

# An introduction to Quantum Information and Quantum Computation (Temas de Física)

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# Outline

1

## Introduction

- Quantum information science

2

## Quantum information science

- Quantum mechanics
- Quantum optics
- Quantum engineering
- Theory of quantum entanglement
- Quantum cryptography
- Quantum metrology
- Quantum computing

# Quantum information science

- Analytic aspects
  - Quantum mechanics
  - Quantum optics
- Constructive aspects
  - Quantum engineering, creating large quantum states, entanglement
  - Quantum cryptography, quantum communication
  - Quantum metrology
  - Quantum computing, quantum simulation

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# Quantum mechanics

- Basic tools have been developed in the 1930's:
  - Schrödinger equation,
  - von Neumann equation  $i\frac{\partial \varrho}{\partial t} = [H, \varrho]$ ,
  - state function, state vector  $|\Psi\rangle$ .
  - density matrix  $\varrho$ ,
  - Dirac equation.
- However, one thing was missing:
  - it was difficult to test this model since individual particles could not be observed.

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## Quantum information science

- Quantum mechanics
- **Quantum optics**
- Quantum engineering
- Theory of quantum entanglement
- Quantum cryptography
- Quantum metrology
- Quantum computing

# Quantum optics

- LASER: a new system in which quantum mechanics was important.
- They developed a formalism to describe light modes in 1960's
  - annihilation, creation operators (like  $\Psi$  and  $\Psi^+$  in field theory)
  - coherent states (light fields we see in practice)
  - Fock states (states with given particle number)
  - Wigner function (even before)  $W(x, p)$
  - light-atom interaction, photon detection, superradiance, etc.
- However, one thing was still missing:
  - they could not observe few particles in a correlated quantum state.
  - They could see only many particles interacting with light, where the particles did nothing with each other.

# Question

- Do individual particles exist?
- Or they are only a tool for modeling?

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# Qubits vs. bits

- A quantum bit (=two-state system, spin- $\frac{1}{2}$  particle) can be in a pure state

$$|q\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle,$$

where  $\alpha_0$  and  $\alpha_1$  are complex numbers, and the normalization condition  $|\alpha_0|^2 + |\alpha_1|^2 = 1$ .

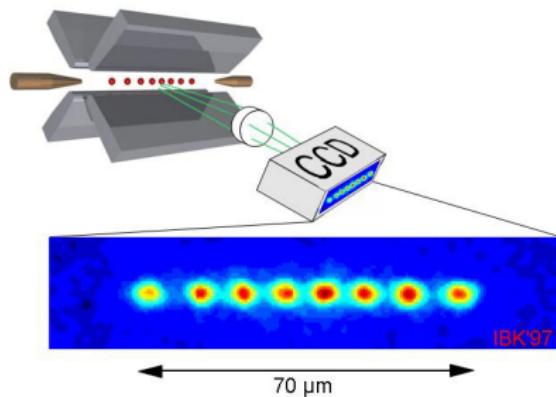
- Two qubits can be in a state

$$|q_1 q_2\rangle = \alpha_{00}|0\rangle \otimes |0\rangle + \alpha_{01}|0\rangle \otimes |1\rangle + \alpha_{10}|1\rangle \otimes |0\rangle + \alpha_{11}|1\rangle \otimes |1\rangle.$$

- $N$  qubits can be in a state that is the superposition of  $2^N$  basis states → **exponential scaling**.

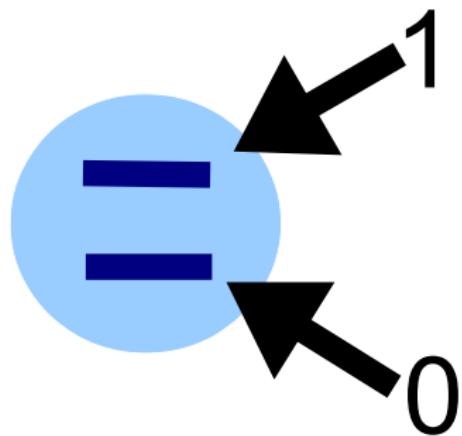
# Trapped cold ions

- Due to the **technological development**, it became possible to manipulate small number of particles, and accessing the particles individually.
- Examples: trapped cold ions



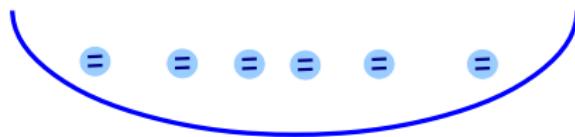
# Trapped cold ions II: Qubits

- Ion as a qubit

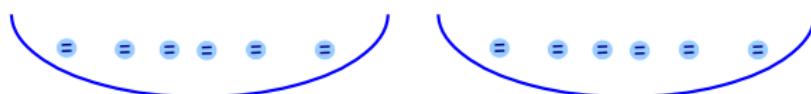


# Trapped cold ions III

- Two-state ions trapped in an electromagnetic field



- Coulomb-repulsion keeps them apart from each other.
- Phonon bus:** the internal states can be coupled



