

My major research topics: Spin squeezing et al.

G. Tóth^{1,2,3}

¹Theoretical Physics, University of the Basque Country UPV/EHU, Bilbao, Spain

²IKERBASQUE, Basque Foundation for Science, Bilbao, Spain

³Wigner Research Centre for Physics, Budapest, Hungary

(For collaborators, please see the transparencies)

ICE-2012, Madrid
19 September 2012





Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Why spin squeezing inequalities for $j > \frac{1}{2}$ is important?

- Many experiments are aiming to create entangled states with many atoms.
- Only collective quantities can be measured.
- Most experiments use atoms with $j > \frac{1}{2}$.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Many-particle systems for $j=1/2$

- For spin- $\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2} \sum_{k=1}^N \sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ a Pauli spin matrices.

- We can also measure the

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2$$

variances.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

The standard spin-squeezing criterion

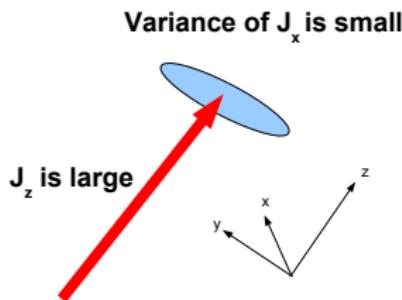
- The spin squeezing criteria for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} \geq \frac{1}{N}.$$

If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature **409**, 63 (2001).]

- States violating it are like this:



Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Generalized spin squeezing criteria for $j = \frac{1}{2}$

- Let us assume that for a system we know only

$$\vec{J} := (\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle),$$
$$\vec{K} := (\langle J_x^2 \rangle, \langle J_y^2 \rangle, \langle J_z^2 \rangle).$$

- Then any state violating the following inequalities is entangled

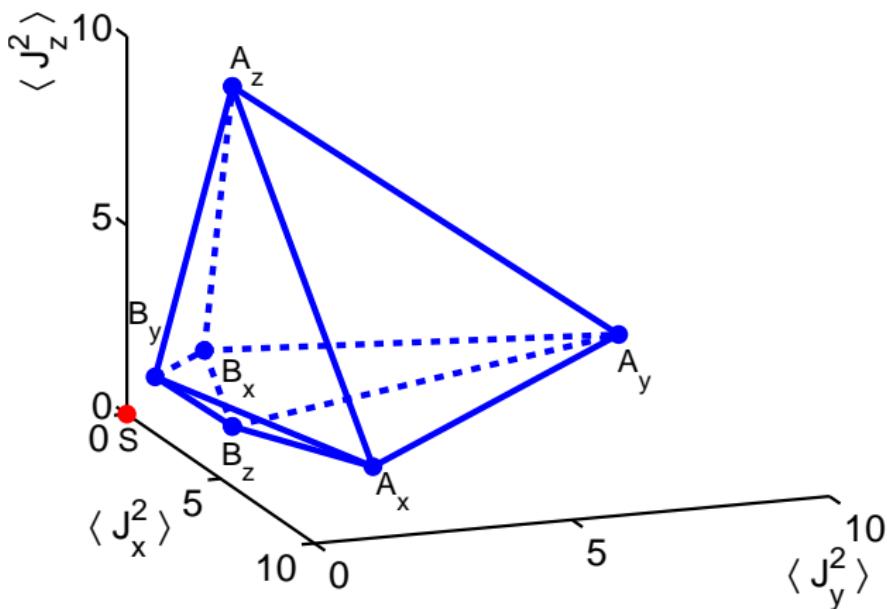
$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq \frac{N(N+2)}{4},$$
$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq \frac{N}{2},$$
$$\langle J_k^2 \rangle + \langle J_l^2 \rangle \leq (N-1)(\Delta J_m)^2 + \frac{N}{2},$$
$$(N-1) [(\Delta J_k)^2 + (\Delta J_l)^2] \geq \langle J_m^2 \rangle + \frac{N(N-2)}{4},$$

where k, l, m take all the possible permutations of x, y, z .

[GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL 99, 250405 (2007);
quant-ph/0702219.]

