Generation of macroscopic singlet states in atomic ensembles and its modeling with Gaussian states

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Jan Mayen (NORWAY)
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Denmark Strait
Norwegian Sea
Barents Sea
White Sea
Arctic Circle
Reykjavík
ICELAND
Hammerfest
Tromsø
Murmansk
Arkhangel'sk
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Strait of Gibraltar
Ceuta (SPAIN)
Alborán Sea
Melilla (SPAIN)
Oran
Algiers
Tunis
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Valletta
MALTA
Crete
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles
Boundary representation not necessarily authoritative

Outline

1. Motivation
2. Spin squeezing and entanglement
3. Spin squeezing with atomic ensembles
4. Formalism based on the covariance matrix

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Motivation

- Qubits cannot be individually accessed. How to create and detect entanglement.

- Entanglement creation and detection is possible through spin squeezing. Use the ideas behind the spin squeezing approach such that
  - Create and detect entanglement in systems of particles with arbitrarily large spin.
  - Engineer quantum states other than the classical spin squeezed state with a large spin, that is, unpolarized states.
  - Generalize the Gaussian approach for describing the dynamics leading to such states.

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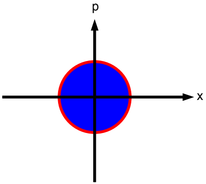
From squeezing to spin squeezing

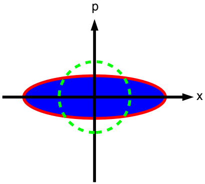
- The variances of the two quadrature components are bounded

$$(\Delta x)^2(\Delta p)^2 \geq const.$$

- Coherent states saturate the inequality.

- Squeezed states are the states for which one of the quadrature components have a smaller variance than for a coherent state.





- Can one use similar ideas for spin systems?

Spin squeezing

Definition

The variances of the angular momentum components are bounded

$$(\Delta J_x)^2(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z\rangle|^2,$$

where the mean spin points to the z direction. If $(\Delta J_x)^2$ is smaller than the standard quantum limit $\frac{|\langle Jz\rangle|}{2}$ then the state is called spin squeezed.

- In practice this means that the angular momentum of the state has a small variance in one direction, while in an orthogonal direction the angular momentum is large.

  [M. Kitagawa and M. Ueda, PRA **47**, 5138 (1993).]

Entanglement

Definition

Fully separable states are states that can be written in the form

$$\rho = \sum_l p_l \rho_l^{(1)} \otimes \rho_l^{(2)} \otimes ... \otimes \rho_l^{(N)},$$

where $\sum_l p_l = 1$ and $p_l > 0$.

Definition

A state is entangled if it is not separable.

- Note that one could also look for other type of entanglement in many-particle systems, e.g., entanglement in the two-qubit reduced density matrix.

The standard spin-squeezing criterion

Definition

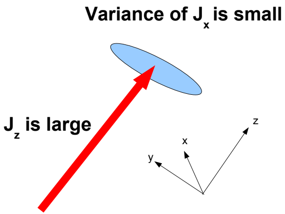
The spin squeezing criterion for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature **409**, 63 (2001).]

- Note that this criterion is for spin-1/2 particles.
- States violating it are like this:



A generalized spin squeezing entanglement criterion

Separable states of N spin-j particles must fulfill

$$\xi_s^2 := (\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj.$$

It is maximally violated by a many-body singlet, e.g., the ground state of an anti-ferromagnetic Heisenberg chain.

[GT, PRA **69**, 052327 (2004);GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL **99**, 250405 (2007).]

- For such a state

$$\langle J_k^m \rangle = 0.$$

- $N\xi_s^2$ gives an upper bound on the number of unentangled spins.

Singlet: Not sensitive to external fields

- The decrease of fidelity of a pure state $|\Psi\rangle$ under an external magnetic field is

$$\Delta F := 1 - |\langle\Psi|U_\phi|\Psi\rangle|^2,$$

where

$$U_\phi = \exp(-i\phi J_{\mathbf{n}}).$$

The spin squeezing parameter has a meaning concerning the sensitivity of the state to external fields

$$\mathrm{avg}_{\mathbf{n}}(\Delta F) = \frac{\phi^2 J}{3}\xi_s^2,$$

for small ϕ, and the averaging is over all directions **n**.

