Entanglement in a Bose-Einstein condensate

B. Lücke[1], J. Peise[1], W. Ertmer[1], and C. Klempt[1]
[1]Institut für Quantenoptik, Leibniz Universität Hannover, 30167 Hannover, Germany.

J. Arlt[2], L. Pezze[3], A. Smerzi[3], F. Deuretzbacher[4], L. Santos[4], G. Vitagliano[5], G. Toth[5]
[2]Center for Quantum Optics (QUANTOP), Institut for Fysik og Astronomi, Aarhus Universitet, 8000 Aarhus C, Denmark.
[3]Istituto Nazionale di Ottica (INO), Consiglio Nazionale delle Ricerche (CNR), and European Laboratory for Non-Linear Spectroscopy (LENS), 50125 Firenze, Italy.
[4]Institut für Theoretische Physik, Leibniz Universität Hannover, 30167 Hannover, Germany.
[5]Department of Theoretical Physics, The University of the Basque Country, 48080 Bilbao, Spain.

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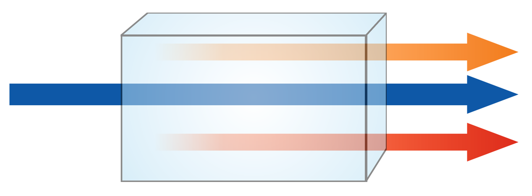
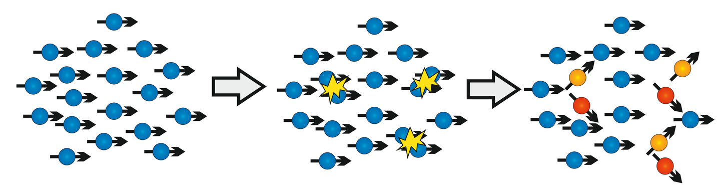
Summary

Matter wave optics with ultra cold samples has reached the point where nonclassical states can be prepared and their fascinating properties can be explored. In optics, parametric down conversion is routinely used to generate light with squeezed observables as well as highly entangled photon pairs. The applications of these nonclassical states range from fundamental tests of quantum mechanics to improved interferometers and quantum computation. Therefore, it is of great interest to realize such nonclassical states with matter waves. Bose-Einstein condensates with non-zero spin can provide a mechanism analogous to parametric down conversion, thus enabling the generation of nonclassical matter waves. The process acts as a two-mode parametric amplifier and generates two clouds with opposite spin orientation consisting of the same number of atoms. At a total of 8000 atoms, we observe a squeezing of the number difference of -7 dB below shot noise, including all noise sources. A microwave coupling between the two modes allows for an investigation of the interferometric sensitivity. We find that the created state is entangled and useful for sub-shot-noise interferometry. We show that the created state has a minimum of 30 entangled atoms.

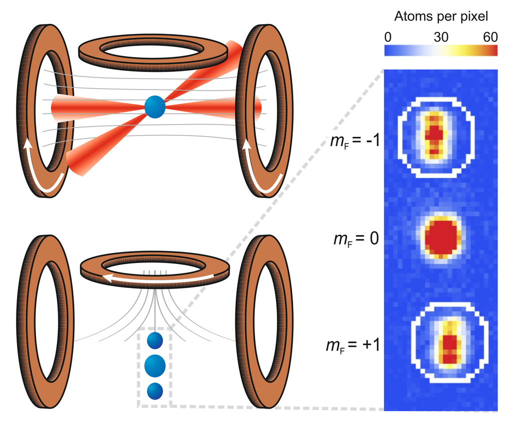
Optical parametric down conversion vs. spin dynamics





