Image histogram

ISTD 50.035

Computer Vision

Acknowledgement: Some images are from various sources: UCF, Stanford cs231n, etc.

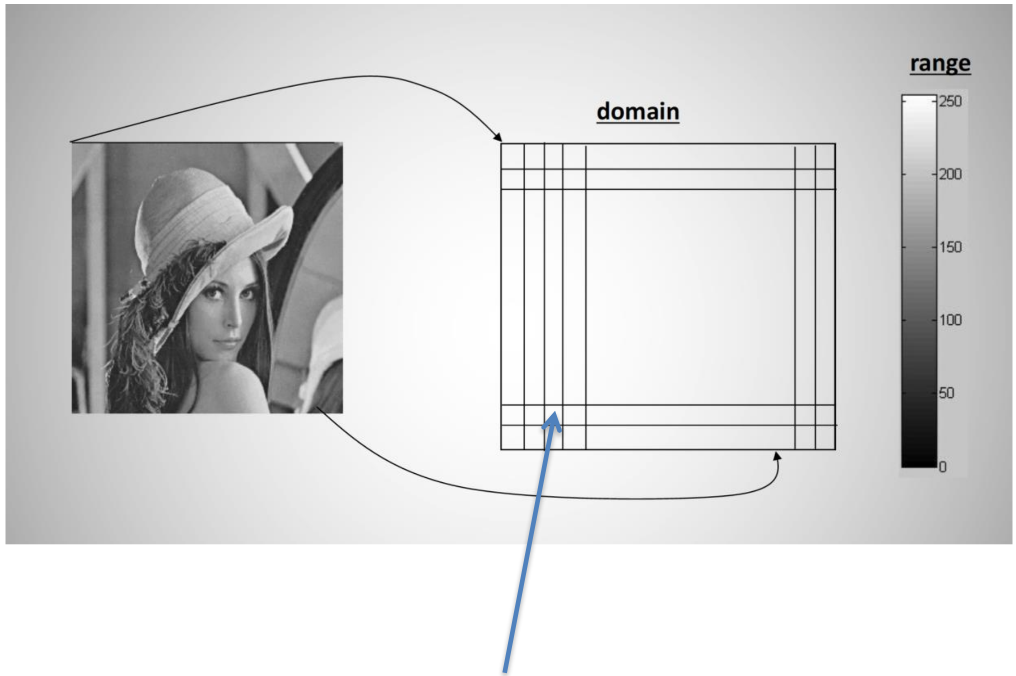
Image is an array of numbers

-Grayscale image

-2D array of numbers (pixels) / matrix

-Number indicates the intensity: [0,255] for 8-bit representation

-Image resolution / number of pixel in an image: 100x100, 1920x1080, etc.



A pixel

0: black, 255: white

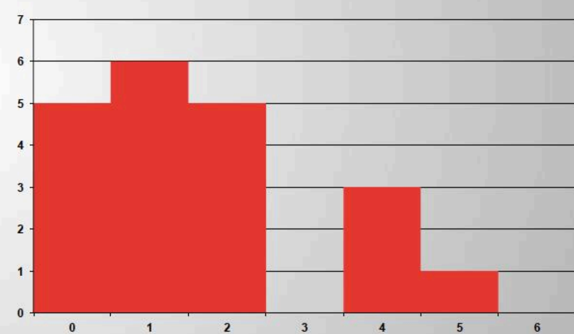
Image Histogram

- Histogram
  - X-axis: bins of possible values
  - Y-axis: frequency of a value (number of samples)
- Normalize Y-axis => probability mass function
- Area = total number of pixels

camera image histogram

0	1	1	2	4
2	1	0	0	2
5	2	0	0	4
1	1	2	4	1

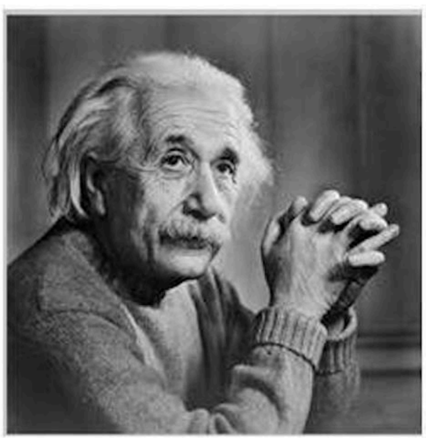
Image (In general, data)

