Multistability in quantum systems and related cavity QED experiments

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National Research, Development and Innovation Office Hungary
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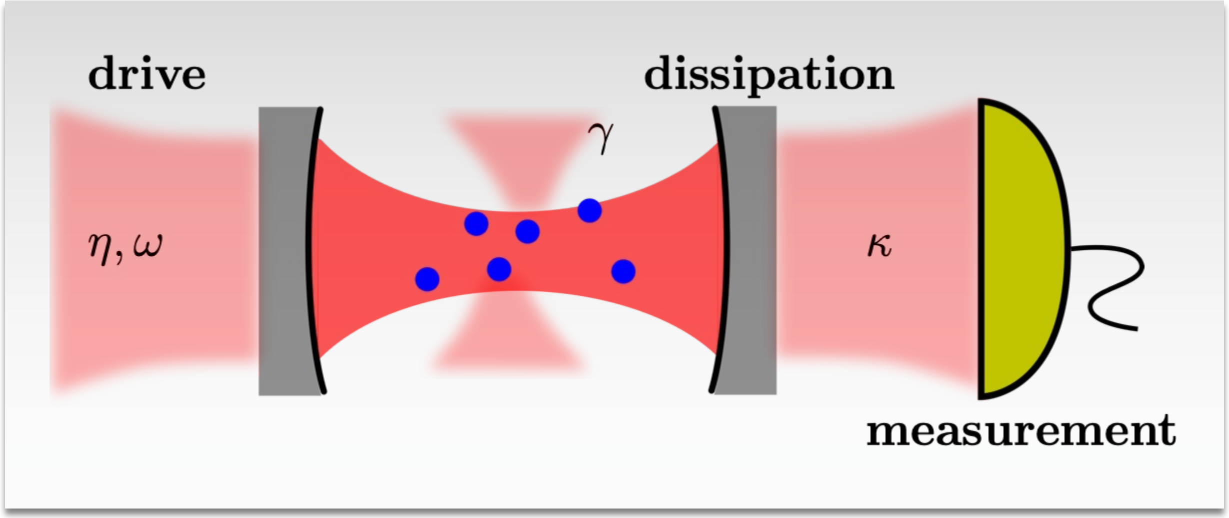
MOMENTUM OF INNOVATION

Institute Seminars, DICP San Sebastian & UPV-EHU Bilbao, 16-18 October 2024

Cavity QED: a driven-dissipative open quantum system



"many-body"

few atoms but large cooperativity

$$\mathcal{C} = \frac{\mathcal{F}}{\pi} \times N \times \frac{\sigma_A}{\mathcal{A}}$$

$$\mathcal{C} \gg 1$$

Phases

- stationary states: dynamical equilibrium of driving and dissipation
- continuous measurement due to dissipation
- order parameter: macroscopic observable

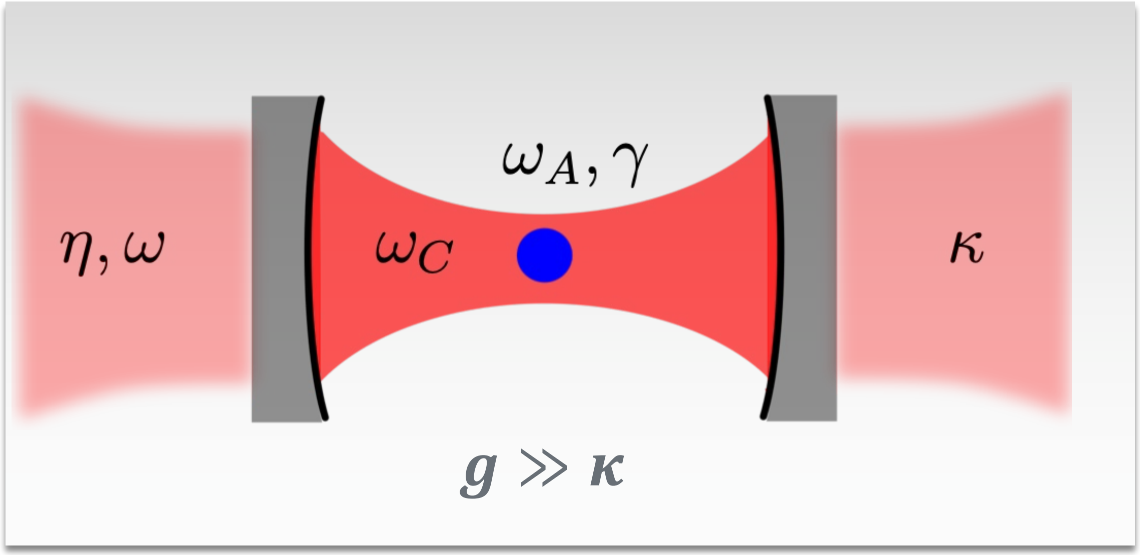
Phase transition

- non-analytic change of the order parameter as a
- control parameter is continuously tuned through
- a critical point

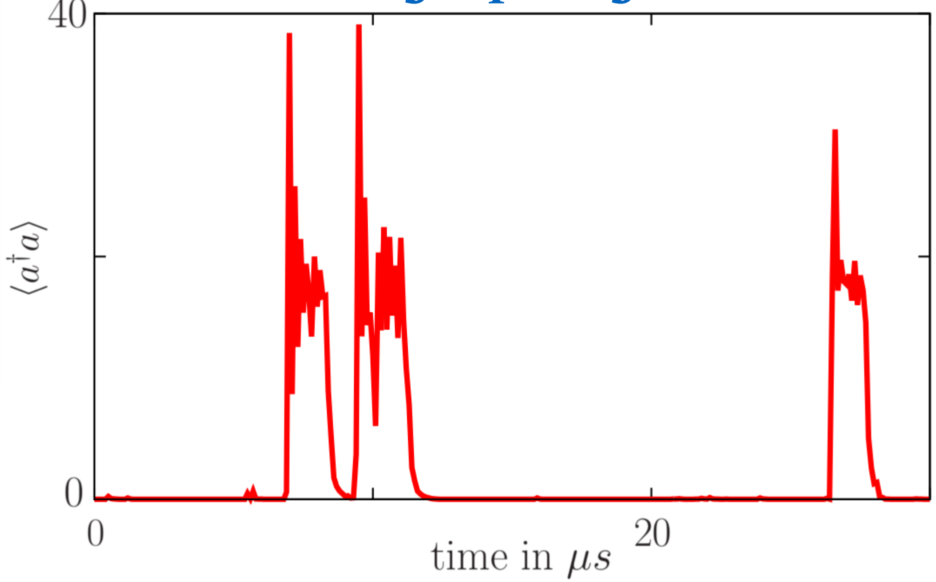
A. Vukics, A. Dombi, J. Fink, P. Domokos, Quantum (2019)

Quantum bistability

Continuously driven cavity with a strongly-coupled single atom



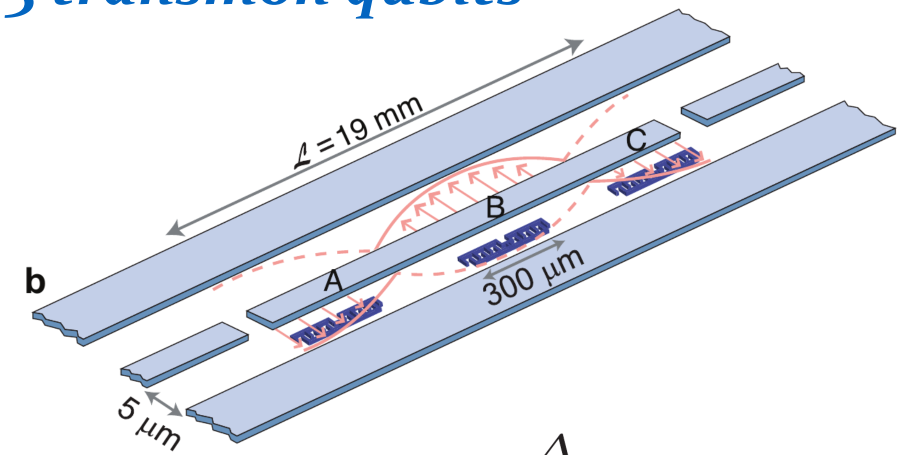
Transmission output is a random telegraph signal



H. J. Carmichael, Phys. Rev. X 5, 031028 (2025)

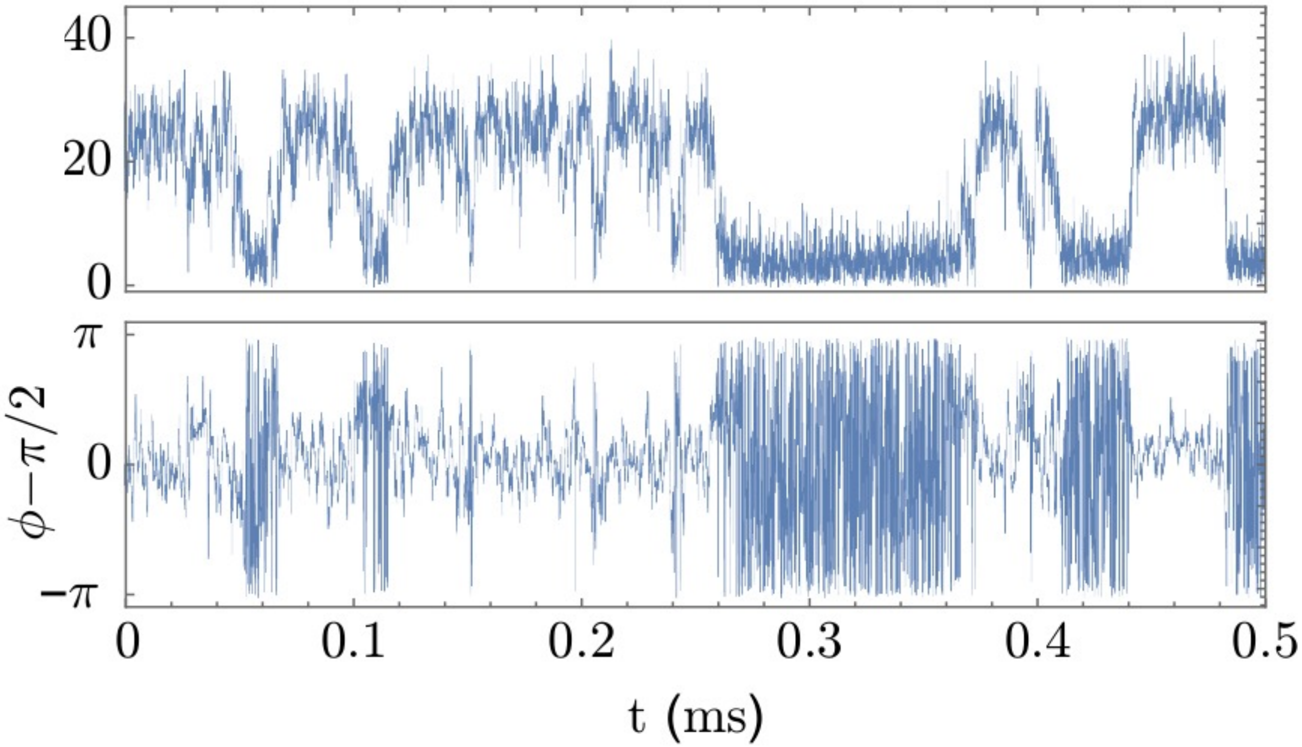
Quantum bistability observed in circuit QED

3 transmon qubits



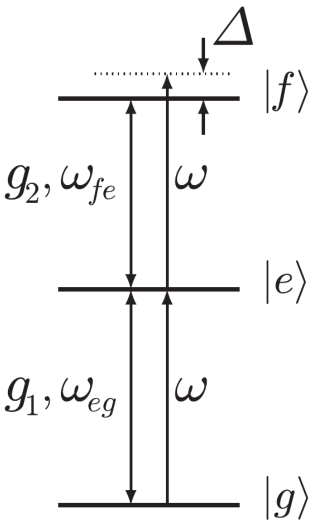
Johannes Fink

Experimental observation



$\Delta = 2\text{GHz}$

$g_2 = \sqrt{2}\, g_1$



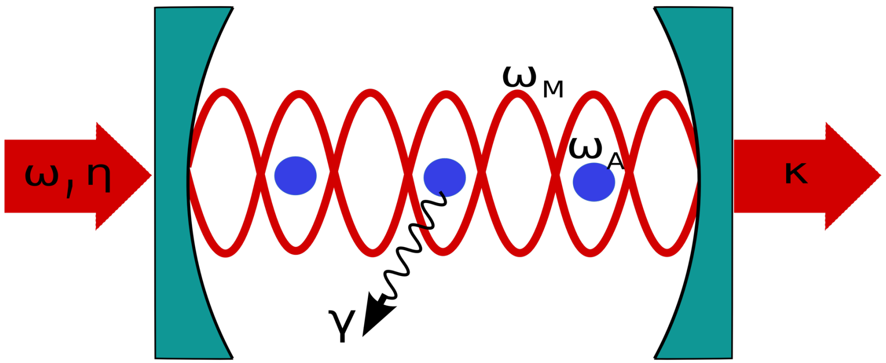
Fink, Dombi ,Vukics, Wallraff, and Domokos, Phys. Rev. X 7 (2017)

Driven Jaynes-Cummings model with damping

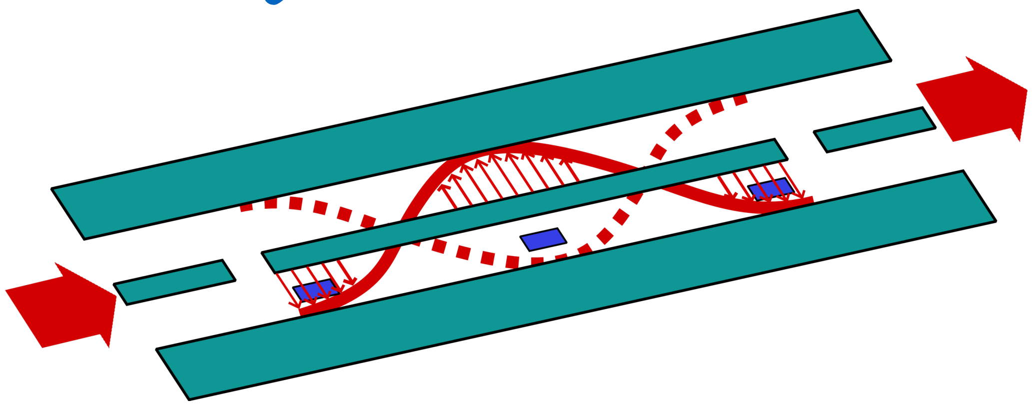
$$H = \omega_M\, a^\dagger a + \omega_A\, \sigma^\dagger \sigma + ig\left(a^\dagger \sigma - \sigma^\dagger a + a^\dagger \sigma^\dagger - \sigma a\right)$$
$$+ i\eta\left(a^\dagger e^{-i\omega t} - a\, e^{i\omega t}\right)$$

$$\mathcal{L}[\hat{\rho}] = \kappa\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$ dissipation