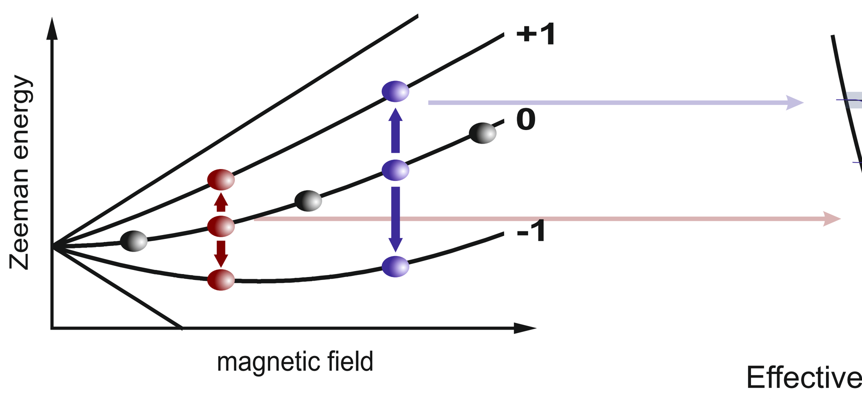
Optics	Spin dynamics
Coherent pump beam	BEC in $m_F = 0$
Non-linear crystal	Spin dynamics
Signal and idler beam	Atoms in $m_F = \pm 1$
Cavity	Trapping potential
Quadratures	Bogoliubov modes
Cavity modes	Spatial modes

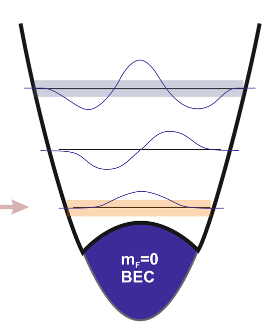
Experimental sequence



- Preparation of a ^{87}Rb BEC in $|F=2, m_F=0\rangle$
- Homogeneous magnetic field during a variable evolution time
- Sequence of microwave pulses can be applied
- Magnetic field gradient separates the Zeeman sublevels during time of flight

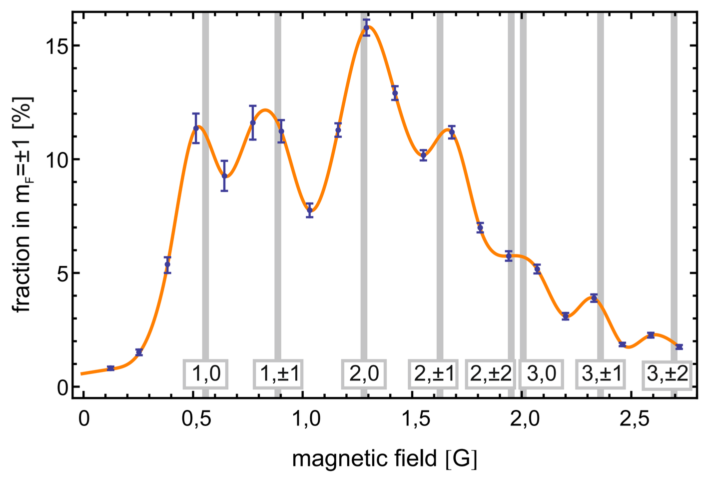
Magnetic field resonances and parametric amplification

