Application of genetic algorithms to the knapsack and travelling salesman problem

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1 Kapsack

1.1 Approach

1.1.1 Genotype Representation and Fitness

For the 0-1 version of the knapsack problem, genotype represented as a list of n entries, where every entry is a 1 if that item is carried and 0 otherwise. every gene has a weight and reward. Weights get initialised randomly uniform between 1 and 10 regardless of the problem size. Rewards are the index of the item + 1. The fitness value of an individual is determined as the dot product of rewards and genotype.

1.1.2 Initialisation

Our case's initialisation is done picking random integers 0 or 1 with equal probability and only accepting the genotype if the weight is below the threshold. Dealing with initialisation as described already puts pressure on the initial population so that all solutions are feasible.

1.1.3 Selection

The selection method evaluated for the knapsack problem is called roulette wheel selection. The roulette wheel is tuned by the temperature parameter where a lower temperature increases representation of fitter individuals in the mating pool. Dependencies between genes are weak, but weight units can only be spent once. Therefore, items in the knapsack exclude others. There is little gene interdependence, so we can hypothesise that diversity is not that important. High pressure will likely lead to faster convergence and stability afterwards.

1.1.4 Crossover

Crossover in this work is done by using half the solution from each parent split in the middle. This could be problematic if no mutation would be done since you would only keep the same starting and ending sequences propagating. With this simple recombination scheme on the knapsack problem, one should always have a mutation to enable new solutions. The algorithm commits to legal solutions only and enforce this by copying the parent in case the proposed child exceeds the weight limit. This slows the effect of crossover, but solutions stay feasible. It is expected crossover rate can likely be safely set close to or at one, because of its mild breaking.

1.1.5 Mutation

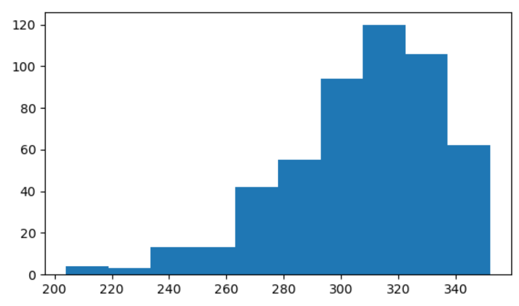
An individual's knapsack mutation is done by setting a gene to its alternative value with some probability. In the mutation, the algorithm also enforces legality. As before, an illegal solution means the mutation will not be done . Mutation can damage solutions since it is not informed by selected blocks beforehand. Knapsack mutation poses a direct trade-off between preservation and exploration, so it is expected that the mutation rate can't be set to high without consequences.

1.2 Experiments

The following settings are used to enable the hypothesis described above:

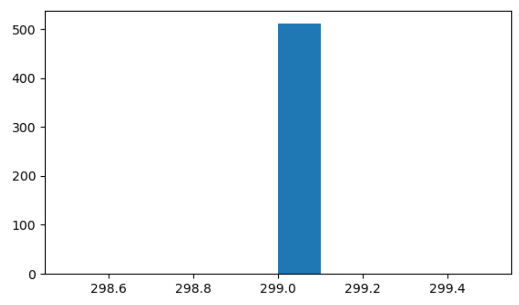
Fixed:

- number of items: 32
- maximum sum of weights: 64
- population size: 512
- maximum generations: 10



```
problem: knapsack            temperature is: 2
generation is: 5             c_prob: 0.8
population_size: 512         m_prob: 0.1
```

Figure 1: initial pop



```
problem: knapsack            temperature is: 2
generation is: 10            c_prob: 0.8
population_size: 512         m_prob: 0
```

Figure 2: no mutation

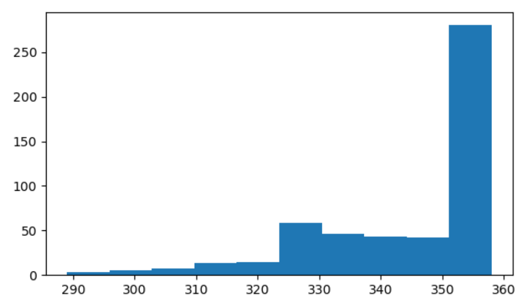
Variable:

