

50.034 – Introduction to Probability and Statistics

January–May Term, 2019

Homework Set 1

Due by: Week 2 Cohort Class (7 Feb 2019 or 8 Feb 2019)

Question 1. Express each of the following events in terms of the events A , B and C , as well as the operations of complements, unions and intersections:

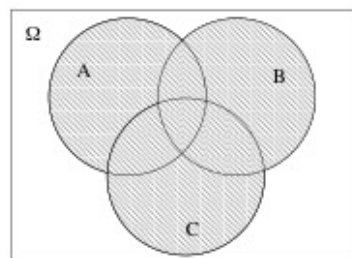
- (i) at least one of the events A , B , C occurs;
- (ii) all three events A , B , C occur;
- (iii) exactly one of the events A , B , C occurs;
- (iv) events A and B occur, but not C ;
- (v) either event A occurs or, if not, then B also does not occur.

In each case, draw the corresponding Venn diagrams.

As an example, part (i) has been done for you:

$$(\text{at least one of events } A, B, C \text{ occurs}) = A \cup B \cup C.$$

The corresponding Venn diagram is shown on the right.



Question 2. This year's SUTD chess champion is to be selected by the following procedure. Brandon and Colin, the leading challengers, first play a two-game match. If one of them wins both games, he gets to play a two-game second round with Amanda, the current champion. Amanda retains her championship unless a second round is required and the challenger beats Amanda in both games. If Amanda wins the initial game of the second round, no more games are played. Furthermore, we know the following:

- $\Pr(\text{Brandon will beat Colin in any particular game}) = 0.6$.
- $\Pr(\text{Amanda will beat Brandon in any particular game}) = 0.5$.
- $\Pr(\text{Amanda will beat Colin in any particular game}) = 0.7$.

Assume no tie games are possible and assume that the outcomes all games are independent of each other. Determine the following probabilities:

- (i) Brandon will win the first round.
- (ii) The second round will be required.
- (iii) Amanda will retain her championship this year.

Question 3. Peter has a peculiar pair of tetrahedron-shaped dice, each with four sides having the numbers 1, 2, 3, 4 respectively. One die is red, and the other is blue. When he rolls both dice, the probability that the sum of the numbers on both dice equals k is proportional to k . For example, it is three times more likely to have a sum $k = 6$ than to have a sum $k = 2$, since 6 is three times of 2. All outcomes that result in a particular sum k are equally likely. For example, rolling a 1 on the red die and a 2 on the blue die, is equally likely as rolling a 2 on the red die and a 1 on the blue die.

- (i) What is the probability that the sum is even?
- (ii) What is the probability of Peter rolling a 3 on the red die and a 4 on the blue die?

Question 4. There are 11 students in a party. Five of them are girls. In how many ways can these 11 students be arranged in a row if

- (i) there are no restrictions?
- (ii) the 5 girls must be together (forming a block)?
- (iii) no 2 girls are adjacent?

Question 5. Of all the students in SUTD, 90% love at least one of the following three snacks: Hello Panda chocolate biscuits (Snack A), Tao Kae Noi crispy seaweed (Snack B), and Huang Fei Hong spicy peanuts (Snack C). 59% of the students love Snack A. 58% of the students love Snack B. 42% of the students love Snack C. Suppose we are given the following additional information:

- 32% of the students love both Snack A and Snack B.
- 29% of the students love both Snack A and Snack C.
- 34% of the students love both Snack B and Snack C.

Determine the probability that a randomly selected student loves Snack B and Snack C but does not love Snack A.