QUANTUM-DOT CELLULAR NONLINEAR NETWORKS: COMPUTING WITH LOCALLY-CONNECTED QUANTUM DOT ARRAYS

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ABSTRACT

We discuss a novel nano-electronic computing paradigm in which cells composed of interacting quantum dots are employed in a locally-interconnected architecture. We develop a network-theoretic description in terms of appropriate local state variables in each cell.

1. INTRODUCTION

Since its inception a few decades ago, silicon ULSI technology has seen an exponential improvement in virtually any figure of merit, as described by Moore's Law. However, there are indications now that this progress will slow, or even come to a still-stand, as fundamental limits are being reached. This slow-down of silicon technology may provide an opportunity for alternative devices. In this paper, we will describe some ideas of the Notre Dame NanoDevices Group on a possible nanoelectronic computing technology based on cells of coupled quantum dots, which we call Quantum-Dot Cellular Automata [1].

The miniaturization of semiconductor devices has resulted in structures which exhibit novel physical effects due to the emerging quantum mechanical nature of the electrons. In a subsequent chapter, we will briefly outline the fabrication techniques which are utilized to create such quantum dots [2]. We will then discuss how these dots may be arranged in cells with interesting computational properties.

The arrangement of quantum-dot cells in locally-connected arrays is similar to the architecture used in Cellular Nonlinear Networks (CNNs) [3}. We will show that the quantum mechanical equations of motion for the quantum-dot cells may be cast in a form which explicitly shows the connection to CNN dynamics.

2. LOW-DIMENSIONAL SEMICONDUCTOR STRUCTURES

Advanced semiconductor growth techniques, such as molecular beam epitaxy (MBE), allow the fabrication of semiconductor sandwich structures with interfaces of virtually atomic precision [4]. This control in the growth direction allows the realization of artificial crystals with desired electronic and optical properties. A schematic picture of such a sandwich structure is shown in Fig. 1(a). The various layers can be made to posses different properties by choosing appropriate material combinations during growth. In particular, it is possible to fabricate layer structures with an effective confining potential for the electrons. This technique is referred to as bandgap engineering and has been used extensively to taylor device structures. The layers can be grown so thin that quantum mechanical confinement effect become important. This may lead to the formation of a quasi two-dimensional electronic gas (2DEG) in the quantum well layer.

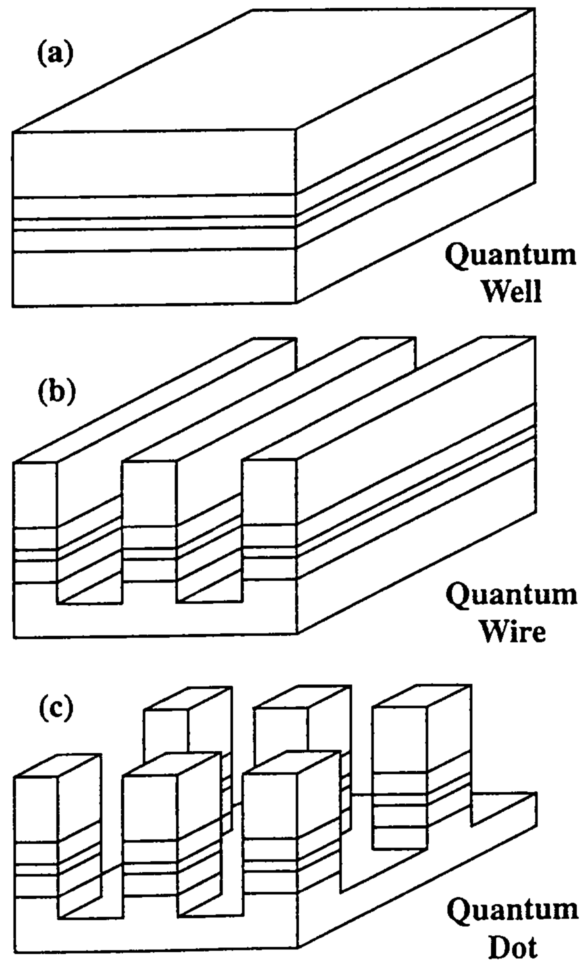

Figure 1: Schematic of semiconductor heterostructures; (a) quantum well, (b) quantum wire, and (c) quantum dot.