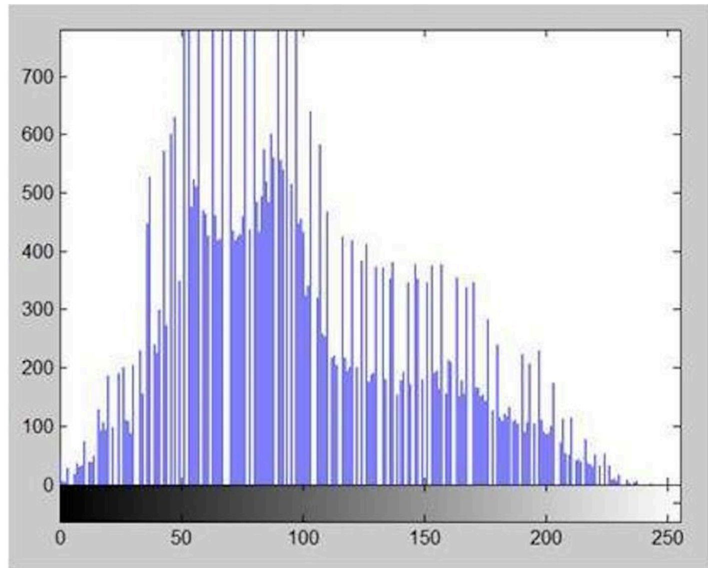


Histogram

Image Histogram

- Image Histogram
- X-axis: pixel values, i.e. 0 to 255
- Y-axis: number of pixels with a certain pixel value





Histogram equalization

- To increase the contrast of an image
  - Over or under-exposed photographs
  - Medical imaging: x-ray images, etc.
- Distribute intensities more evenly over the range: spread out the most frequent intensity values

Image with low contrast, what would be the histogram?

Histogram equalization

- To increase the contrast of an image
  - Over or under-exposed photographs
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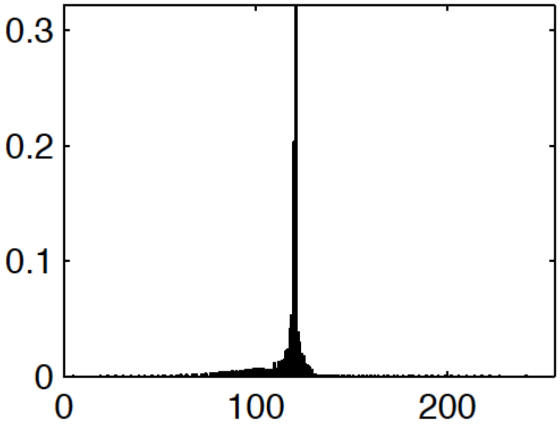
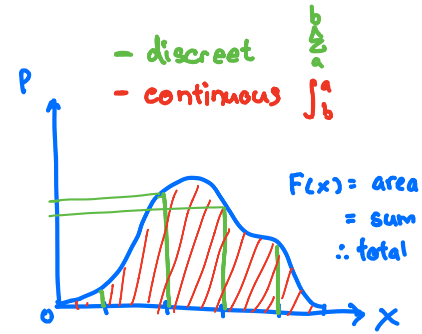


Image with low contrast, what would be the histogram?

Cumulative distribution / density function (cdf)



- The cdf of a random variable X is given by

$$F_X(x) = P(X \leq x)$$

- If X is a continuous r.v., cdf is given by ($f_X(x)$ is the probability density function, pdf)

area of a range of value.
Resulting in cumulative probability

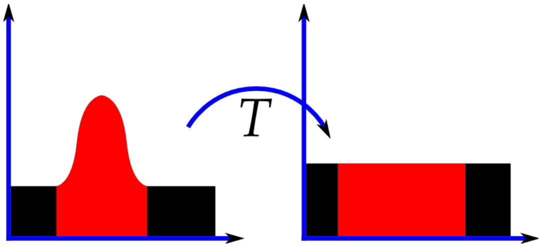
$$F_X(x) = \int_{-\infty}^{x} f_X(w)dw$$