- Possible application: Quantum memory protected from decoherence.


Corsetería
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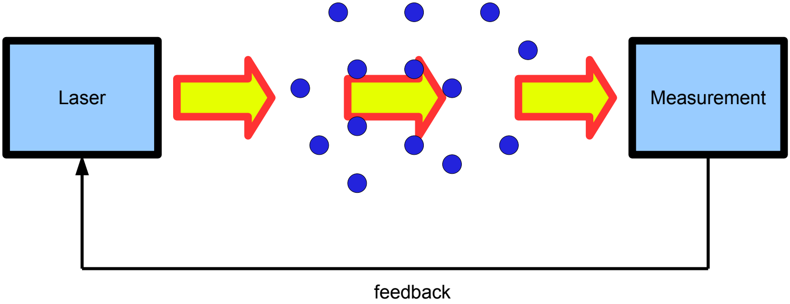
The physical system: atoms + light

- Atoms interacting with light. [B. Julsgaard, A. Kozhekin, and E.S. Polzik, Nature **413**, 400 (2001); J. Appel, P.J. Windpassinger, D. Oblak, U.B. Hoff, N. Kjaergaard, and E.S. Polzik, arXiv:0810.3545.]
- The light is measured and the atoms are projected into an entangled state.



The physical system: atoms + light II

- Atoms: J_x, J_y, J_z.
- Light: S_x, S_y, S_z.
- N is the number of atoms, $J = N/2$.
- S_0 is the maximum of S_l.

Quantum non-demolition measurement (QND) of the ensemble

The steps the QND measurement of J_k :

- 1. Set the light to
$$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
- 2. The atoms interact with the light for time t
$$H = \Omega J_k S_z$$
- 3. Measurement of S_y.

- The QND measurements decrease $(\Delta J_k)^2$.
- Timescale of the dynamics, for $J := Nj$, is
$$t \sim \tau := \frac{1}{\Omega \sqrt{S_0 J}}.$$

The proposed protocol

1. Initial state
   - Atoms
     $$\varrho_0 := \frac{\mathbb{1}}{(2j+1)^N}$$
   - Light
     $$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
2. Measurement of J_x + feedback or postselection.
3. Measurement of J_y + feedback or postselection.
4. Measurement of J_z + feedback or postselection.

- We consider 10^6 spin-1 ^{87}Rb atoms and $S_0 = 0.5 \times 10^8$.
- Initial state of the atoms has $(\Delta J_k)^2 \sim N$ for $k = x, y, z$.
- After squeezing, we obtain $\xi_s < 1$.
- Thus, we get a state close to a singlet state.

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How to model the dynamics of the atom-light system?

- A: 10^6 atoms: does the density matrix fit into MATLAB?
- Q: No.

Gaussian states

- Gaussian states are quantum states for which all third and higher order correlations are trivial.

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- The dynamics of Gaussian systems for the CV case: can be followed by dynamical equations for the covariance matrix and the expectation values of x_k and p_k.

- For a single mode, this matrix looks like

$$\Gamma_{xp} \propto \begin{pmatrix} \langle x^2 \rangle - \langle x \rangle^2 & \langle xp + px \rangle/2 - \langle x \rangle \langle p \rangle \\ \langle xp + px \rangle/2 - \langle x \rangle \langle p \rangle & \langle p^2 \rangle - \langle p \rangle^2 \end{pmatrix}.$$

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- How does it appear for spin states? We assume that

$$\langle J_z \rangle = \frac{N}{2},$$

and

$$\left[\frac{J_x}{\sqrt{N/2}}, \frac{J_y}{\sqrt{N/2}} \right] = i.$$

Thus, $\frac{J_{x/y}}{\sqrt{N/2}}$ play the role of x/p.

Gaussian states for unpolarized ensembles

- Spin systems: Such ideas can be used if one of the spin components is large.