Cavity QED



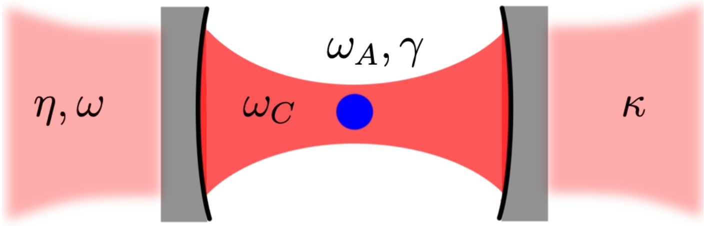
Circuit QED



Transmission spectrum of single-atom CQED systems

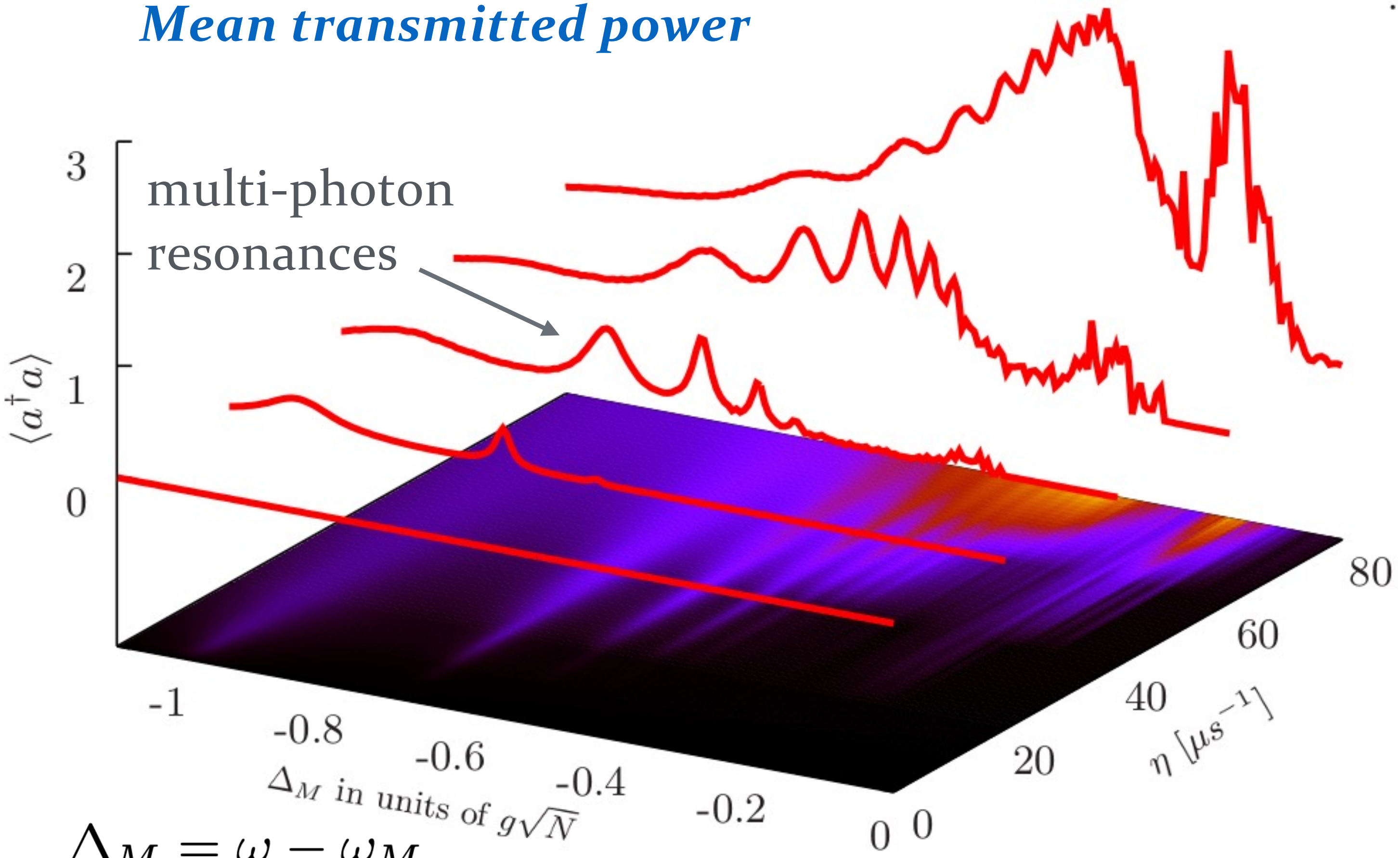
Strong coupling

$$g = 100\kappa$$



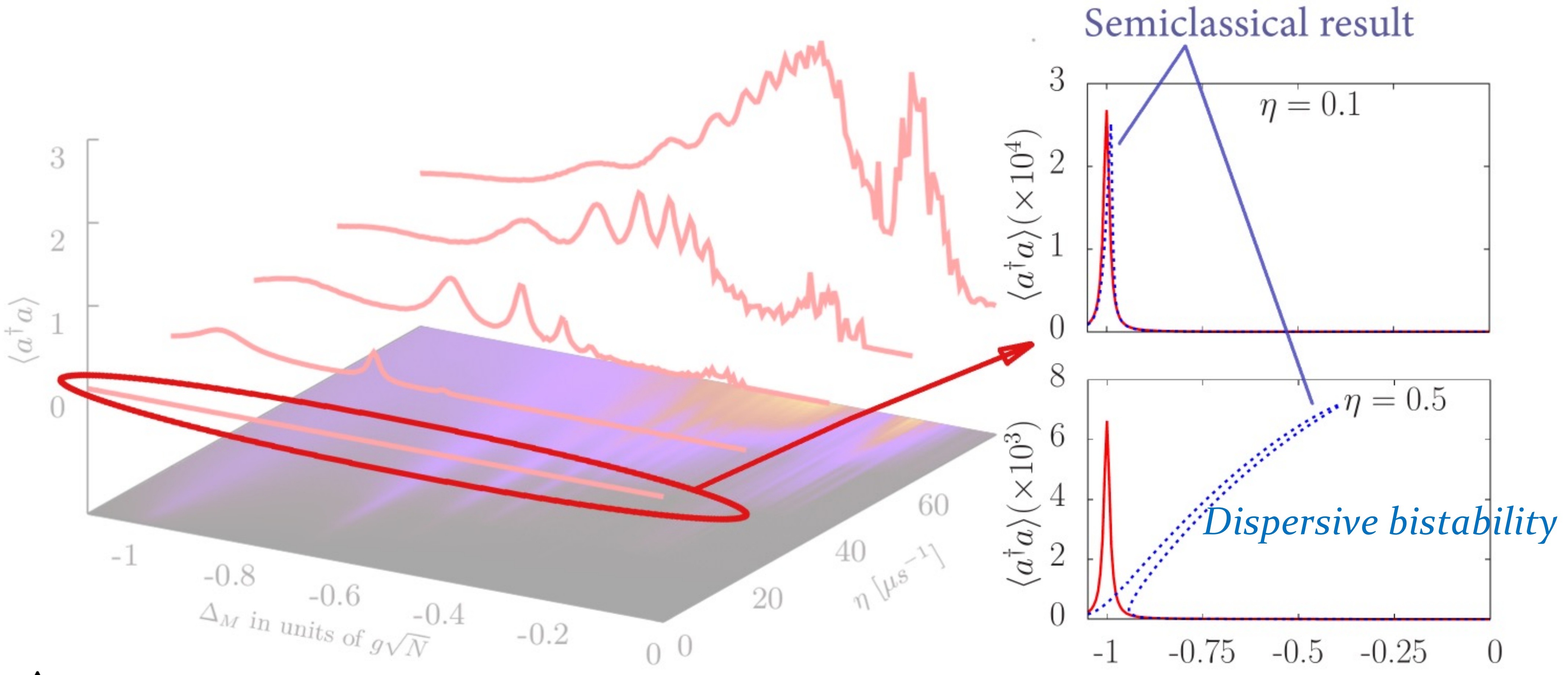
$$\omega_A = \omega_M$$

Mean transmitted power



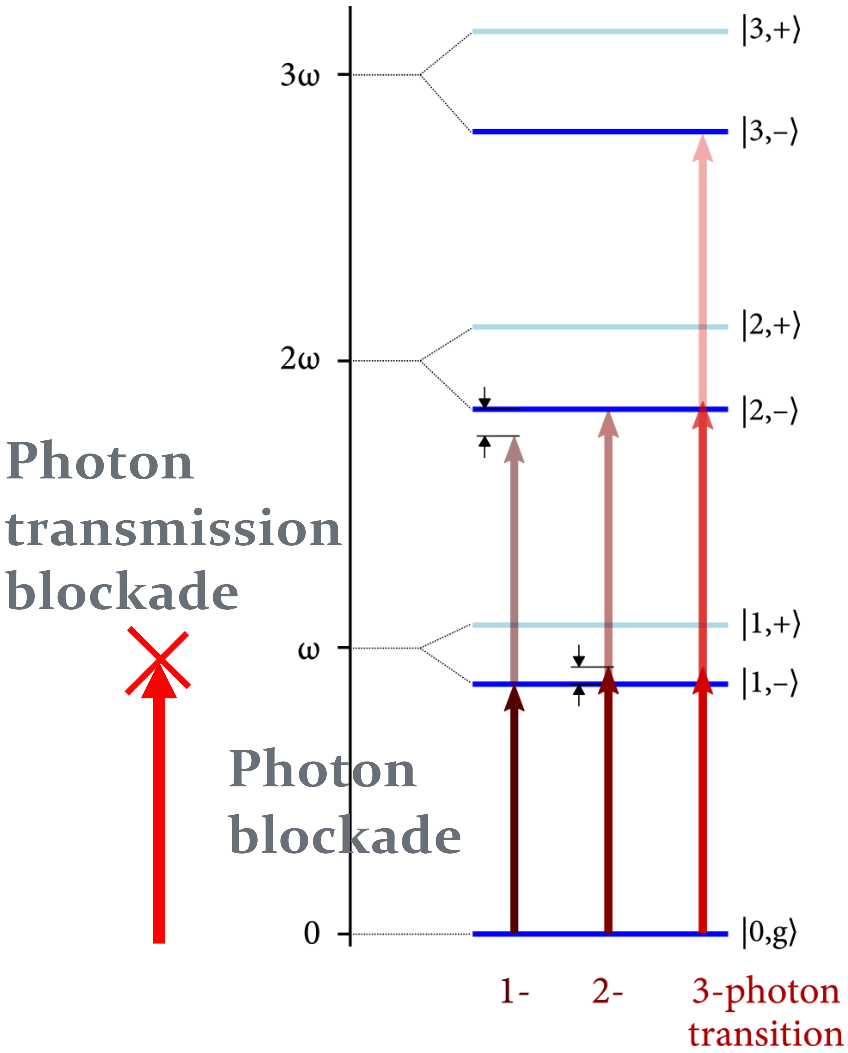
$$\Delta_M = \omega - \omega_M$$

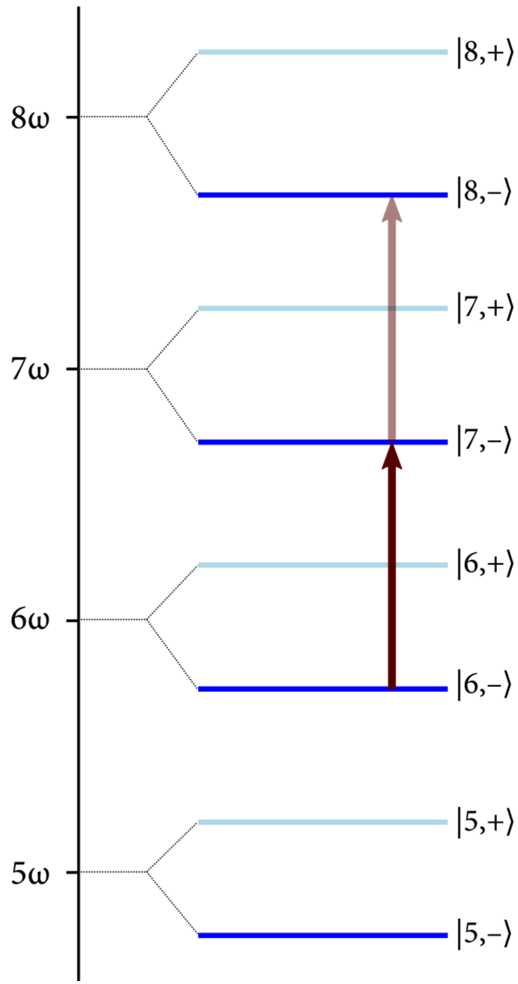
Photon-blockade



$$\Delta_M = \omega - \omega_M \qquad \omega_A = \omega_M$$

Photon-blockade and its breakdown mechanism



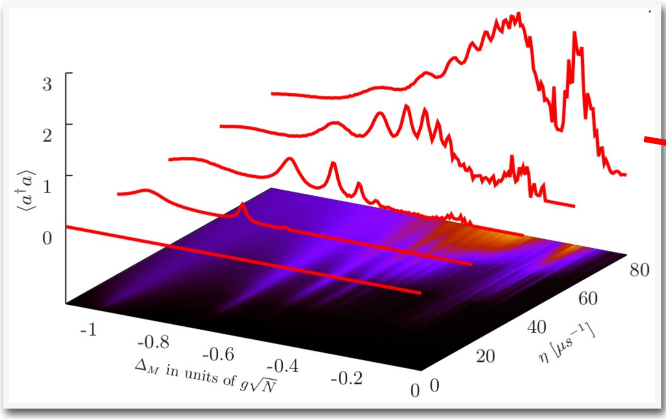


$$E_{n,\pm} = n\,\hbar\omega \pm \sqrt{n}\,g$$

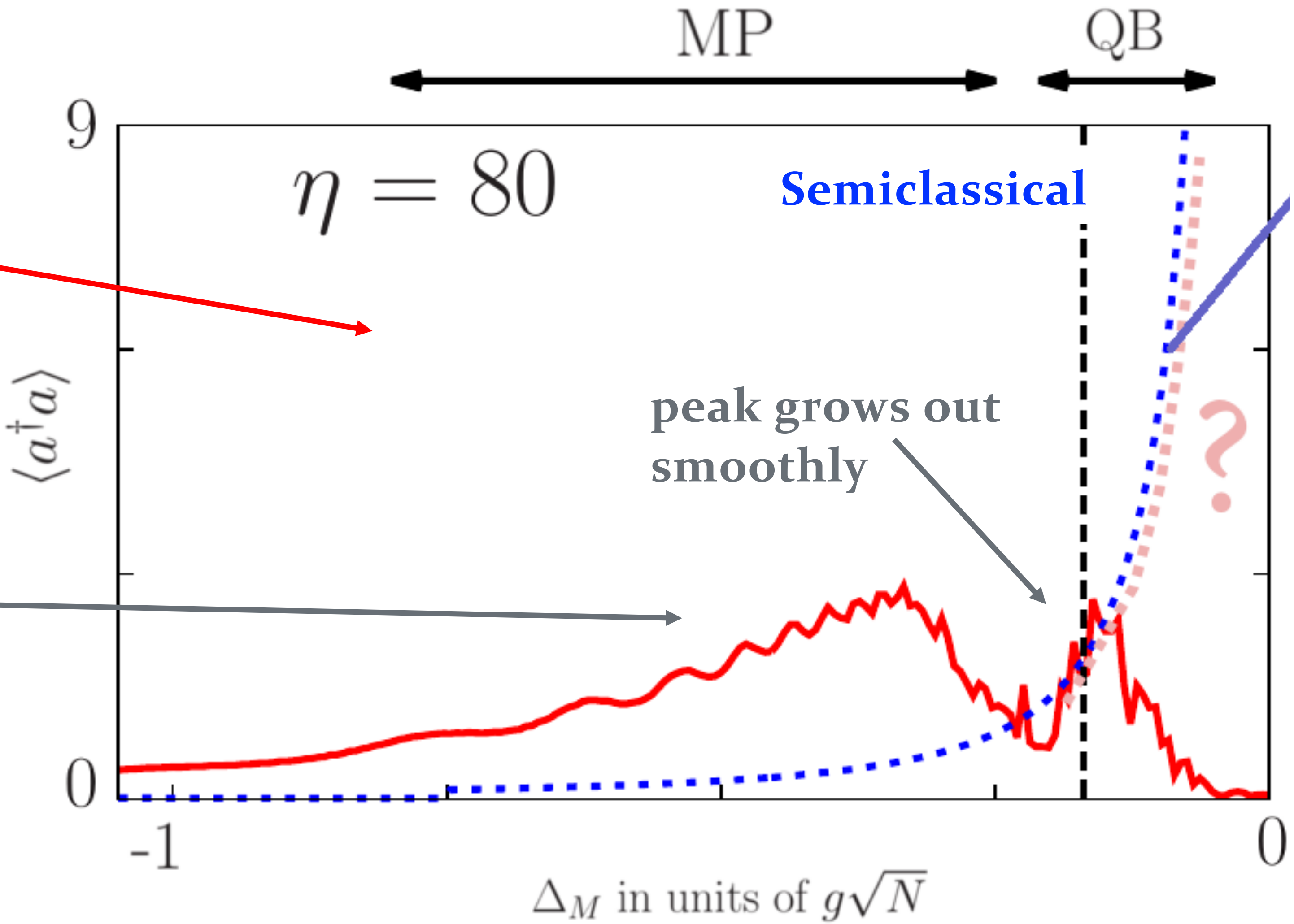
$$\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \propto n^{-3/2} \to 0$$

equidistant ladder
→ hosts quasi-coherent states
→ attractor of a driven lossy oscillator