- If X is a discrete r.v., cdf is given by (p_i is the probability mass of X at i)

$$F_X(k) = \sum_{i=-\infty}^{k} p_i$$

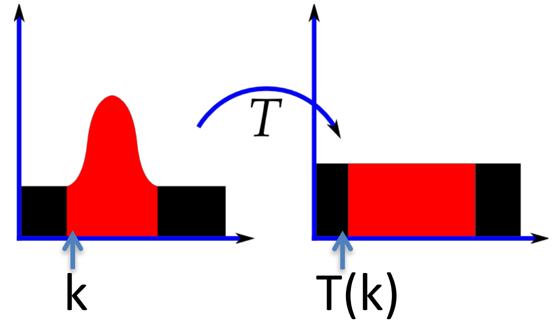
Histogram equalization

- Apply a transformation T to distribute intensities evenly over the range -> increase contrast
- A mapping of pixel value
- Note area (no. of pixel) in the histogram remains the same after transformation



Histogram equalization

- A mapping of pixel value
- For a pixel with intensity k, transform it using (L = number of level = 256)



$$T(k) = \text{floor}((L-1)\sum_{i=0}^{k} p_i) = \text{floor}((L-1)F_X(k))$$

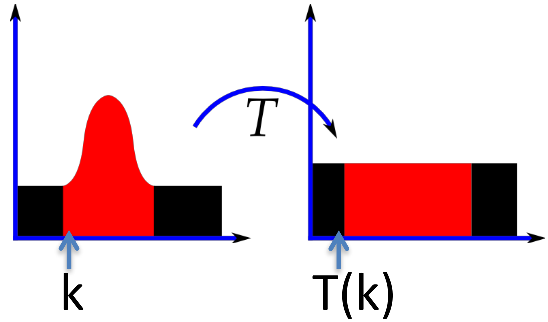
A pixel with value k

A pixel with value T(k)

Histogram equalization

- A mapping of pixel value
- For a pixel with intensity k, transform it using (L = number of level = 256)

Grey saturation , 0 - 255



$$T(k) = \text{floor}((L-1)\sum_{i=0}^{k} p_i) = \text{floor}((L-1)F_X(k))$$

- Algorithm:
  1. Compute cdf at k [*]
  2. Multiply by L-1, then floor(.)
  3. The result is the new intensity value

[*] normalize cdf to [0,1] by:

[acc – acc.min()] / [acc.max() – acc.min()]

Check that acc.max() equals to number of pixels

Histogram equalization

- Cohort exercise

$$T(k) = \text{floor}((L-1)\sum_{i=0}^{k} p_i) = \text{floor}((L-1)F_X(k))$$

k	count	accumulate	cdf	T(k) = ⌊255 × cdf⌋
1	3	3	0	0
2	3	6	3/13	58
3	6	12	9/13	176
10	2	14	11/13	215
11	2	16	1	255

Input:

[[1 3 1 3]
[2 3 10 11]
[11 10 2 3]
[1 2 3 3]]

Output:

[[0 176 0 176]
[58 176 215 255]
[255 215 58 176]
[0 58 176 176]]

Input:

[[52 52 53 72]
[72 72 53 53]
[88 72 52 52]
[88 88 53 53]]

