L01.02 Coding Cost model Document Distance

50.004 Introduction to Algorithm

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(slides adapted from Dr. Simon LUI)

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About the coding requirement of the course

coding requirement of this course

- we will be reading and understanding code / pseudo code
- You need to do some coding in exercise/ homework/problem set

- You need to read/write code in exam
 - E.g. read a code, tell me if it is O(n) or O(nlogn)

coding requirement of this course

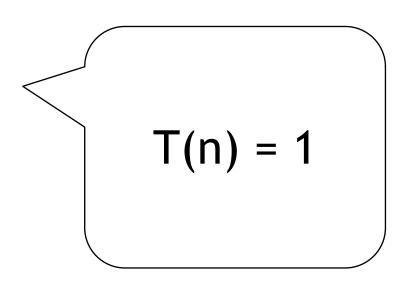
What is pseudo code if a is divisible by 3 print "OH YEAH"

What is code

```
if (a%3 == 0)
  print "OH YEAH";
```

How to read code and find its T(n)

Print("GREAT");



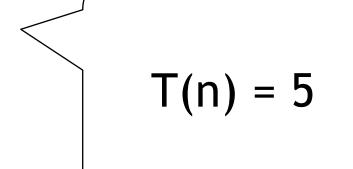
i.e. the number of steps required to complete this program is 1

This code is $\Theta(1)$

```
Print("GREAT");
Print("GREAT");
T(n) = 4
Print("GREAT");
```

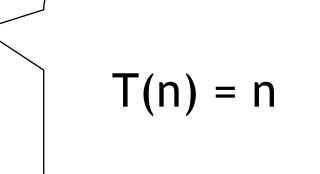
i.e. the number of steps required to complete this program is 4

This code is $\Theta(1)$



i.e. the number of steps required to complete this program is 5

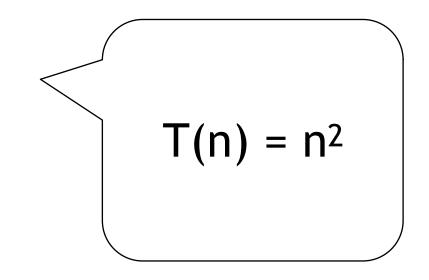
This code is $\Theta(1)$



i.e. the number of steps required to complete this program is n

This code is $\Theta(n)$

```
For (int i=0;i<n;i++)
For (int j=0;j<n;j++)
Print("GREAT");
```



i.e. the number of steps required to complete this program is n²

This code is $\Theta(n^2)$

```
For (i=0;i<n;i++)
    For (j=0;j<n;j++)
        Print("GREAT");
For (k=0;k<n;k++)
    print("GREAT");</pre>
```

$$T(n) = n^2 + n$$

i.e. the number of steps required to complete this program is n²+n

> This code is Θ(n²)

```
For (i=0;i<n;i++)
  For (j=0;j<n;j++)
     Print("GREAT");
For (k=0;k<n;k++)
  print("GREAT");
print("GREAT");
print("GREAT");
print("GREAT");
```

i.e. the number of steps required to complete this program is n²+n

This code is $\Theta(n^2)$

The cost model of code

Python cost model

$$L = [a_1, a_2, ..., a_n]$$
 L is a list

Commands	complexity (time)	
L[i]=x	Θ(1)	
L.append(x)	Θ(1)	Add x to L
L_1 .extend(L_2)	$\Theta(L_2)$	Add L ₂ to L ₁
$A = L_1 + L_2$	$\Theta(L_1 + L_2)$	Add L ₂ and L ₁ to A
x in L	Θ(L)	Search for x

$$D = \{x_1 : y_1, x_2 : y_2, ..., x_n : y_n\}$$
 D is a dictionary

Commands	complexity (time)	
$D[x_i]=y_i$	Θ(1)	X is index, y is value
x in D	Θ(1)	Search for x

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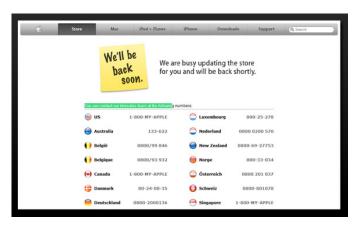
Document distance

