# Interesting quantum states (Lecture of the Quantum Information class of the Master in Quantum Science and Technology)

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# Outline

1. **Interesting quantum states**
   - Motivation
   - A. Single particle states
   - B. Bipartite singlet state

# Which quantum states are interesting?

- We have infinite possibilities to pick a quantum state in a multi-qubit system.
- We would like to find useful ones or states that have interesting symmetries.

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# Single particle states

- Pure states. The von Neumann entropy $S = 0$.
- Completely mixed state

$$\varrho_{\mathrm{cm}} = \frac{1}{d} \sum_{k=1}^{d} |k\rangle\langle k|.$$

The von Neumann entropy $S = \log d$, maximal.

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# Bipartite singlet state

- The two-qubit singlet state looks like
$$|\Psi_{\rm singlet}\rangle = \frac{1}{\sqrt{2}}\,(|01\rangle - |10\rangle).$$
- We get the same form after any basis transformation ( if we transform the bases of the two qubits in the same way).This can be seen as follows. Let us choose two vectors as
$$\begin{aligned} |v\rangle &= \alpha|0\rangle + \beta|1\rangle, \\ |v_\perp\rangle &= \beta^*|0\rangle - \alpha^*|1\rangle. \end{aligned}$$
Clearly,
$$\langle v|v_\perp\rangle = 0,$$
Then, simple algebra yields
$$\frac{1}{\sqrt{2}}(|v\rangle \otimes |v_\perp\rangle - |v_\perp\rangle \otimes |v\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$
This is true for any $|v\rangle$ and $|v_\perp\rangle$.

# Bipartite singlet state II

- Due to the independence from the choice of the local basis, it is invariant under a transformation of the type $U \otimes U$, apart from a global phase $\phi$.

$$U \otimes U|\Psi_{\text{singlet}}\rangle = |\Psi_{\text{singlet}}\rangle \exp(-i\phi).$$

We can also say that

$$U \otimes U|\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|(U \otimes U)^{\dagger} = |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|.$$

Hence,

$$U \otimes U|\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}| = |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|U \otimes U.$$

Thus, the density matrices of such states will commute with all $U \otimes U$ :

$$[U \otimes U, |\Psi_{\text{singlet}}\rangle\langle\Psi_{\text{singlet}}|] = 0 \tag{1}$$

for any $U$.