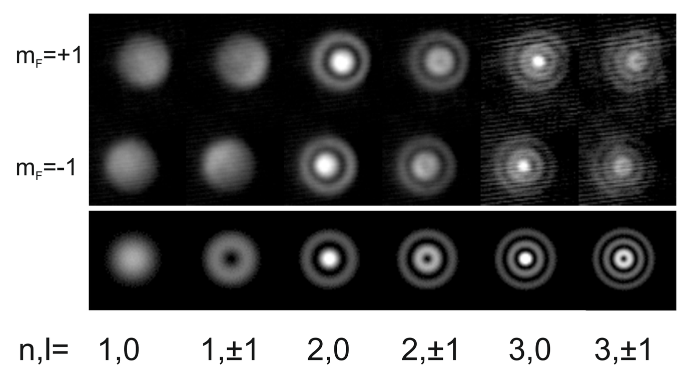


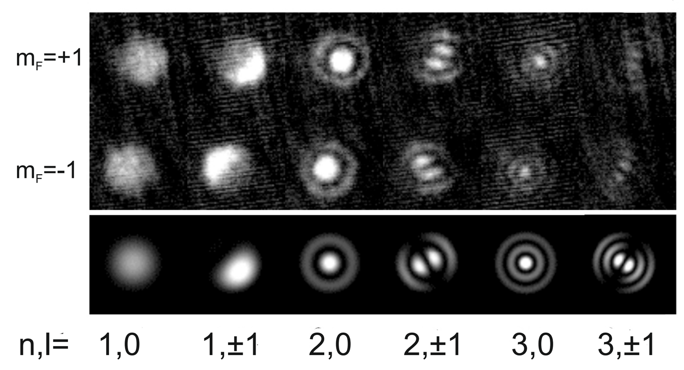


Effective potential and eigenstates of the $m_F=\pm1$ atoms

The number of atoms that change their spin to $m_F=\pm1$ depends on the applied homogeneous magnetic field. Resonances occur when the difference in Zeeman energies in ±1 fits to an eigen energy of the effective potential.





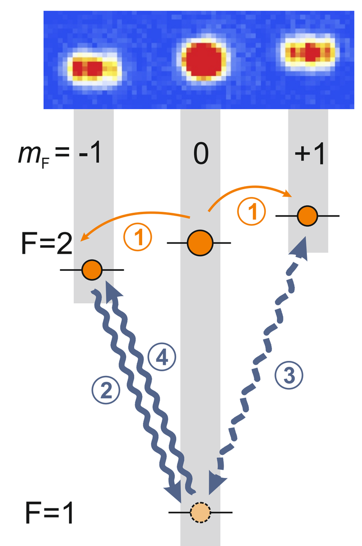


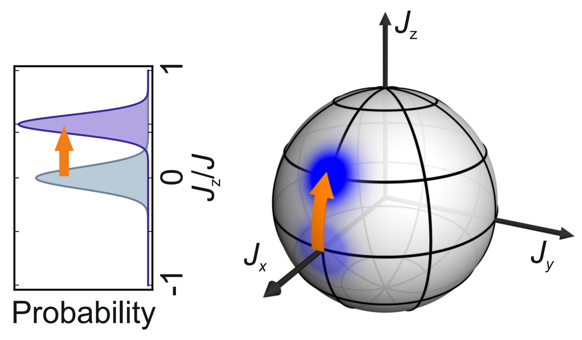
In a cylindrical trap the effective potential for the $m_F=\pm1$ atoms can be approximated by a roundish box potential. The eigenmodes for such a potential are Bessel functions which describe the observed density profiles and their energies fit to the resonance positions.

Scherer, et al. Phys. Rev. Lett., 105(13):135302, Sep 2010.

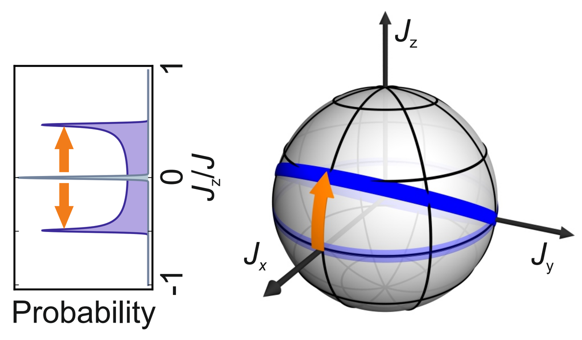
The twin Fock interferometer

Coupling between $|2,-1\rangle$ and $|2,+1\rangle$ is achieved by three resonant microwave pulses.