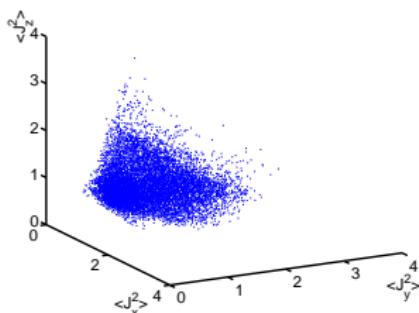
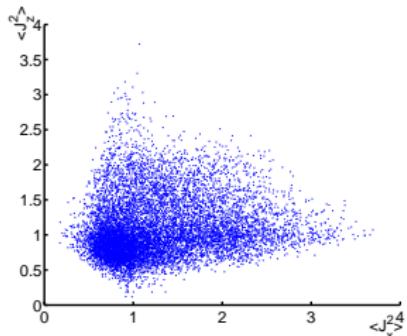
Generalized spin squeezing criteria for $j = \frac{1}{2}$

- The previous inequalities, *for fixed* $\langle J_{x/y/z} \rangle$, describe a polytope in the $\langle J_{x/y/z}^2 \rangle$ space. The polytope has six extreme points: $A_{x/y/z}$ and $B_{x/y/z}$.
- For $\langle \vec{J} \rangle = 0$ and $N = 6$ the polytope is the following:



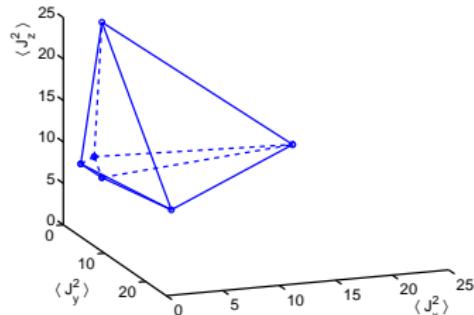
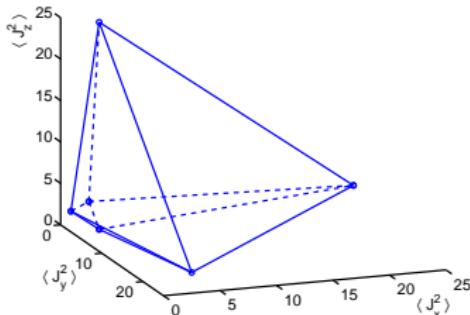
Completeness

- Random separable states:

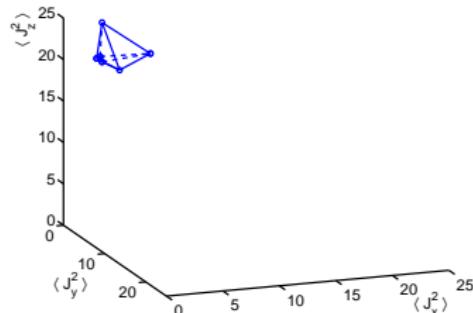


- The completeness can be proved for large N .

- The polytope for $N = 10$ and
 $J = (0, 0, 0)$, $J = (0, 0, 2.5)$,



and $J = (0, 0, 4.5)$.



Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Basic ideas for the $j > \frac{1}{2}$ case

- Particles with $d > 2$ internal states.
- a_k for $k = 1, 2, \dots, M$ denote single-particle operators with the property $\text{Tr}(a_k a_l) = C\delta_{kl}$, where C is a constant.
- We need the upper bound K for the inequality $\sum_{k=1}^M \langle a_k^{(n)} \rangle^2 \leq K$.
- The N -qudit collective operators used in our criteria will be denoted by

$$A_k = \sum_n a_k^{(n)}.$$

“Modified” quantities for $j > \frac{1}{2}$

- For the $j = \frac{1}{2}$ case, the SSIs were developed based on the first and second moments and variances of the such collective operators.
- For the $j > \frac{1}{2}$ case, we define the modified second moment

$$\langle \tilde{A}_k^2 \rangle := \langle A_k^2 \rangle - \langle \sum_n (a_k^{(n)})^2 \rangle = \sum_{m \neq n} \langle a_k^{(n)} a_k^{(m)} \rangle$$

and the modified variance

$$(\tilde{\Delta} A_k)^2 := (\Delta A_k)^2 - \langle \sum_n (a_k^{(n)})^2 \rangle.$$

- These are essential to get tight equations for $j > \frac{1}{2}$.