- Document means sequence of words
- The Problem: Given 2 documents, find out how similar are they
- The Algorithm: we will talk about it

Document distance - usage

- 1. detect plagiarism
- 2. Web update check
- Simon loves iPhone. He wants to be the world first person to buy it
- So he write a DOCUMENT DISTANCE code to check if the apple webpage has updated
 - If updated, probably there are new product.
 - So, send a email to notify Simon

Document distance - usage



```
<!DOCTYPE html>
<html class="singapore sg nois en en-sg apac" lang="en-SG">
    <head>
      <meta name="viewport" content="width=1024" />
  <title>Pre-order iPhone 6s and iPhone 6s Plus - Apple (SG)</
title>
  <meta charset="utf-8" />
  <meta http-equiv="X-UA-Compatible"
content="IE=edge,chrome=1" />
  <meta name="format-detection" content="telephone=no" />
  <meta http-equiv="Set-Cookie"
content="as sfa=MnxzZ3xzZ3x8ZW5fU0d8Y29uc3VtZXJ8aW50ZXJ
uZXR8MHwwfDE=; path=/; domain=.apple.com; expires=Mon,
15-Sep-2025 04:18:11 GMT;" />
                   <meta property="og:type"
content="product" />
```

Apple.com/iphone.html ...at 11:58am



Apple.com/iphone.html ...at 11:59am

COMPARE

Problem definition

- Document D: sequence of words
- Word w: sequence of characters
- Word frequency D(w): number of occurrences of w in D
- For example
 - D: "good morning good good"
 - w: {"good", "morning", "good", "good"}
 - w[0] = "good"
 - w[1] = "morning"
 - w[2]= "good"
 - w[3] = "good"
 - D("good") = 3

What we will do today

- I will show you some VERY COMPLICATED CODE
- No need to understand them (out of syllabus)
- I just want see
 - the running time of O(1), O(n) and O(n²) function can be very different
 - How to improve the speed of a code

- Treat each document as a vector of its words
 - one coordinate per word of the English dictionary

The dot product between D1 and D2

represent their similarity

w

$$D_1 \circ D_2 \equiv \sum D_1(w) \cdot D_2(w)$$

```
e.g.
"the cat"。 "the dog"
= [the x the + cat x dog]
= 1 + 0
= 1
```

```
e.g.
"the the cat cat". "the the dog dog"
= the x the + the x the + cat x dog + cat x dog
= 1 + 1 + 0 + 0
= 2
```

... wait,

A: "the cat" vs "the dog"

B: "the the cat cat" vs "the the dog dog"

Both problem A and B are 50% similar. Why B score more (2) than A (1)? So, we need to normalize the result

```
e.g.
["the cat". "the dog"] / sqrt(2 x 2)
= [the x the + cat x dog] / 2
= [1 + 0] / 2
= 50%
```

```
e.g.
["the the cat cat", "the the dog dog"] / sqrt(4)
\times 4
= [the x the + the x the + cat x dog + cat x dog]
/ 4
= [1 + 1 + 0 + 0]/4
= 50%
```

Vector space model

- Normalization
 - divide by the length of the vectors

$$\frac{D_1 \circ D_2}{||D_1|| \cdot ||D_2||} \qquad \qquad D_1 = aD_2$$

- measure distance by angle between vectors:

$$\theta(D_1, D_2) = a\cos\left(\frac{D_1 \circ D_2}{||D_1|| \cdot ||D_2||}\right)$$

e.g. θ =0 documents "identical" (if of the same size, permutations of each other)

 $\theta = \pi/2$ not even share a word

aabcvbnlbgbaanlaaubaanlcvbxfeof

Read file

aabcvbnlbgbaanlaaubaanlcvbxfeof

- Read file
- Make word list (divide file into words)

[aa, cv, bg, aa, aau, aa, cv,xf]

- Read file
- Make word list (divide file into words)
- Count frequencies of words

```
[aa, cv, bg, aa, aau, aa, cv,xf]  
[(aa,3), (cv,2), (bg,1), (aau,1),(xf,1)]
```