Photon-blockade-breakdown at strong drive

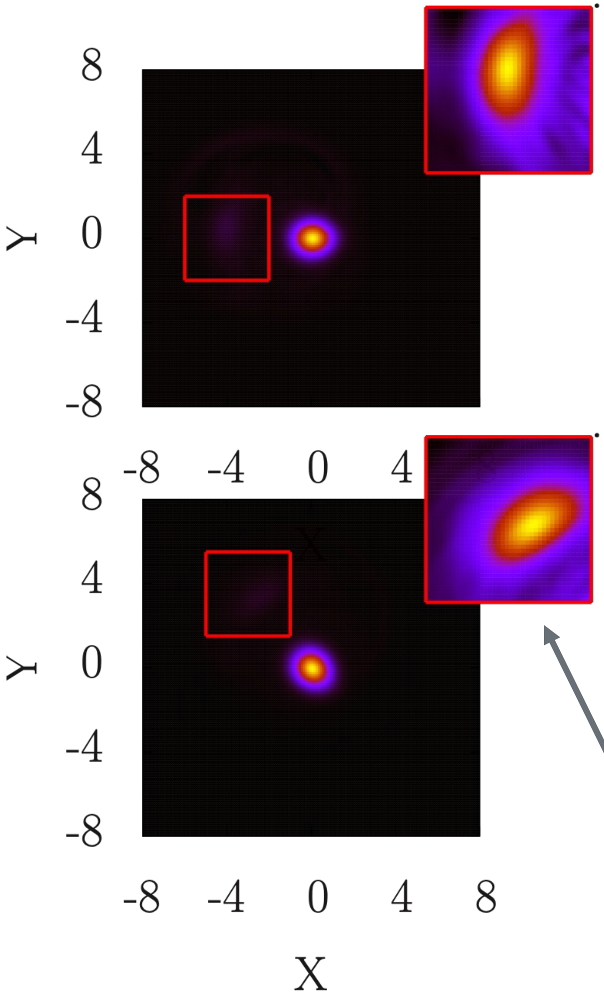


merging power broadened multi-photon resonances

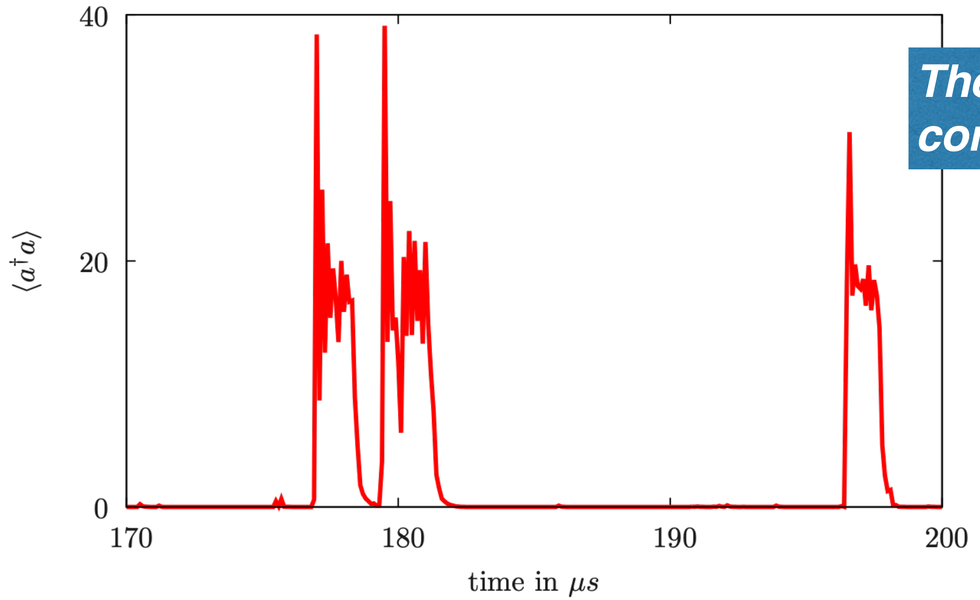


Bimodal density matrix

Phase space distribution: mixture of two semiclassical attractors



Random telegraph signal



These states are continuously measured

drive phase is reflected

Dombi, Vukics, Domokos, Eur. Phys. J. D (2015)

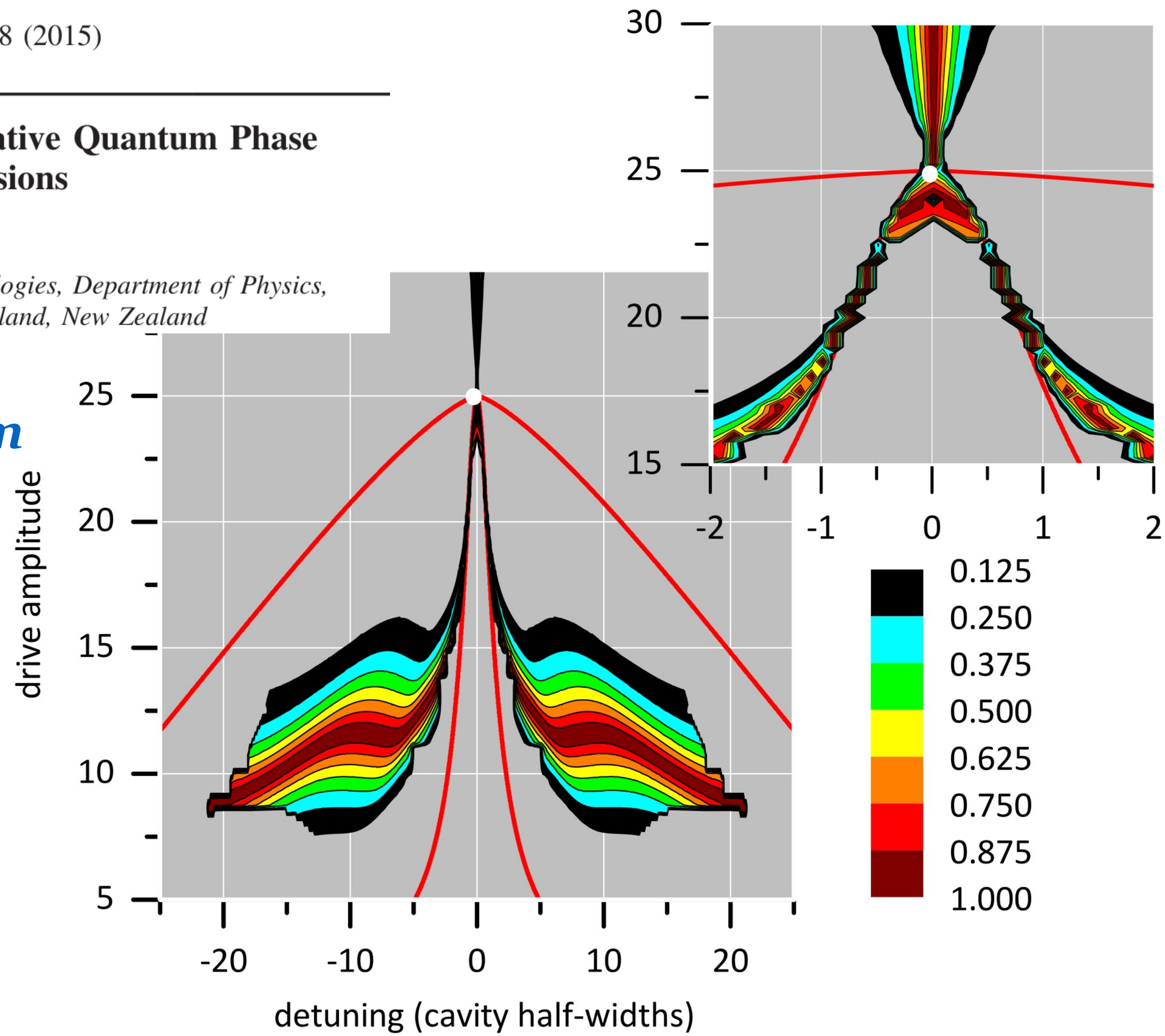
PHYSICAL REVIEW X 5, 031028 (2015)

Breakdown of Photon Blockade: A Dissipative Quantum Phase Transition in Zero Dimensions

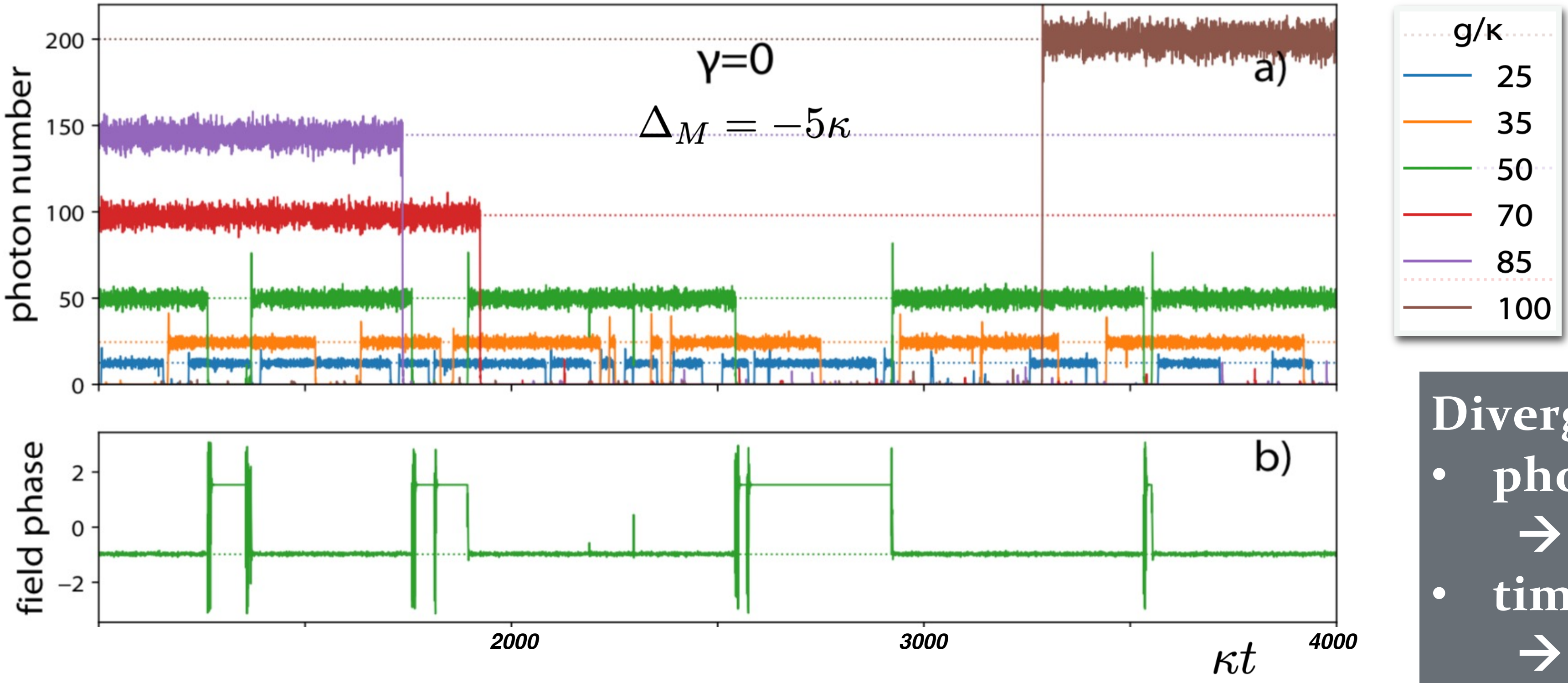
H. J. Carmichael

The Dodd-Walls Centre for Photonic and Quantum Technologies, Department of Physics, University of Auckland, Private Bag 92019 Auckland, New Zealand