Basic equation

- For separable states, i.e., for states that can be written as a mixture of product states,

$$(N - 1) \sum_{k \in I} (\tilde{\Delta} A_k)^2 - \sum_{k \notin I} \langle (\tilde{A}_k)^2 \rangle \geq -N(N - 1)K$$

holds, where each index set $I \subseteq \{1, 2, \dots, M\}$ defines one of the 2^M inequalities.

- Note that $I = \emptyset$ and $I = \{1, 2, \dots, M\}$ are among the possibilities.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

The inequalities for $j > \frac{1}{2}$ with the angular momentum coordinates

- Application 1:

$$a_k = \{j_x, j_y, j_z\}.$$

- For spin- j particles for $j > \frac{1}{2}$, we can measure the collective angular momentum operators:

$$J_l := \sum_{k=1}^N j_l^{(k)},$$

where $l = x, y, z$ and $j_l^{(k)}$ are the angular momentum coordinates [i.e., SU(2) generators].

- We can also measure the

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2$$

variances.

The inequalities for $j > \frac{1}{2}$ with the angular momentum coordinates II

- For fully separable states of spin- j particles, all the following inequalities are fulfilled

$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq Nj(Nj + 1),$$

$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj,$$

$$\langle \tilde{J}_k^2 \rangle + \langle \tilde{J}_l^2 \rangle - N(N-1)j^2 \leq (N-1)(\tilde{\Delta}J_m)^2,$$

$$(N-1)[(\tilde{\Delta}J_k)^2 + (\tilde{\Delta}J_l)^2] \geq \langle \tilde{J}_m^2 \rangle - N(N-1)j^2,$$

where k, l, m take all possible permutations of x, y, z .

- Violation of any of the inequalities implies entanglement.

Completeness

- In the large N limit, the inequalities with the angular momentum are **complete**.
- That is, it is not possible to come up with a new entanglement conditions with based on $\langle J_k \rangle$ and $\langle \tilde{J}_k^2 \rangle$ that detect states not detected by these inequalities.

Mapping qubit inequalities to qudits

- An entanglement condition for qubits can be transformed to a criterion for a system of N spin- j particles by the substitution

$$\langle J_I \rangle \rightarrow \frac{1}{2j} \langle J_I \rangle, \quad \langle \tilde{J}_I^2 \rangle \rightarrow \frac{1}{4j^2} (\langle \tilde{J}_I^2 \rangle).$$

The usual spin squeezing inequality for $j > \frac{1}{2}$

- The standard spin-squeezing inequality becomes

$$\frac{(\Delta J_x)^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} + \frac{\sum_n (j^2 - \langle (j_x^{(n)})^2 \rangle)}{\langle J_y \rangle^2 + \langle J_z \rangle^2} \geq \frac{1}{N}.$$

This inequality is violated only if there is entanglement between the spin- j particles.

- With the original inequality, there is “spin squeezing” without entanglement between the particles.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- **SU(d) generators**
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

The inequalities for $j > \frac{1}{2}$ with the G_k 's

- Application 2:

$$a_k = SU(d) \text{ generators.}$$

- For spin- j particles for $j > 1/2$, we can measure the collective operators:

$$G_l := \sum_{k=1}^N g_l^{(k)},$$

where $l = 1, 2, \dots, d^2 - 1$ and $g_l^{(k)}$ are the $SU(d)$ generators.

- We can also measure the

$$(\Delta G_l)^2 := \langle G_l^2 \rangle - \langle G_l \rangle^2$$

variances.

The inequalities for $j > \frac{1}{2}$ with the G_k 's

- The SSIs for G_k have the general form

$$(N-1) \sum_{k \in I} (\tilde{\Delta} G_k)^2 - \sum_{k \notin I} \langle (\tilde{G}_k)^2 \rangle \geq -2N(N-1) \frac{(d-1)}{d}.$$

- For the $d = 3$ case, the SU(d) generators can be the eight Gell-Mann matrices.
- I is a subset of indices between 1 and M . We have 2^M equations!