- We extend this approach to states for which the spin is not large. Our covariance matrix for a single spin is

$$\Gamma_J \propto \begin{pmatrix} \langle \Delta J_x \Delta J_x \rangle & \langle (\Delta J_y \Delta J_x)_{\rm sym} \rangle & \langle (\Delta J_z \Delta J_x)_{\rm sym} \rangle \\ \langle (\Delta J_x \Delta J_y)_{\rm sym} \rangle & \langle \Delta J_y \Delta J_y \rangle & \langle (\Delta J_z \Delta J_y)_{\rm sym} \rangle \\ \langle (\Delta J_x \Delta J_z)_{\rm sym} \rangle & \langle (\Delta J_y \Delta J_z)_{\rm sym} \rangle & \langle \Delta J_z \Delta J_z \rangle \end{pmatrix},$$

where

$$(\Delta J_k \Delta J_l)_{\rm sym} = (\Delta J_k \Delta J_l + \Delta J_l \Delta J_k)/2.$$

- The word "Gaussian" in our case: all the higher order cumulants of J_k are trivial.

Examples for Gaussian states

- Fully polarized state, for large N.

$$|\Psi\rangle_{\text{full}} = |+\frac{1}{2}\rangle_x^{\otimes N}.$$

For this,

$$\langle (J_x^k J_y^l J_z^m)_{\text{sym}} \rangle = \left(\frac{N}{2}\right)^k \langle (J_y^l J_z^m)_{\text{sym}} \rangle.$$

- Completely mixed state, for qubits, for large N.

$$\langle J_{k_1}^{m_1} J_{k_2}^{m_2} ... \rangle_{\text{sym}} \sim \left(\frac{N}{4}\right)^{\sum_k m_k/2} \frac{\sum_k m_k!}{2^{(\sum m_k)/2} \sum_k (m_k/2)!}.$$

Examples for Gaussian states II

- Example:
$$\langle J_x^4 \rangle = \sum_{k_1,k_2,k_3,k_4} \langle j_x^{(k_1)} j_x^{(k_2)} j_x^{(k_3)} j_x^{(k_4)} \rangle.$$
  Nonzero terms come out from the case when ($k_1 = k_2$ and $k_3 = k_4$), or ($k_1 = k_3$ and $k_2 = k_4$), or ($k_1 = k_4$ and $k_2 = k_3$), that is, every index appears even times.
- This happens $\sim 3N^2$ times. The 3 is there since we found 3 different pairings of the qubits.
- If ϱ_1 and ϱ_2 are Gaussian, then $\varrho_1 \otimes \varrho_2$ is also Gaussian. (Here, ϱ_k describes a many-qubit system.)

Examples for Gaussian states III

Mixing two Gaussian states does not always lead to Gaussian states. For example,

$$\varrho = \frac{1}{2}\left(|\tfrac{+1}{2}\rangle\langle\tfrac{+1}{2}|^{\otimes N} + |\tfrac{-1}{2}\rangle\langle\tfrac{-1}{2}|^{\otimes N}\right).$$

- We have

$$\begin{aligned} (\Delta J_z)^2 &= \frac{N^2}{4}, \\ (\Delta J_z)^4 &= 0. \end{aligned} \tag{1}$$

However, for Gaussian states we need

$$(\Delta J_z)^4 = 3(\Delta J_z)^2.$$

Covariance matrix - Unitary Dynamics

Covariance matrix

We define the set of operators

$$R = \{\frac{J_x}{\sqrt{J}}, \frac{J_y}{\sqrt{J}}, \frac{J_z}{\sqrt{J}}, \frac{S_x}{\sqrt{S_0}}, \frac{S_y}{\sqrt{S_0}}, \frac{S_z}{\sqrt{S_0}}\}$$

and covariance matrix as

$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

Covariance matrix - Unitary Dynamics

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and covariance matrix as

$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

- For short times, the dynamics of an operator O_0 is given by

$$O_P = O_0 - it[O_0, H],$$

where we assumed $\hbar = 1$.