Phase diagram



Thermodynamic limit: $g \to \infty, \quad \eta / g = const.$

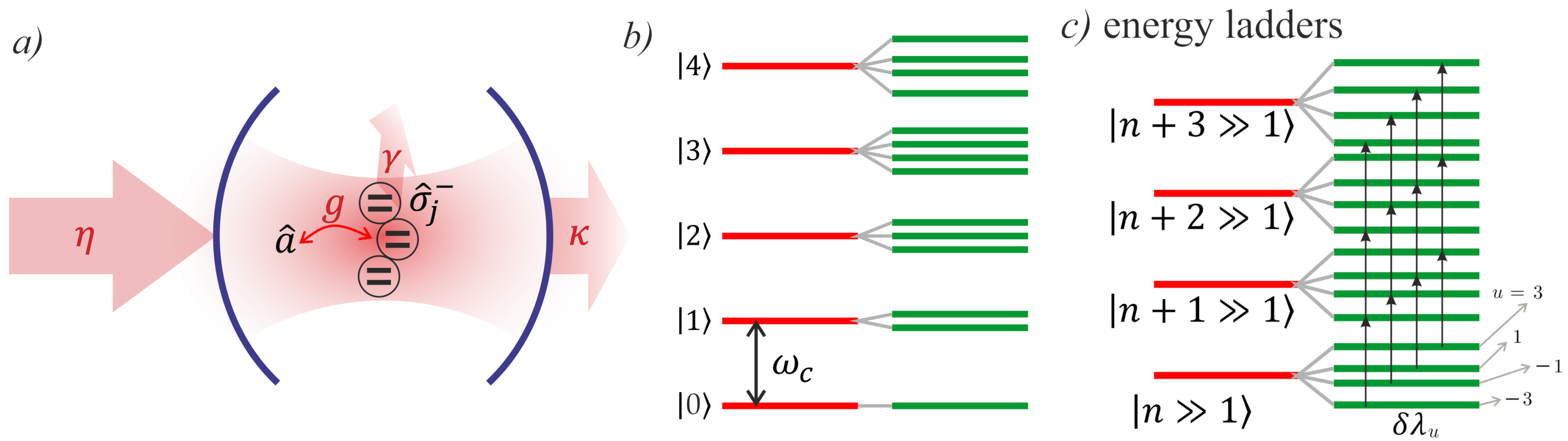


Diverging
- photon number
  → phases
- time scale
  → hysteresis

‚Zero dimension' : no increase in system size

Vukics, Dombi, Fink, Domokos, Quantum 2019

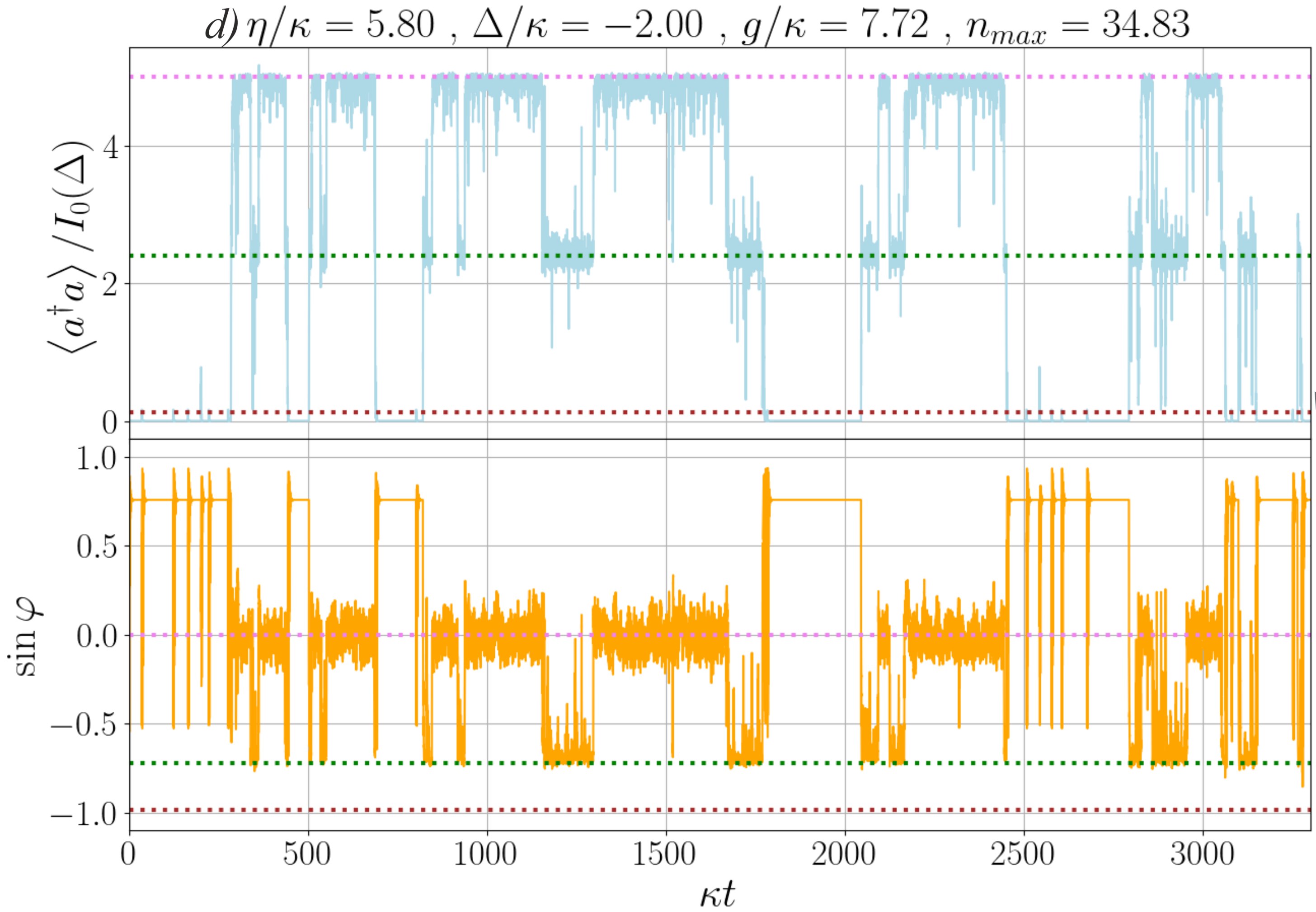
Super-quantisation rule for multistability



Self-consistent equation for the quasi-coherent state amplitude

$$\alpha_u = \frac{\eta/\kappa}{1 - i\left(\delta - \frac{ug}{2\kappa|\alpha_u|}\right)}$$

Quantum trajectories in the multistability domain



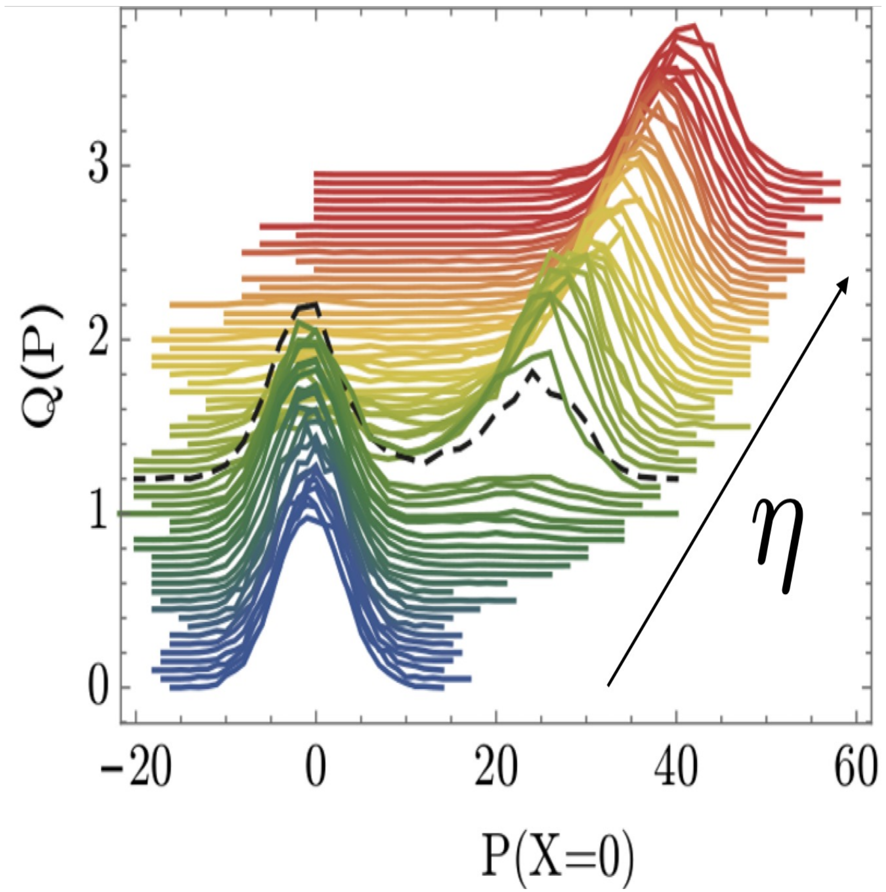
Super-quantisation rule predictions

Német, Kurkó, Vukics, Domokos, New Journal of Physics 2024

Quantum bistability without singularity

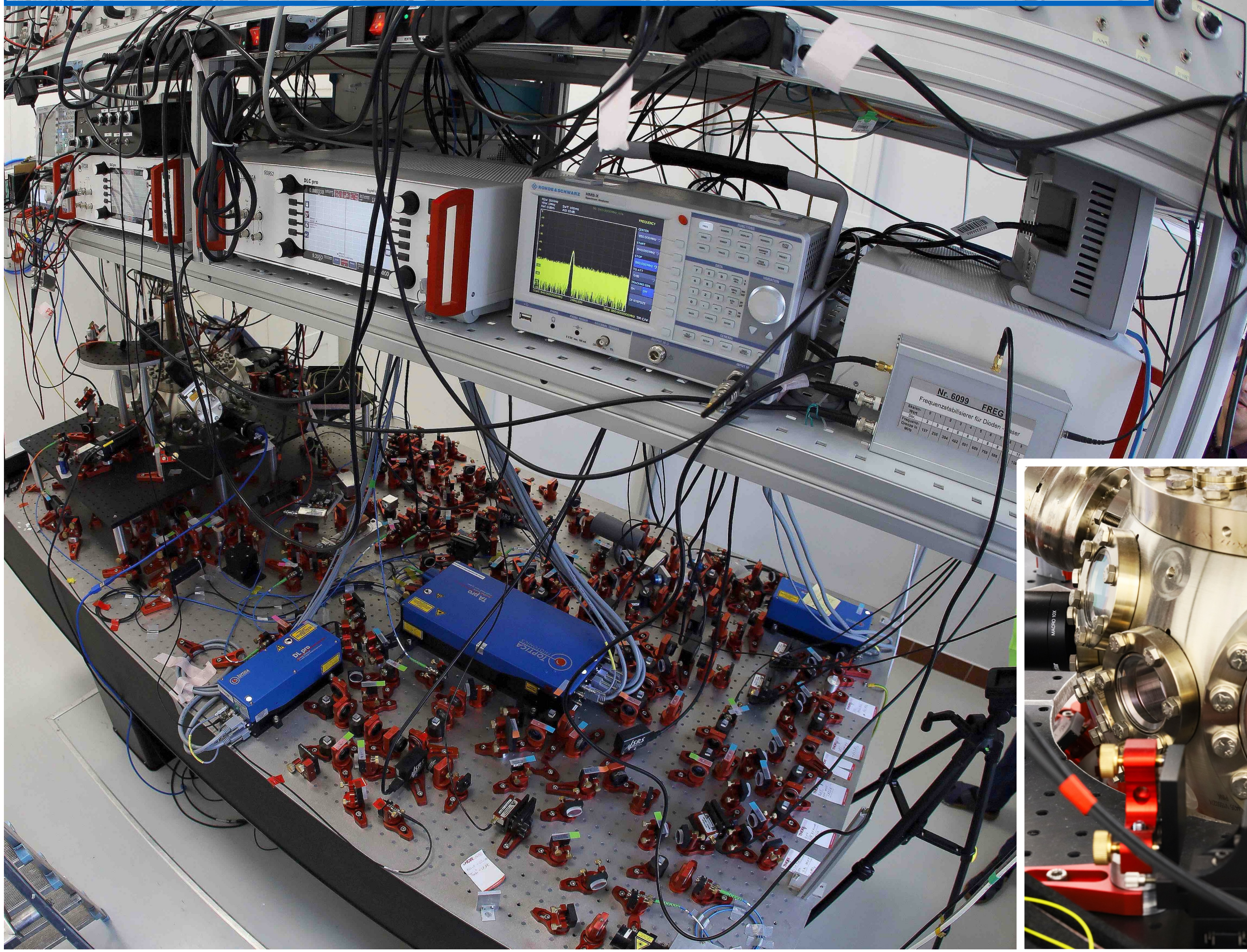
- *1st order ~ discontinuity*
- $\frac{d}{dt}\rho = L\,\rho = 0$
- *density matrix is a continuous function of all system parameters*
- *Co-existence of phases*
- $\rho_{ss} = (1 - F) \cdot \rho_{dim} + F \cdot \rho_{bright}$