- Read file
- Make word list (divide file into words)
- Count frequencies of words
- Compute dot product

```
D<sub>1</sub>:[(aa,3), (aau,1), (bg,1), (cv,2)]
D<sub>2</sub>:[(aa,2), (ba,2), (bg,2), (ca,1), (fv,2)]
```

- Read file
- Make word list (divide file into words)
- Count frequencies of words
- Compute dot product
 - for every word in the first document, check if it appears in the other document; if yes, multiply their frequencies and add to the dot product
 - worst case time: order of #words(D₁) x #words(D₂)

- Read file
- Make word list (divide file into words)
- Count frequencies of words
- Compute dot product
 - for every word in the first document, check if it appears in the other document; if yes, multiply their frequencies and add to the dot product
 - worst case time: order of #words(D₁) x #words(D₂)
 - micro-optimization:
 - sort documents into word order (alphabetically)
 - compute inner product in time

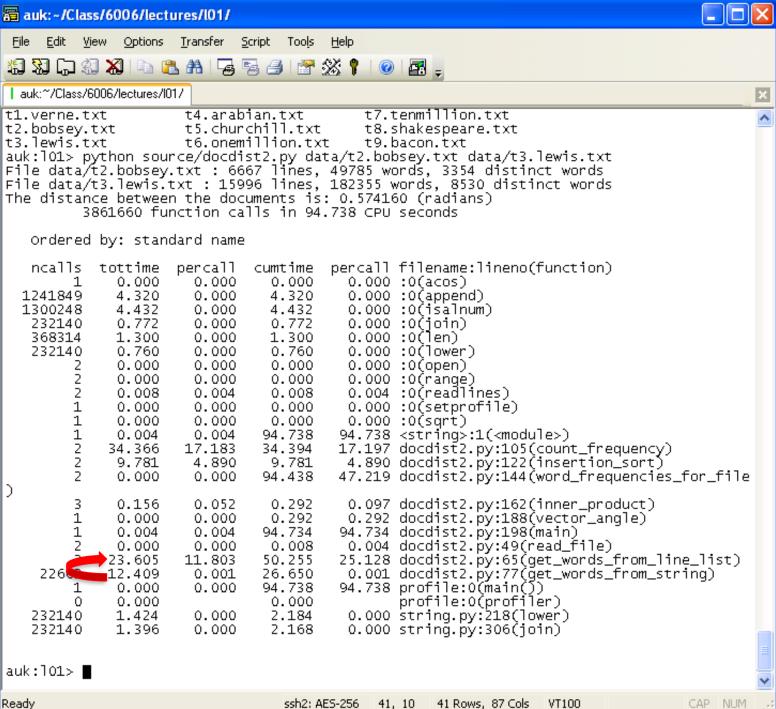
$$\#words(D_1) + \#words(D_2)$$

Python Implementation

- Docdist1.py (see handout)
- Read file: read_file(filename)
 - Output: list of lines (strings)
- Make word list: get_words_from_line_list(L)
 - Output: list of words (array)
- Count frequencies: count_frequency(word list)
 - Output: list of word-frequency pairs
- Dot product: inner_product(D1, D2)
 - Output: number

cProfiling.run()

- return T(n)
 - import profile
 - profile.run("main()")

