

L04.01

Binary Search Trees (BST)

50.004 Introduction to Algorithm

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(slides adapted from Dr. Simon LUI)

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L04.01

Binary Search Trees (BST)

Chapter 12 and 14 of CLRS book

Overview

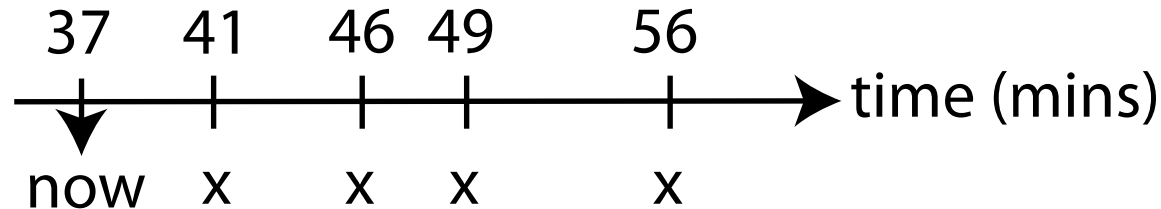
- We will talk about a **runway reservation system**
- Use an array/list to solve it
 - In $O(n)$ time
- Use **Binary Search Tree (BST)** to solve it
 - In $O(\text{height})$ time
 - usually height = $\log n$,
 - but in the worst case height = $n-1$

Runway reservation system

- Problem definition:
 - Single (busy) runway
 - Reservations for landings
 - maintain a set of future landing times (R)
 - a new request to land at time t
 - add t to R if no other landings are scheduled within < 3 minutes from t
 - when a plane lands, remove its reservation from the set

Runway reservation system

- Example



- $R = (41, 46, 49, 56)$ - reserved landing times
- requests for time:
 - 40 => reject (crash with 41)
 - 42 => reject (crash with 41)
 - 53 => ok
 - 20 => not allowed (already past)

Proposed algorithm

init: $R = []$

req(t): if $t < \text{now}$: return "error" if in past, reject

for i in range (len(R)):

 if $\text{abs}(t - R[i]) < 3$: return "error" if there is at least 1 violation,
 reject

$R.\text{append}(t)$

$R = \text{sorted}(R)$

land: $t = R[0]$

if ($t \neq \text{now}$) return;

$R = R[1:]$ (drop $R[0]$ from R)

- Complexity? $O(n)$ + sorting complexity
- Can we do better?

Some options:

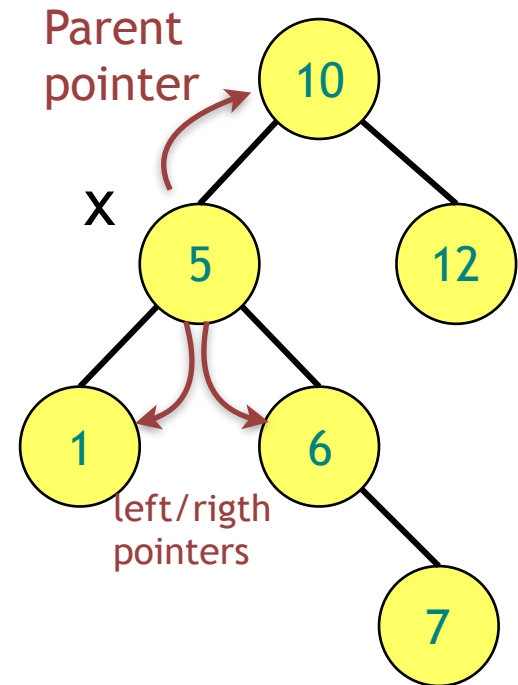
- Keep R as a sorted list:
 - insert new element in proper place: $O(n)$
 - <3 minute check: $O(1)$
- Keep R as a sorted array:
 - Find the place to insert new element: $O(\log n)$
 - <3 minute check: $O(1)$
 - Insert (shifting of elements): $O(n)$
- Keep R in unsorted order (min-heap):
 - search for collisions: $O(n)$
 - insert new element: $O(\log n)$

To improve it, we need fast insertion

Binary Search Trees (BSTs)

Binary Search tree:

- Each node x has:
 - $\text{key}[x]$
 - Pointers:
 - $\text{left}[x]$
 - $\text{right}[x]$
 - $\text{parent}[x]$