Oscillates

Does not oscillate

# Trapped cold ions IV

- Q: How can the internal states of the ions interact?
- (The Coulomb interaction is not sensitive to the internal state of the neighbor.)
- A: Through the phonon bus.  
[ J. I. Cirac and P. Zoller, Phys. Rev. Lett. 1995 ]

## Trapped cold ions V

- Phonon bus=a bosonic mode $\approx$  extra qubit.
- It can be even in a **superposition state**

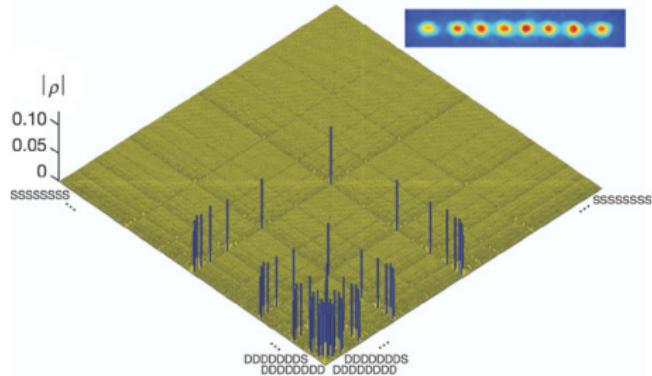
$$\frac{1}{\sqrt{2}}(|\text{Oscillates}\rangle + |\text{Does not oscillate}\rangle).$$

- The Hamiltonian for a single ions is (using RWA)

$$H \propto |0\rangle\langle 1|a + |1\rangle\langle 0|a^\dagger.$$

# Trapped cold ions VI

- Quantum tomography of an eight ion quantum state giving the density matrix:



- The state is the state that they wanted to create:

$$|W\rangle = \frac{1}{\sqrt{6}} (|10000000\rangle + |01000000\rangle + \dots + |00000001\rangle).$$

# Trapped cold ions VII

- Greenberger-Horn-Zeilinger (GHZ) state

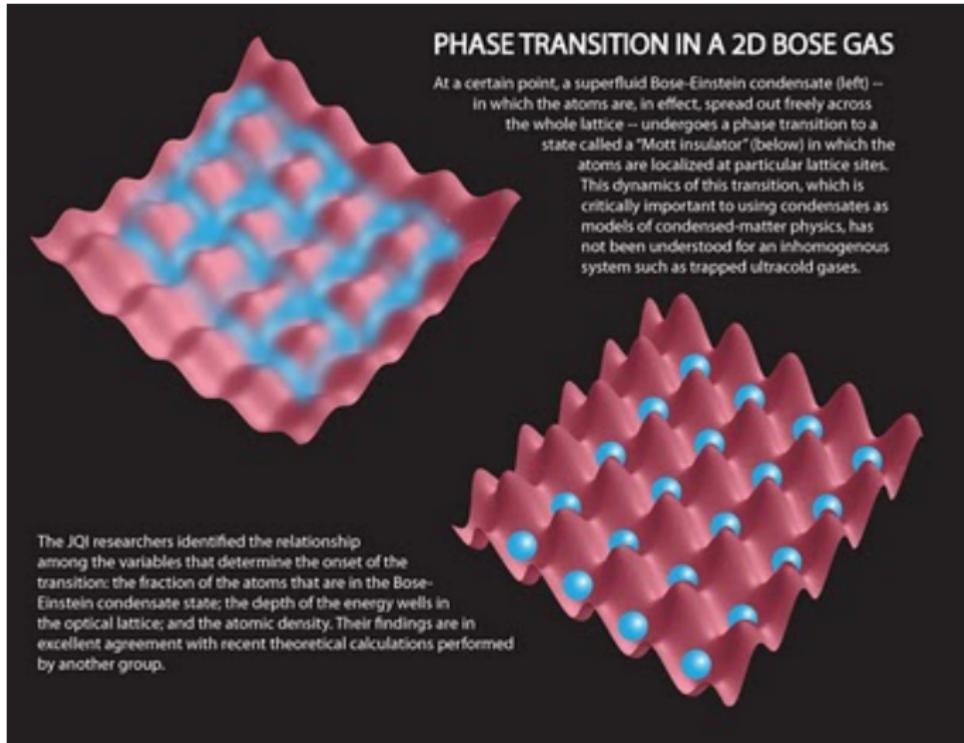
$$|GHZ\rangle = \frac{1}{\sqrt{2}} (|00\dots 00\rangle + |11\dots 11\rangle)$$

- In another context, Schrödinger's cat state.
- Some experiments:
  - 3 particles, Nature 2001.(NIST, Boulder, Colorado)
  - 14 particles, Phys. Rev. Lett 2013. (Innsbruck, Austria)

# Questions

- Experiments are noisy.
- How well quantum systems can be realized in an experiment.
- Is there a fundamental limit that does not allow certain, otherwise not physical quantum states? (i.e., Schrödinger cats)
- Is universal quantum computing possible?