- mutation rate
- temperature

1.3 Results

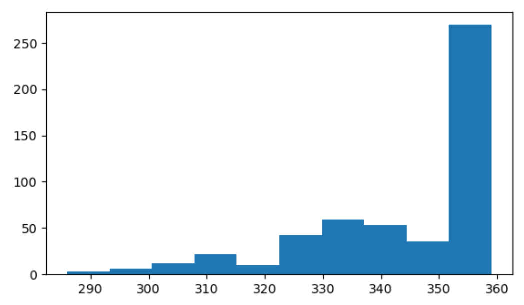
1.4 Discussion

The intuition that the fixed point recombination would not cause instability was correct. setting the crossover rate at 1 Figure 7 gives the highest value solution in generation 5, which is the fastest to reach this of all settings. A higher mutation rate Figure 8 on a relatively long genome seems to degrade too many solutions. This also conforms with the expectations of mutation. There is a $(1 - MR)^n$ chance that a solution stays the same after mutation. Improving a max weight solution takes turning off one or more genes and turning on one or more high rewarding genes. The feasibility filter does not allow temporarily having overweight solutions. With too much elitism, the solutions



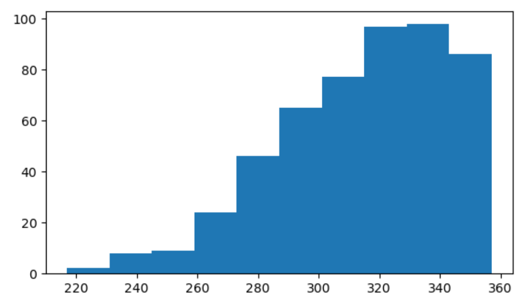
```
problem: knapsack            temperature is: 2
generation is: 10            c_prob: 0.8
population_size: 512         m_prob: 0.03125
```

Figure 3: small mutation



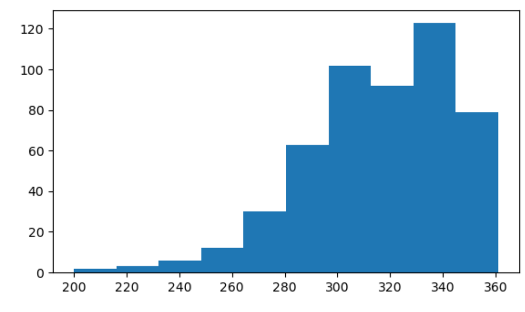
```
problem: knapsack            temperature is: 2
generation is: 20            c_prob: 0.8
population_size: 512         m_prob: 0.03125
```

Figure 4: small mutation on last iteration



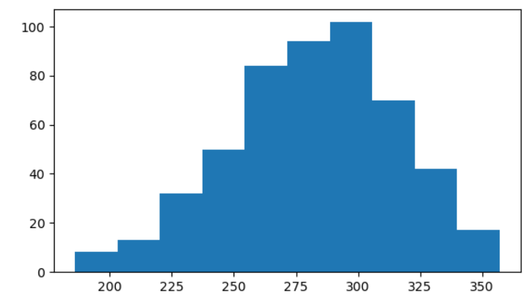
```
problem: knapsack            temperature is: 1
generation is: 10            c_prob: 0.8
population_size: 512         m_prob: 0.1
```

Figure 5: 10% mutation in gen 10



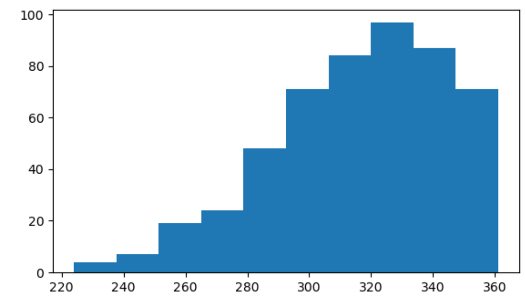
problem: knapsack
generation is: 20
population_size: 512
temperature is: 2
c_prob: 0.8
m_prob: 0.1

Figure 6: 10% mutation in gen 10



problem: knapsack
generation is: 15
population_size: 512
temperature is: 4
c_prob: 1
m_prob: 0.2