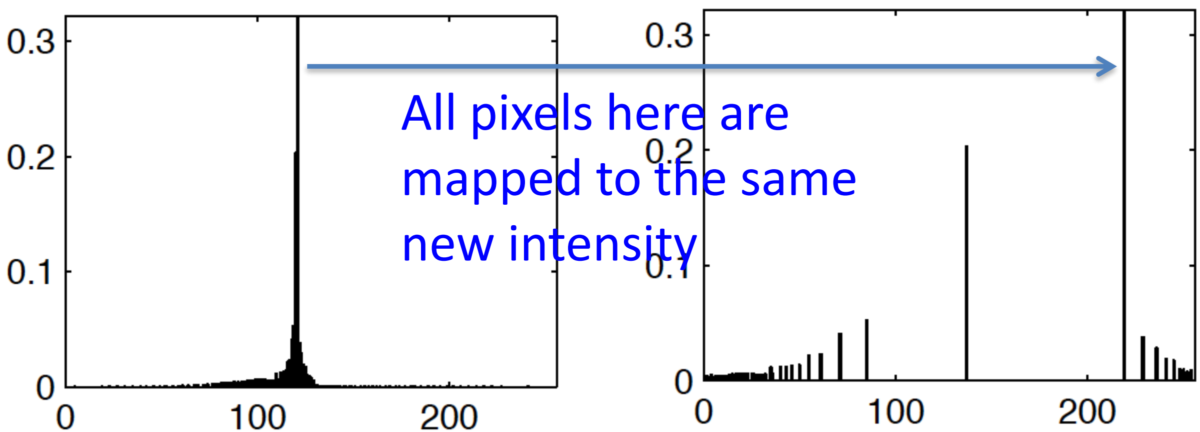
k	count	acc	cdf	T(k)
52	4	4	0	0
53	5	9	5/12	106
72	4	13	9/12	191
88	3	16	1	255

Output:

[[0 0 106 191]
191 191 106 106
255 191 0 0
255 255 106 106]]