Starting from such a 2DEG, control in the lateral directions can be achieved by conventional patterning techniques, such as optical or electron-beam lithography. Subsequent processing steps, such as etching, can then selectively remove material to define line- or dot-patterns, as schematically shown in Figs. 1(b) and 1(c). This processing further confines the 2DEG into quasi one-dimensional systems (so-called quantum wires) or even quasi zero-dimensional systems (so-called quantum dots).

A different approach of further confining a 2DEG is to use electrostatic confinement. As schematically shown in Fig. 2, one may use lateral patterning techniques to structure a metallic layer on top of the MBE-grown structure. Applying a negative bias to the gates will deplete the 2DEG underneath the metallic electrodes. In this fashion, one may create quantum wires by using two gates as schematically shown. Using this so-called split-gate design, quantum dots may be realized in a similar fashion by appropriately-shaped electrodes.

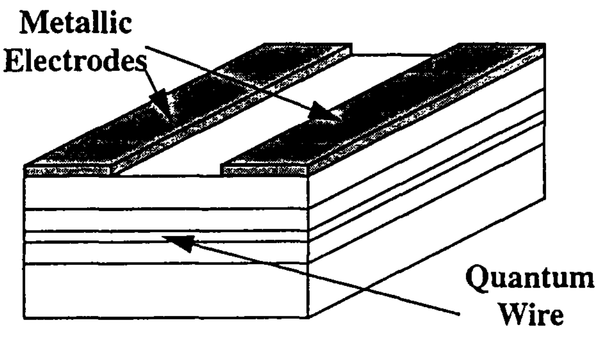

Figure 2: Schematic diagram of split-gate technology using metallic electrodes for the lateral patterning of a 2DEG.

3. QUANTUM-DOT CELLULAR AUTOMATA

Based upon the emerging technology of quantum-dot fabrication, the Notre Dame group has proposed a scheme for computing with cells of coupled dots [1], which will be described below. The coupling between the cells is given by their physical interaction, and not by wires. The mechanisms available for the interactions between nanoelectronic structures are the Coulomb interaction and quantum-mechanical tunneling. To our knowledge, this is the first concrete proposal to utilize quantum dots for computing.

3.1 A Quantum-Dot Cell:

The Notre Dame proposal is based on a cell which contains five quantum dots, as schematically shown in Fig. 3. The dots are shown as the circles which represent the confining electronic potential. In the ideal case, this cell is occupied by two electrons, which are schematically shown as the solid dots. The electrons are allowed to "jump" between the individual dots in a cell by the mechanism of quantum mechanical tunneling. Tunneling is possible on the nano-meter scale when there is sufficient leaking of the electronic wavefunction out of the confining potential of each dot, and the rate of these jumps may be controlled during fabrication by the physical separation between neighboring dots.

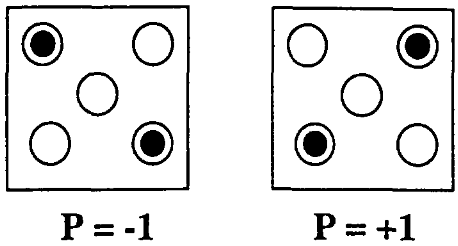

Figure 3: Schematic diagram of cells (squares) composed of quantum dots (circles). Each cell is occupied by two electrons (solid dots). There are two ground-state configurations, corresponding to "polarizations" of +1 and -1.

This quantum-dot cell represents an interesting dynamical system. The two electrons experience their mutual Coulombic repulsion, yet they are constrained to occupy the dots. If left alone, they will seek the configuration corresponding to the physical ground state of the cell. It is easy to see that the ground state of the system will be an equal superposition of the two basic configurations with electrons at opposite corners, as shown in Fig. 3. We may associate a "polarization" of P=+1 and P=-1 with either arrangement.

3.2 Cell-Cell Coupling

The properties of an isolated cell were discussed above. Here, we study the interactions between two cells. The electrons are allowed to tunnel between the dots in the same cell, but not between different cells. Coupling between the two cells is provided by the Coulomb interaction between the electrons in different cells.

Figure 4 shows how one cell is influenced by the state of its neighbor. The inset shows two cells where the polarization of cell 1 (P_1) is determined by the polarization of its neighbor (P_2). The polarization of cell 2 is presumed to be fixed at a given value, corresponding to a certain arrangement of charges in cell 2, and this charge distribution exerts its influence on cell 1, thus determining its polarization P_1. The figure shows that cell 1 is almost completely polarized even though cell 2 might only be partially polarized. The abruptness of the cell-cell response function depends upon the ratio of the strength of the tunneling energy to the Coulomb energy for electrons on neighboring sites [5].