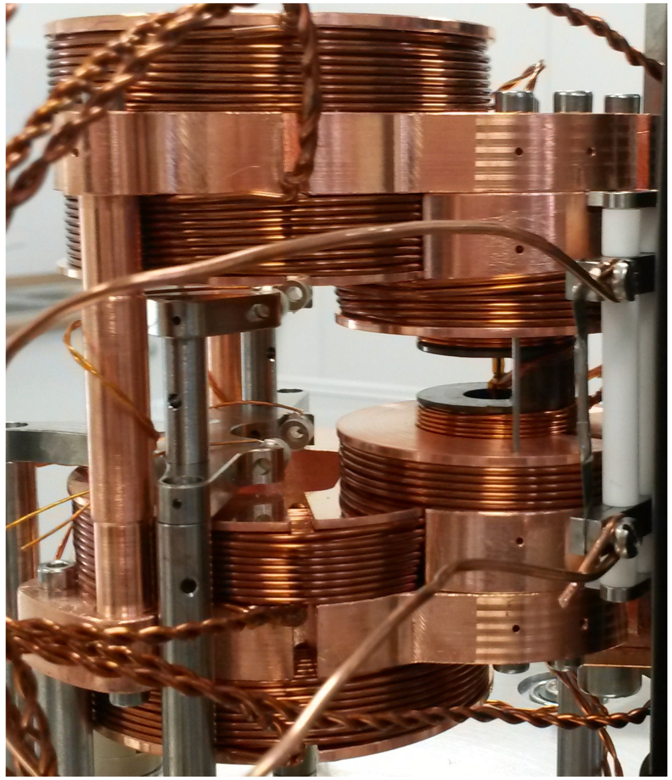
Bimodal density matrix

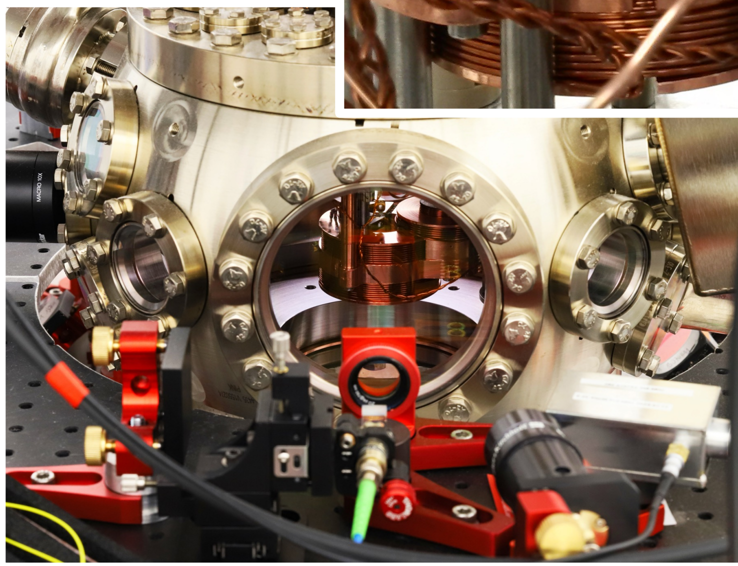


Fink, Dombi, Vukics, Walraff, Domokos, Phys Rev X 2017

Quantum optics lab @ Wigner



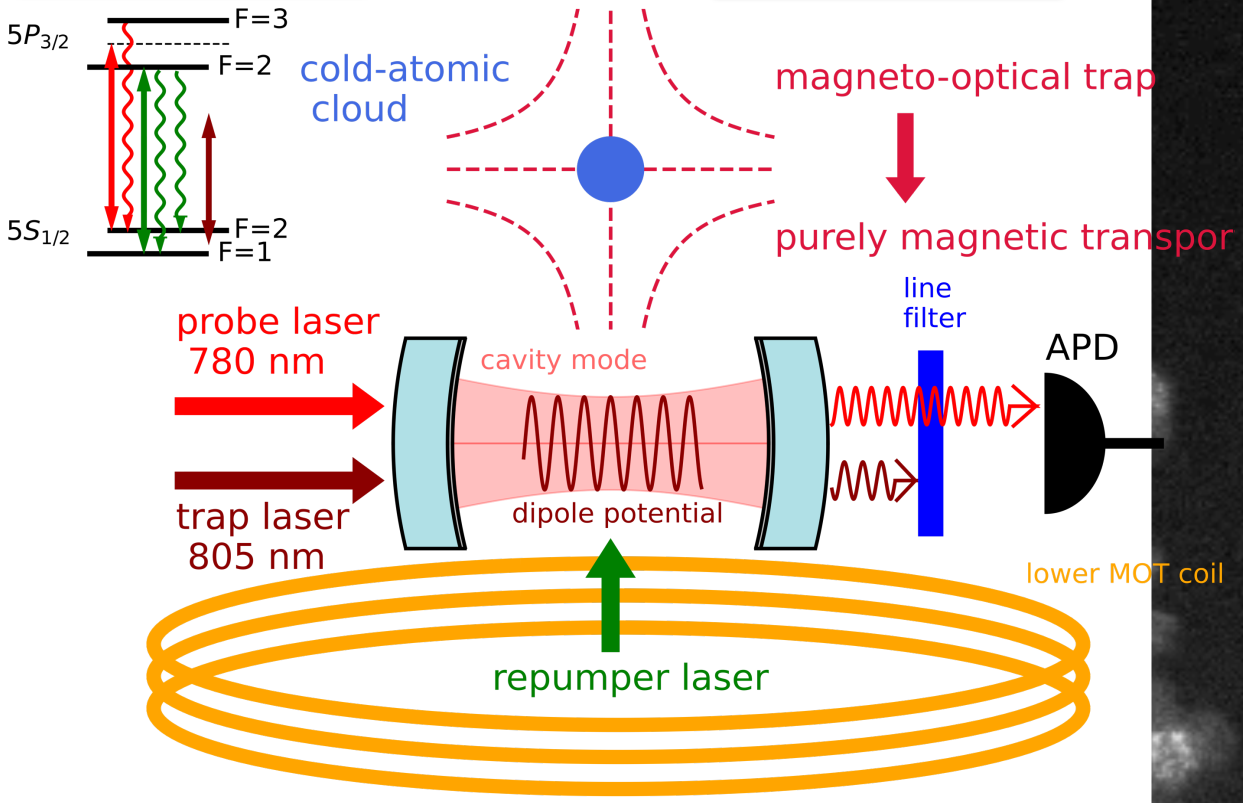


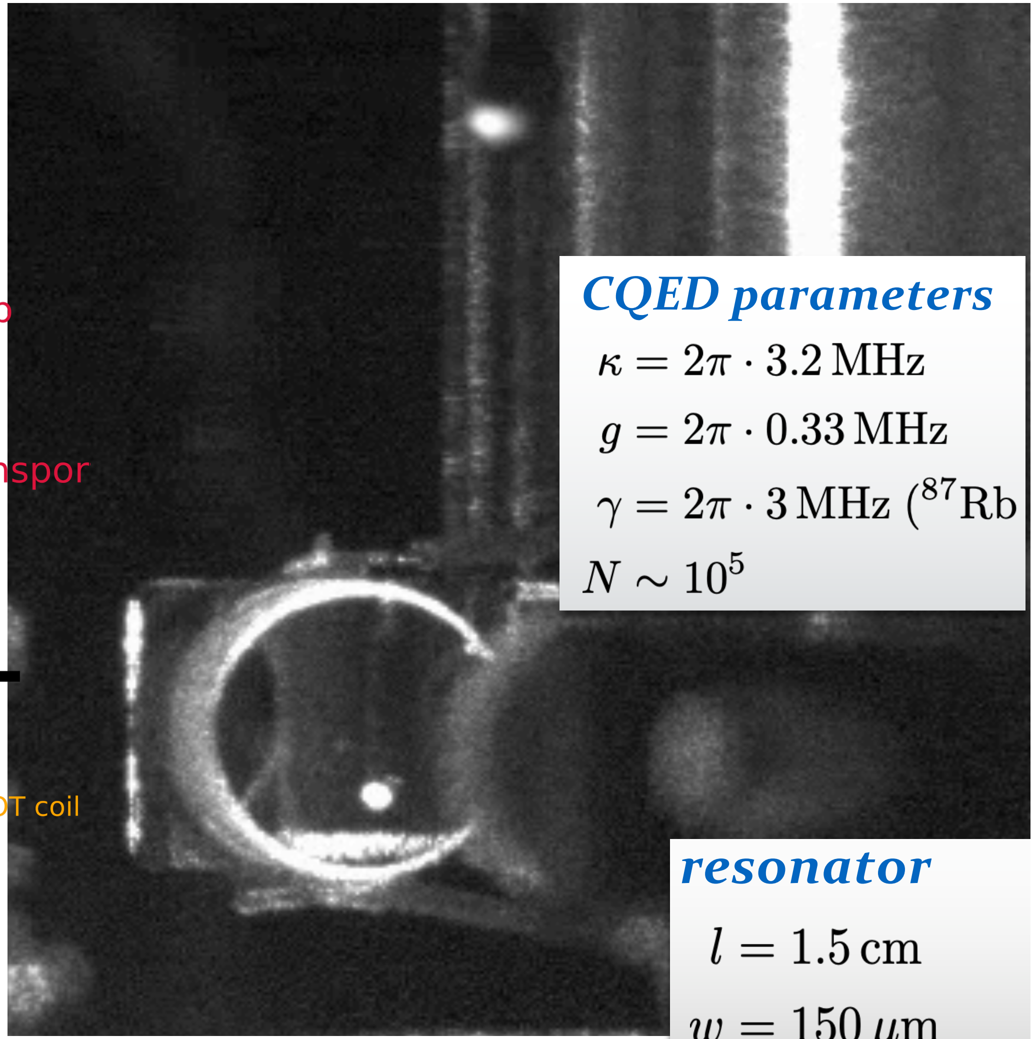


Experimental setup

^{87}Rb

$T \geq 10\ \mu K$





CQED parameters

$\kappa = 2\pi \cdot 3.2\,\mathrm{MHz}$

$g = 2\pi \cdot 0.33\,\mathrm{MHz}$

$\gamma = 2\pi \cdot 3\,\mathrm{MHz}$ (^{87}Rb

$N \sim 10^5$

resonator

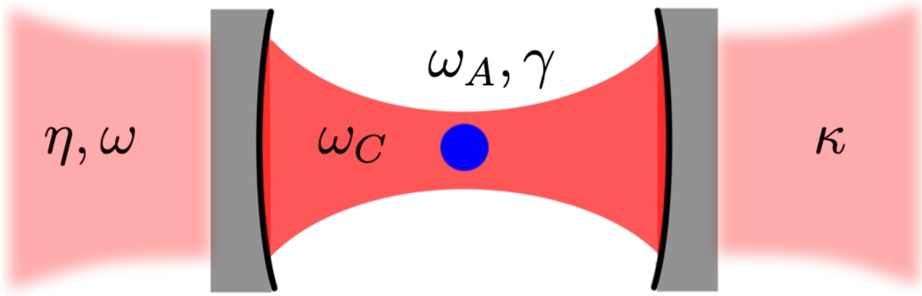
$l = 1.5\,\mathrm{cm}$

$w = 150\,\mu\mathrm{m}$

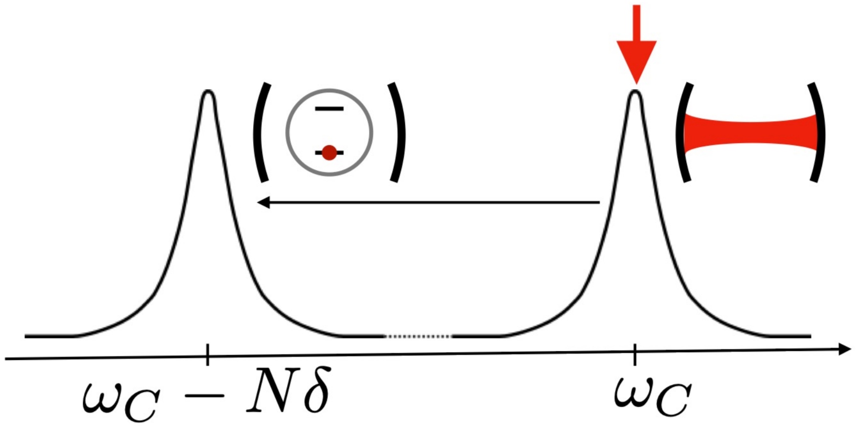
plano-concave

D. Varga et al, Physics Letters A, 2024

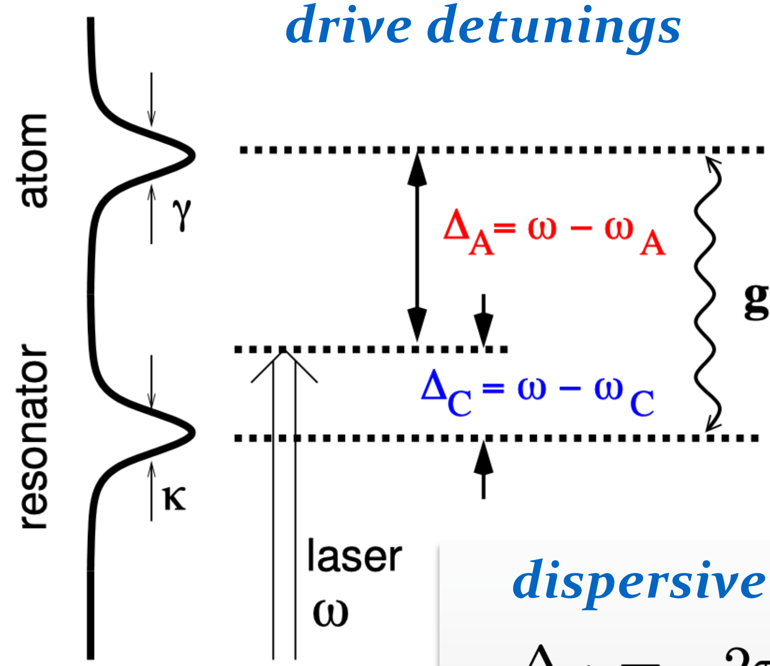
Simple transmission blockading mechanism



resonance shift



drive detunings



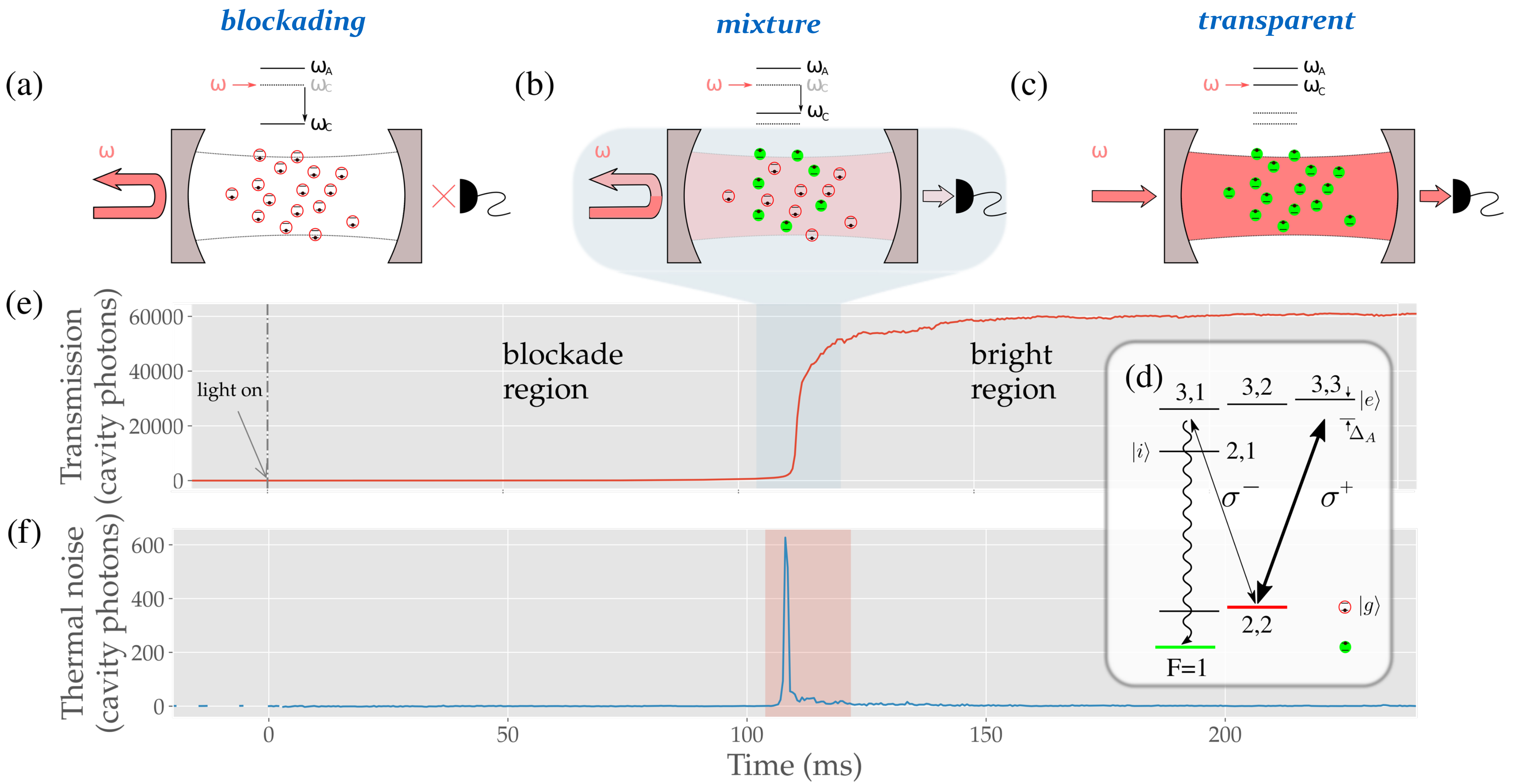
dispersive limit

$$\Delta_A = -2\pi \cdot 35\,\mathrm{MHz}$$

$$\delta \approx \frac{g^2}{\Delta_A} = -2\pi \cdot 3\,\mathrm{kHz}$$

$$N \approx 10^4$$

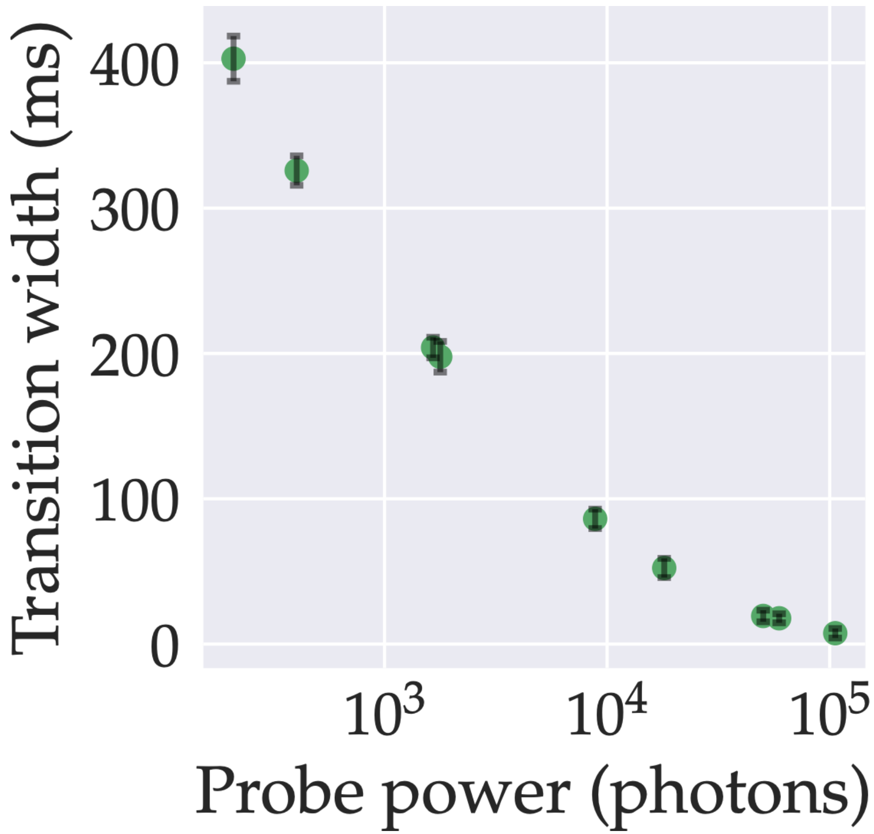
Time-resolved observation of the transmission blockade breakdown