# Optical lattices of cold atoms



Superfluid-Mott insulator phase transition, MPQ, Munich.  
[ Greiner, Mandel, Esslinger, Hänsch & Bloch, Nature 2002 ]

# Optical lattices of cold atoms II

- Hamiltonian: Bose-Hubbard model for two-state atoms:

$$\begin{aligned} H = & J_a \sum_k a_k a_{k+1}^\dagger + a_k^\dagger a_{k+1} \\ & + J_b \sum_k b_k b_{k+1}^\dagger + b_k^\dagger b_{k+1} \\ & + \sum_k U_a n_{a,k} (n_{a,k} - 1) \\ & + U_b n_{b,k} (n_{b,k} - 1) + U_{ab} n_{a,k} n_{b,k}. \end{aligned}$$

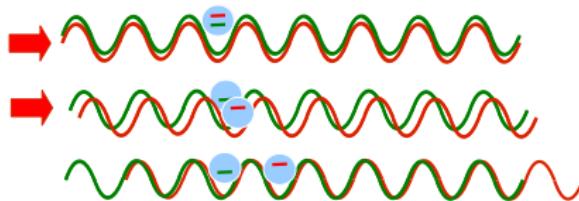
- Tunneling between sites for species  $a$  and  $b$ , self-interaction for species  $a$  and  $b$ , and interaction between the two species.

# Optical lattices of cold atoms III

- Two species, two potentials



- Atoms in the two basis states can be trapped by different potentials



- An atom can be delocalized by several lattice sites. MPQ, Munich, 2003.

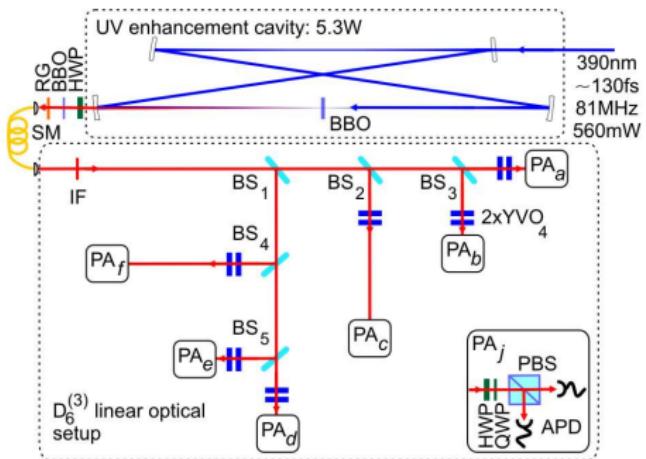
# Optical lattices of cold atoms IV

- They could realize an Ising spin chain Hamiltonian with this technique. MPQ, Munich, 2003.

# Photons

- A photon can have a horizontal and a vertical polarization.
- H/V can take the role of 0 and 1.
- Problem: photons do not interact with each other.

# Photons II

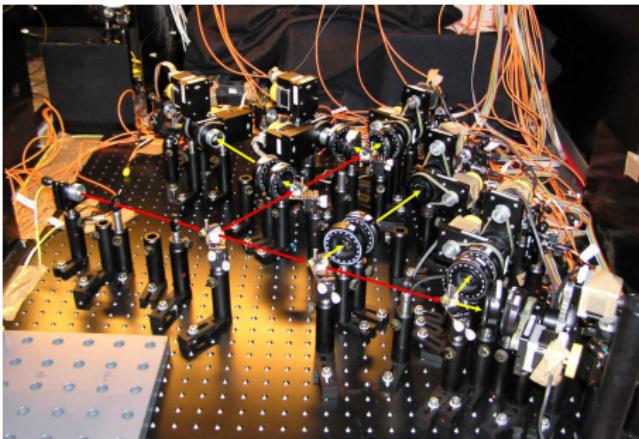
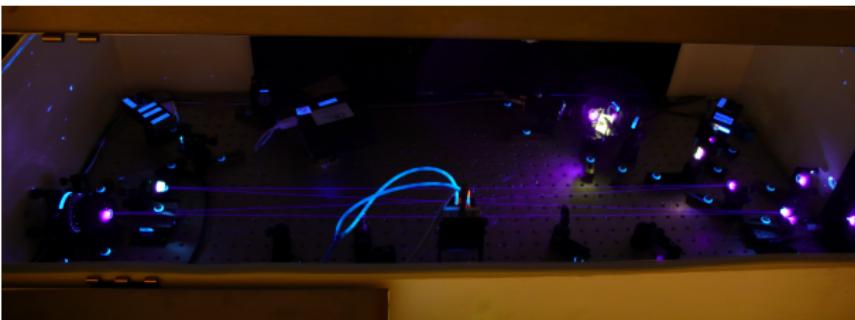


MPQ, Munich. Experiments with 6 photons.