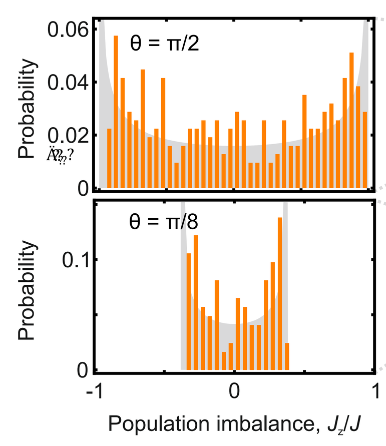


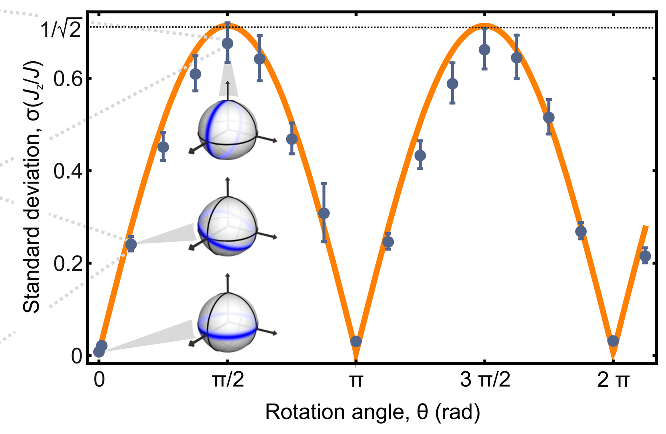


Classical state
The rotation angle is mapped onto $\langle J_z\rangle$.

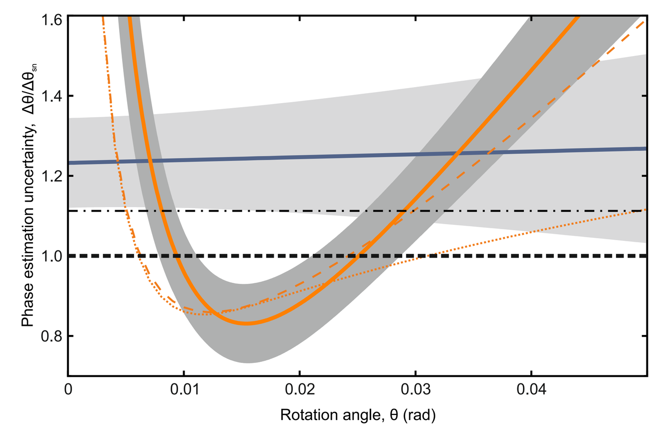


Nonclassical state
For all rotation angles $\langle J_z\rangle=0$, but $\langle J_z^2\rangle$ contains the quantity of interest.





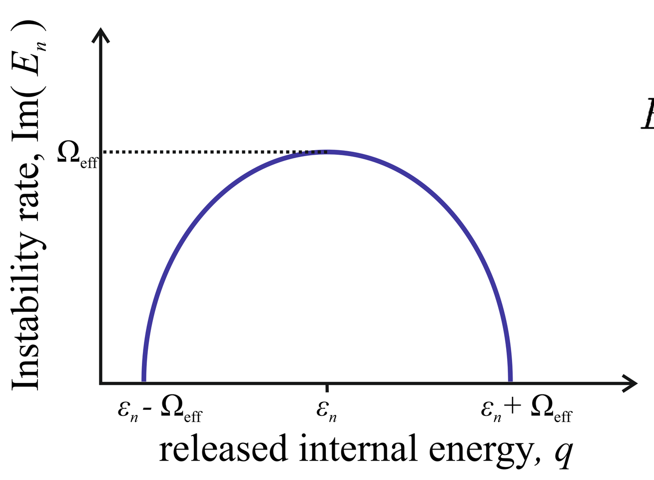
Standard deviation of the normalized population imbalance $\sigma(J_z/J)$ as a function of the rotation angle θ, which can be used as a signal of the interferometer.



At the optimal point (θ = 0.015), we reach a measurement uncertainty $\Delta\theta/\Delta\theta_{sn}$ = 0.83, which is $-1.61^{+0.98}_{-1.1}$ dB below the shot noise limit $\Delta\theta_{sn} = 1/\sqrt{n\langle N_{tot}\rangle}$

Lücke et al. Science, 334(6057):773776, Nov 2011.