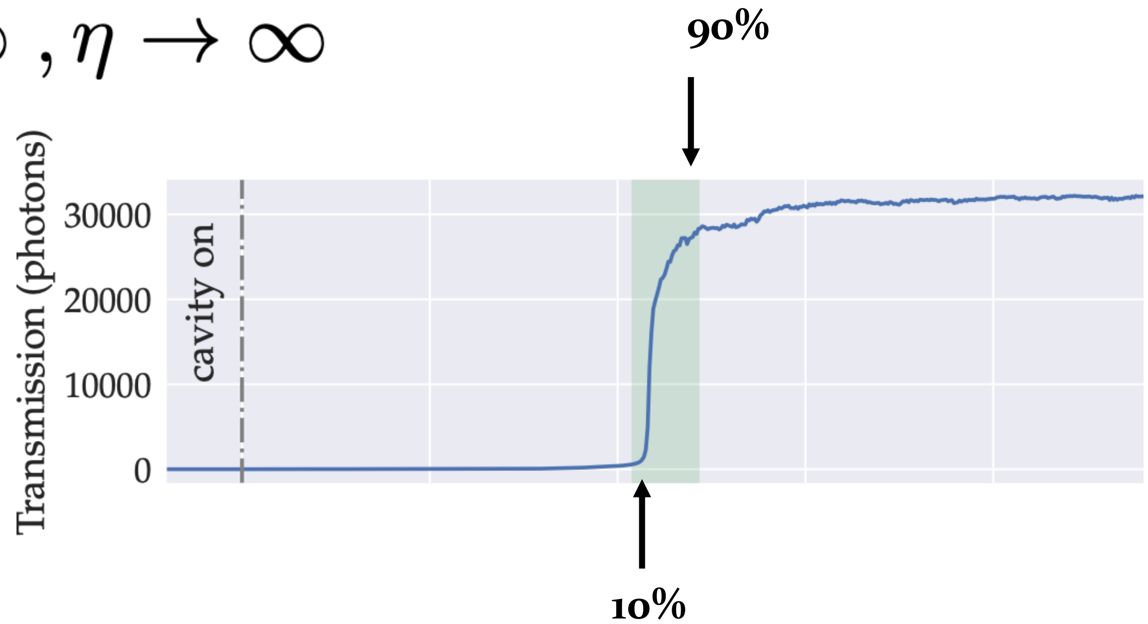


T. W. Clark, A. Dombi et al, Phys. Rev. A 105, 063712 (2022)

Defining and calibrating a finite-size measure

Thermodynamic limit: $\mathcal{C} \to \infty \,, \eta \to \infty$

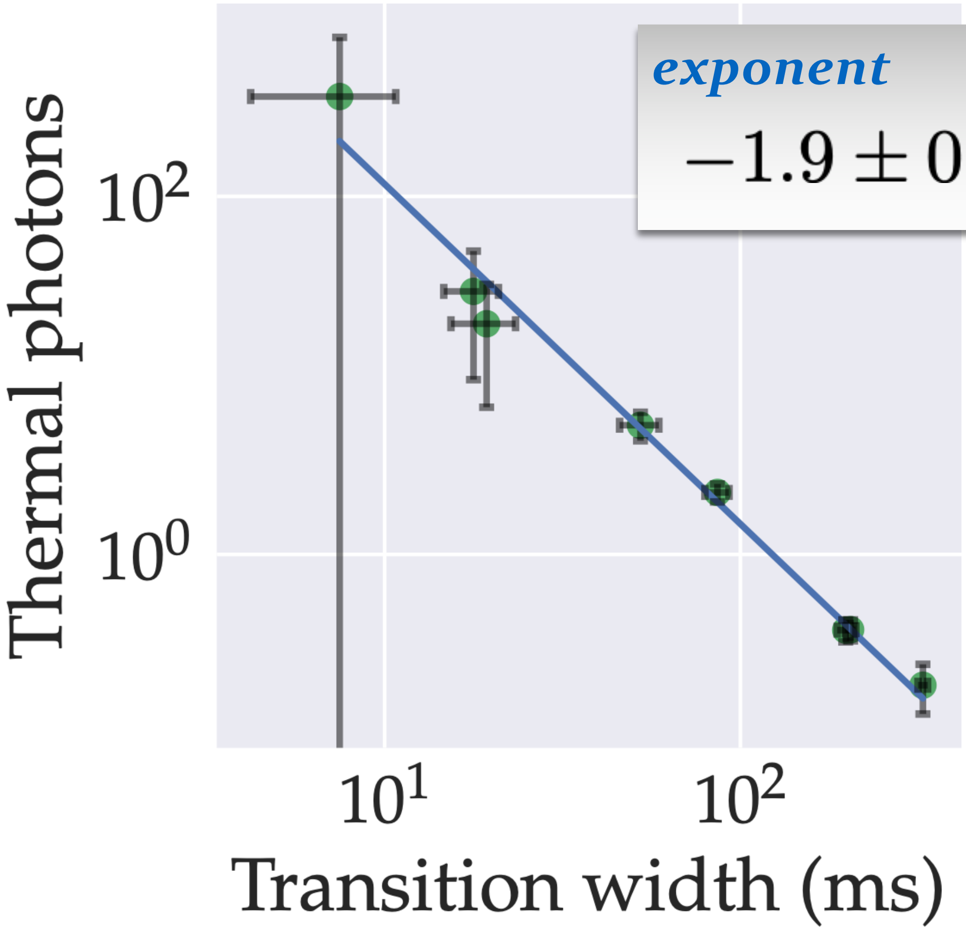




Two orders of magnitude covered!

T. W. Clark, A. Dombi et al, Phys. Rev. A 105, 063712 (2022)

Finite-size scaling of fluctuations



exponent

$$-1.9 \pm 0.1$$

measured
photo-current noise

displaced thermal state

$$P_{\mathrm{th,disp}}(\alpha) = \frac{1}{\pi n_{\mathrm{th}}} \exp\left(-|\alpha - \beta|^2 / n_{\mathrm{th}}\right)$$

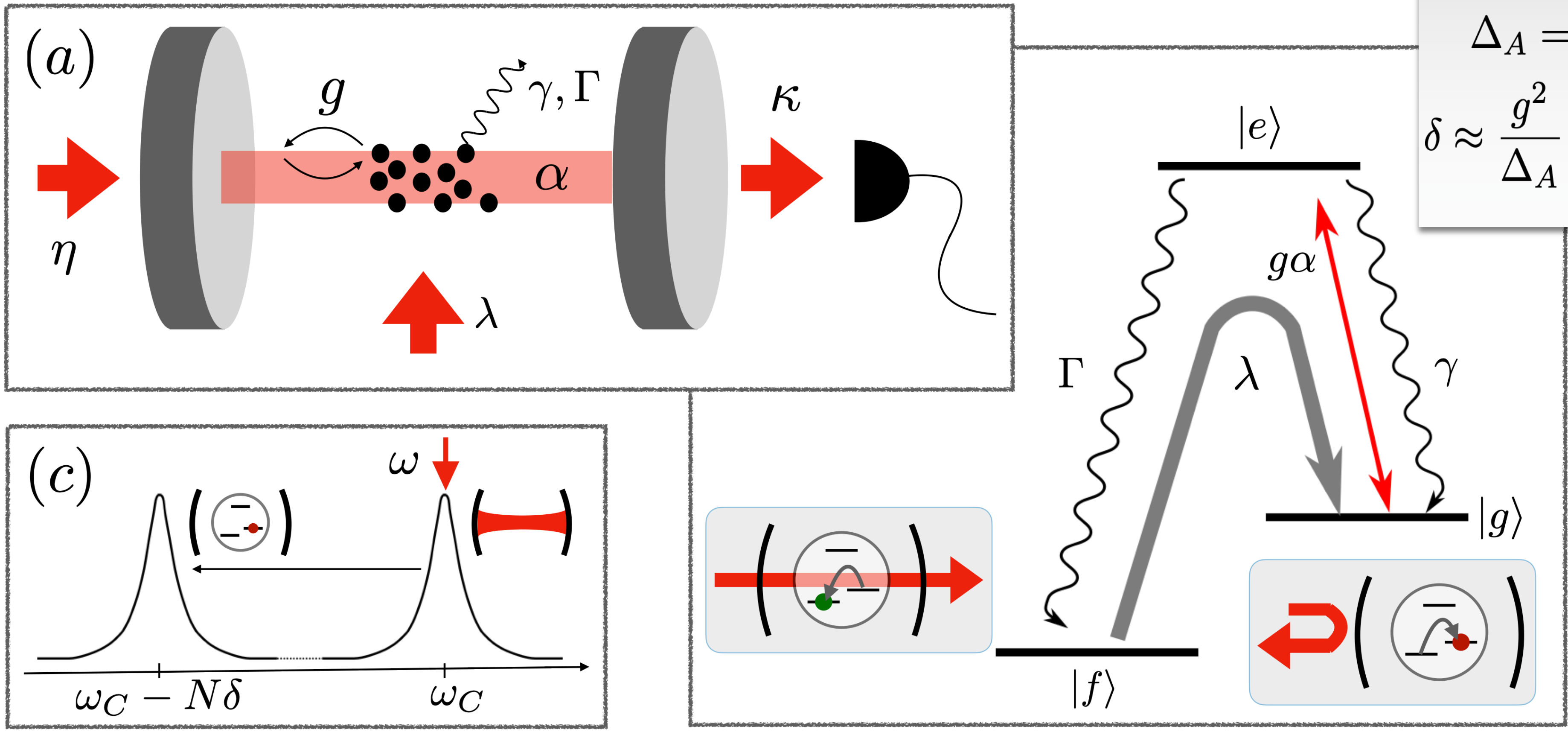
phase transition $\beta : 0 \to \eta/\kappa$