[ W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, G. Tóth, and H. Weinfurter, Phys. Rev. Lett. 2009. ]

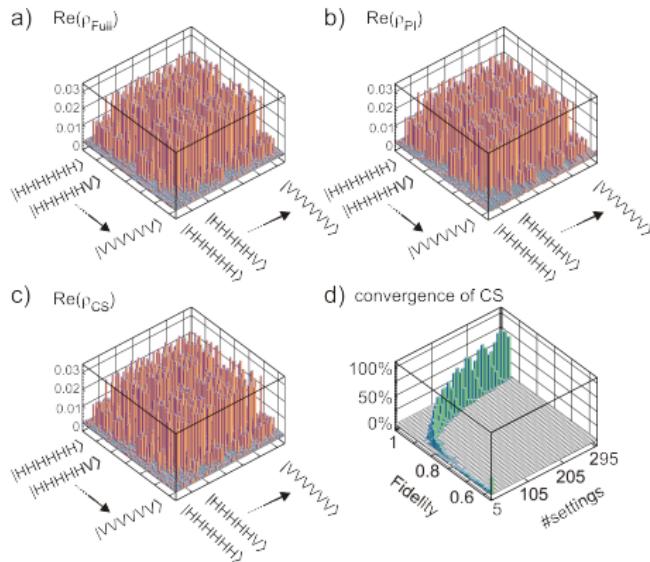
$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}} (|111000\rangle + |110100\rangle + \dots + |000111\rangle).$$

# Photons III



# Photons IV

## 6-qubit Quantum state tomography



[ C. Schwemmer, G. Tóth, A. Niggebaum, T. Moroder, D. Gross, O. Gühne, and H. Weinfurter, Efficient Tomographic Analysis of a Six Photon State, arxiv:1401.7526. ]

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# Theory of quantum entanglement

- Full tomography is not possible for large systems. What can we still say about the state? We can still say entangled/not entangled.
- Pure states
  - A pure product state is **separable**. All states that are not product states are **entangled**.
- Mixed states
  - A quantum state is called **separable** if it can be written as a convex sum of product states as [Werner, 1989]

$$\varrho = \sum_k p_k \varrho_1^{(k)} \otimes \varrho_2^{(k)},$$

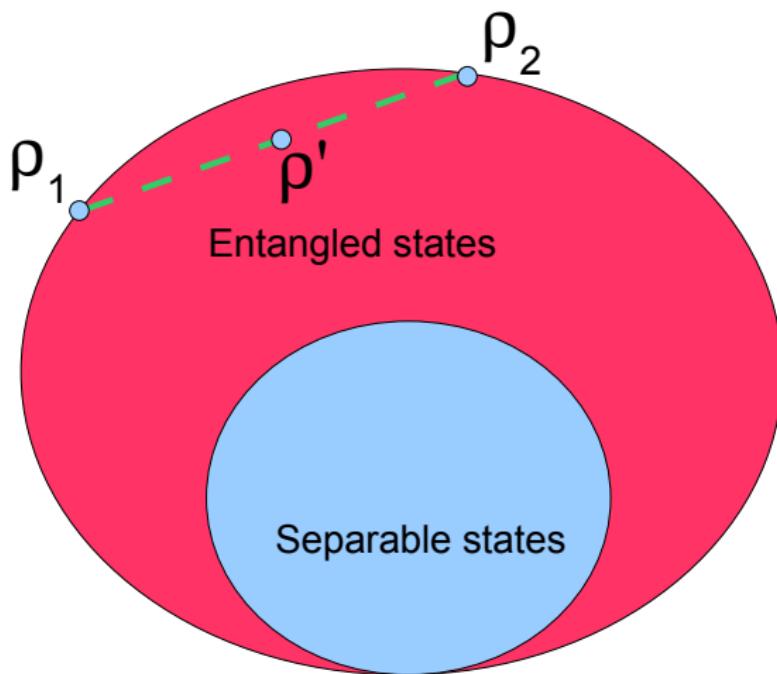
where  $p_k$  form a probability distribution ( $p_k > 0$ ,  $\sum_k p_k = 1$ ), and  $\varrho_n^{(k)}$  are single-qudit density matrices. A state that is not separable is called **entangled**.

# Theory of quantum entanglement II

- Entangled states are more useful than separable ones
  - in certain quantum information processing tasks.
  - in certain metrological tasks.
- It is difficult to decide whether a quantum state is entangled or not.
- For example, Bell inequalities can be used to detect entangled states.

# Theory of quantum entanglement III

- Separable states form a convex set.