Covariance matrix - Unitary Dynamics II

- Dynamical equations for Γ_{kl} in terms of other correlation terms Γ_{mn} and higher order correlations.
- Consider dynamics for $t \sim \tau$. Then variances stay small during squeezing
$$|\langle(\prod_{k=1}^{K} \Delta J_{a_k})(\prod_{l=1}^{L} \Delta S_{b_l})\rangle| \ll J^K S_0^L.$$

Unitary dynamics

Hence one arrives to
$$\Gamma_P = M \Gamma_0 M^T,$$
where M is the identity matrix, apart from $M_{5,1} = \frac{\langle S_x \rangle}{S_0}\kappa$, and $\kappa := t/\tau = \Omega t \sqrt{J S_0}$.

Covariance matrix - Unitary Dynamics III

- In more detail: We denote the normalized variables by $j_k = J_k / \sqrt{J}$ and $s_k = S_k / \sqrt{S_0}$. The dynamics of the operators are

$$\begin{aligned}
j_x^{(out)} &= j_x, \\
j_y^{(out)} &= j_y - \frac{\kappa}{\sqrt{J}} s_z j_z, \\
j_z^{(out)} &= j_z + \frac{\kappa}{\sqrt{J}} s_z j_y, \\
s_x^{(out)} &= s_x - \frac{\kappa}{\sqrt{S_0}} s_y j_x, \\
s_y^{(out)} &= s_y + \frac{\kappa}{\sqrt{S_0}} s_x j_x, \\
s_z^{(out)} &= s_z,
\end{aligned}$$

where $\kappa := t/\tau = \Omega t \sqrt{J S_0} \sim 1$ for our dynamics.

Covariance matrix - Unitary Dynamics IV

- Dynamics of second moments:

$$
\begin{aligned}
(j_x^2)^{(out)} &= j_x^2, \\
(j_y^2)^{(out)} &= j_y^2 + (\kappa^2/J) s_z^2 j_z^2 - (\kappa/\sqrt{J}) s_z \{j_y, j_z\}, \\
(j_z^2)^{(out)} &= j_z^2 + (\kappa^2/J) s_z^2 j_y^2 + (\kappa/\sqrt{J}) s_z \{j_y, j_z\}, \\
(s_x^2)^{(out)} &= s_x^2 + (\kappa^2/S_0) s_y^2 j_x^2 - (\kappa/\sqrt{S_0}) j_x \{s_x, s_y\}, \\
(s_y^2)^{(out)} &= s_y^2 + (\kappa^2/S_0) s_x^2 j_x^2 + (\kappa/\sqrt{S_0}) j_x \{s_x, s_y\}, \\
(s_z^2)^{(out)} &= s_z^2,
\end{aligned}
$$

where $\{A, B\} := AB + BA$.

Covariance matrix - Unitary Dynamics IV

- Take the case of s_y^2. Then,

$$\langle (s_y^2)^{(out)} \rangle = \langle s_y^2 \rangle + (\kappa^2/S_0)\langle s_x^2 j_x^2 \rangle + (\kappa/\sqrt{S_0})\langle j_x\{s_x, s_y\}\rangle.$$

- Write the equation for the variances, rather than for the moments. Note that $\langle s_x \rangle = \sqrt{S_0}$ at the beginning and it remains nonzero, while all other expectation values remain zero, that is, $\langle s_y \rangle = \langle s_z \rangle = \langle j_k \rangle = 0$.
- We obtain

$$\begin{aligned} \langle (\Delta s_y^{(out)})^2 \rangle &= \langle (\Delta s_y)^2 \rangle + (\kappa^2/S_0)\langle (\Delta s_x)^2 (\Delta j_x)^2 \rangle \\ &+ (\kappa^2/S_0)\langle s_x \rangle^2 \langle (\Delta j_x)^2 \rangle + (\kappa/\sqrt{S_0})\langle \Delta j_x \{\Delta s_x, \Delta s_y\}\rangle \\ &+ (2(\kappa/\sqrt{S_0})\langle \Delta j_x \Delta s_y \rangle \langle s_x \rangle. \end{aligned}$$

Here we used that $\Delta s_x = s_x - \langle s_x \rangle$, while $\Delta s_y = s_y$, etc.