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

One of the generalized spin squeezing criteria

A condition for separability is

$$\sum_k (\Delta G_k)^2 \geq 2N(d - 1).$$

- Three-body entanglement can also be detected based on a higher violation of the inequality.

[G. Vitagliano, P. Hyllus, I.L. Egusquiza, and G. Tóth,
Optimal spin squeezing inequalities for arbitrary spin,
arXiv:1104.3147.]

Advantages of criteria for $j > \frac{1}{2}$

- Most atoms have $j > \frac{1}{2}$. No need to create spin-1/2 subsystems artificially
- Manipulation is possible with magnetic fields rather than with lasers.
- New experiments can be proposed.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Singlet creation with cold atomic clouds



Generation of macroscopic singlet states in atomic ensembles

Géza Tóth^{1,2,3,5} and Morgan W Mitchell⁴

¹ Department of Theoretical Physics, The University of the Basque Country,
PO Box 644, E-48080 Bilbao, Spain

² IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain

³ Research Institute for Solid State Physics and Optics,
Hungarian Academy of Sciences, PO Box 49, H-1525 Budapest, Hungary

⁴ ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park,
E-08860 Castelldefels (Barcelona), Spain

E-mail: toth@alumni.nd.edu

New Journal of Physics **12** (2010) 053007 (12pp)

Received 10 November 2009

Published 6 May 2010

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/12/5/053007

Abstract. We study squeezing of the spin uncertainties by quantum non-demolition (QND) measurement in non-polarized spin ensembles. Unlike the case of polarized ensembles, the QND measurements can be performed with

Singlet creation with cold atomic clouds

3

IOP Institute of Physics Φ DEUTSCHE PHYSIKALISCHE GESELLSCHAFT

robust to decoherence, and produces a many-atom singlet state. Unlike standard spin squeezing, the method creates entanglement even in the limit of very strong interaction, which might be used in experimental implementations with cavities [20, 21], or in any multi-atomic system in which a von Neumann measurement of the collective spin is possible. We demonstrate the validity of the Gaussian approximation for unpolarized spin states. For the lossless case, we confirm our finding with comparison to the exact model.

The paper is organized as follows. In section 2, we present the spin squeezing parameter to detect the entanglement of many-body singlet states and also discuss the properties of the singlets we aim to prepare. In section 3, we describe the squeezing process. First, we consider the lossless case and present a model based on a Gaussian approximation. Later, we include decoherence in the model. For the lossless case, we compare our results to the results of the exact model. In the [appendix](#), we present the details of the calculations for the exact model.

2. Detecting the entanglement of singlet states

In this paper, we will use the generalized spin squeezing parameter

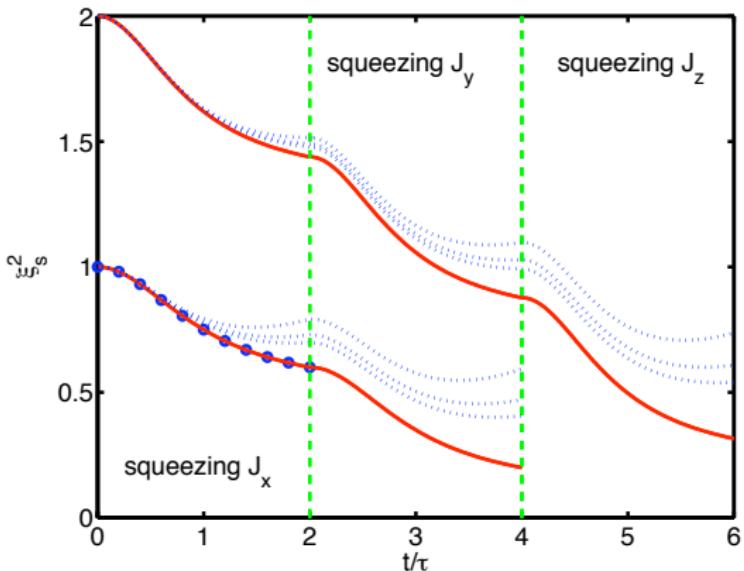
$$\xi_s^2 := \frac{(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2}{J}, \quad (1)$$