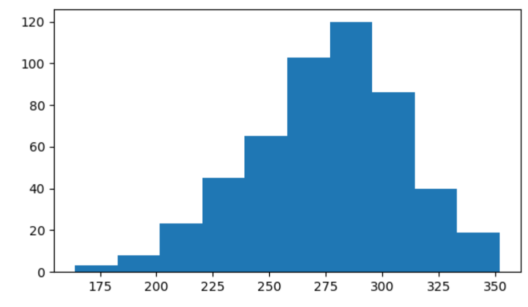
Figure 8: 20% mutation 100% crossover



problem: knapsack
generation is: 10
population_size: 512
temperature is: 4
c_prob: 1
m_prob: 0.1

Figure 7: 10% mutation 100% crossover

that don't use up the weight budget might not survive, which would cause the algorithm to get stuck without help from a crossover. Even at low temperature Figure9, the selection pressure can't cope with excessive mutation, and optima can easily be lost. As expected, a zero mutation rate Figure 2combined with fixed point recombination gives no information to the gene pool and can therefore not find better solutions unless the parts already existed in the initial population. It appeared that the mild recombination of fixed point crossover is sufficient to improve the population as long as it is combined with a form of mutation to introduce new information. Faster convergence might be achieved with initialisation heuristics that favour a high reward to weight ratio. Additionally, the initialisation could be informed by the weight budget parameter to produce relatively more feasible starting solutions and reduce the overhead from rejection.



problem: knapsack
generation is: 15
population_size: 512
temperature is: 1
c_prob: 1
m_prob: 0.2

Figure 9: 20% mutation 100% crossover low temp

2 Travelling Salesman Problem

2.1 Approach

2.1.1 Genotype Representation and Fitness

The TSP solution is represented as a list of cities to visit sequentially. The representation is chosen for compatibility with the desired crossover and mutation operators.

2.1.2 Initialisation

All cities in the experimentation instance lie equidistantly on the unit circle. Individuals are initialised with a random order of visiting cities.

2.1.3 Selection

Tournament selection is used for the TSP. The advantage of tournament selection for the travelling salesmen problem is the indifference to the magnitude and sign of the fitness value. In the implementation, only the size of k is varied. It is expected that family and mutation elitism will allow for different tournament sizes to be effective.

2.1.4 Crossover

With the use of the order crossover operator (Hussain et al.) A sequence or random length from one parent is fixed in place. The remaining slots from the second cut of point get the remaining cities not in the sequence in the order they appear in the other parent. The cut points are the same for both children in the crossover. The option is built in to enable family elitism where only the two best of four survive. This elitism seems advantageous in TSP as we can crossover aggressively without any downside to the children's fitness value. It is expected that a rate of 1 will be performing best,

2.1.5 Mutation

By the use of inversion, a sub tour happens in reverse (Pullan) . Just like with family elitism, the option is built-in only to accept the mutation if it is an improvement. If there is a search path that requires an inversion to a strictly worse solution to reach the global optimum, the algorithm could get stuck, because it won't accept this change. However, there is a crossover in place that changes the tour differently, so this shouldn't be a problem. The mutation rate can be cranked up to 1 without risking losing the best solution by having this mutation elitism enabled.

2.2 Experiments

One experiment of interest was to see whether it is beneficial to have crossover rates of 1 with family elitism or relatively lower rates with or without elitism. Likewise, it will be tested for the mutation, whether a rate of 1 and only accepting the solution if it is better than the non-mutated individual. Additionally, selection pressure will be varied by adjusting the tournament size. Note that the algorithm can be terminated once a solution appears with all cities ordered as 0,1,2,. . . ,n-1,0 or 0,n-1,n-2,. . . ,1,0 as this is the known optimum for traversing points on a unit circle. Indeed the algorithm stops once it reaches this solution.