Covariance matrix - Unitary Dynamics V

- Take short time dynamics

$$t \sim \tau := \frac{1}{\Omega\sqrt{S_0 J}} \ll \frac{1}{\Omega\sqrt{S_0}}.$$

- Then

$$\begin{aligned}
\langle \Delta s_y^2 \rangle &\sim 1, \\
(\kappa^2/S_0)\langle (\Delta s_x)^2 (\Delta j_x)^2 \rangle &\sim \frac{1}{S_0} \ll 1, \\
(\kappa^2/S_0)\langle s_x \rangle^2 \langle (\Delta j_x)^2 \rangle &\sim 1, \\
(\kappa/\sqrt{S_0})\langle \Delta j_x \{\Delta s_x, \Delta s_y\} \rangle &\sim \frac{1}{\sqrt{S_0}} \ll 1, \\
2(\kappa/\sqrt{S_0})\langle \Delta j_x \Delta s_y \rangle \langle s_x \rangle &\sim 1.
\end{aligned}$$

- We obtain

$$\langle (\Delta s_y^2)^{(out)} \rangle = \langle \Delta s_y^2 \rangle + (\kappa^2/S_0)\langle s_x \rangle^2 \langle (\Delta j_x)^2 \rangle + 2(\kappa/\sqrt{S_0})\langle s_x \rangle \langle \Delta j_x \Delta s_y \rangle.$$

Covariance matrix VI

We obtain that for $\kappa \sim 1$ (or $\kappa \ll \sqrt{J}$) we can use

$$
\begin{aligned}
j_x^{(out)} &= j_x, \\
j_y^{(out)} &= j_y, \\
j_z^{(out)} &= j_z, \\
s_x^{(out)} &= s_x, \\
s_y^{(out)} &= s_y + \frac{\kappa}{\sqrt{S_0}} \langle s_x \rangle j_x, \\
s_z^{(out)} &= s_z.
\end{aligned}
$$

- Note again that we used that for short times

$$
|\langle (\prod_{k=1}^{K} \Delta j_{a_k})(\prod_{l=1}^{L} \Delta s_{b_l}) \rangle| \ll J^{K/2} S_0^{L/2}.
$$

Covariance matrix - Unitary Dynamics VII

- There is some evidence that the dynamics might stay Gaussian even if we consider all the terms. (Have to be checked.)

Covariance matrix - Measurement

For large N, for the completely mixed state

$$\langle (J_x^k J_y^l J_z^m)_{sym} \rangle = \int dxdydz (x^k y^l z^m) f(x,y,z),$$

where f is a quasi-probability distribution playing the role of a Wigner function.

- For the completely mixed state, it is

$$f(x,y,z) \propto \exp(-\frac{1}{2} v \Gamma^{-1} v),$$

  where Γ is the correlation matrix.

- For two ensembles a similar function is $f(x_1, y_1, z_1, x_2, y_2, z_2)$. Moreover, a linear transformation of the variables x_k and y_k does not change the Gaussian nature of f.

Covariance matrix - Measurement

- Let us see, how to compute the value of the projector to $S_y = 0$. (We measured S_y and had a result $S_y = 0$)

$$\langle P_{Sy=0} \rangle = \mathrm{Tr}(\varrho P_{S_y=0}).$$

  The projector is diagonal in the basis of S_y eigenstates and can be given as $g(S_y)$, where $g(x)$ is some polynomial.

- In practice, $g(0) = 1$, and $g(x) = 0$ for $x \geq 1$. Then $g(S_y)$ is a projector to $S_y = 0$. (Here the subtlety comes that S_y does not have a continuous spectrum while x has.)

Ongoing discussion with Géza Giedke, MPQ, München.

Covariance matrix - Measurement II

- Thus,

$$\langle P_{S_y=0}\rangle = \langle g(S_y)\rangle = \mathrm{Tr}(\varrho g(S_y)) \qquad (*1)$$

  The expectation value with the Wigner function is

$$\langle g(S_y)\rangle = \int f(j_x, j_y, j_z, s_x, s_y, s_z) g(s_y) djds \qquad (*2)$$

  Note that (*1) and (*2) are equal.