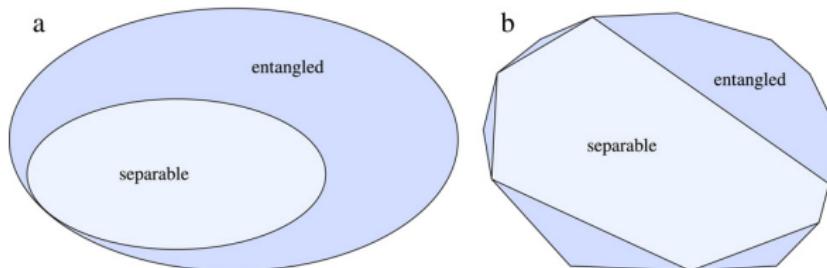


# Theory of quantum entanglement IV

- A more accurate picture (Guhne, Toth, Phys. Rep. 2009):

6

O. Gühne, G. Tóth / Physics Reports 474 (2009) 1–75



**Fig. 1.** (a) Schematic picture of the set of all states as a convex set and the set of separable states as a convex subset. (b) Different schematic picture of the same sets. Here, it is stressed that the extremal points of the separable states (the pure product states), are also extremal points of the set of all states, hence they are located on the border of the total set.

The state is called *separable*, if there are convex weights  $p_i$  and product states  $\varrho_i^A \otimes \varrho_i^B$  such that

$$\varrho = \sum_i p_i \varrho_i^A \otimes \varrho_i^B \quad (6)$$

holds. Otherwise the state is called *entangled*.

Physically, this definition discriminates between three scenarios. First, a product state is an uncorrelated state, where Alice and Bob own each a separate state. For non-product states there are two different kinds of correlation. Separable states are classically correlated. This means that for the production of a separable state only local operations and classical communication (LOCC) are necessary. Alice and Bob can, by classical communication, share a random number generator that produces the outcomes  $i$  with probabilities  $p_i$ . For each of the outcomes, they can agree to produce the state  $\varrho_i^A \otimes \varrho_i^B$  locally. By this procedure they produce the state  $\varrho = \sum_i p_i \varrho_i^A \otimes \varrho_i^B$ . This procedure is not specific for quantum theory, which justifies the notion of *classical* correlations. Otherwise, if a state is entangled, the correlations cannot originate from the classical procedure described above. In this sense entangled states are a typical feature of quantum mechanics.

# Theory of quantum entanglement V

- Concrete example (Verstraete, 2001):

$$\min_t (1-t)\rho^{PT} + \frac{t}{4}I_4 \geq 0 \quad (8)$$

This problem is readily solved, and the solution is

$$t = \frac{|d_{\min}|}{|d_{\min}| + \frac{1}{4}} \quad (9)$$

where  $d_{\min}$  is the minimal negative eigenvalue of  $\rho^{PT}$ . The minimal  $t$  is therefore only a function of the negative eigenvalues. A geometrical implication of this fact is that all surfaces of constant  $d_{\min}$  are similar to the boundary of separable and entangled states: the set of all states with constant  $d_{\min}$  can be generated by extrapolating all lines from the identity to the boundary of separable states such that the distance of the extrapolated state to the identity is a constant factor ( $> 1$ ) of the distance of the separable state to the identity.

Let us now move to the case of two qubits. In this case  $\rho^{PT}$  has at most one negative eigenvalue [9]. Numerical investigations indicate that in a vast majority of the states the optimal rank of  $E^2$  is equal to three, and if the rank is equal to two it implies that  $\rho_s$  has a negative eigenvalue. For the states for which  $E^2$  is rank 3, it follows that their distance to the set of partially transposed states is given by

$$\|\rho - \rho_s\| = \frac{2}{\sqrt{3}}|d_{\min}| \quad (10)$$

where  $d_{\min}$  is the negative eigenvalue of  $\rho^{PT}$ . Surfaces of two-qubit states with constant negativity, defined as  $N = 2|d_{\min}|$ , have therefore two distinct properties: they are all similar to each other and the Hilbert-Schmidt distance between them is almost everywhere constant.

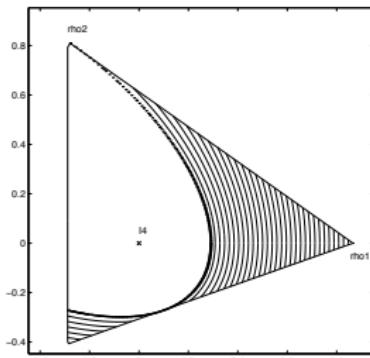


FIG. 2. Intersection of the convex set of all states including states (11) and the maximally mixed state. The contours represent surfaces of constant negativity, the starred line is the boundary between separable and entangled states.