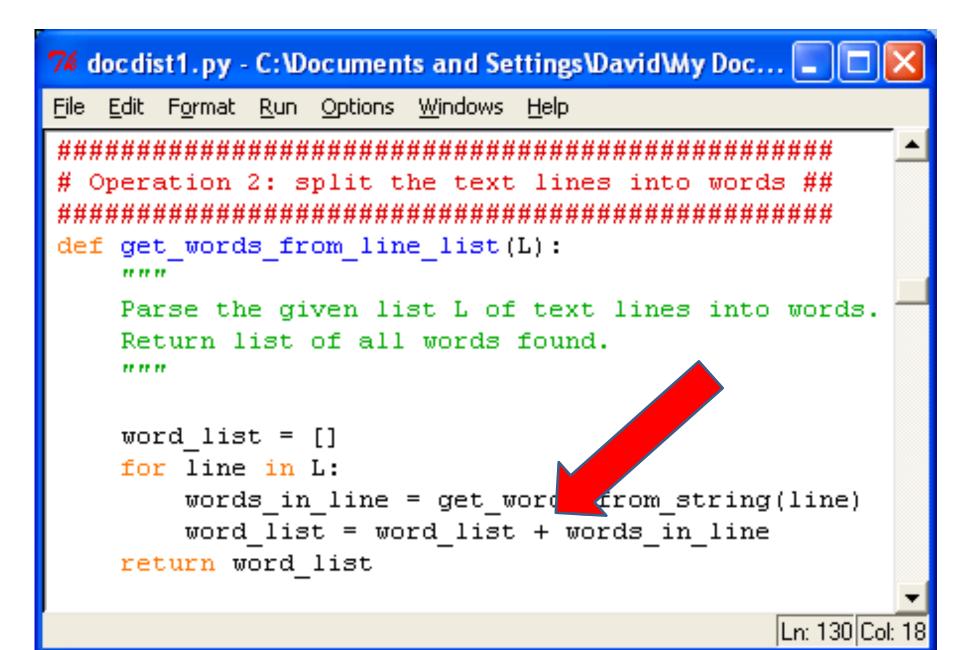


ssh2: AES-256 41, 10 41 Rows, 87 Cols VT100

docdist1

- Docdist1.py is slow
- Docdist2.py is faster
- Docdist3.py is even faster
- •
- Docdist8.py is the best

Now let's look at docdist1.py



+ is slow, need to improve it

- Look at word_list = word_list + words_in_line
- + : concatenation of arrays
- running time = T(k)
 - k = length (word_list) + length(words_in_line)
- Total T(n) is $1+2+...+n = n(n+1)/2 = \Theta(n^2)$

```
for line in L:
    words_in_line = get_word_from_string(line)
    word_list = word_list + words_in_line
```

Solution

- word_list = word_list + words_in_line: Θ(n²)
 - docdist1.py (3.7s)
- word_list.extend(words_in_line) : Θ(n)
 - docdist2.py (2.722s)
- How to run:
 - Python docdist1.py file5.txt file4.txt

Further Improvements

- Docdist3.py: sort the data first (2.657s)
- Docdist4.py: Instead of inserting words in list, insert in dictionary: (0.735s)
- Docdist5.py: Process words instead of chars (0.138s)
- Docdist6.py: better sorting algorithm (0.069s)
 - merge sort instead of insertion sort
- Docdist7.py: dictionary instead of sort.
 (0.052s)

Appendix - how to optimize the code

docdist<1-8>.py - Iterative improvements

Objective

 To arrive at the most efficient implementation in terms of running time

Approach

- At each iteration, analyze the running times of various parts of the code
- Based on the biggest consumer of running time, refactor the code by
 - Introducing faster algorithms
 - Using features of the Python language
 - Simplifying and optimizing code

```
1 def main():
2   if len(sys.argv) != 3:
3     print "Usage: docdist1.py filename_1 filename_2"
4   else:
5     filename_1 = sys.argv[1]
6     filename_2 = sys.argv[2]
7     document_vector_1 = word_frequencies_for_file(filename_1)
8     document_vector_2 = word_frequencies_for_file(filename_2)
9     distance = vector_angle(document_vector_1,document_vector_2)
10     print "The distance between the documents is: %0.6f (radians)"%distance
```

```
def main():
    if len(sys.argv) != 3:
      print "Usage: docdist1.py filename 1 filename 2"
    else:
      filename_1 = sys.argv[1]
      filename_2 = sys.argv[2]
      document_vector_1 = word_frequencies_for_file(filename_1)
      document_vector_2 = word_frequencies_for_file(filename_2)
      distance = vector_angle(document_vector_1, document_vector_2)
10
      print "The distance between the documents is: %0.6f (radians)"%distance
     def word_frequencies_for_file(filename):
       line_list = read_file(filename)
       word_list = get_words_from_line_list(line_list)
       freq_mapping = count_frequency(word_list)
       return freq mapping
```