where J_l are the components of the collective angular momentum, $(\Delta J_l)^2 = \langle J_l^2 \rangle - \langle J_l \rangle^2$, and for a system of N spin- j particles we define $J := Nj$. It has already been shown in [22]–[26] that any state giving $\xi_s < 1$ is entangled (i.e. not fully separable). For completeness, we present briefly the proof for (1). For pure product states of the form $|\Psi_p\rangle = \otimes_{k=1}^N |\psi_k\rangle$, we have

$$\sum_{l=x,y,z} (\Delta J_l)^2 = \sum_{l=x,y,z} \sum_{k=1}^N (\Delta j_l^{(k)})^2 |_{\psi_k} \geq Nj, \quad (2)$$

where $j_l^{(k)}$ denotes the spin coordinates of particle (k) for $l = x, y$ and z . Here, we used the fact that $\sum_l (\Delta j_l^{(k)})^2 \geq j$. For a mixture of pure product states, i.e. for separable states, (2) remains

Singlet creation with cold atomic clouds



Upper curve: Spin squeezing of the white noise.

3x3 correlation matrices instead of 2x2 ones

For modeling the QND pulse, the atoms are described by the J_l operators, while the light pulse is characterized by the Stokes operators S_l [13, 17]. We choose the initial state to be the completely mixed atomic state, $\varrho_0 := (1/(2j+1))^N \mathbb{1}$, and a fully polarized optical state with $\langle \mathbf{S} \rangle = (S_0, 0, 0)$. The full system is described by the operators

$$R = \left\{ \frac{J_x}{\sqrt{J}}, \frac{J_y}{\sqrt{J}}, \frac{J_z}{\sqrt{J}}, \frac{S_x}{\sqrt{S_0}}, \frac{S_y}{\sqrt{S_0}}, \frac{S_z}{\sqrt{S_0}} \right\} \quad (7)$$

with a covariance matrix

$$\Gamma_{mn} := \frac{1}{2} \langle R_m R_n + R_n R_m \rangle - \langle R_m \rangle \langle R_n \rangle. \quad (8)$$

As shown by simple calculations, for large N the initial state is Gaussian for the R_k operators. That is, symmetric moments with order higher than the second can be obtained from lower-order ones according to the theory of Gaussian distributions, knowing that cumulants with order three and higher are zero [41]. In other words, concerning the moments of R_k , the state is completely characterized by Γ , $\langle \mathbf{S} \rangle$ and $\langle \mathbf{J} \rangle$.

The first step of the QND measurement of J_x is interaction between the atoms and light via the Hamiltonian

$$H = \hbar \Omega J_x S_z. \quad (9)$$

This suggests a characteristic time scale [12]

$$\tau := \frac{1}{\Omega \sqrt{S_0 J}}. \quad (10)$$

The dynamical equations of Γ_{mn} can be obtained from the Heisenberg equation of motion for the operators R_k given as

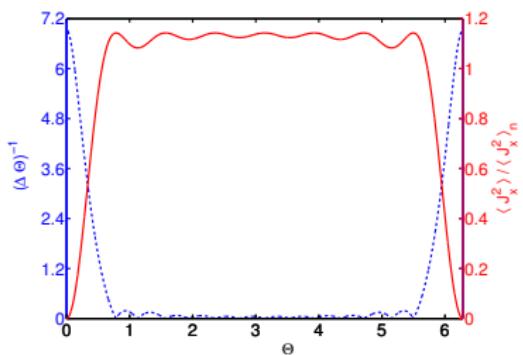
$$R_k^{(\text{out})} = R_k^{(\text{in})} - i\tau [R_k^{(\text{in})}, H], \quad (11)$$

with $\hbar = 1$. For example, the dynamics of R_5 is obtained as

$$R_5^{(\text{out})} = R_5^{(\text{in})} - \frac{\kappa}{\tau} R_5^{(\text{in})} R_5^{(\text{in})}. \quad (12)$$

Differential magnetometry with singlets

- The singlet is invariant under homogenous magnetic fields. It can be used to measure the field gradient with a single cloud.
- I. Uriar-Lanz, P. Hyllus, I. Egusquiza, M.W. Mitchell, G. Tóth, Differential magnetometry with multiparticle singlets, arxiv:1203.3797.