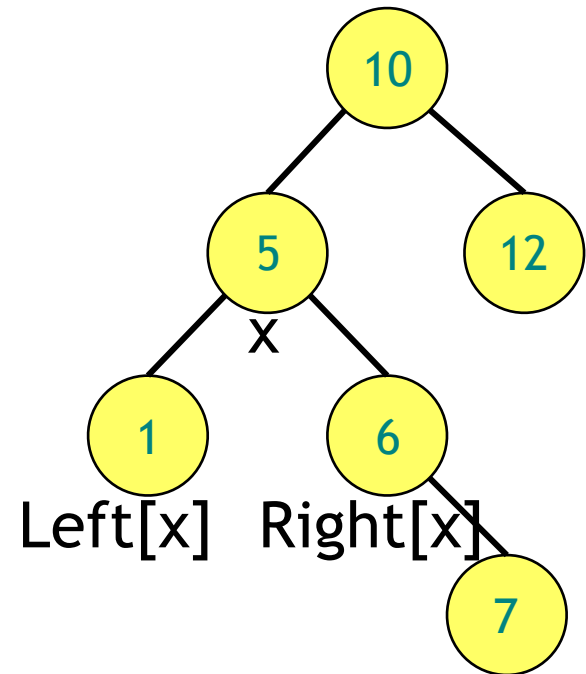


Binary Search Trees (BSTs)

Binary Search tree:

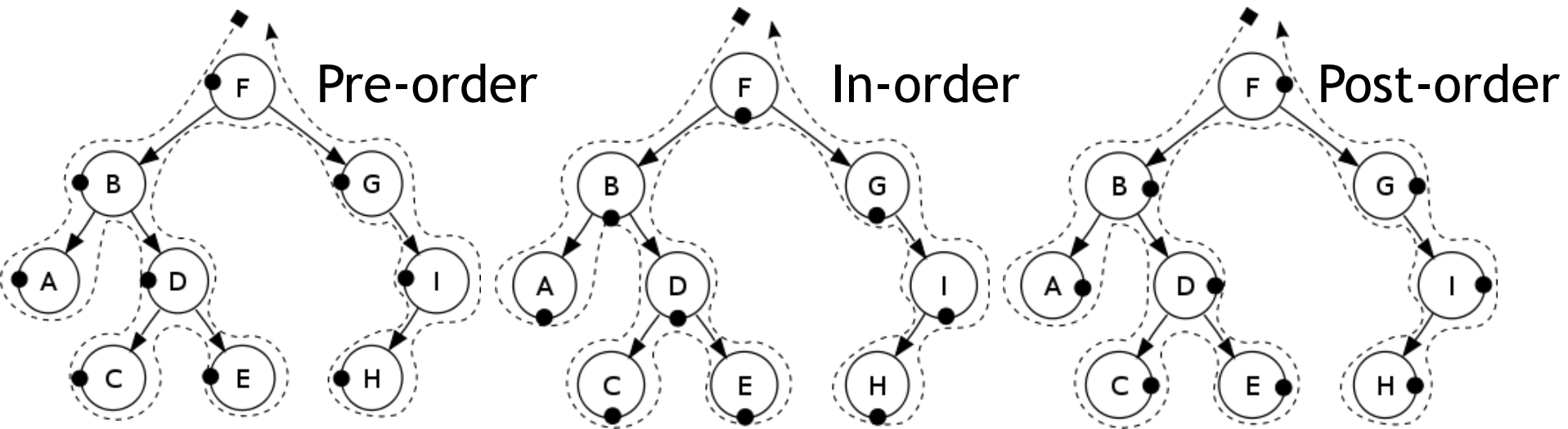
Property:

- $\text{Key}[\text{left}[x]] < \text{key}[x]$
- $\text{Key}[x] < \text{key}[\text{right}[x]]$