Fixed:

- number of cities(on the unit circle): 50
- population size: 512
- maximum generations: 100

Variable:

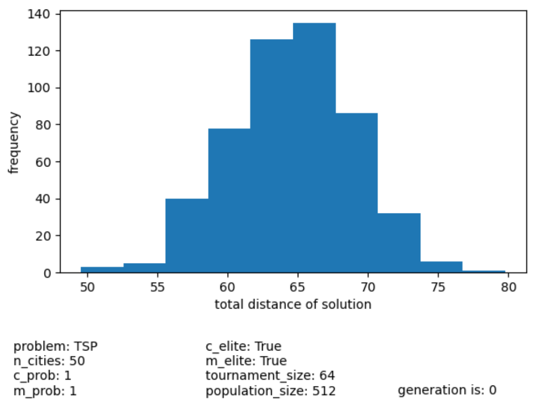
- mutation rate and elitism
- crossover rate and elitism
- tournament size

2.3 Results

Each generations histogram plots are created before selection. The line plots are created at termination. Termination takes place whenever either the best solution is found, or 100 generations passed.

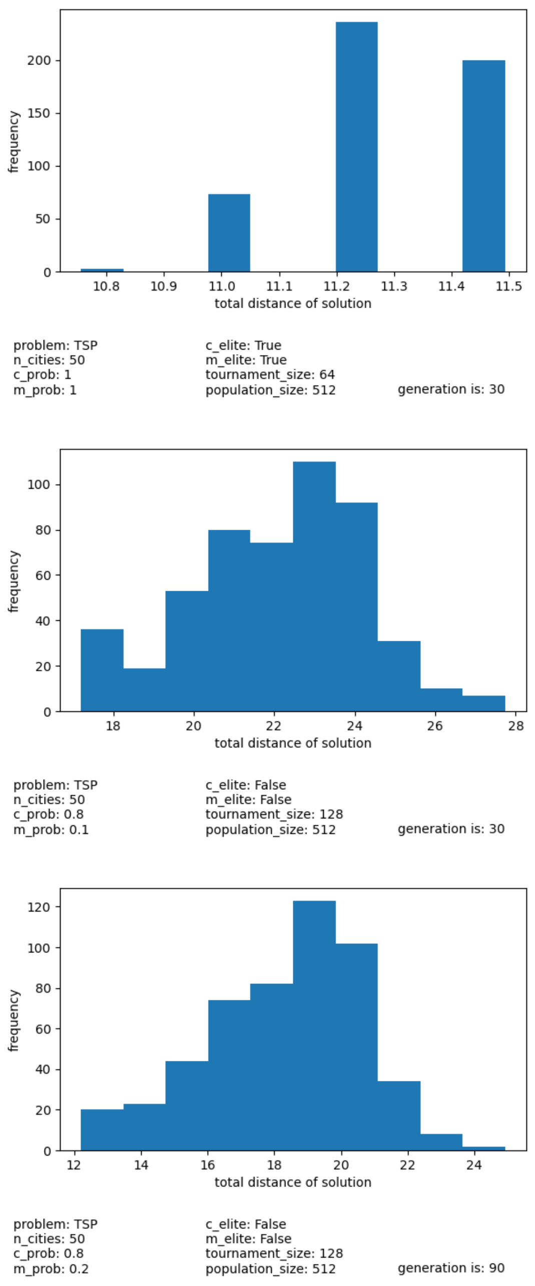
2.4 Discussion

The initial population in Figure 10 seems roughly normally distributed as expected. The Fastest convergence among the experiments happen when mutation and crossover elitism are enabled, their rates at 1 and the tournament size at a quarter of the population see Figure 14. The hypothesis that the rate of crossover and mutation can be set to maximum in the presence of family and mutation elitism seems to hold. The tournament size is large but not as big as the population Figure 14; if that where the case, the crossover operator would be rendered ineffective. When the tournament size is n/2, there is very little diversity, as the



```
problem: TSP            c_elite: True
n_cities: 50            m_elite: True
c_prob: 1               tournament_size: 64
m_prob: 1               population_size: 512        generation is: 0
```