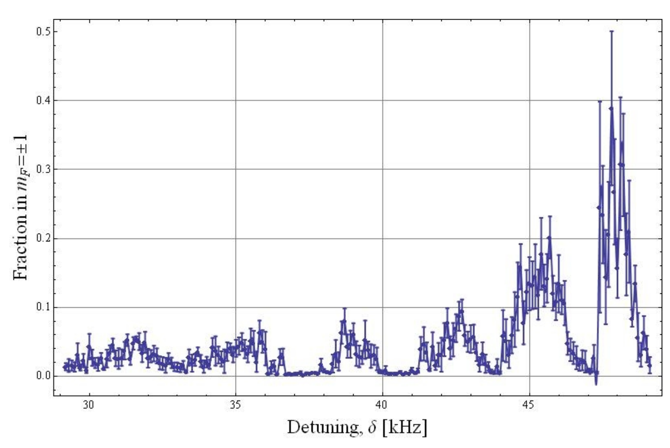
Spin dynamics in F=1

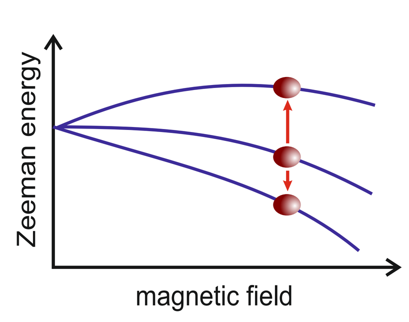


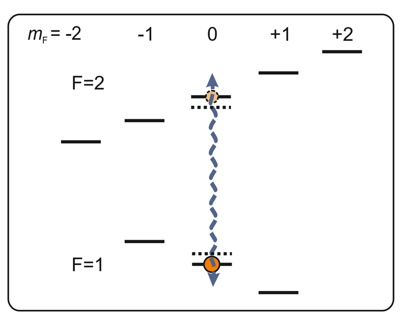
$$E_n = \sqrt{(\varepsilon_n - q)^2 - \Omega_{\text{eff}}^2} \qquad \Omega_{\text{eff}} = n_0 U_1$$

$$|U_1| \xrightarrow{F=1} \frac{U_1}{10} \longrightarrow \begin{cases} t \to 10\,t \\ \frac{\Delta\text{resonance}}{10} \end{cases}$$

In the Bogoliubov approximation the imaginary part of the eigenenergies is called instability rate and gives rise to an exponential amplification of the corresponding mode. The instability rate in dependence of the released internal energy describes a half circle. Its radius is given by the effective interaction energy Ω_{eff}.







Since the interaction strength U_1 and thus the effective interaction energy Ω_{eff} in F=1 is an order of magnitude smaller compared to F=2 the different resonances can be nicely resolved.

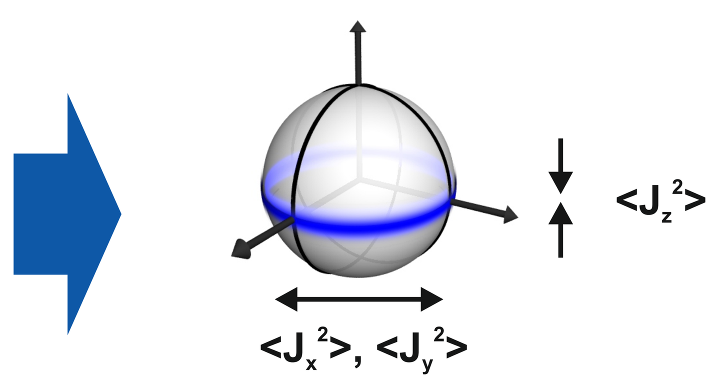
In F=1 energy is needed to change the internal state of two $m_F=0$ atoms to $m_F=\pm1$. Therefore we need to shift the Zeeman energies using a microwave dressing field to be able to populate higher modes.