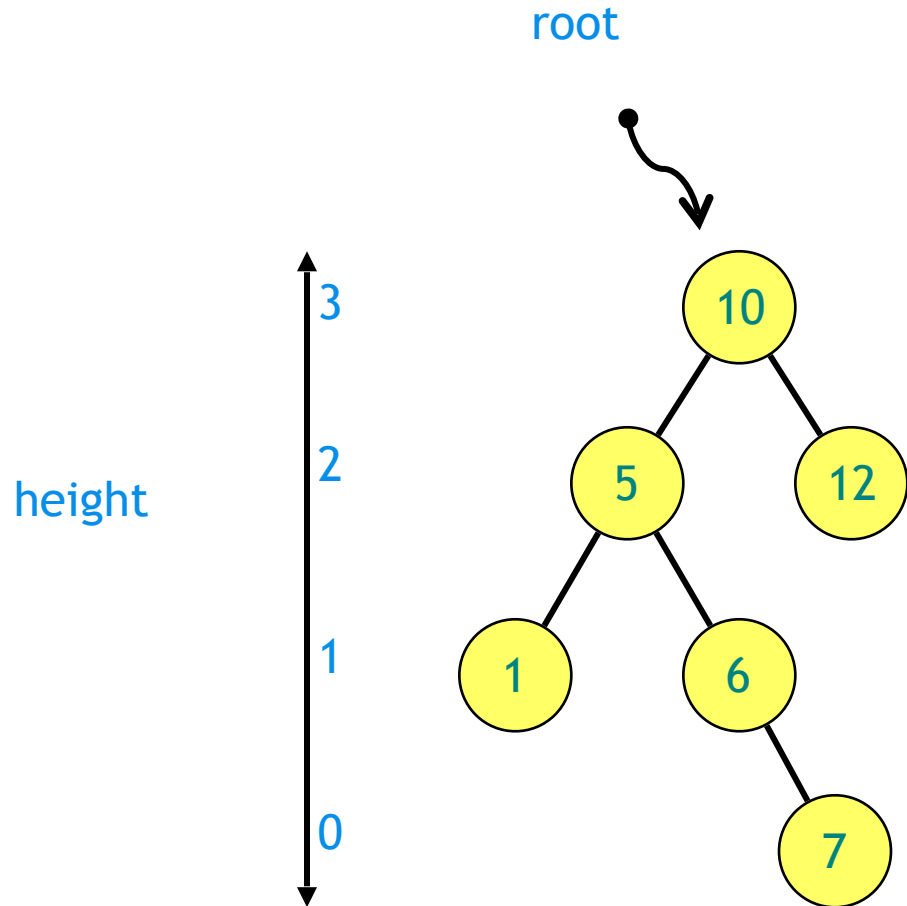
Tree traversals

- Algorithms to **output** the tree nodes
- Pre-order: FBADCEGIH
- In-order: ABCDEFGHI (In-order = sorted list of BST)
- Post-order: ACEDBHI GF

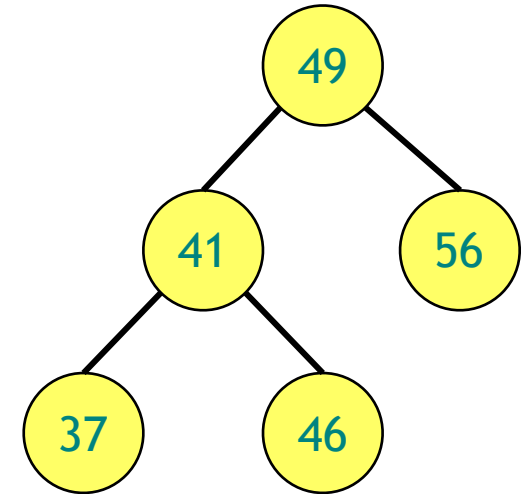
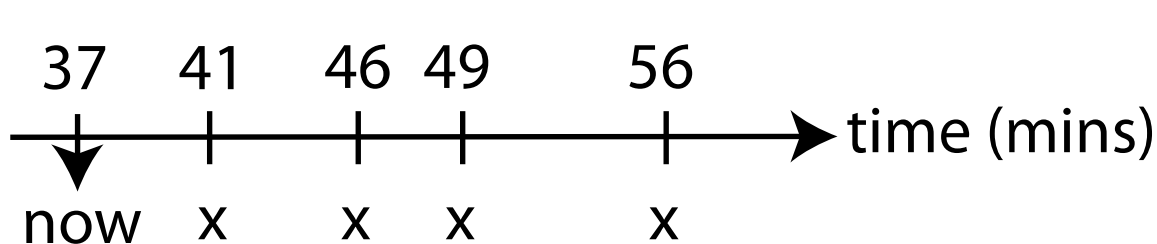