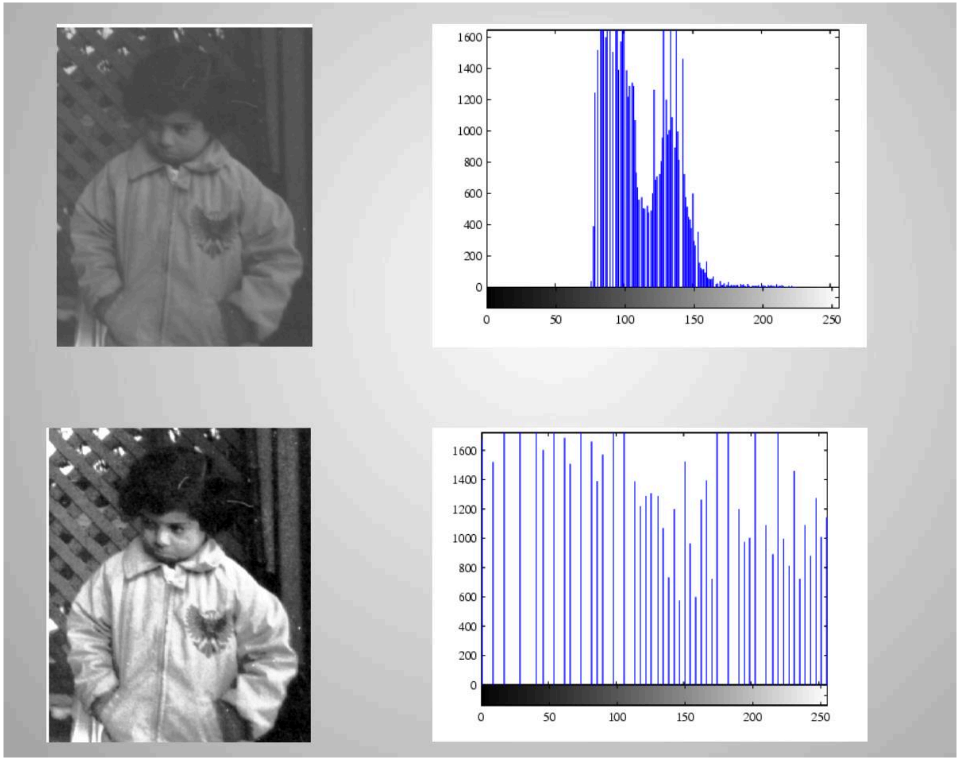
Histogram equalization

- An example



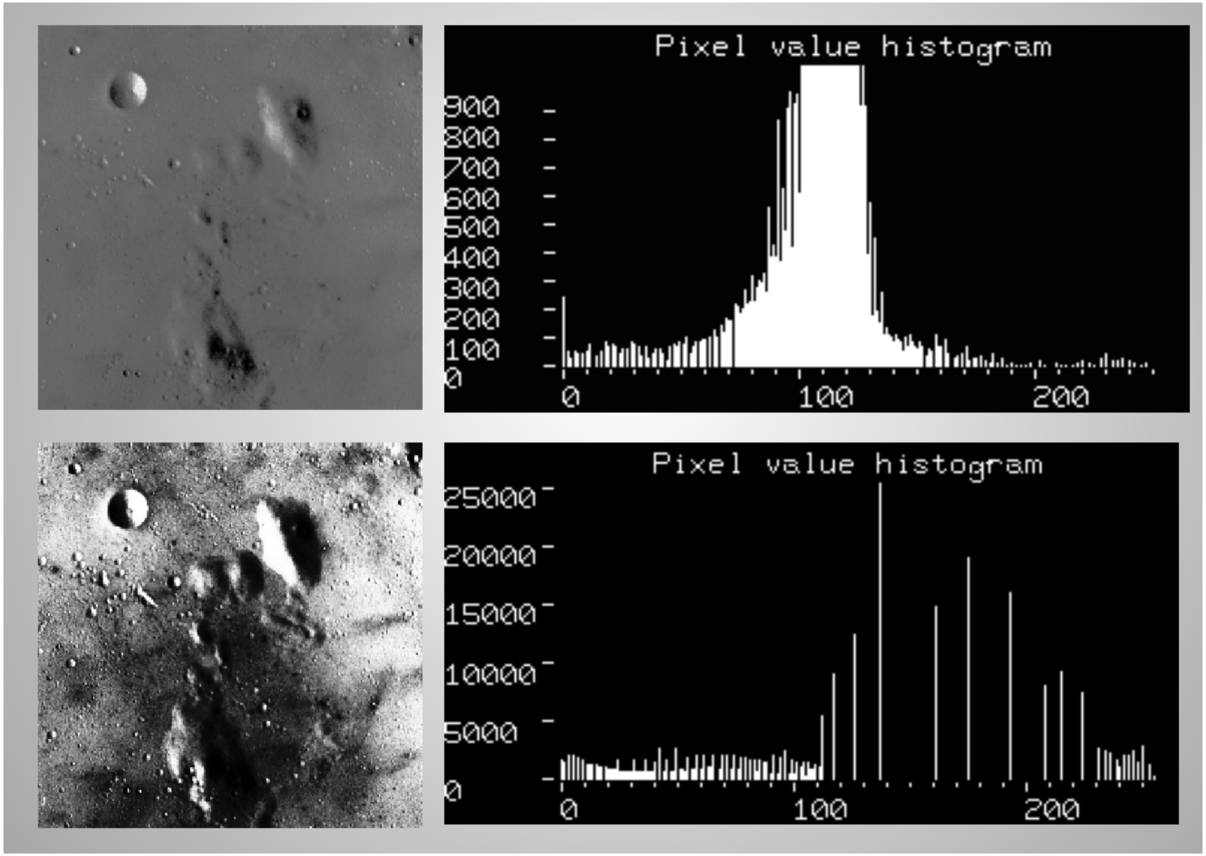
Histogram equalization

- An example

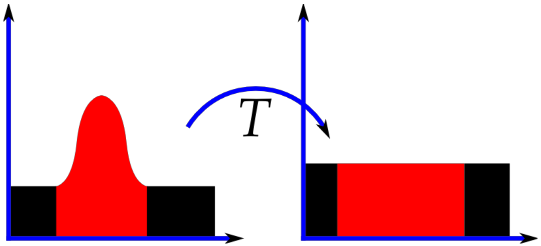


Histogram equalization

- An example



Histogram equalization - Justification



- Discrete case:

$$T(k) = \text{floor}((L-1)\sum_{i=0}^{k} p_i) = \text{floor}((L-1)F_X(k))$$

- Continuous case: T(.) transforms a continuous r.v. X ~ $p_X(x)$ into Y ~ $p_Y(y)$, so that $p_Y(y)$ = U[0,L-1]

$$Y = T(X) = (L-1)F_X(X)$$

- Note that for any X ~ $p_X(x)$, Y is U[0,1] when the transformation is the cdf of X