Multipartite entanglement - preliminary

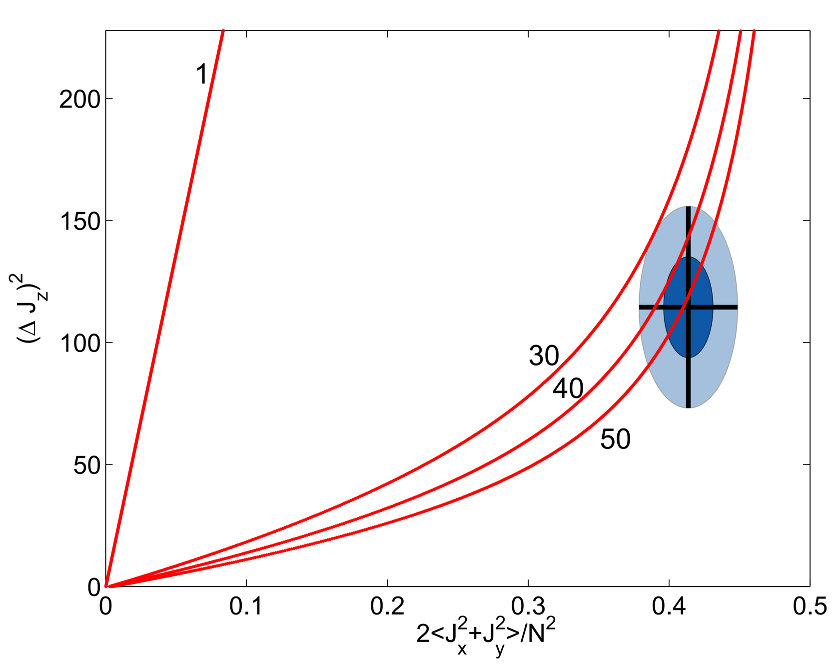
Violation of any of the following inequalities implies entanglement:

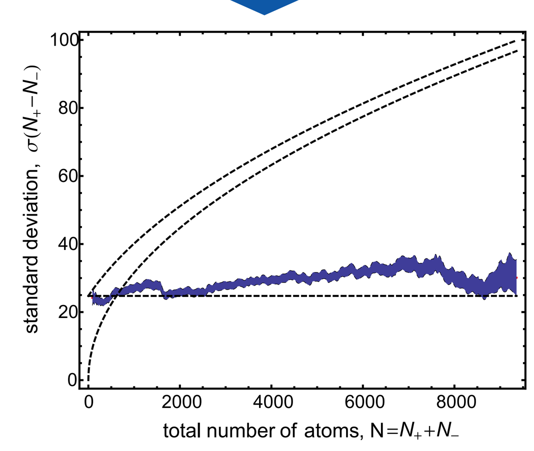
$$\langle J_x^2\rangle + \langle J_y^2\rangle + \langle J_z^2\rangle \le N(N+2)/4,$$
$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \ge N/2,$$
$$\boxed{\langle J_i^2\rangle + \langle J_j^2\rangle - N/2 \le (N-1)(\Delta J_k)^2,}$$
$$(N-1)[(\Delta J_i)^2 + (\Delta J_j)^2] \ge \langle J_k^2\rangle + N(N-2)/4,$$

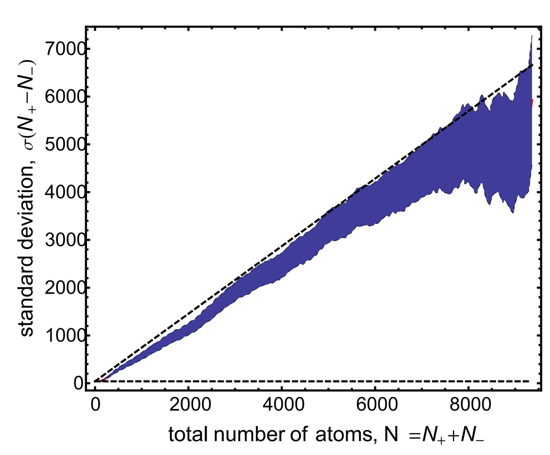
G. Tóth et al., PRL **99**, 250405 (2007)



Measure $\langle J_z^2\rangle$ and $\langle J_x^2\rangle$, $\langle J_y^2\rangle$







At least 30 particles are entangled!