Figure 10: initial population

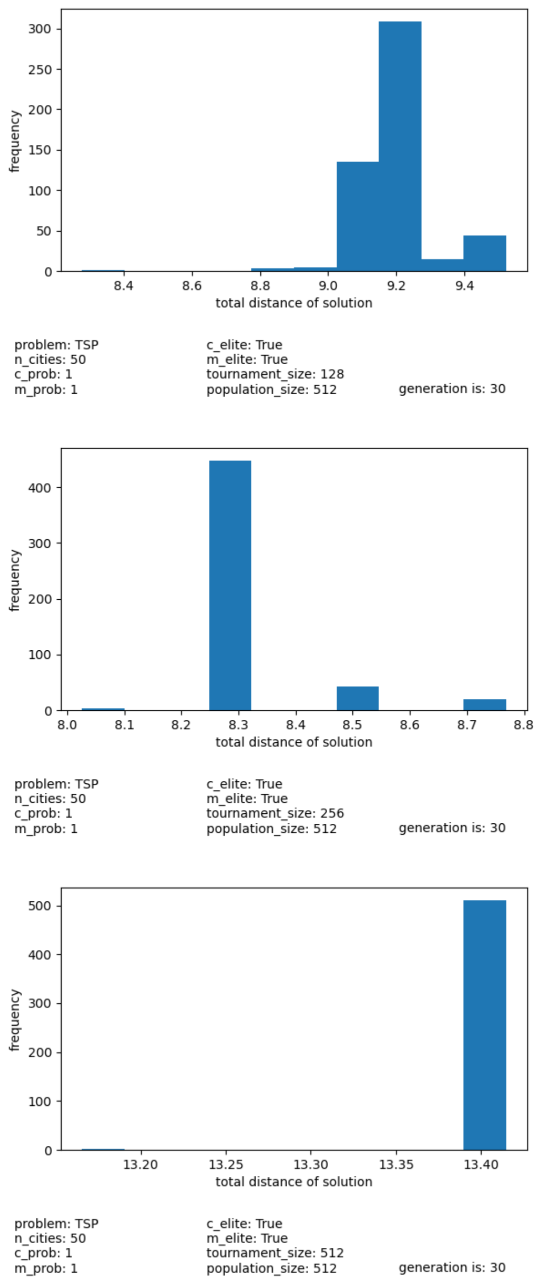


```
problem: TSP            c_elite: True
n_cities: 50            m_elite: True
c_prob: 1               tournament_size: 64
m_prob: 1               population_size: 512        generation is: 30
```

```
problem: TSP            c_elite: False
n_cities: 50            m_elite: False
c_prob: 0.8             tournament_size: 128
m_prob: 0.1             population_size: 512        generation is: 30
```

```
problem: TSP            c_elite: False
n_cities: 50            m_elite: False
c_prob: 0.8             tournament_size: 128
m_prob: 0.2             population_size: 512        generation is: 90
```

Figure 11: no elitism crossover and mutation

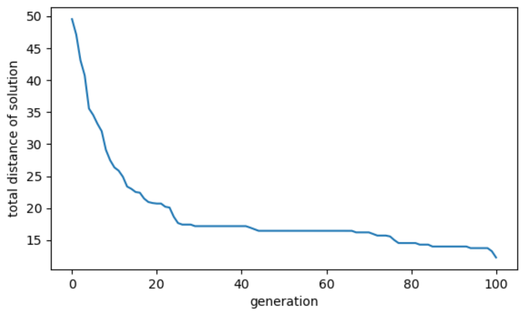


```
problem: TSP            c_elite: True
n_cities: 50            m_elite: True
c_prob: 1               tournament_size: 128
m_prob: 1               population_size: 512        generation is: 30
```

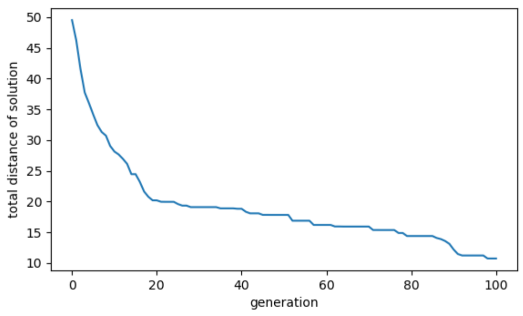
```
problem: TSP            c_elite: True
n_cities: 50            m_elite: True
c_prob: 1               tournament_size: 256
m_prob: 1               population_size: 512        generation is: 30
```

```
problem: TSP            c_elite: True
n_cities: 50            m_elite: True
c_prob: 1               tournament_size: 512
m_prob: 1               population_size: 512        generation is: 30
```