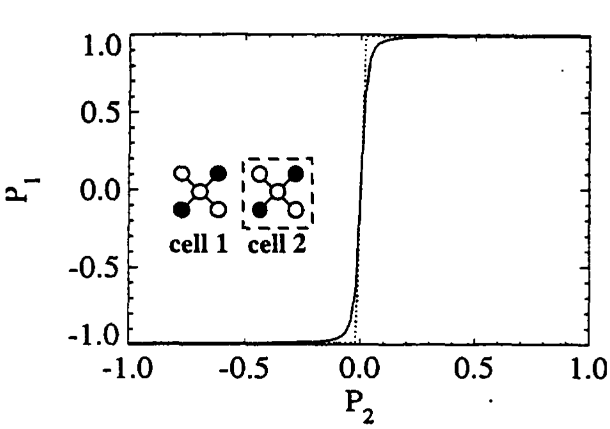

Figure 4: Cell-cell interaction. Note the highly nonlinear nature of the coupling, which represents gain.

This bistable saturation is the basis for the application of such quantum-dot cells for computing structures. The nonlinear saturation replaces the gain in conventional circuits. Note that no power dissipation is required in this case. One can think of the saturation levels of the polarization as the "signal rails."

These general conclusions regarding cell behavior and cell-cell coupling are not specific to the five-dot cell discussed so far. Similar behavior is also found for alternate cell designs, such as with four dots in the corners as opposed to the five discussed so far [5].

3.3 QCA Logic

Based upon the bistable behavior of the cell-cell coupling, the cell polarization can be used to encode binary information. The physical interactions between cells may be used to realize elementary Boolean logic functions [6].

Figure 5 shows examples of simple cell arrays. In each case, the polarization of the cell at the edge of the array is kept fixed; this is the so-called driver cell and it

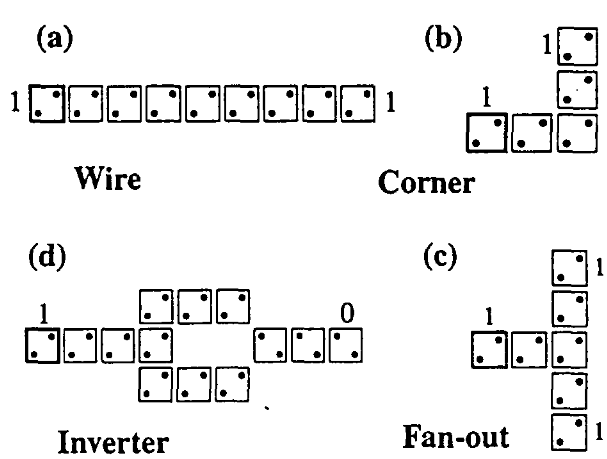

Figure 5: Examples of simple QCA structures.

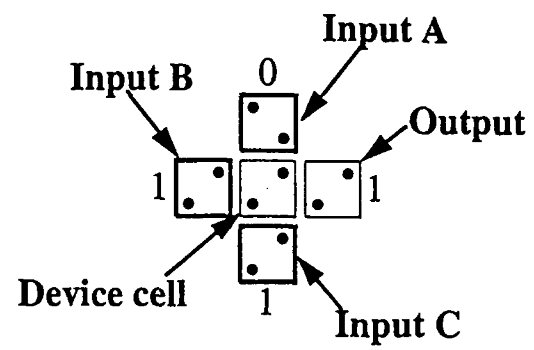

Figure 6: QCA majority logic structure.

is plotted with a thick border. We call it the driver since it determines the state of the whole array. Each figure shows the cell polarizations for the physical ground state configuration of the whole array.

Fig. 5(a) shows that a line of cells allows the propagation of information, thus realizing a binary wire. Note that only information but no electric current flows down the line, which results in low power dissipation. Information can also flow around corners, as shown in Fig 5(b), and fan-out is possible, compare Fig. 5(c). A specific arrangement of cells, such as the one shown in Fig. 5(d), may be used to realize an inverter.