The plane is plotted in figure (2) and the boundary of all (rank-deficient) states is given by the solid envelope. The starred line is the boundary between the convex set of separable states and the convex set of all states. The surfaces of constant negativity are indeed all similar to this boundary. The fact that the distance between these surfaces is not constant throughout the picture indicates that the closest separable states lie in other planes. Note that the Werner states lie along the line between the maximally mixed state and the maximally entangled state  $\rho_1$ .

# Theory of quantum entanglement VI: Example

PRL 112, 155304 (2014)

PHYSICAL REVIEW LETTERS

week ending  
18 APRIL 2014



## Detecting Multiparticle Entanglement of Dicke States

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Recent experiments demonstrate the production of many thousands of neutral atoms entangled in their spin degrees of freedom. We present a criterion for estimating the amount of entanglement based on a measurement of the global spin. It outperforms previous criteria and applies to a wider class of entangled states, including Dicke states. Experimentally, we produce a Dicke-like state using spin dynamics in a Bose-Einstein condensate. Our criterion proves that it contains at least genuine 28-particle entanglement. We infer a generalized squeezing parameter of  $-11.4(5)$  dB.

DOI: 10.1103/PhysRevLett.112.155304

PACS numbers: 67.85.-d, 03.67.Bg, 03.67.Mn, 03.75.Mn

Entanglement, one of the most intriguing features of quantum mechanics, is nowadays a key ingredient for many applications in quantum information science [1,2], quantum simulation [3,4], and quantum-enhanced metrology [5]. Entangled states with a large number of particles cannot be characterized via full state tomography [6], which is routinely used in the case of photons [7,8], trapped ions [9], or superconducting circuits [10,11]. A reconstruction of the full density matrix is hindered and finally prevented by the exponential increase of the required number of measurements. Furthermore, it is technically impossible to address all individual particles or even fundamentally forbidden if the particles occupy the same quantum state. Therefore, the entanglement of many-particle states is best characterized by measuring the expectation values and variances of the components of the collective spin  $\mathbf{J} = (J_x, J_y, J_z)^T = \sum_i \mathbf{s}_i$ , the sum of all individual spins  $\mathbf{s}_i$  in the ensemble.

In particular, the spin-squeezing parameter  $\xi^2 = N(\Delta J_z)^2 / (\langle J_x \rangle^2 + \langle J_y \rangle^2)$  defines the class of spin-squeezed states for  $\xi^2 < 1$ . This inequality can be used to verify the presence of entanglement, since all spin-squeezed states are entangled [12]. Large clouds of entangled neutral atoms are typically prepared in such spin-squeezed states, as shown in thermal gas cells [13].

quantified by means of the so-called entanglement depth, defined as the number of particles in the largest nonseparable subset [see Fig. 1(a)]. There have been numerous experiments detecting multiparticle entanglement involving up to 14 qubits in systems, where the particles can be addressed individually [9,20–24]. Large ensembles of neutral atoms

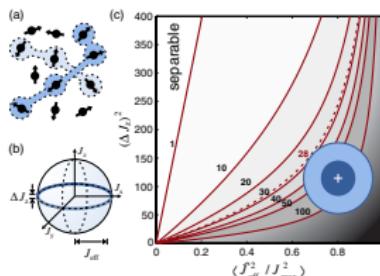


FIG. 1 (color online). Measurement of the entanglement depth for a total number of 8000 atoms. (a) The entanglement depth is given by the number of atoms in the largest nonseparable subset

## Theory of quantum entanglement VII: Example

- We define

$$F_j(X) := \frac{1}{J} \min_{\substack{\langle j_x \rangle = X \\ J}} (\Delta j_z)^2.$$

- States for states with at most  $k$ -particle entanglement, we have

$$(\Delta j_z)^2 \geq \frac{N}{2} F_{\frac{k}{2}} \left( \frac{\sqrt{\langle j_x^2 + j_y^2 \rangle - \frac{N}{2} \left( \frac{k}{2} + 1 \right)}}{\frac{N}{2}} \right)$$

- Any state that violates this has at least  $(k + 1)$ -particle entanglement.

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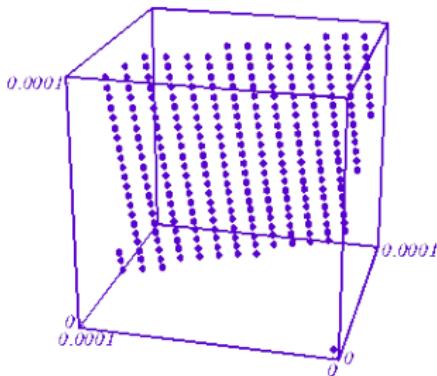
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# True randomness

- Pseudo-random numbers have unexpected correlations. Example from Karl Entacher:



- Solution: measure in the  $|0\rangle/|1\rangle$  basis the state

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

- Commercially available random number generators based on this idea.

## No-cloning theorem

We are looking for a mechanism that clones quantum states

$$U|\Psi\rangle \otimes |0\rangle = |\Psi\rangle \otimes |\Psi\rangle,$$

where  $U$  is a unitary dynamics.