- Now, how is the **unnormalized** expectation value of correlations like J_x^2 after the measurement?

$$\langle P_{S_y=0} J_x^2 P_{S_y=0}\rangle = \mathrm{Tr}(\varrho g(S_y) J_x^2 g(S_y)) \qquad (**1)$$

  The same with the Wigner functions

$$\langle P_{S_y=0} J_x^2 P_{Sy=0}\rangle = \int j_x^2 f(j_x, j_y, jz, s_x = 0, s_y, s_z) djds_y ds_z \qquad (**2)$$

Covariance matrix - Measurement III

- That is, the Wigner function corresponding to the reduced state of the atoms is

$$f_a(j_x, j_y, j_z) = \int f(j_x, j_y, j_z, s_x, s_y = 0, s_z) ds_x ds_z \qquad (***)$$

  With this

$$\langle P_{S_y=0} J_x^2 P_{S_y=0} \rangle = \int j_x^2 f_a(j_x, j_y, j_z) dj_x dj_y dj_z.$$

  Here s_y is set to zero, and s_x and s_z are integrated out.

- (***) shows that after the von Neumann measurement the Wigner function of the reduced state was obtained by setting one variable zero, i.e, getting a slice from the Wigner function.

Covariance matrix - Measurement IV

- Note that in general f can be negative. However, for our case, f is positive! It is really a probability distribution.
- Our formula is exactly the same, as one would use if s_k and j_k were classical variables, and f were a probability distribution. From the original probability distribution we get a "slice", after the von Neumann measurement.
- Because of that, a formula very similar to the CV case can be used. It is based on classical probability distributions.
- The measurement of the light can be modeled with a projection

$$\Gamma_M = \Gamma_P - \Gamma_P (P_y \Gamma_P P_y)^{\mathrm{MP}} \Gamma_P^T, \tag{3}$$

where MP denotes the Moore-Penrose pseudoinverse, and P_y is $(0,0,0,0,1,0)$. Like in [G. Giedke and J.I. Cirac, Phys. Rev. A **66**, 032316 (2002)].

Dictionary

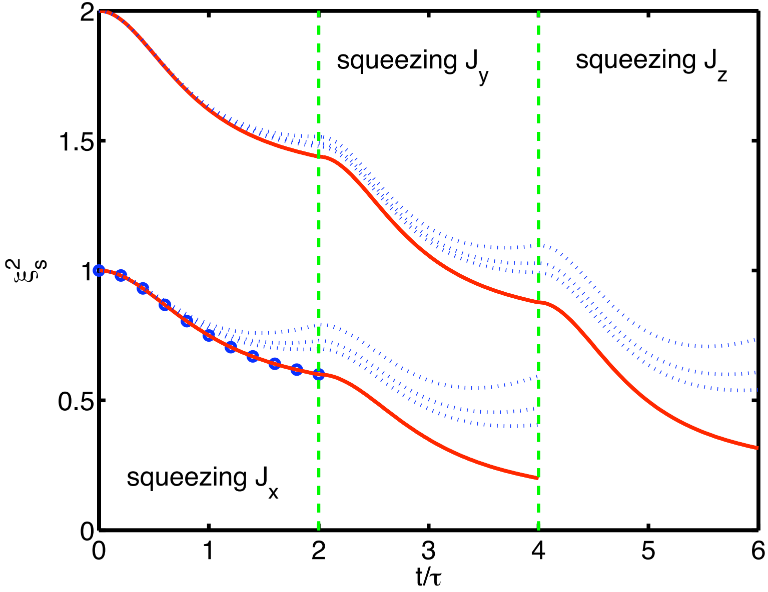
- M. Keyl, D.-M. Schlingemann, Z. Zimborás, *On the bosonic behavior of mean-field fluctuations in atomic ensembles*, arXiv:1006.3461.