$$Y = F_X(X)$$

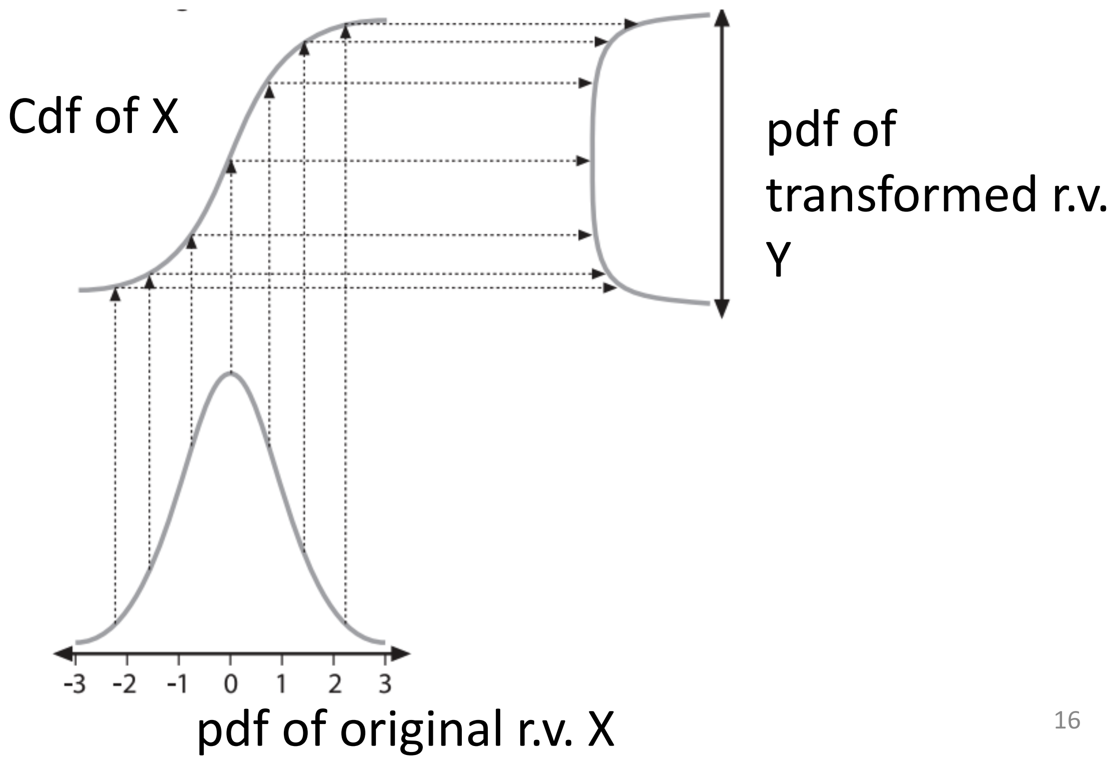
claim: $Y \sim u[0,1]$ y is uniformly distributed between 0 and 1

proof:

$$
\begin{aligned}
F_y(y) &= P(Y \le y) \\
&= P(F_x(x) \le y) \\
&= P(x \le F_x^{-1}(y)) \\
&= F_x(F_x^{-1}(y)) \\
&= y
\end{aligned}
$$

Histogram equalization

- Cdf: transform a r.v. to a uniform one, ~U[0,1]



Histogram equalization

- Linear?

  Non-linear cause needs some complicated operations

- Invertible?

  It's not because of the floor operation

Histogram equalization

- Cohort exercise

$$T(k) = \text{floor}((L-1)\sum_{i=0}^{k} p_i) = \text{floor}((L-1)F_X(k))$$

Input:

```
[[ 1  3  1  3]
 [ 2  3 10 11]
 [11 10  2  3]
 [ 1   2   3  3]]
```

Output:

```
[[  0  176   0     176]
 [ 58 176   215 255]
 [255 215  58   176]
 [  0   58    176 176]]
```

Input:

```
[[52  52  53  72]
  [72  72  53  53]
  [88  72  52  52]
  [88  88  53  53]]
```

Output:

```
[[    0      0  106  191]
  [191  191  106  106]
  [255  191      0      0]
  [255  255  106  106]]
```