Growing BSTs

- Insert 10
- Insert 12
- Insert 5
- Insert 1
- Insert 6
- Insert 7



Use BST for the runway problem



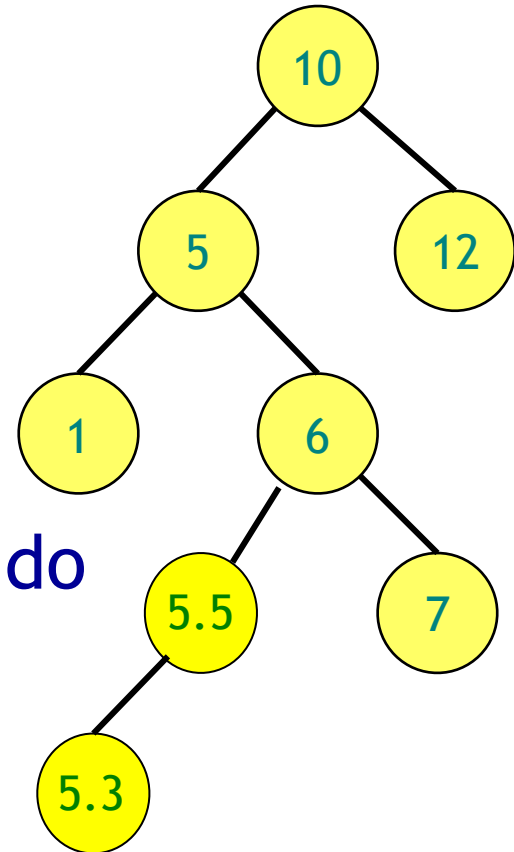
Operations:

- `insert(k)` (with "< 3 minute check")
- `find(k)`: finds the node containing key `k` (if it exists)
- `findmin(x)`: finds the minimum of the tree rooted at `x`
- `deletemin()`: finds the minimum of the tree and delete it
- `next-larger(x)`: finds the next element after element `x`

Next-larger

3 cases to consider!

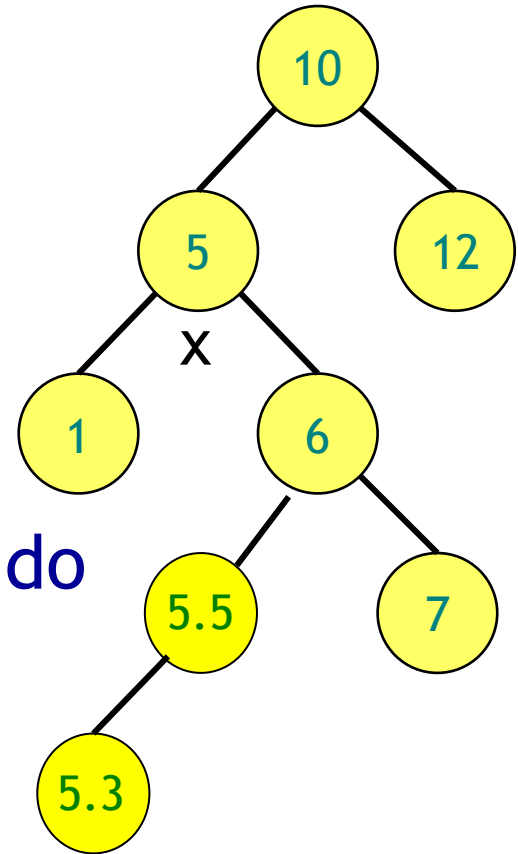
```
next-larger(x):  
  if right[x] ≠ NIL then  
    return findmin(right[x])  
  else  
    y = parent[x]  
    while y ≠ NIL and x == right[y] do  
      x = y  
      y = parent[y]  
    return y
```



Next-larger

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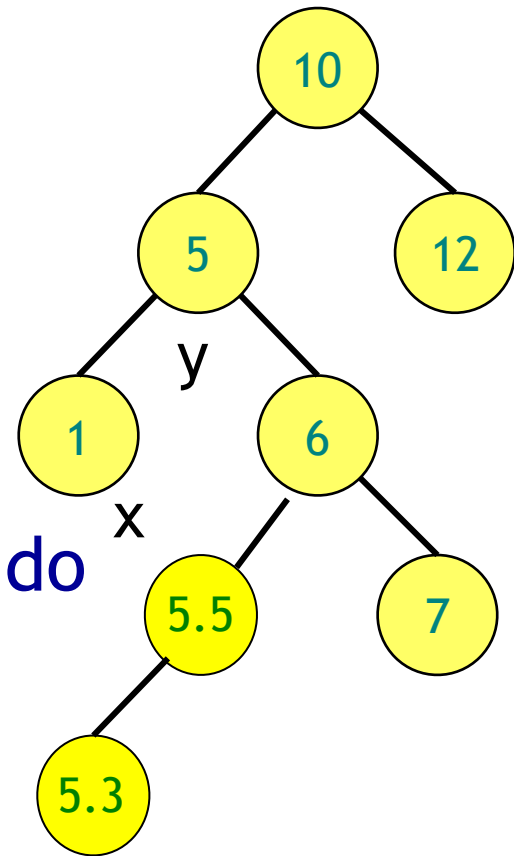