$$\Phi_N(a) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} a_i - \langle a \rangle \equiv j_l, s_l.$$

$$\Gamma_J \propto \begin{pmatrix} \langle \Phi_N(\hat{j}_x)\Phi_N(\hat{j}_x) \rangle & \langle \Phi_N(\hat{j}_y)\Phi_N(\hat{j}_x) \rangle & \langle \Phi_N(\hat{j}_z)\Phi_N(\hat{j}_x) \rangle \\ \langle \Phi_N(\hat{j}_x)\Phi_N(\hat{j}_y) \rangle & \langle \Phi_N(\hat{j}_y)\Phi_N(\hat{j}_y) \rangle & \langle \Phi_N(\hat{j}_z)\Phi_N(\hat{j}_y) \rangle \\ \langle \Phi_N(\hat{j}_x)\Phi_N(\hat{j}_z) \rangle & \langle \Phi_N(\hat{j}_y)\Phi_N(\hat{j}_z) \rangle & \langle \Phi_N(\hat{j}_z)\Phi_N(\hat{j}_z) \rangle \end{pmatrix}.$$

Note: $\hat{j}_k$ and $\hat{s}_k$ are now single particle operators.

Spin squeezing dynamics (top curve, solid)



Modeling losses

The dynamics of the covariance matric for the case of losses

$$\Gamma_P' = (\mathbb{1} - \eta D) M \Gamma_0 M^T (\mathbb{1} - \eta D) + \eta(2-\eta) D \Gamma_{\mathrm{noise}},$$

where $D = \mathrm{diag}(1,1,1,0,0,0)$ and $\Gamma_{\mathrm{noise}} = \mathrm{diag}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0, 0)$.

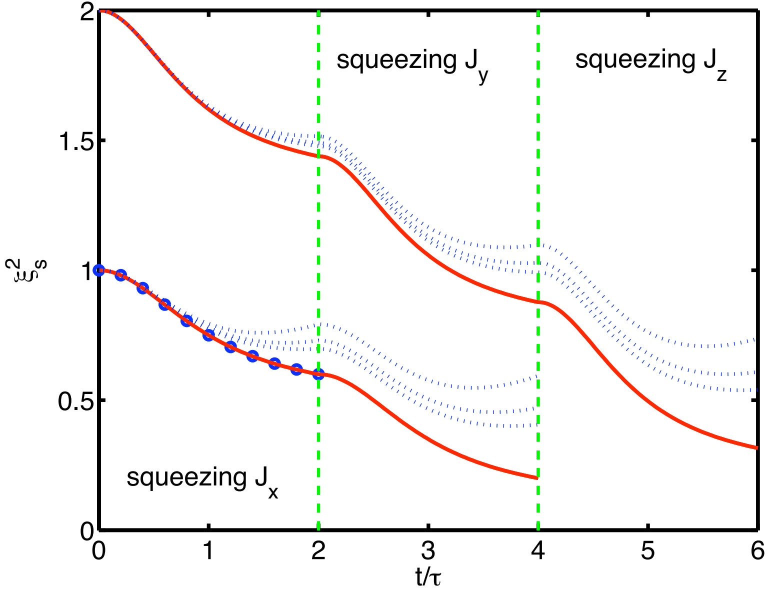
- η is the probability of spontaneous excitation by the off-resonant probe, that is, the fraction of atoms that decoherence during the QND process.
- The losses are connected to κ through

$$\eta = Q\kappa^2/\alpha,$$

where α is the resonant optical depth of the sample and $Q = \frac{8}{9}$

[L.B. Madsen and K. Mølmer, Phys. Rev. A **70**, 052324 (2004).]

Spin squeezing dynamics: $\alpha = 50, 75, 100$ (dotted)



Conclusions

- We presented a method for creating and detecting entanglement in an ensemble of atoms with spin $j > \frac{1}{2}$.
- Our experimental proposal aims to create a many-body singlet state through squeezing the uncertainties of the collective angular momenta.
- We showed how to use an extension of the usual Gaussian formalism for modeling the experiment.
- Presentation based on: GT and M.W. Mitchell, New J. Phys. 2010.

*** THANK YOU ***

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