3

4

5

6

7

```
def main():
  2
       if len(sys.argv) != 3:
        print "Usage: docdist1.py filename 1 filename 2"
  4
      else:
  5
         filename_1 = sys.argv[1]
  6
         filename_2 = sys.argv[2]
  7
         document_vector_1 = word_frequencies_for_file(filename_1)
  8
         document_vector_2 = word_frequencies_for_file(filename_2)
  9
         distance = vector angle (document vector 1, document vector 2)
 10
        print "The distance between the documents is: %0.6f (radians)"%distance
            def word frequencies for file (filename):
              line_list = read_file(filename)
              word list = get words from line list(line list)
              freq mapping = count frequency (word list)
              return freq mapping
                                                     word_list = []
 def get_words_from_line_list(L):
                                                     character_list = []
                                                10
    word list = []
                                                11
                                                     for c in line:
    for line in L:
                                                12
                                                       if c.isalnum():
      words_in_line = get_words_from_string(lin
                                                13
                                                         character_list.append(c)
      word_list = word_list + words_in_line
                                                14
                                                       elif len(character_list)>0:
    return word list
                                                15
                                                         word = "".join(character_list)
                                                         word = word.lower()
                                                16
8 def get_words_from_string(line):
                                                17
                                                         word_list.append(word)
                                                18
                                                         character list = []
                                                19
                                                     if len(character_list)>0:
                                                20
                                                       word = "".join(character_list)
                                                21
                                                       word = word.lower()
                                                22
                                                       word_list.append(word)
                                                23
                                                     return word list
                            SUTD ISTD 50.004 Intro to Algorithms
```

```
def main():
     if len(sys.argv) != 3:
       print "Usage: docdist1.py filename 1 filename 2"
4
     else:
5
       filename_1 = sys.argv[1]
       filename_2 = sys.argv[2]
6
7
       document_vector_1 = word_frequencies_for_file(filename_1)
8
       document_vector_2 = word_frequencies_for_file(filename_2)
9
       distance = vector angle (document vector 1, document vector 2)
10
       print "The distance between the documents is: %0.6f (radians)"%distance
         def word frequencies for file (filename):
            line_list = read_file(filename)
            word_list = get_words_from_line_list(line_list)
            freq mapping = count_frequency(word_list)
            return freq_mapping
                         1 def count_frequency(word_list):
                         L = []
                             for new word in word list:
                               for entry in L:
                                 if new_word == entry[0]:
                         6
                                   entry[1] = entry[1] + 1
                                   break
                         8
                               else:
                                 L.append([new_word,1])
                        10
                             return L
```

3

4

```
def main():
    if len(sys.argv) != 3:
      print "Usage: docdist1.py filename 1 filename 2"
    else:
5
      filename_1 = sys.argv[1]
6
      filename_2 = sys.argv[2]
      document_vector_1 = word_frequencies_for_file(filename_1)
8
      document_vector_2 = word_frequencies_for_file(filename_2)
9
      distance = vector_angle(document_vector_1, document_vector_2)
      print "The distance between the documents is: %0.6f (radians)"%distance
10
 def vector_angle(L1,L2):
   numerator = inner_product(L1,L2)
   denominator = math.sqrt(inner_product(L1,L1)*inner_product(L2,L2))
   return math.acos(numerator/denominator)
```

docdist1 vs. docdist2

def get_words_from_line_list(L):

6

word list = []

words in line = get words from string(line)

word list.extend(words in line)

return word list

docdist1 vs. docdist2

docdist1 Performance Scorecard

Method	Time
get_words_from_line_list	$O(\frac{W^2}{k}) = O(W^2)$
count_frequency	O(WL)
word_frequencies_for_file	$O(W^2)$
inner_product	$O(L_1L_2)$
vector_angle	$O(L_1L_2 + L_1^2 + L_2^2) = O(L_1^2 + L_2^2)$
main	$O(W_1^2 + W_2^2)$