Figure 6 shows a majority logic gate, which just consists of an intersection of lines and the "device cell" is simply the one in the center. If we view three of the neighbors as inputs (kept fixed), then the polarization of the output cell is the one which "computes" the majority votes of the inputs. Using conventional circuitry, the design of a majority logic gate would be significantly more complicated. The new physics of quantum mechanics gives rise to new functionality, which allows a very compact realization of majority logic.

Note that conventional AND and OR gates are hidden in the majority logic gate. Inspection of the majority-logic truth table reveals, that if input A is kept fixed at 0, the remaining two inputs B and C realize an AND gate. Conversely, if A is held at 1, B and C realize a binary OR gate. In other words, majority logic gates may be viewed as programmable AND and OR gates.

4. QUANTUM-DOT CELLULAR NONLINEAR NETWORKS

We consider here a simple model for the quantum states in each cell and show how the quantum dynamics can be transformed into a CNN-style description [7].

4.1 Quantum Model of Cell Array

We describe the quantum state in each cell using two basis states $|\phi_1\rangle$ and $|\phi_2\rangle$ which are completely polarized.

$$|\Psi\rangle = \alpha\, |\phi_1\rangle + \beta\, |\phi_2\rangle$$

Within this two-state model, the properties of each cell are completely specified by the quantum mechanical amplitudes α and β. In particular, P, the cell polarization is given by:

$$P = |\alpha|^2 - |\beta|^2$$

The Coulomb interaction between adjacent cells increases the energy of the configuration if the two cell polarizations differ. This can be accounted for by an energy shift corresponding to the weighted sum of the neighboring polarizations, which we denote by $\bar{P}$ E. The cell dynamics is then given by the Schrödinger equation,

$$i\hbar\, \delta/\delta t\ |\Psi\rangle = H\ |\Psi\rangle$$

where H represents the cell Hamiltonian.

4.2 Formulating CNN-Like Quantum Dynamics

In order to transform the quantum mechanical description of an array into a CNN-style description, we perform a transformation from the quantum-mechanical state variables to a set of state variables which contains the classical cell polarization, P, and a quantum mechanical phase angle, φ :

$$|\Psi\rangle = (\alpha, \beta) \longrightarrow |\Psi\rangle = (P, \varphi)$$

This is accomplished by the following relations:

$$\alpha = \sqrt{(1+P)/2}$$

$$\beta = \sqrt{(1-P)/2}\ e^{i\varphi}$$

With this, the dynamical equations derived from the Schrödinger can be rewritten as equations for the new state variables P and φ

$$\hbar\, \delta/\delta t\, P = -2\gamma \sin\varphi \sqrt{(1-P)^2}$$

$$\hbar\, \delta/\delta t\, \varphi = -\bar{P} E + 2\gamma \cos\varphi\ P / \sqrt{(1-P)^2}$$

The term $\bar{P}$ E accounts for the cell-cell interaction and γ is the tunneling matrix element between dots.

It can be shown that the resulting dynamics for each cell is governed by a Liapounov function V(P, φ) which is given by:

$$V(P, \varphi) = 2\gamma \cos\varphi \sqrt{(1-P)^2} + P\bar{P}E$$

4.3 Cellular Network Model of Quantum-Dot Array

In case of a two-dimensional array, each cell possesses an equivalent CNN-cell model described by the differential equations given above, thus this array is a special case of cellular nonlinear networks [8]. The equivalent circuit describing a cell is composed of two linear capacitors, four nonlinear controlled sources and eight linear controlled sources representing the interactions between the cell and its eight neighbors. This network model simulates the dynamics of the polarization and the phase of the coupled cellular array. If the polarization of the driver cells of an array (plotted with thick border on Figures 5 and 6) in equilibrium is changed in time, a dynamics of the polarizations and phases of the cells is launched in the whole array. This dynamics of different arrays has been studied, and a class of spatio-temporal wave-phenomena [9] was identified and explored.

In the framework of the CNN model, ground-state computing by the Quantum Cellular Array corresponds to transients between equilibrium states. Let us assume that in an equilibrium state the configuration of the array is a binary string s. If at t=0 the polarization of a few driver cells is abruptly changed from -1 to +1 or from +1 to -1, then a transient emerges. If we wait till the a new equilibrium is reached, we get a new configuration f(s) of the binary cells. We can say that the array "mapped" s to f(s). In this sense the cellular network model simulates the functions of the quantum-dot cellular automata, including the QCA Logic described in 3.3.

5. References

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