Let us see why this is not possible. For the two basis states we have

$$U|0\rangle \otimes |0\rangle = |0\rangle \otimes |0\rangle,$$

$$U|1\rangle \otimes |0\rangle = |1\rangle \otimes |1\rangle.$$

Then, due to the linearity of quantum mechanics

$$U\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle).$$

However, we would have needed

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Thus, a quantum state cannot be cloned.

## Coding in the 0/1 or on the (0+1)/(0-1) basis

- Let us code a classical bit  $b \in \{0, 1\}$  in a qubit. We can use the 0/1 basis as before:

$$|q\rangle = (1 - b)|0\rangle + b|1\rangle.$$

- We can also use another basis, the 0 + 1/0 - 1 basis:

$$|q'\rangle = (1 - b)\frac{|0\rangle + |1\rangle}{\sqrt{2}} + b\frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

- If we do not know the basis, we cannot recover the bit  $b$ .

## Coding in the 0/1 or on the (0+1)/(0-1) basis II

- Let us assume we used the 0/1 to code the bit

$$|q\rangle = (1 - b)|0\rangle + b|1\rangle.$$

- Then, a *single* measurement of

$$M = 0 \cdot |0\rangle\langle 0| + 1 \cdot |1\rangle\langle 1|$$

will give the bit exactly.

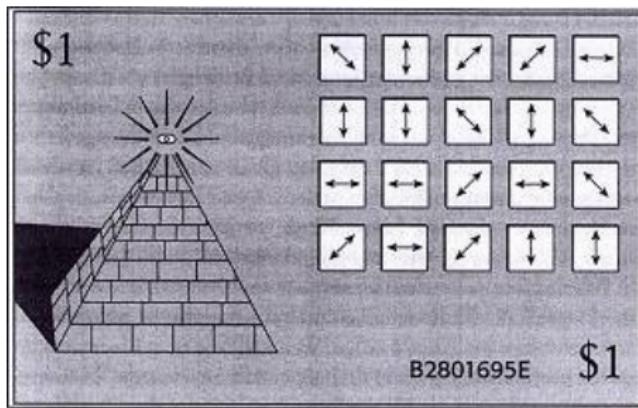
- If the bit was encoded in the 0/1 basis, then we get with 50% probability 0, 50% probability 1, independently from  $b$ .

## Coding in the 0/1 or on the (0+1)/(0-1) basis III

- Note: if the quantum state could be copied, we could just copy the state many times. From many copies, we could guess, which basis was used.
- Thus, it is very important that the quantum states cannot be copied.

# Quantum money

- S. Wiesner 1970, a graduate student at Columbia University, published in 1983.



- Every banknote has a code, a series of bits.
- The bits are encoded either in the 0/1 basis or in the 0+1/0-1 basis.
- The bank has the list of bases.
- The banknote cannot be copied.
- Its validity can be verified by the bank.

# Quantum cryptography (BB84)

- Alice sends the secret message in qubits, randomly choosing the bases: 0/1 or (0+1)/(0-1).
- Bob receives the qubits and measures them in randomly chosen bases.
- Alice and Bob decides, using a public classical channel, for which qubits they used the same bases.

Valores de bit enviados	0	1	1	0	1	0	0	1
Fotones enviados	↔	↔	↓	↑	↓	↑	↓	↔
Bases elegidas en recepción	⊗	+	⊗	⊗	⊗	+	+	⊗
Fotones detectados	↓	↔	↑	↑	↓	↑	↑	↓
Valores de bit recibidos	1	1	0	0	1	0	0	1
Clave final	-	1	-	0	1	-	0	-

## Quantum cryptography (BB84) II

- In 2004, the world's first bank transfer using QKD was carried in Vienna, Austria. (Zeilinger group, Vienna)
- Quantum encryption technology provided by the Swiss company Id Quantique was used in the Swiss canton (state) of Geneva to transmit ballot results to the capital in the national election occurring on 21 October 2007. (Gisin group, Geneva)
- In 2013, Battelle Memorial Institute installed a QKD system built by ID Quantique between their main campus in Columbus, Ohio and their manufacturing facility in nearby Dublin.

(Wikipedia)

# Quantum teleportation

- A quantum state cannot be copied.
- But, it can be transferred from one particle to another one such that the state of the original particle is destroyed.

# Quantum teleportation II

- Initial state:

$$|\Psi\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \otimes (\alpha|0\rangle_C + \beta|1\rangle_C).$$

Alice and Bob want to teleport. Alice has two particles: A and C. She wants to teleport the C particle to the B particle of Bob. Particle A is helping the teleportation.