Example 1:
next-larger(5) = 5.3

Next-larger

3 cases to consider!

```
next-larger(x):  
  if right[x]  $\neq$  NIL then  
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  else  
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    return y
```



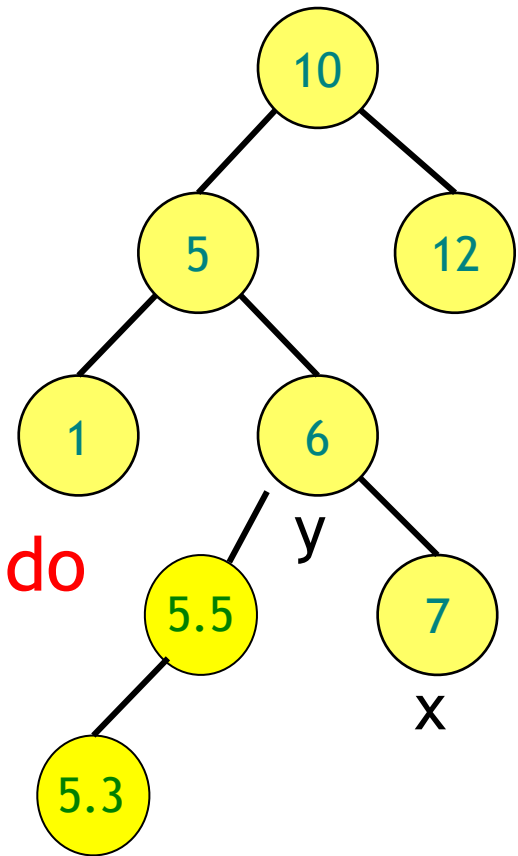
Example 2:
next-larger(1) = 5

Next-larger

3 cases to consider!

```
next-larger(x):  
  if right[x]  $\neq$  NIL then  
    return findmin(right[x])  
  else
```

```
    y = parent[x]  
    while y  $\neq$  NIL and x == right[y] do  
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    return y
```