Dynamics of the variance of J_x and the precision for an 8 particle chain.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Permutationally invariant tomography

PRL 105, 250403 (2010)

PHYSICAL REVIEW LETTERS

week ending
17 DECEMBER 2010

Permutationally Invariant Quantum Tomography

G. Tóth,^{1,2,3} W. Wieczorek,^{4,5,*} D. Gross,⁶ R. Krischek,^{4,5} C. Schwemmer,^{4,5} and H. Weinfurter^{4,5}

¹*Department of Theoretical Physics, The University of the Basque Country, P.O. Box 644, E-48080 Bilbao, Spain*
²*IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain*

³*Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary*
⁴*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*

⁵*Fakultät für Physik, Ludwig-Maximilians-Universität, D-80797 München, Germany*

⁶*Institute for Theoretical Physics, Leibniz University Hannover, D-30167 Hannover, Germany*
(Received 4 June 2010; revised manuscript received 30 August 2010; published 16 December 2010)

We present a scalable method for the tomography of large multiqubit quantum registers. It acquires information about the permutationally invariant part of the density operator, which is a good approximation to the true state in many relevant cases. Our method gives the best measurement strategy to minimize the experimental effort as well as the uncertainties of the reconstructed density matrix. We apply our method to the experimental tomography of a photonic four-qubit symmetric Dicke state.

DOI: 10.1103/PhysRevLett.105.250403

PACS numbers: 03.65.Wj, 03.65.Ud, 42.50.Dv

Because of the rapid development of quantum experiments, it is now possible to create highly entangled multiqubit states using photons [1–5], trapped ions [6], and cold atoms [7]. So far, the largest implementations that allow for an individual readout of the particles involve on the order of 10 qubits. This number will soon be overcome, for example, by using several degrees of freedom within each particle to store quantum information [8]. Thus, a new regime will be reached in which a complete state tomography is impossible even from the point of view of the storage place needed on a classical computer. At this point the question arises: Can we still extract useful information about the quantum state created?

In this Letter we propose permutationally invariant (PI) tomography in multiqubit quantum experiments [9]. Concretely, instead of the density matrix ϱ , we propose to determine the PI part of the density matrix defined as

$$\varrho_{\text{PI}} = \frac{1}{N!} \sum \Pi_k \varrho \Pi_k, \quad (1)$$

for both density matrices and are thus obtained exactly from PI tomography [2–4]. Finally, if ϱ_{PI} is entangled, so is the state ϱ of the system, which makes PI tomography a useful and efficient tool for entanglement detection.

Below, we summarize the four main contributions of this Letter. We restrict our attention to the case of N qubits—higher-dimensional systems can be treated similarly.

(1) In most experiments, the qubits can be individually addressed whereas nonlocal quantities cannot be measured directly. The experimental effort is then characterized by the number of local measurement settings needed, where “setting” refers to the choice of one observable per qubit, and repeated von Neumann measurements in the observables’ eigenbases [13]. Here, we compute the minimal number of measurement settings required to recover ϱ_{PI} .

(2) The requirement that the number of settings be minimal does not uniquely specify the tomographic protocol. On the one hand, there are infinitely many possible choices for the local settings that are both minimal and

Permutationally invariant tomography II

- For full state tomography, the number of measurements scales exponentially in N .

Permutationally invariant part of the density matrix:

$$\varrho_{\text{PI}} = \frac{1}{N!} \sum \Pi_k \varrho \Pi_k^\dagger,$$

where Π_k are all the permutations of the qubits.

