

## Student Information

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Due Date: 02 Oct 11:59pm.

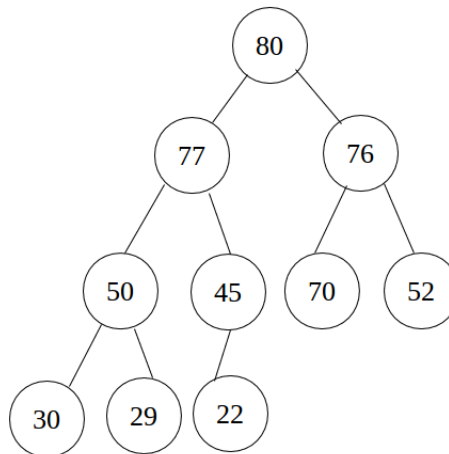
Submit answers on eDimension in pdf format. Submission without student information will **NOT** be marked! Any questions regarding the homework can be directed to the TA through email (contact information on eDimension).

## Week 3

Note: Please read and understand the Heap operations before doing the following questions.

1. The array [80, 77, 76, 50, 45, 70, 52, 30, 29, 22] forms a heap [True/False].  
Show explanation by drawing. **Only half of the full marks will be awarded if answer is correct without explanation.**

**Solution:** True.



**Figure 1:** q1 ans

2. We have a max heap of  $n$  elements and we want to insert  $m$  more elements to this heap. Assume that all the  $m$  elements are inserted at the same time and the end result must also be a max heap. The entire operation takes  $O(m + n)$  total time [True/False].  
Will the time complexity change if the  $m$  elements are inserted one by one to a max heap

containing  $n$  elements? If it changes, what would be the time complexity? **Only half of the full marks will be awarded if answer is correct without explanation.**

**Solution:** True. Building a heap given  $n$  elements all at once takes a linear time :  $O(n)$  (see lecture notes). So we can think of it as: we are building a heap out of  $m+n$  elements, instead of inserting the elements 1 by 1. Thus, the time complexity is  $O(m+n)$ .

The complexity changes if we insert the  $m$  elements one by one. Recall from lecture notes that inserting a new element to a heap size of  $n$  is  $O(\log n)$ . Note that the height of the heap changes as more and more elements are inserted into the heap. Therefore, given that the heap already has  $n$  elements and we insert  $m$  elements one by one, it would take  $O(m \log(n+m))$  total time.

3. Consider the heap created from the array [80, 77, 76, 50, 45, 70, 52, 30, 29, 22]. If the node with value 29 has its value increased to 79, how many swaps must occur to convert the heap into a max heap? Provide answer and show explanation by drawing. **Only half of the full marks will be awarded if answer is correct without explanation.**

**Solution:** Two swaps.

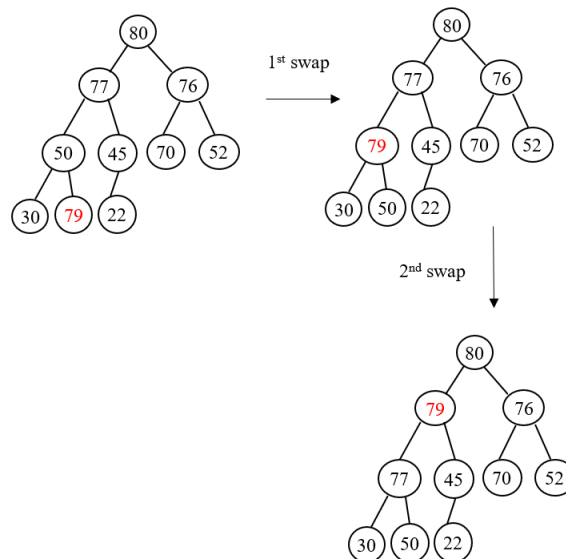


Figure 2: q3 ans

4. In the worst case scenario, what is the time complexity of finding the smallest item from a max heap?

- A.  $O(1)$
- B.  $O(n)$
- C.  $O(\log n)$
- D.  $O(n \log n)$

**Solution:** B.

5. What is the number of swaps needed to construct a max heap from the array [9, 19, 50, 7, 8, 10, 25, 2, 5, 17, 12, 8]?
- A. 6 swaps
  - B. 3 swaps
  - C. 1 swap
  - D. 4 swaps

**Solution:** B.