docdist2 Performance Scorecard

Method	Time
get_words_from_line_list	O(W)
count_frequency	O(WL)
word_frequencies_for_file	O(WL)
inner_product	$O(L_1L_2)$
vector_angle	$O(L_1^2 + L_2^2)$
main	$O(W_1L_1 + W_2L_2)$

docdist2 vs. docdist3

```
1 def word_frequencies_for_file(filename):
2  line_list = read_file(filename)
3  word list = get words from line list(line list)
4  freq_mapping = count_frequency(word_list)
5  return freq_mapping
```

```
1 def word_frequencies_for_file(filename):
2    line_list = read_file(filename)
3    word_list = get_words_from_line_list(line_list)
4    freq_mapping = count_frequency(word_list)
5    insertion_sort(freq_mapping)
6    return freq_mapping
```

docdist3 - Implementing insertion_sort()

```
1 def insertion_sort(A):
2   for j in range(len(A)):
3     key = A[j]
4     i = j-1
5     while i>-1 and A[i]>key:
6         A[i+1] = A[i]
7     i = i-1
8     A[i+1] = key
9   return A
```

docdist3 - Reimplementing inner_product()

```
def inner_product(L1,L2):
    sum = 0.0
    for word1, count1 in L1:
4
      for word2, count2 in L2:
5
        if word1 == word2:
6
           sum += count1 * count2
    return sum
                              def inner_product(L1,L2):
                                 sum = 0.0
                               i = 0
                            4
                                 i = 0
                                while i<len(L1) and j<len(L2):
                                   # L1[i:] and L2[j:] yet to be processed
                            6
                                   if L1[i][0] == L2[j][0]:
                           8
                                     # both vectors have this word
                                     sum += L1[i][1] * L2[j][1]
                           10
                                     i += 1
                           11
                                    i += 1
                                  elif L1[i][0] < L2[j][0]:
                           13
                                     # word L1[i][0] is in L1 but not L2
                           14
                                     i += 1
                           15
                                  else:
                           16
                                     # word L2[j][0] is in L2 but not L1
                           17
                                     j += 1
```

docdist2 vs. docdist3

docdist2 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(WL)
word frequencies for file	O(WL)
inner_product	$O(L_1L_2)$
vector_angle	$O(L_1^2 + L_2^2)$
main	$O(W_1L_1 + W_2L_2)$

docdist3 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(WL)
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(WL + L^2) = O(WL)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1L_1 + W_2L_2)$

docdist4 - Refining count_frequency()

```
def count frequency (word list):
     L = []
     for new word in word list:
       for entry in L:
4
5
         if new word == entry[0]:
6
            entry[1] = entry[1] + 1
           break
8
       else:
9
         L.append([new_word, 1])
10
     return L
```

```
1 def count_frequency(word_list):
2   D = {}
3   for new_word in word_list:
4    if new_word in D:
5     D[new_word] = D[new_word]+1
6   else:
7     D[new_word] = 1
8   return D.items()
```

docdist3 vs. docdist4

docdist3 Performance Scorecard

Method	Time
get words from line list	O(W)
count_frequency	$O(\widetilde{WL})$
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(WL + L^2) = O(WL)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1L_1 + W_2L_2)$

docdist4 Performance Scorecard

Method	Time
get words from line list	O(W)
count_frequency	O(W)
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(W + L^2) = O(L^2)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2 + L_1^2 + L_2^2)$

docdist5 - Simplifying get_words_from_string()

```
def get words from string(line):
 9
      word list = []
10
      character list = []
      for c in line:
11
12
        if c.isalnum():
13
           character list.append(c)
14
        elif len(character list)>0:
           word = "".join(character_list)
15
16
           word = word.lower()
17
           word list.append(word)
18
           character list = []
19
      if len(character list)>0:
20
        word = "".join(character list)
21
        word = word.lower()
22
        word list.append(word)
23
      return word list
                       1 translation table = string.maketrans(string.punctuation+string.uppercase,
                       2
                                         " "*len(string.punctuation)+string.lowercase)
                       3
                       4 def get words from string(line):
                       5 line = line.translate(translation table)
                       6 word_list = line.split()
                           return word list
```

docdist4 vs. docdist5 (same as docdist4)