$$g^{(2)}(0) = 2 - \frac{|\beta|^4}{(n_{\mathrm{th}} + |\beta|^2)^2}$$

derived from measurement

Competing optical pumping processes

Three-level scheme with repumper



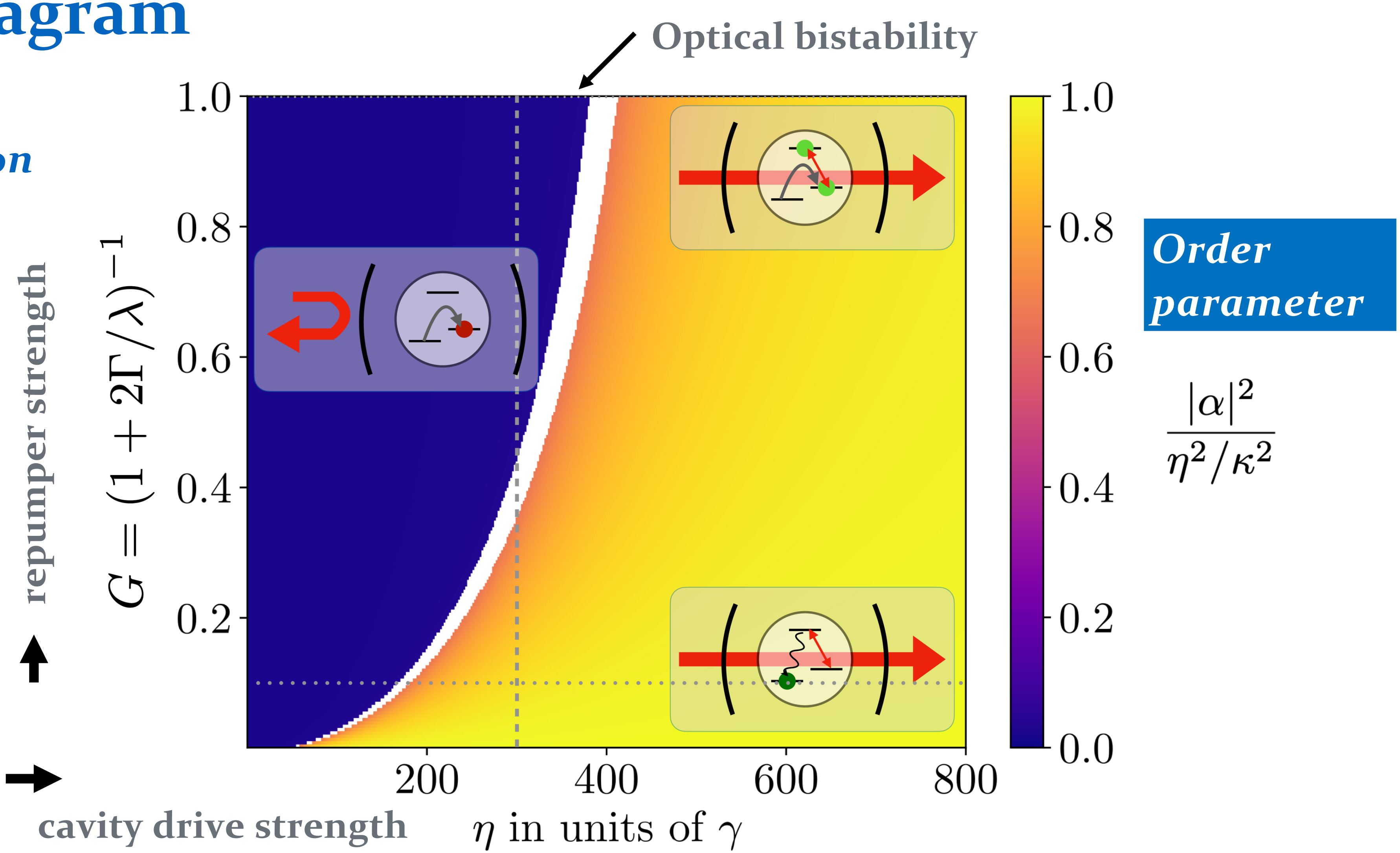
dispersive limit

$$\Delta_A = -2\pi \cdot 35\,\mathrm{MHz}$$

$$\delta \approx \frac{g^2}{\Delta_A} = -2\pi \cdot 3\,\mathrm{kHz}$$

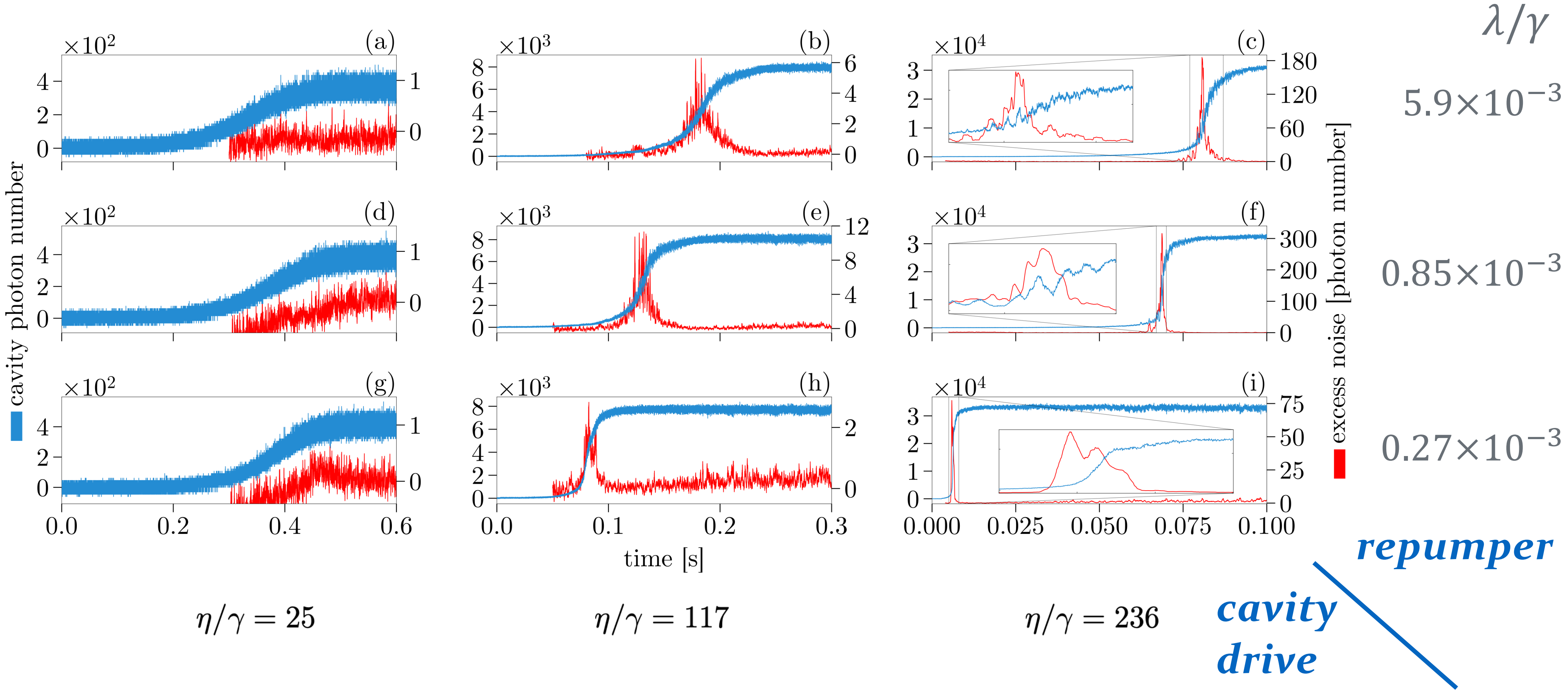
Phase diagram

mean-field approximation



Control parameters

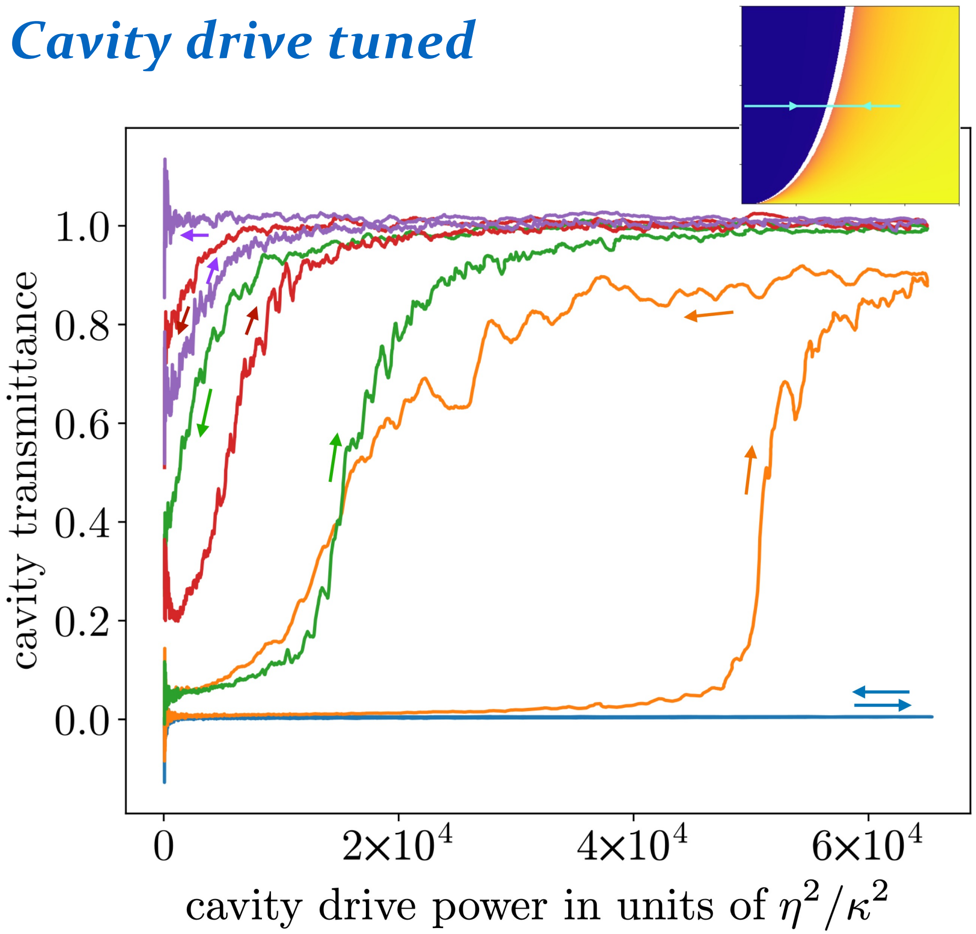
Time evolution from different points in phase space



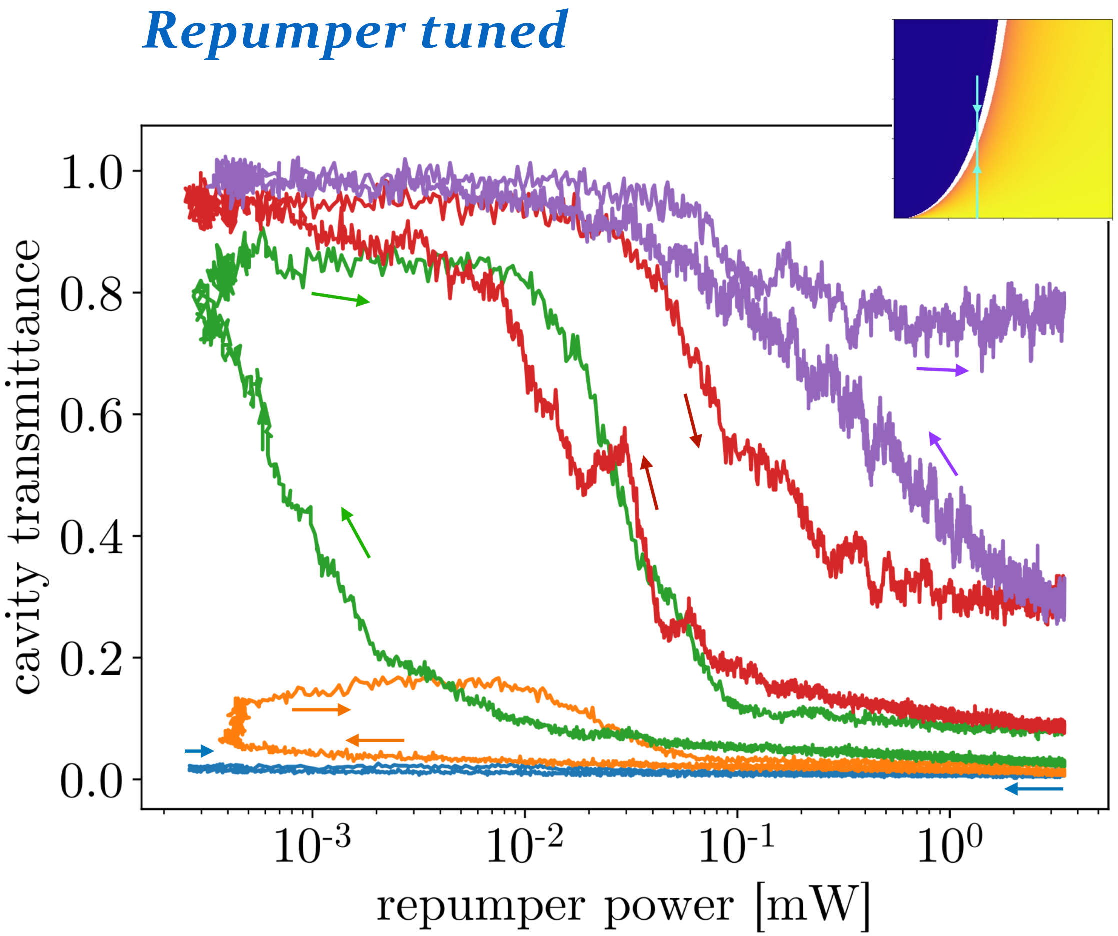
B. Gábor et al, Phys. Rev. A 107, 023713 (2023)

Demonstration of the hysteresis

Cavity drive tuned

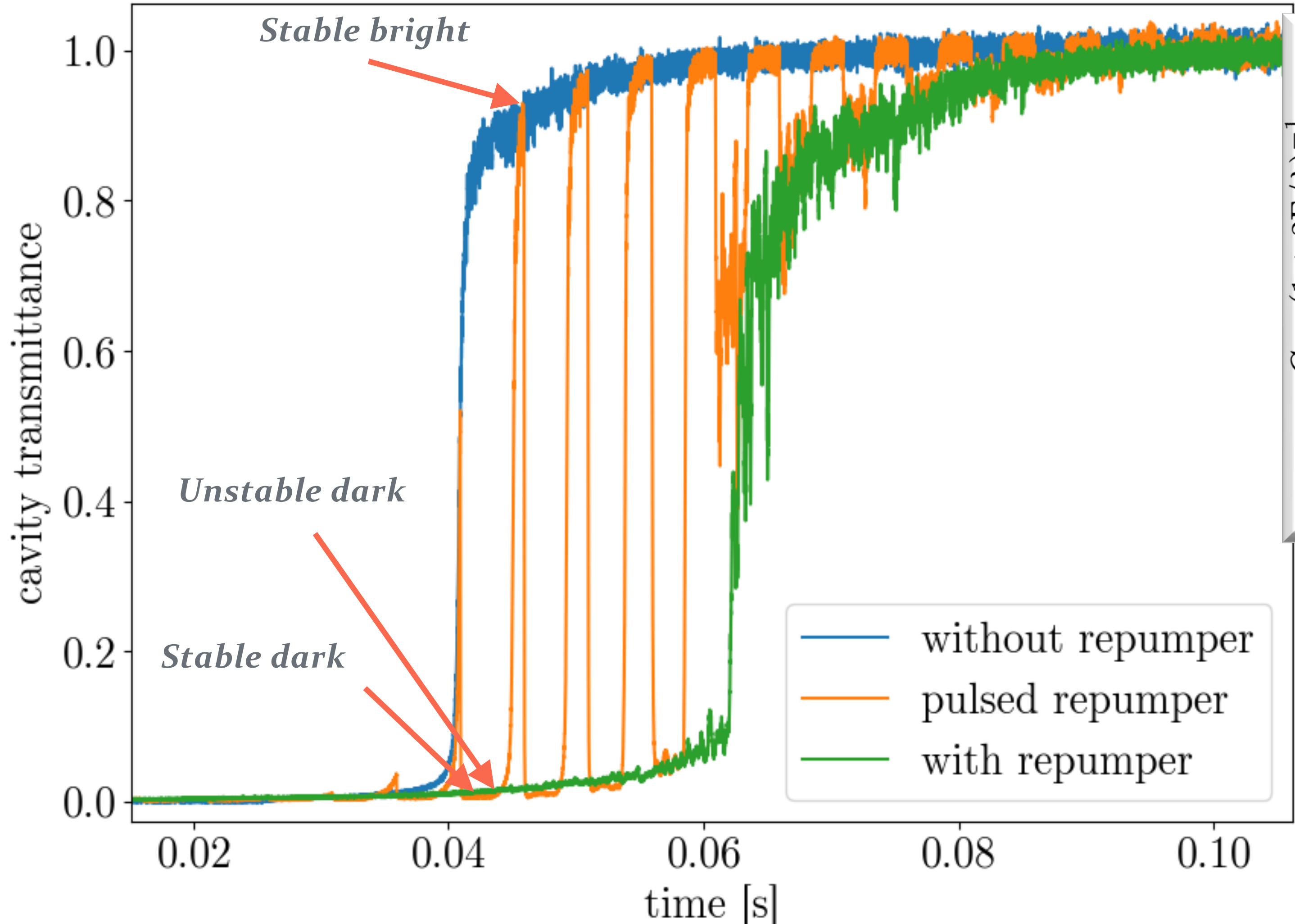


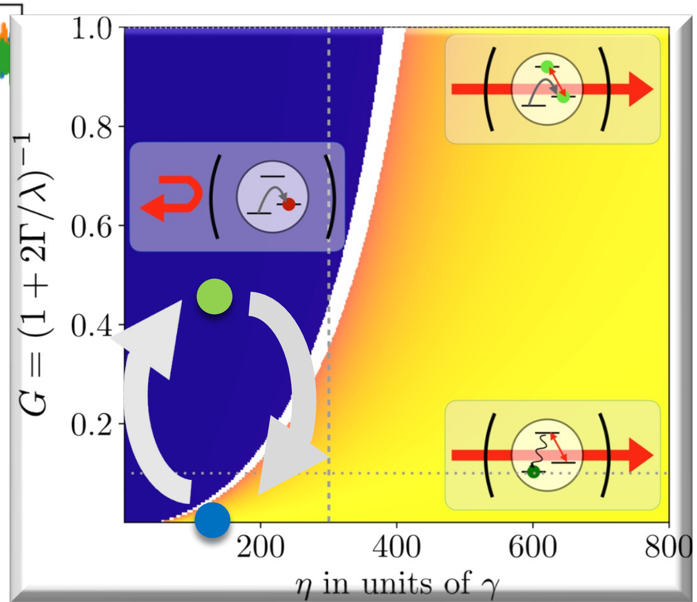
Repumper tuned



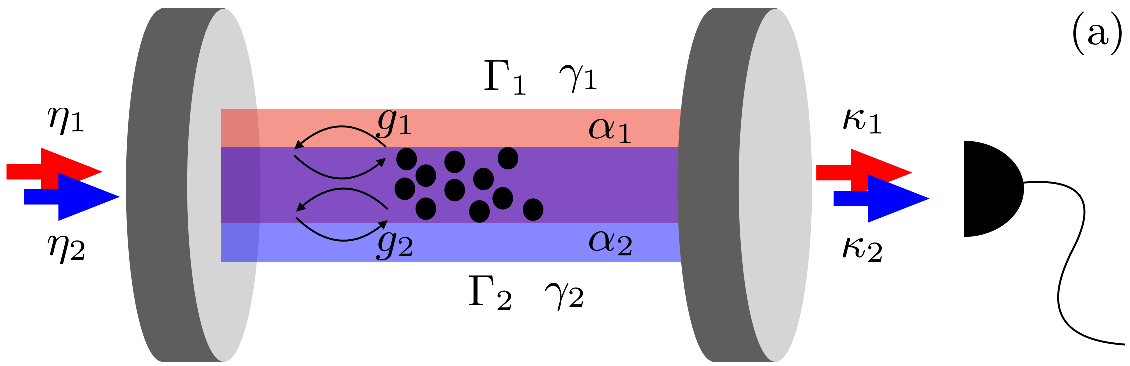
B. Gábor et al, Phys. Rev. A 107, 023713 (2023)

