Quantum entanglement of oscillator subsystems in open quantum systems

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Abstract

The concept of self-adjoint integral operators has been in the center of mathematical and physical research over the last century. Fredholm, Hilbert, Mercer, Schmidt and Neumann were the first ones who established the cornerstones of this area, which is still an active field of research. One of the most important mathematical objects of quantum information theory and quantum physics is the density operator, which is a positive semidefinite, self-adjoint, trace-class operator with trace one [1].

A new toy model to study is given by two oscillators coupled to each other harmonically and interacting with the harmonic oscillator bath [2]. Here a possible initial state can be a polynomial Gaussian operator, which remains polynomial Gaussian during the time evolution.

Paper [3] gives tools to handle the entanglement problem of this model where we derive a useful sufficient condition, which often allows us to consider only the Gaussian part of the operator. Also, our preorder makes testing positivity and NPT entanglement more sensitive and efficient. Generalizations for more than two harmonically coupled harmonic oscillators or two harmonically coupled harmonic oscillators are straightforward.

Keywords: Entanglement; Separability; Positive semidefinite operators; Integral operators; Gaussian operators; Polynomial Gaussian Density operators; Quantum theory; Open quantum systems; Master equations; Phase space; Position representation.

References

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