Phase transition in a nonlinear quantum protocol

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Extended abstract

Post-selective quantum operations can lead to nonlinear quantum evolution if applied on more initial systems in the same quantum state [1]. Iterated application of this type of protocols leads to chaotic quantum evolution equivalent to a complex dynamical system, characterized by the iteration of a rational complex function [2]. The emerging time-evolution can be exponentially sensitive to the initial pure quantum state for single qubit systems [3] as well as for qubit pairs [4]. In the latter case, chaoticity emerges also in the entanglement properties of the qubit pair. One can construct an operation for any complex nth order rational map [3].

Noisy initial states are characterized by a density matrix, described by three real variables for a qubit. The iterated nonlinear protocol thereby naturally extends complex dynamical systems on the the Riemann sphere (equivalent to the surface of the Bloch sphere in physics) to the internal volume of the sphere. Somewhat surprisingly, noise does not destroy the fractals belonging to the Julia set on the surface. Instead, a phase transition occurs at a critical noise level. Above the corresponding critical purity, the fractal dimension of the borders between convergence regions remains constant [6]. One can identify a repelling fixed point inside the sphere at the critical purity. Backward iterations of the dynamics starting from points close to the Julia set on the sphere will all converge to this fixed point. A similar phase transition is present in protocols corresponding to all higher order rational maps by applying the same construction [6].

We show that small coherent errors in the quantum gates do not affect substantially the characteristic features of the system [6]. The robustness of the discussed phenomenon suggests that it can be observed in physical realizations, e.g. in quantum computers, where such protocols may be useful for testing and benchmarking purposes. Two steps of the ergodic protocol have been realized in an optical experiment [7]

Keywords: Iterated quantum protocols; Nonlinear quantum dynamics; Chaotic quantum evolution; Phase transition.

References

[1] H. Bechmann-Pasquinucci, B. Huttner, and N. Gisin, Phys. Lett. A 242, 198 (1998); D. R. Terno, Phys. Rev. A 59, 3320 (1999)

[2] Kiss, T., Jex, I., Alber, G., and Vymĕtal, S.: Complex chaos in the conditional dynamics of qubits. Physical Review A 74(4), 040301 (2006)

[3] Gilyén, A., Kiss, T., and Jex, I..: Exponential sensitivity and its cost in quantum physics. Scientific Reports 6(1), 20076 (2016)

[4] Kiss, T., Vymĕtal, S., Tóth, L. D., Gábris, A., Jex, I., & Alber, G.: Measurement-induced chaos with entangled states. Physical Review Letters, 107(10), 100501 (2011)

[5] Malachov, M., Jex, I., Kálmán, O., and Kiss, T.: Phase transition in iterated quantum protocols for noisy inputs. Chaos: An Interdisciplinary Journal of Nonlinear Science 29(3) (2019)

[6] Portik, A., Kálmán, O., Jex, I., & Kiss, T.: Iterated nth order nonlinear quantum dynamics with mixed initial states. Physics Letters A, 431, 127999 (2022); Portik, A., Kálmán, O., Jex, I., and Kiss, T.: Robustness of chaotic behavior in iterated quantum protocols. Physical Review A 109(4), 042410 (2024)

[7] Qu, D., Kálmán, O., Zhu, G., Xiao, L., Wang, K., Kiss, T. and Xue, P.: Observation of the dynamics of an ergodic quantum protocol in a photonic realization. New Journal of Physics, 23(8), p.083008 (2021)