No-cloning theorem and related issues (Lecture of the Quantum Information class of the Master in Quantum Science and Technology)

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Outline

1. **No-cloning theorem and related issues**
   - No-cloning theorem
   - Measurement problem
   - Quantum teleportation

No-cloning theorem

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LETTERS TO NATURE

A single quantum cannot be cloned

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom

on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \quad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle \quad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \quad (3)$$

No-cloning theorem II

We are looking for a mechanism that clones quantum states

$$U|\Psi\rangle \otimes |0\rangle = |\Psi\rangle \otimes |\Psi\rangle,$$

where U is a unitary dynamics.
Let us see why this is not possible. For the two basis states we have

$$U|0\rangle \otimes |0\rangle = |0\rangle \otimes |0\rangle,$$

$$U|1\rangle \otimes |0\rangle = |1\rangle \otimes |1\rangle.$$

Then, due to the linearity of quantum mechanics

$$U\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle).$$

However, we wanted

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Thus, a quantum state cannot be cloned.

Outline

1. **No-cloning theorem and related issues**
   - No-cloning theorem
   - Measurement problem
   - Quantum teleportation

Measurement problem

- Von Neumann postulated two types of quntum dynamics: unitary dynamics for closed systems and the dynamics under measurement, if the system is connected to a measuement device.
- We would expect that the dynamical description for closed systems can be used even for the case of a quantum measuement, if the measured particle and the measuring device are in one closed system.
- However, this is not the case. A unitary dynamics cannot describe the dynamics of the measured particle, the device (D) and the envoronment (E).

Measurement problem II

- We have the spin, the measurement device and the environment. The measurement dynamics should be

$$U|s = +\frac{1}{2}\rangle \otimes |D_0\rangle \otimes |E_0\rangle = |s = +\frac{1}{2}\rangle \otimes |D_{+1/2}\rangle \otimes |E'\rangle,$$

  and

$$U|s = -\frac{1}{2}\rangle \otimes |D_0\rangle \otimes |E_0\rangle = |s = -\frac{1}{2}\rangle \otimes |D_{-1/2}\rangle \otimes |E''\rangle.$$

Measurement problem III

- If the spin is in a superposition of $s = +1/2$ and $s = -1/2$, then we get

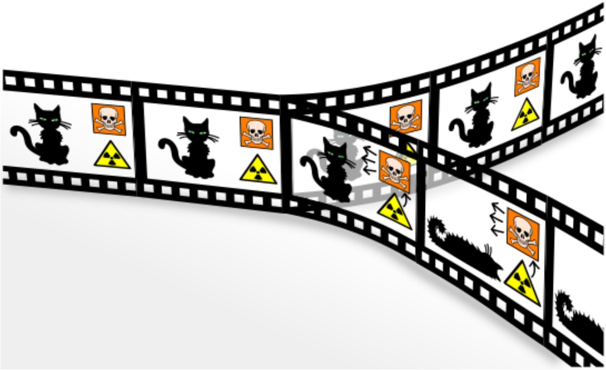
$$U\frac{1}{\sqrt{2}}\left(|s = +\frac{1}{2}\rangle + |s = + - \frac{1}{2}\rangle\right) \otimes |D_0\rangle \otimes |E_0\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|s = +\frac{1}{2}\rangle \otimes |D_{+1/2}\rangle \otimes |E'\rangle + |s = -\frac{1}{2}\rangle \otimes |D_{-1/2}\rangle \otimes |E''\rangle\right)$$

- We get a superposition of two states, rather than one or the other.
- This is a fundamental problem in quantum mechanics. A possible solution is the many-world interpretation.

Measurement problem IV

- A possible solution is the many-world interpretation.

- The idea of MWI originated in the Ph. D. thesis of Everett at Princeton in 1957, with the title "The Theory of the Universal Wavefunction", developed under his thesis advisor John Archibald Wheeler.