- Alice makes a measurement on particles A and C in the Bell basis. The Bell basis consists of the states:

$$|\Phi^\pm\rangle_{AC} = \frac{1}{\sqrt{2}}(|00\rangle_{AC} \pm |11\rangle_{AC})$$

and

$$|\Psi^\pm\rangle_{AC} = \frac{1}{\sqrt{2}}(|01\rangle_{AC} \pm |10\rangle_{AC}).$$

# Quantum teleportation III

- To see how this works, one can rewrite

$$\begin{aligned} |\Psi\rangle_{AB} \otimes |\Psi\rangle_C \\ = \frac{1}{2} [ & |\Phi^+\rangle_{AC} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + |\Phi^-\rangle_{AC} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) \\ & + |\Psi^+\rangle_{AC} \otimes (\beta|0\rangle_B + \alpha|1\rangle_B) + |\Psi^-\rangle_{AC} \otimes (\beta|0\rangle_B - \alpha|1\rangle_B ) ]. \end{aligned}$$

- Hence, measurement of AC in the Bell basis results in one of the four possibilities above for particle B. Knowing the result of the measurement, we can obtain

$$(\alpha|0\rangle_B + \beta|1\rangle_B).$$

Thus, we successfully teleported the state of particle C to particle B.

- Note that this does not make possible faster than light communication, since the result of the Bell measurement has to be sent classically.

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# Quantum Metrology

- Let us take a GHZ state.
- Let us employ the dynamics

$$U = e^{-iJ_z\theta}.$$

- Basic task of metrology: we want to estimate  $\theta$  based on measuring the state after the evolution.
- The dynamics is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|000..000\rangle + |111..111\rangle e^{-iN\theta}).$$

# Quantum Metrology II

- Let us measure

$$M = \sigma_x^{\otimes N}.$$

- With this,

$$\langle M \rangle = \cos(N\theta), \quad (\Delta M)^2 = \sin^2(N\theta).$$

- Precision is

$$(\Delta\theta)^{-2}|_{\theta=0} = \frac{(\Delta M)^2}{|\partial_\theta \langle M \rangle|^2} = N^2.$$

- Tested for 3 qubits: Nature 2001.(NIST, Boulder, Colorado).
- One can show that for separable states, for any measurements,

$$(\Delta\theta)^{-2}|_{\theta=0} \leq F_Q[\rho, J_z] \leq N.$$

[Pezzé, Smerzi, Phys. Rev. Lett. 2007]

# Quantum Metrology III

- For states with  $k$ -particle entanglement:

$$(\Delta\theta)^{-2}|_{\theta=0} \leq F_Q[\varrho, J_z] \leq \sim kN.$$

[Tóth, 2012, Hyllus et al., 2012.]

- Thus, full entanglement is needed for maximal precision.
- One can show that states that lead to a larger precision, are also less stable under the same dynamics. Hence, the very unstable states must be entangled.

[del Campo, Egusquiza, Plenio, Huelga, Phys. Rev. Lett. 2013;  
Escher et al, Phys. Rev. Lett. 2013.]

# Outline

1

## Introduction

- Quantum information science

2

## Quantum information science

- Quantum mechanics
- Quantum optics
- Quantum engineering
- Theory of quantum entanglement
- Quantum cryptography
- Quantum metrology
- Quantum computing

# Computing in “parallel”

- Quantum mechanics is linear

$$U|\Psi_1\rangle = |\Phi_1\rangle,$$

$$U|\Psi_2\rangle = |\Phi_2\rangle,$$

hence

$$U(|\Psi_1\rangle + |\Psi_2\rangle) = |\Phi_1\rangle + |\Phi_2\rangle.$$

- Not so simple, since we cannot separate the results.

# Factoring of primes

- Usual encryption is based on the fact that it is very difficult to find prime factors for a number.
- Quantum computers can efficiently factor primes: Shor's algorithm.
- To factor an integer  $N$ , the execution time is
  - $O((\log N)^3)$  for a quantum computer,
  - $O(e^{1.9(\log N)^{1/3}(\log \log N)^{2/3}})$  for the best classical algorithm.
- Thus, for large  $N$  the quantum algorithm must be faster.

## Search in a database

- Quantum computers can search efficiently in a database: Grover's algorithm.
- Task: find  $x$  for which

$$f(x) = 1,$$

where  $x$  is an  $N$ -bit non-negative integer.

(Assume that  $f(x) = -1$  for all other cases.)

- To factor an integer  $N$ , the execution time is
  - $O(N^{\frac{1}{2}})$  for a quantum computer,
  - $\frac{N}{2}$  classically.

Thus, again, for large  $N$  the quantum algorithm must be faster.

# Quantum simulation

- If quantum computing with thousands of qubits is not possible, we can still be interested in specific problems.
- Spin chains of 30-40 particles: already, we cannot simulate them on a classical computer.

# Conclusions

- We discussed several aspects of quantum information and quantum computation. For the transparencies, see

[www.gtoth.eu](http://www.gtoth.eu)

THANK YOU FOR YOUR ATTENTION!



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