Institute of
Pervasive Computing

Social Graphs

Algorithms and Data Structures 2, 340300
Lecture – 2023W
Univ.-Prof. Dr. Alois Ferscha, teaching@pervasive.jku.at

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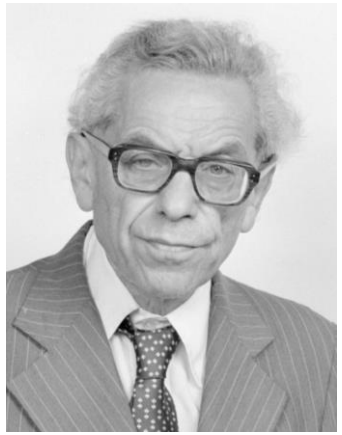
4040 Linz, Austria
jku.at

Networks

Erdös Number: “Collaborative Distance” among mathematical authors



Erdős Pál
(1913-1996)

Leitseite | Präferenzen | **Freie Werkzeuge** | Allgemeines | Librarians | Reviewers | Terms of Use | Blog

AMERICAN MATHEMATICAL SOCIETY MATHSCINET MATHEMATICAL REVIEWS

Johannes Kepler University

MSC Suche | Abstand zwischen Autoren | Aktuelle Zeitschriften | Aktuelle Publikationen

ISSN 2167-5163

MR Erdos Number = 3

Heinz W. Engl	coauthored with	Charles W. Groetsch	MR1381290
Charles W. Groetsch	coauthored with	Oved Shisha	MR0380193
Oved Shisha	coauthored with	Paul Erdős[1]	MR0775969

Change First Author | Change Second Author | New Search

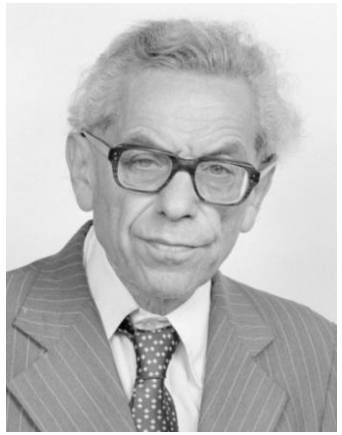
Freie Werkzeuge

AMS American Mathematical Society

Statistics on Mathematical Collaboration, 1903-2016

	#Laureates	#Erdős	%Erdős	Min	Max	Average	Median
Nobel Physics	200	159	79.50%	2	12	5.63	5
Fields Medal	56	56	100.0%	2	6	3.36	3
Nobel Economics	76	47	61.84%	2	8	4.11	4
Nobel Medicine	210	58	27.62%	3	12	5.50	5
Nobel Chemistry	172	42	24.42%	3	10	5.48	5

„Small-World“ Networks



Erdős Pál
(1913-1996)

Small-world network

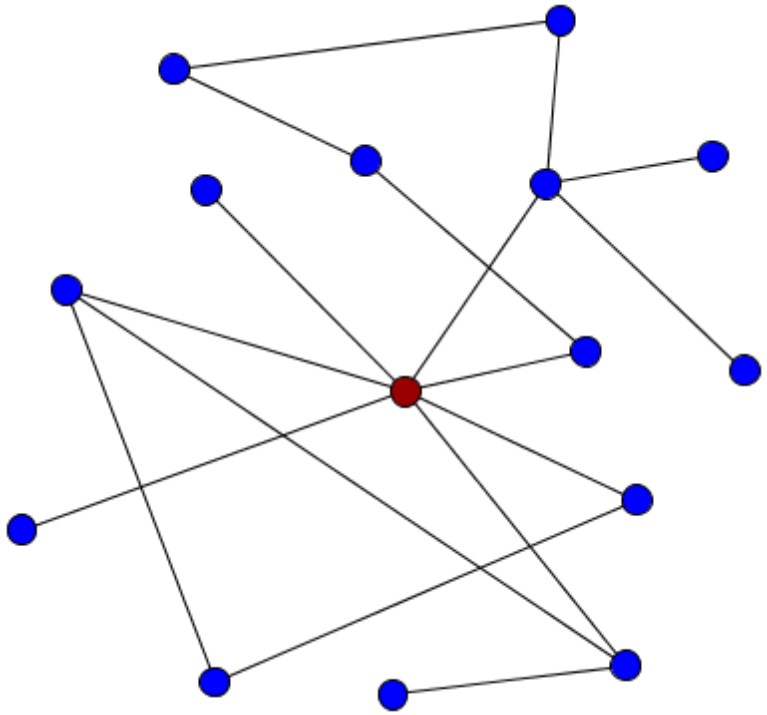
- **most nodes** are **not neighbors** of one another
- but **most nodes** can be **reached from every** other by a **small number** of **hops** or steps.

distance L between two randomly chosen nodes **grows** proportionally to the **logarithm** of the **number of nodes N** in the network (=**society**)

human society is a small-world-type network

characterized by short path-lengths.