Figure 12: different temperatures with family and mutation elitism

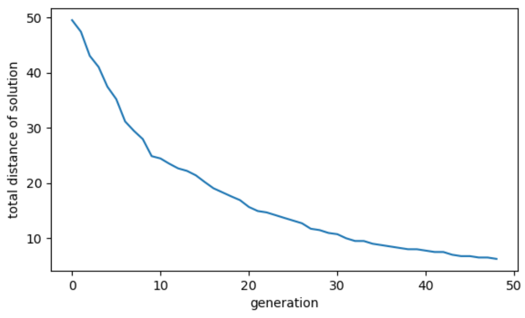


problem = TSP	c_elite: False
n_cities: 50	m_elite: False
c_prob: 0.8	tournament_size: 128
m_prob: 0.1	population_size: 512

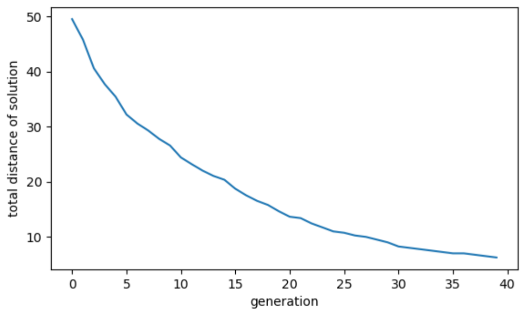


problem = TSP	c_elite: False
n_cities: 50	m_elite: False
c_prob: 0.8	tournament_size: 128
m_prob: 0.2	population_size: 512

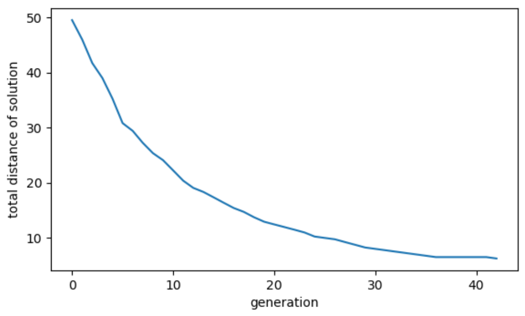
Figure 13: fitness progressions no elitism



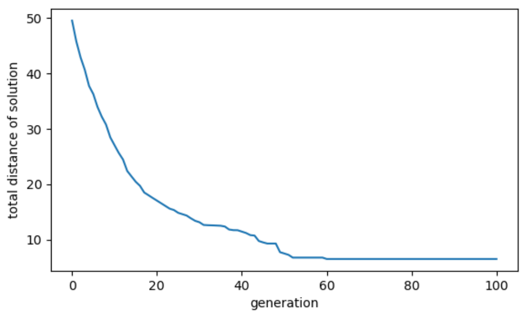
problem = TSP	c_elite: True
n_cities: 50	m_elite: True
c_prob: 1	tournament_size: 64
m_prob: 1	population_size: 512



problem = TSP	c_elite: True
n_cities: 50	m_elite: True
c_prob: 1	tournament_size: 128
m_prob: 1	population_size: 512



problem = TSP	c_elite: True
n_cities: 50	m_elite: True
c_prob: 1	tournament_size: 256
m_prob: 1	population_size: 512



problem = TSP	c_elite: True
n_cities: 50	m_elite: True
c_prob: 1	tournament_size: 512
m_prob: 1	population_size: 512

Figure 14: fitness progressions with elitism

histograms show in Figure 12. With full mutation, the algorithm has the best chance to find beneficial inversions. Since it will discard any non-beneficial inversion mutation, there is no risk of obtaining a worse solution; this is visible in the non-increasing best distance in each iteration in Figures 14. That the approach works so well on the unit circle instance is likely helped by the fact that there are no locally optimal connections that are not globally optimal. If a city has found its closest neighbours, the algorithm does not need to break this connection to find something better.

References

Abid Hussain, Yousaf Shad Muhammad, M. Nauman Sajid, Ijaz Hussain, Alaa Mohamd Shoukry, and Showkat Gani. Genetic algorithm for traveling salesman problem with modified cycle crossover operator. 2017:1–7.

W. Pullan. Adapting the genetic algorithm to the travelling salesman problem. In *The 2003 Congress on Evolutionary Computation, 2003. CEC '03.*, volume 2, pages 1029–1035. IEEE.