[G. Tóth, W. Wieczorek, D. Gross, R. Krischek, C. Schwemmer, and H. Weinfurter, Permutationally invariant quantum tomography, Phys. Rev. Lett. 105, 250403 (2010); arxiv:1005.3313.]

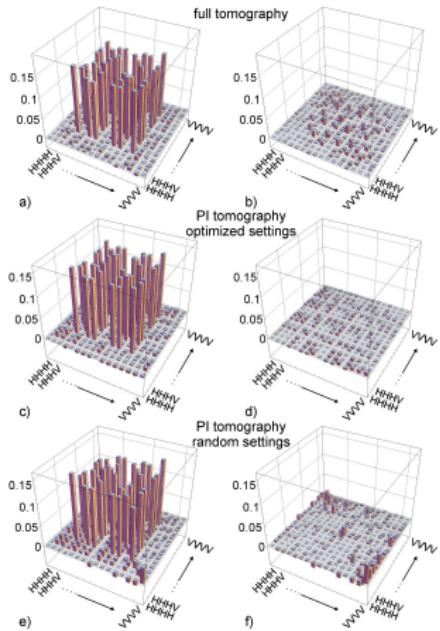
Main results

Features of our method:

- ① Is for spatially separated qubits.
- ② Needs the minimal number of measurement settings.
- ③ Uses the measurements that lead to the smallest uncertainty possible of the elements of ϱ_{PI} .
- ④ Gives an uncertainty for the recovered expectation values and density matrix elements.
- ⑤ If ϱ_{PI} is entangled, so is ϱ . Can be used for entanglement detection.
- ⑥ Fitting of physical states can also be scaleable.

[T. Moroder, P. Hyllus, G. Tóth, C. Schwemmer, A. Niggebaum, S. Gaile, O. Gühne, and H. Weinfurter, Permutationally invariant state reconstruction, New J. Phys, Focus issue on Quantum Tomography, in press; arxiv:1205.4941.]

Experiments: 4 qubits (PRL 2010) and 6 qubits (in preparation)



Experiments in the Weinfurther group, München.

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Quantum metrology

- Quantum Fisher information and multipartite entanglement

G. Tóth, Multipartite entanglement and high precision metrology, Phys. Rev. A 85, 022322 (2012); arxiv:1006.4368. See similar work of P. Hyllus et al.

- Quantum Fisher information as the convex roof of the variance.
Collaboration with Dénes Petz, Rényi Institute for Mathematics.

G. Tóth and D. Petz, Optimal generalized variance and quantum Fisher information, arxiv:1109.2831.

$$F_Q[\varrho, A] = \inf_{p_k, \Psi_k} \sum_k p_k (\Delta A)_{\Psi_k}^2. \quad (3)$$

Outline

1 Motivation

- Why spin squeezing inequalities are important?

2 Spin squeezing criteria for $j = 1/2$

- Collective measurements
- The original criterion
- Generalized criteria for $j = \frac{1}{2}$

3 Spin squeezing inequality for an ensemble of spin- j atoms

- Basic idea for $j > \frac{1}{2}$
- Angular momentum
- SU(d) generators
- Detection of singlets

4 Other topics

- Singlet creation and magnetometry
- PI tomography
- Quantum metrology
- Group

Group

Philipp Hyllus	Research Fellow
Zoltán Zimborás	Research Fellow
Iñigo Urizar-Lanz	2nd year Ph.D. Student
Giuseppe Vitagliano	2nd year Ph.D. Student

- Topics
 - Multipartite entanglement and its detection
 - Metrology, cold gases
 - Collaborating on experiments:
 - Weinfurter group, Munich, (photons)
 - Mitchell group, Barcelona, (cold gases)
- Funding:
 - European Research Council starting grant GEDENTQOPT, 2011-2016, 1.3 million euros
 - CHIST-ERA QUASAR collaborative EU project, 90000 euros
 - Grants of the Spanish Government and the Basque Government

Summary

- We presented a full set of generalized spin squeezing inequalities with the angular momentum coordinates for $j > 1/2$.
- We presented a large set of inequalities with the other collective operators that can be measured.

www.gedentqopt.eu

www.gtoth.eu