Switching between stable and unstable phases

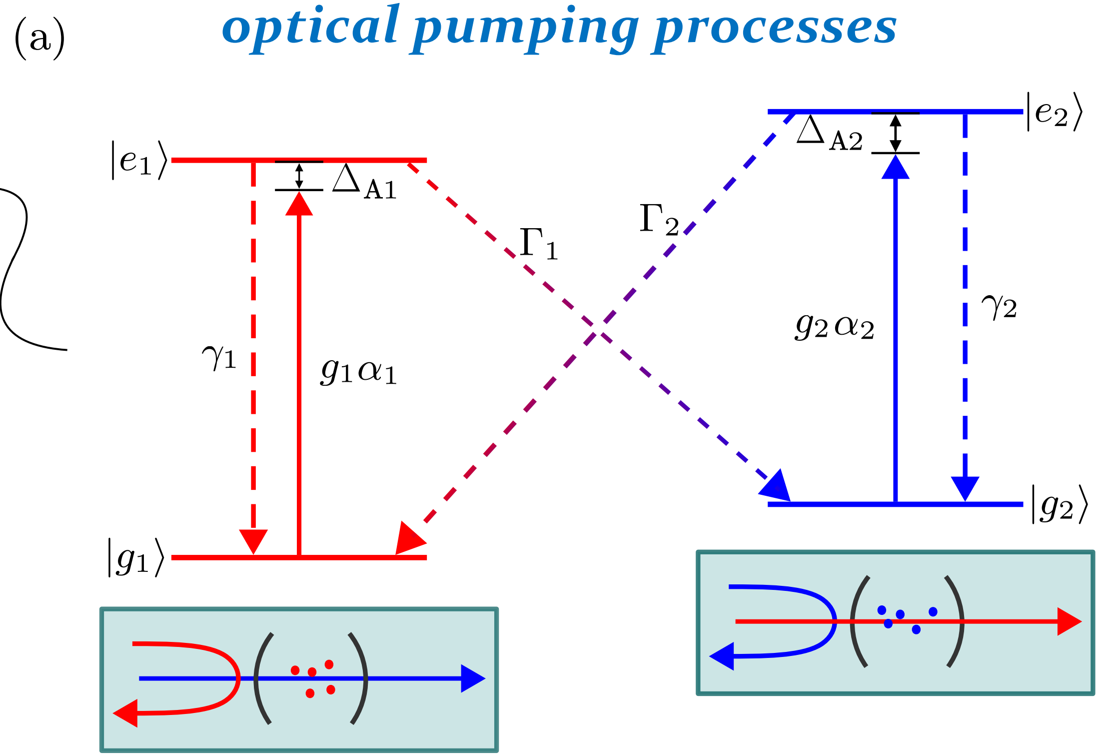




Ground state bistability with two cavity modes



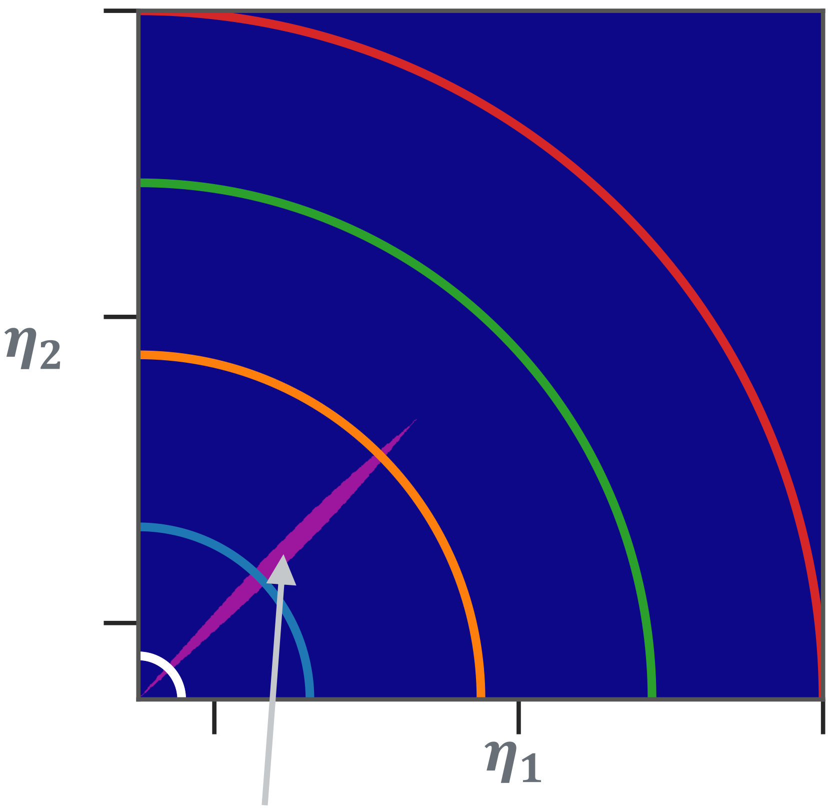
Competing non-linear optical pumping processes



B. Gábor, D. Nagy, A. Vukics, P. Domokos, Phys. Rev. Research 5, L042038 (2023)

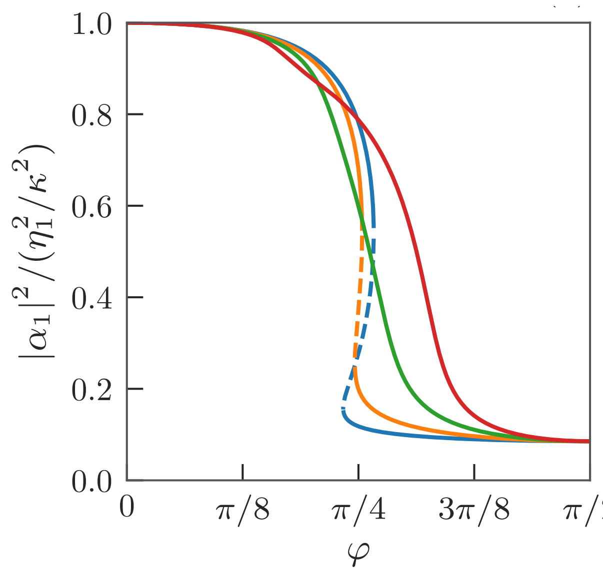
Phase diagram of the ground state bistability

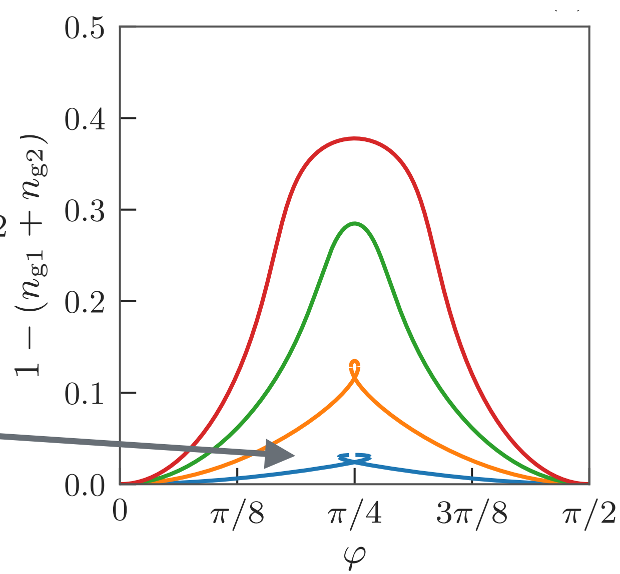
$N = 5 \times 10^3 \quad g = \gamma/10$



Bistability domain

Varying the relative drive stregths at fixed total power



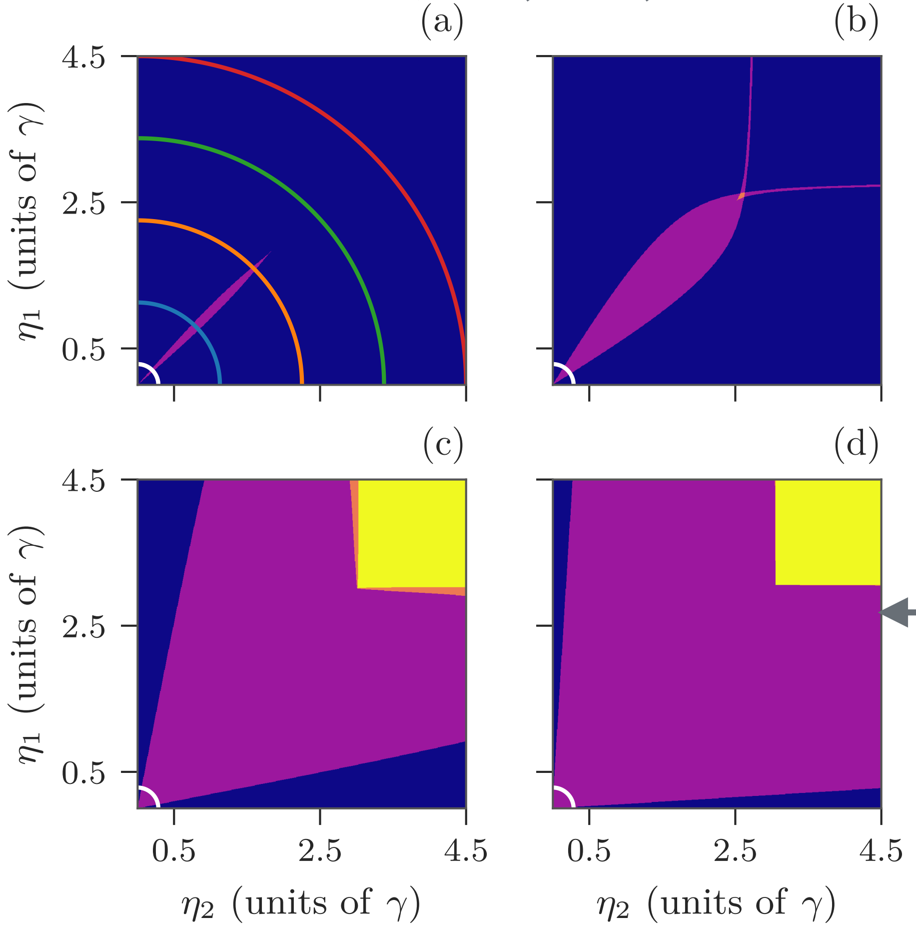


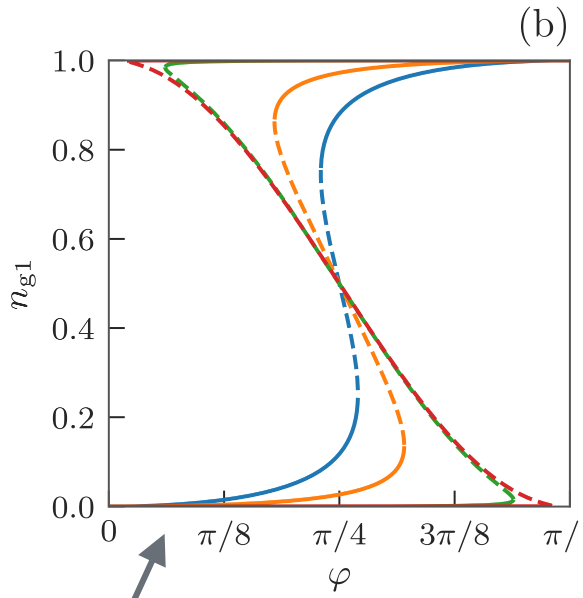
Low excitation

B. Gábor, D. Nagy, A. Vukics, P. Domokos, Phys. Rev. Research 5, L042038 (2023)

Thermodynamic limit: cooperativity C → ∞

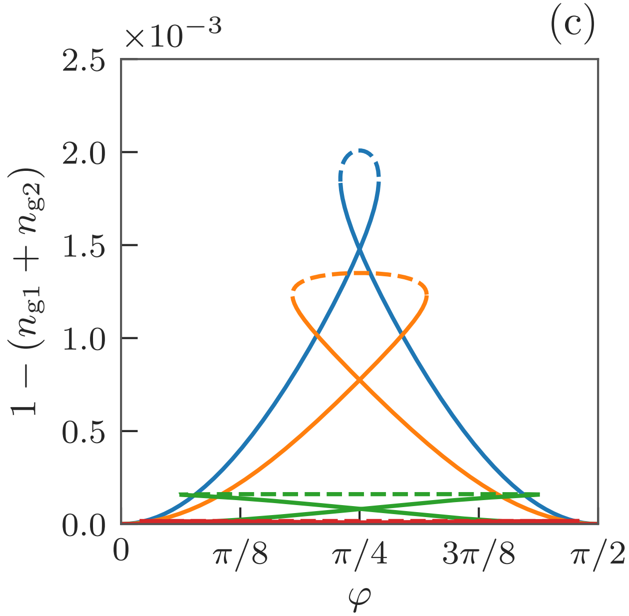
***N* = 5 × 10³, 10⁴, 10⁵, 10⁶**





Both ground states are stable

Vanishing excitation



B. Gábor, D. Nagy, A. Vukics, P. Domokos, Phys. Rev. Research 5, L042038 (2023)

Conclusions

- Zero dimensional quantum systems under continuous measurement can host `macroscopic' phases and can undergo phase transitions
- Cavity QED systems are paradigmatic driven-dissipative open quantum systems where a single or a few atoms in strongly coupled to a cavity mode can produce bistability
- The breakdown of the transmission blockade has been observed with time resolution and finite-size scaling of the fluctuations has been performed
- We demonstrated experimentally hysteresis in a first-order phase transition
- There is a limit of cavity-induced bistability in whihc the phases correspond to pure ground states

Acknowledgements

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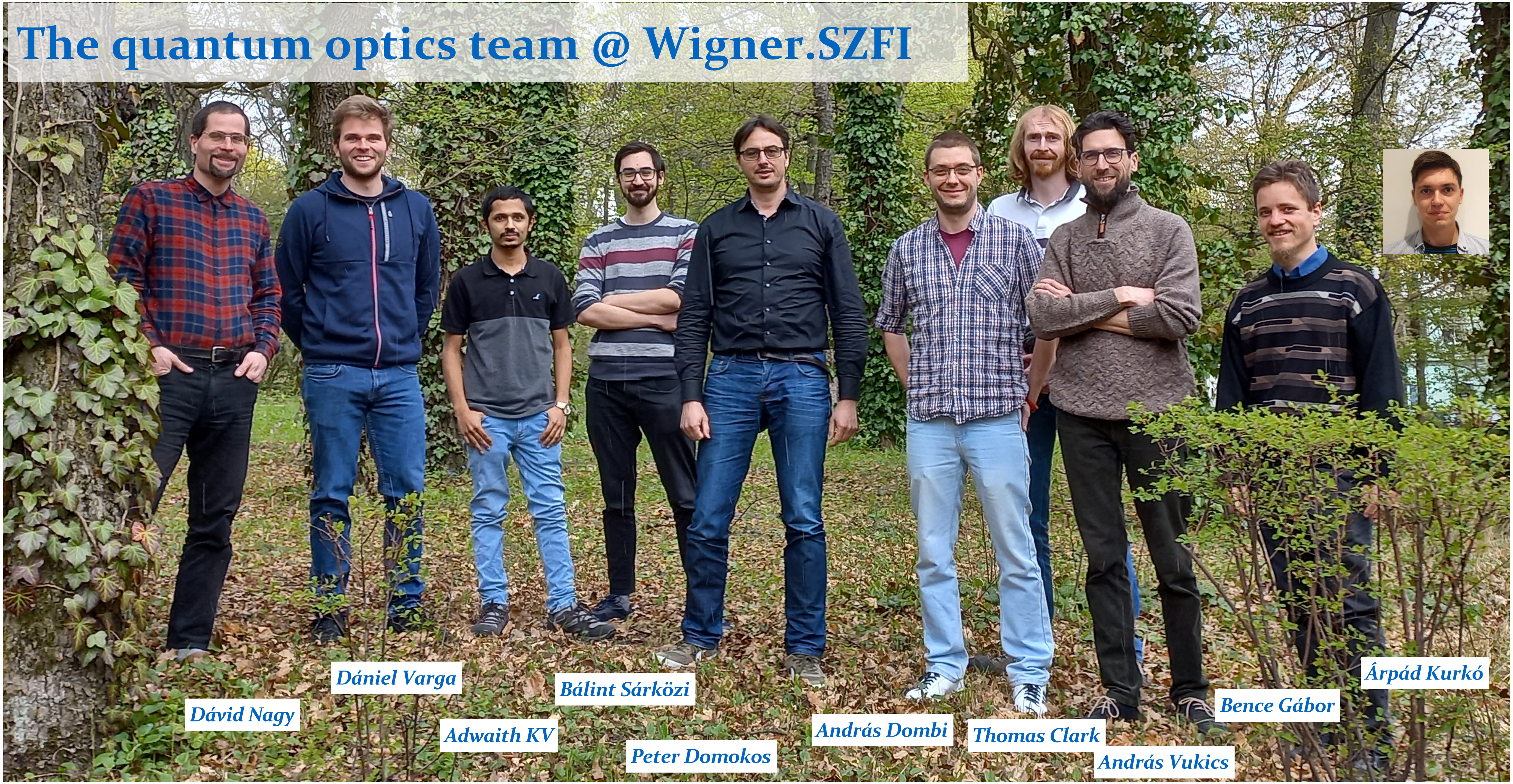
The quantum optics team @ Wigner.SZFI
Dávid Nagy
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Bence Gábor
Árpád Kurkó