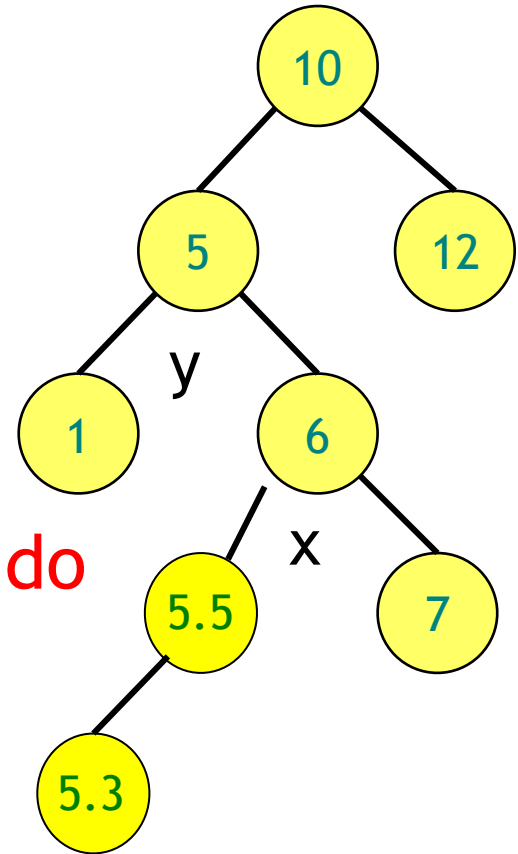
Example 3:
next-larger(7) =

Next-larger

3 cases to consider!

```
next-larger(x):  
  if right[x]  $\neq$  NIL then  
    return findmin(right[x])  
  else
```

```
    y = parent[x]  
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```



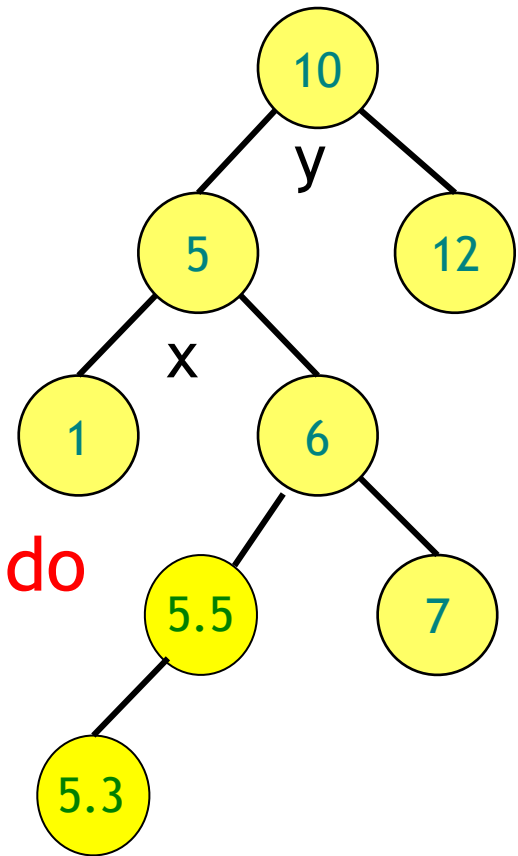
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Next-larger

3 cases to consider!

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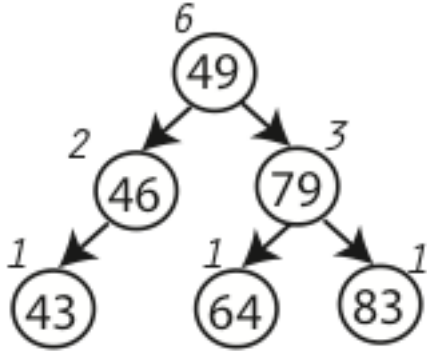
```
    y = parent[x]  
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      y = parent[y]  
    return y
```



Example 3:
next-larger(7) = 10

Augmenting a BST

1. Keep the total size of sub-trees at each node



Back to runway reservation system

- This can help us to answer “How many planes will land before t=79?”

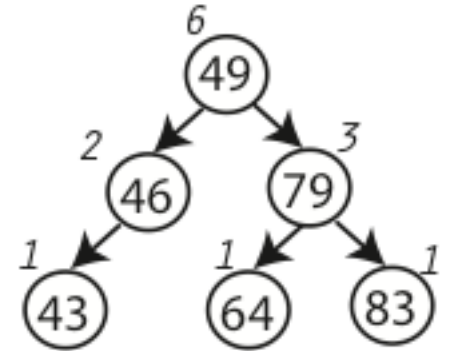
Answer:

$\text{leftsubtree}(79) + \text{parentnode}(79) +$
 $\text{leftsubtree}(\text{parent of } 79)$

$= (64) + (49) + \text{left}(49)$

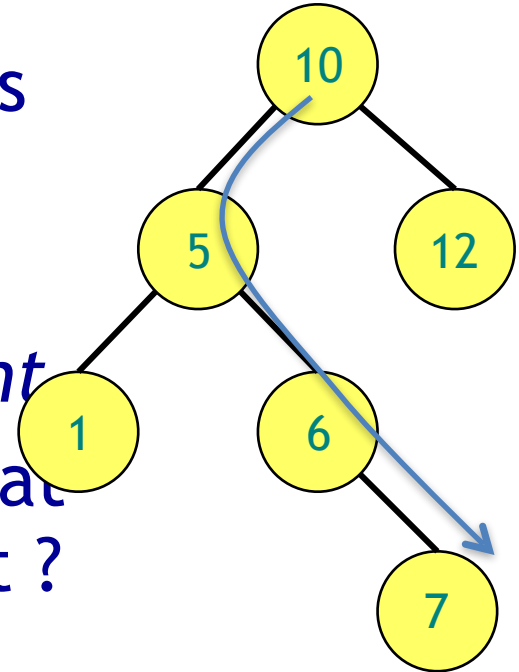
$= 1 + 1 + 2$

$= 4$



Analysis

- We have seen insertion, search, findmin...
- How much time does any of this take ?
- Worst case: $O(\text{height})$
=> *height is really important*
- After we insert n elements, what is the worst possible BST height ?



Analysis

- $n-1$
- so, still $O(n)$ for the runway reservation system operations in the **worst case**, if the height is bad