Measurement problem V

$$U\frac{1}{\sqrt{2}}\left(|s=+\frac{1}{2}\rangle+|s=+-\frac{1}{2}\rangle\right)\otimes|D_0\rangle\otimes|E_0\rangle\otimes|\mathrm{MIND}_0\rangle.$$

$$=\frac{1}{\sqrt{2}}\Big(|s=+\frac{1}{2}\rangle\otimes|D_{+1/2}\rangle\otimes|E'\rangle\otimes|\mathrm{MIND}_{+1/2}\rangle$$

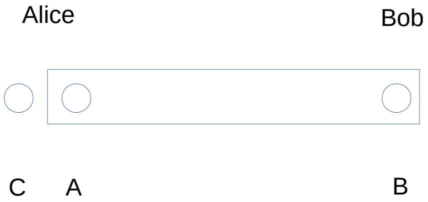
$$+|s=-\frac{1}{2}\rangle\otimes|D_{-1/2}\rangle\otimes|E''\rangle\otimes|\mathrm{MIND}_{-1/2}\rangle\Big)$$

Outline

Quantum teleportation

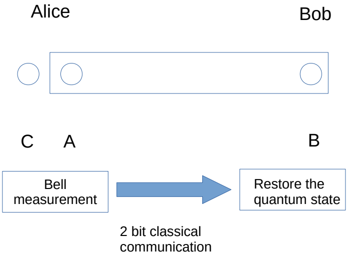
- A quantum state cannot be copied/cloned.
- But, it can be transferred from one particle to another one such that the state of the original particle is destroyed.

Quantum teleportation II



- A,B,C are spin-1/2 particles ≡ qubits.
- Alice wants to send the state of particle C to Bob.
- AB is in a singlet state $(|00\rangle + |11\rangle)/\sqrt{2}$.

Quantum teleportation II



- 1. Measurement of AC in the Bell basis
- 2. Alice sends the two-bit result to Bob
- 3. Depending on the result, Bob carries out $\varrho \rightarrow \sigma_l \varrho \sigma_l$, where $l \in \{0, x, y, z\}$.
- 4. The state of B is the same as the state of C was at the beginning.

Quantum teleportation III

- Initial state:

$$|\Psi\rangle_{AB} \otimes |\Psi\rangle_C = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \otimes (\alpha|0\rangle_C + \beta|1\rangle_C).$$

Alice and Bob want to teleport. Alice has two particles: A and C. She wants to teleport the C particle to the B particle of Bob. Particle A is helping the teleportation.

- Alice makes a measurement on particles A and C in the Bell basis. The Bell basis consists of the states:

$$|\Phi^{\pm}\rangle_{AC} = \frac{1}{\sqrt{2}}(|00\rangle_{AC} \pm |11\rangle_{AC})$$

and

$$|\Psi^{\pm}\rangle_{AC} = \frac{1}{\sqrt{2}}(|01\rangle_{AC} \pm |10\rangle_{AC}).$$

Quantum teleportation IV

- To see how this works, one can rewrite

$$
\begin{aligned}
&|\Psi\rangle_{AB} \otimes |\Psi\rangle_C \\
&\quad = \tfrac{1}{2}[|\Phi^+\rangle_{AC} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B) + |\Phi^-\rangle_{AC} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B) \\
&\quad + |\Psi^+\rangle_{AC} \otimes (\beta|0\rangle_B + \alpha|1\rangle_B) + |\Psi^-\rangle_{AC} \otimes (\beta|0\rangle_B - \alpha|1\rangle_B)].
\end{aligned}
$$

- Hence, measurement of AC in the Bell basis results in one of the four possibilities above for particle B. Knowing the result of the measurement, we can obtain

$$(\alpha|0\rangle_B + \beta|1\rangle_B).$$

  Thus, we successfully teleported the state of particle C to particle B.

- Note that this does not make possible faster than light communication, since the result of the Bell measurement has to be sent classically.

Quantum teleportation V

- Experiment: Experimental quantum teleportation Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger, Nature 390, 575-579 (11 December 1997).

Quantum teleportation VI

- 143 km, employing an optical free-space link between the two Canary Islands of La Palma and Tenerife, Zeilinger's group, 2012.

(figure from www.iqoqi-vienna.at)