E=e)P(B=b)P(A=a|E=e,B=b)P(R=r|E=e)$$

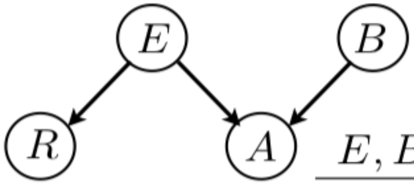
Bayesian Networks

E:

T	F
0.01	0.99

B:

T	F
0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$P(B = T | A = T) = ?$$

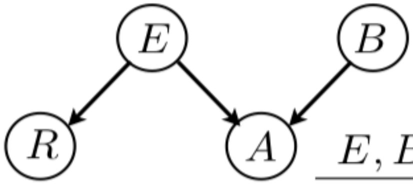
Bayesian Networks

$E:$

T	F
0.01	0.99

$B:$

T	F
0.01	0.99



$R:$

E	T	F
T	1	0
F	0	1

$A|E,B:$

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$P(B=T|A=T) = \frac{P(B{=}T, A{=}T)}{\sum_{b\in\{T,F\}} P(B{=}b, A{=}T)}$$

Let us look at this problem

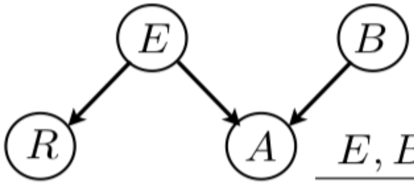
Bayesian Networks

E :

T	F
0.01	0.99

B :

T	F
0.01	0.99



R :

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$
\begin{aligned}
& P(B=b, A=T) \\
= \quad & \textstyle\sum_{e\in\{T,F\}} \sum_{r\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b)P(R=r|E=e)
\end{aligned}
$$

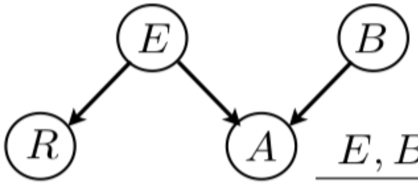
Bayesian Networks

E:

	T	F
	0.01	0.99

B:

	T	F
	0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$\begin{aligned} & P(B=b, A=T) \\ = \;& \textstyle\sum_{e\in\{T,F\}} \sum_{r\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b)P(R=r|E=e) \\ = \;& \textstyle\sum_{e\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b) \sum_{r\in\{T,F\}} P(R=r|E=e) \end{aligned}$$

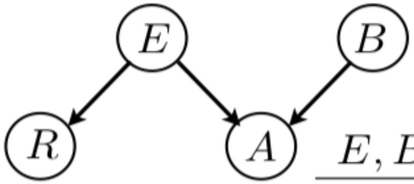
Bayesian Networks

E :

T	F
0.01	0.99

B :

T	F
0.01	0.99



R :

E	T	F
T	1	0
F	0	1

$A \mid E, B$:

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$
\begin{aligned}
& P(B=b, A=T) \\
= \;& \textstyle\sum_{e\in\{T,F\}} \sum_{r\in\{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b)P(R=r \mid E=e) \\
= \;& \textstyle\sum_{e\in\{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b) \sum_{r\in\{T,F\}} P(R=r \mid E=e) \\
= \;& \textstyle\sum_{e\in\{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b)
\end{aligned}
$$

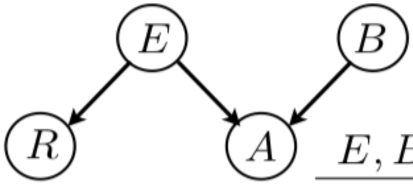
Bayesian Networks

$E:$

	T	F
	0.01	0.99

$B:$

	T	F
	0.01	0.99



$R:$

E	T	F
T	1	0
F	0	1

$A \mid E, B:$

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$
\begin{aligned}
& P(B=b, A=T) \\
= \quad & \textstyle\sum_{e \in \{T,F\}} \sum_{r \in \{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b)P(R=r \mid E=e) \\
= \quad & \textstyle\sum_{e \in \{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b) \sum_{r \in \{T,F\}} P(R=r \mid E=e) \\
= \quad & \textstyle\sum_{e \in \{T,F\}} P(E=e)P(B=b)P(A=T \mid E=e, B=b) \\
= \quad & \textstyle P(B=b) \sum_{e \in \{T,F\}} P(E=e)P(A=T \mid E=e, B=b)
\end{aligned}
$$

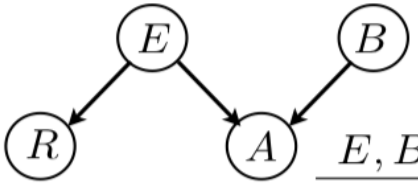
Bayesian Networks

E:

T	F
0.01	0.99

B:

T	F
0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$\begin{aligned}
& P(B=b, A=T) \\
= \;& \sum_{e\in\{T,F\}} \sum_{r\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b)P(R=r|E=e) \\
= \;& \sum_{e\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b) \sum_{r\in\{T,F\}} P(R=r|E=e) \\
= \;& \sum_{e\in\{T,F\}} P(E=e)P(B=b)P(A=T|E=e,B=b) \\
= \;& P(B=b) \sum_{e\in\{T,F\}} P(E=e)P(A=T|E=e,B=b)
\end{aligned}$$

$$P(B=T|A=T) = \frac{P(B{=}T, A{=}T)}{\sum_{b\in\{T,F\}} P(B{=}b, A{=}T)} = ?$$

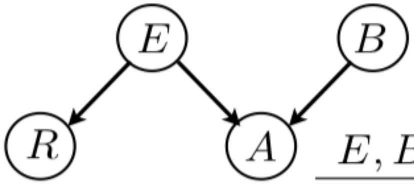
Bayesian Networks

E:

	T	F
	0.01	0.99

B:

	T	F
	0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A \mid E, B$:

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

$$P(B = T \mid A = T, R = T) = ?$$

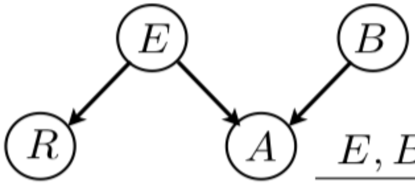
Bayesian Networks

E:

	T	F
	0.01	0.99

B:

	T	F
	0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$P(B=T|A=T,R=T)=?$$

$$P(B=T|A=T,E=T)=?$$

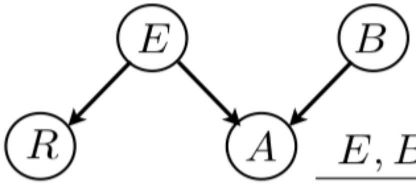
Bayesian Networks

E:

T	F
0.01	0.99

B:

T	F
0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

$$P(B=T|A=T,R=T)=?$$

$$P(B=T|A=T,E=T)=0.01$$

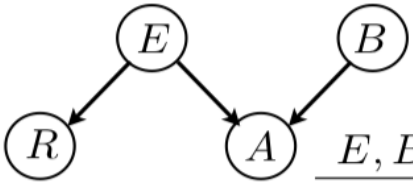
Bayesian Networks

E:

T	F
0.01	0.99

B:

T	F
0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E, B	T	F
T, T	1	0
T, F	1	0
F, T	1	0
F, F	0	1

≈ 0.5 $\quad$ 0.01

$$P(B = T | A = T) \neq P(B = T | A = T, E = T)$$

$$\boxed{B \text{ and } E \text{ are not independent given } A}$$

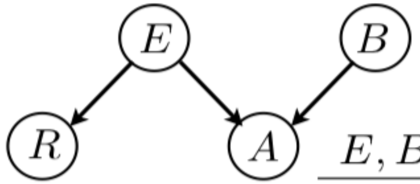
Explaining Away

E:

T	F
0.01	0.99

B:

T	F
0.01	0.99



R:

E	T	F
T	1	0
F	0	1

$A|E,B$:

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0
F,F	0	1

≈ 0.5 $\qquad\qquad$ 0.01

$$P(B=T|A=T) \neq P(B=T|A=T,E=T)$$

$$\boxed{B \text{ and } E \text{ are not independent given } A}$$

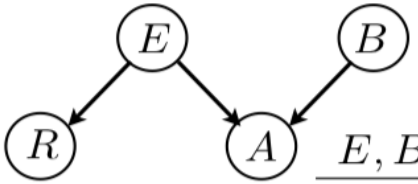
Bayesian Networks

$E:$

	T	F
	0.01	0.99

$B:$

	T	F
	0.01	0.99



$R:$

E	T	F
T	1	0
F	0	1

$A|E,B:$

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0

Can we read off such independence information from the network directly without involving calculation?

$$P(B=T|A=T) \neq P(B=T|A=T, E=T)$$

$$\boxed{B \text{ and } E \text{ are not independent given } A}$$

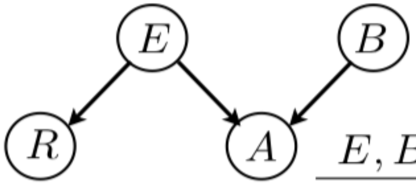
Bayesian Networks

$E:$

T	F
0.01	0.99

$B:$

T	F
0.01	0.99



$R:$

E	T	F
T	1	0
F	0	1

$A|E,B:$

E,B	T	F
T,T	1	0
T,F	1	0
F,T	1	0

Can we read off such independence information from the netw rk directly without invol tion?

Next Lecture!

$$P(B = T|A \;\ldots\;) \neq P(B = T|A = T, E = T)$$

$$\boxed{B \text{ and } E \text{ are not independent given } A}$$