docdist4 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(W + L^2) = O(L^2)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2 + L_1^2 + L_2^2)$

docdist4 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(W + L^2) = O(L^2)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2 + L_1^2 + L_2^2)$

docdist6 - Replacing insertion sort by merge sort

```
def word frequencies for file(filename):
     line list = read file(filename)
3
     word_list = get_words_from_line_list(line_list)
4
     freq_mapping = count_frequency(word_list)
5
     freq_mapping = merge_sort(freq_mapping)
6
                                                         def merge_sort(A):
     return freq mapping
                                                          n = len(A)
                                                           if n==1:
                                                             return A
                                                          mid = n//2
                                                          L = merge_sort(A[:mid])
                                                           R = merge sort(A[mid:])
                                                           return merge (L,R)
                                                      10 def merge(L,R):
                                                      11
                                                           i = 0
                                                           \dot{j} = 0
                                                           answer = []
                                                          while i<len(L) and j<len(R):</pre>
                                                      15
                                                             if L[i]<R[j]:
                                                             answer.append(L[i])
                                                      17
                                                               i += 1
                                                             else:
                                                      19
                                                               answer.append(R[j])
                                                      20
                                                               j += 1
                                                           if i<len(L):</pre>
                                                      21
                                                             answer.extend(L[i:])
                                                      23
                                                           if j<len(R):</pre>
                                                      24
                                                             answer.extend(R[j:])
                                                      25
                                                           return answer
```

docdist5 vs. docdist6

docdist4 Performance Scorecard

Method	Time
get_words_from_line_list	O(W)
count frequency	O(W)
insertion_sort	$O(L^2)$
word_frequencies_for_file	$O(W + L^2) = O(L^2)$
<pre>word_frequencies_for_file inner_product</pre>	$O(W + L^2) = O(L^2)$ $O(L_1 + L_2)$
<u> </u>	$O(W + L^{2}) = O(L^{2})$ $O(L_{1} + L_{2})$ $O(L_{1} + L_{2})$

docdist6 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
merge_sort	$O(L \log L)$
word_frequencies_for_file	$O(W + L \log L) = O(L \log L)$
inner_product	$O(W + L \log L) = O(L \log L)$ $O(L_1 + L_2)$
•	

docdist7 - From sorted list to document vectors

```
1 def count frequency (word list):
    D = \{\}
    for new word in word list:
      if new word in D:
        D[new_word] = D[new_word]+1
      else:
        D[new word] = 1
    return D
  def word frequencies for file (filename):
    line_list = read_file(filename)
    word_list = get_words_from_line_list(line_list)
    freq mapping = count frequency(word list)
    return freq_mapping
  def inner_product(D1,D2):
    sum = 0.0
    for key in D1:
      if key in D2:
         sum += D1[key] * D2[key]
    return sum
```

docdist6 vs. docdist7

docdist6 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
merge_sort	$O(L \log L)$
word_frequencies_for_file	$O(W + L \log L) = O(L \log L)$
inner_product	$O(L_1 + L_2)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2 + L_1 \log L_1 + L_2 \log L_2)$

docdist7 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
word_frequencies_for_file	O(W)
inner_product	$O(L_1)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2)$

docdist8 - Simplifying the code

```
1 def get_words_from_string(string):
2    string = string.translate(translation_table)
3    word_list = string.split()
4    return word_list
5
6    def word_frequencies_for_file(filename):
7     text = read_file(filename)
8    word_list = get_words_from_string(text)
9    freq_mapping = count_frequency(word_list)
10    return freq_mapping
```

docdist7 vs. docdist8

docdist7 Performance Scorecard

Method	Time
<pre>get_words_from_line_list</pre>	O(W)
count_frequency	O(W)
word_frequencies_for_file	O(W)
inner_product	$O(L_1)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2)$

docdist8 Performance Scorecard

Method	Time
get_words_from_text	O(W)
count_frequency	O(W)
word_frequencies_for_file	O(W)
inner_product	$O(L_1)$
vector_angle	$O(L_1 + L_2)$
main	$O(W_1 + W_2)$