50.034 – Introduction to Probability and Statistics

January–May Term, 2019

Homework Set 2

Due by: Week 3 Cohort Class (14 Feb 2019 or 15 Feb 2019)

Reminder: There is Mini-quiz 1 in Week 3 during your cohort class.

Question 1. An old magnetic tape storing information in binary form has been corrupted. Suppose you are trying to save as much information as possible from this magnetic tape. Due to the damage on the tape, you know that there will be errors in the reading. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Given that each digit is either a 0 or a 1 with equal probability, and given that your reading is a 1, what is the probability that this is a correct reading?

Question 2. You are given three coins. The first coin is a fair coin painted blue on the heads side and white on the tails side. The other two coins are biased, such that the probability of getting heads is p. Both biased coins are painted blue on the tails side and red on the heads side. You conduct the following experiment: You select two of the three coins at random, then you toss both selected coins.

(i) Describe the outcomes in the sample space of this experiment.
(ii) It was experimentally determined that the probability that the sides that land face up have the same color is $\frac{29}{96}$. What are the possible values of p?

Question 3. Let A and B be events contained in a sample space Ω, such that $A \subseteq B$. Can A and B be independent events? Justify your answer with as much details as possible.

Question 4. Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} kx^3 \ln x, & \text{if } 2 \leq x \leq 4; \\ 0, & \text{otherwise;} \end{cases}$$

where k is an unspecified constant.

(i) Find the value of k.
(ii) Find the value of $\Pr(X \geq 1)$.

Question 5. Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} \frac{k}{x^2+2x+2}, & \text{if } -2 \leq x \leq 0; \\ 0, & \text{otherwise;} \end{cases}$$

where k is an unspecified constant.

(i) Find the value of k.
(ii) Find the value of $\Pr(X \leq -\frac{1}{\sqrt{3}})$.