Stanley Milgram

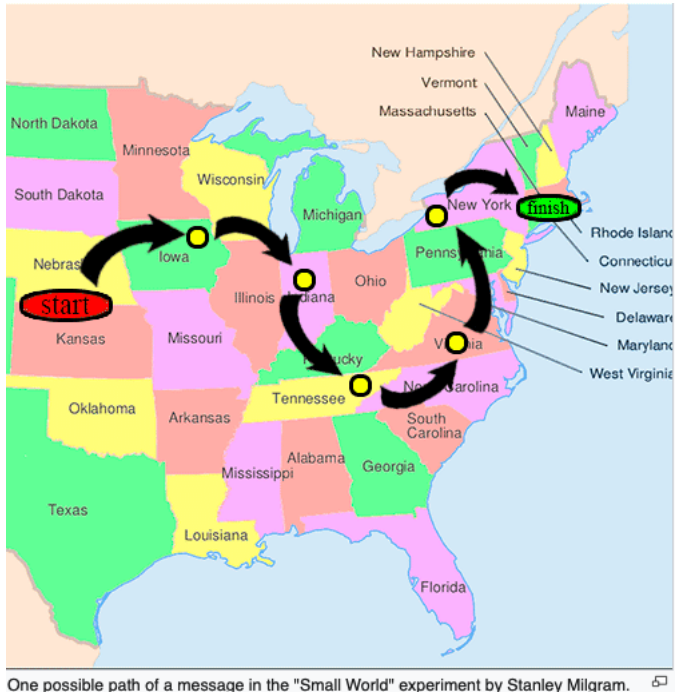


$$L \propto \log N$$

„Small-World“ Networks

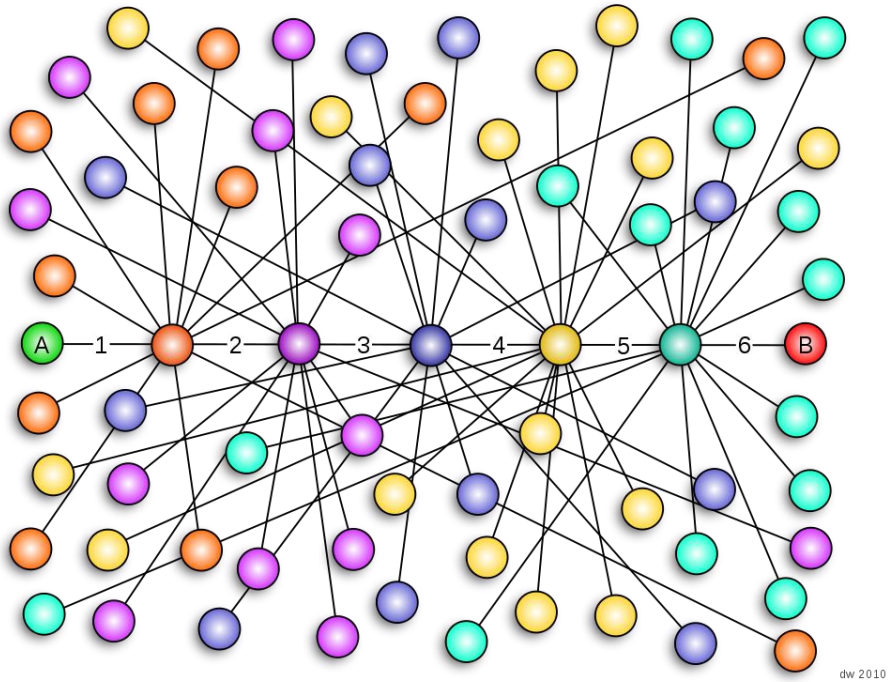
Stanley Milgram
(1933-1984)

1960
„lost letter technique“



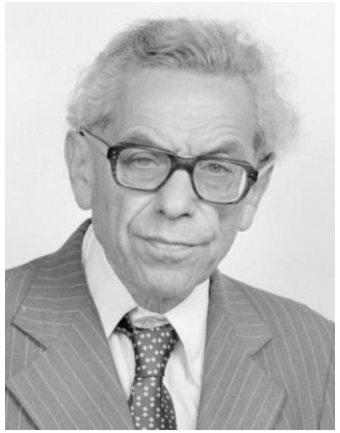
One possible path of a message in the "Small World" experiment by Stanley Milgram.

Took U.S. cities of
Omaha, Nebraska, and Wichita, Kansas,
to be the **starting points**
and Boston, Massachusetts,
to be the **end point**
of a **chain of correspondence**.



The "six degrees of separation" model

Random Networks (1959)

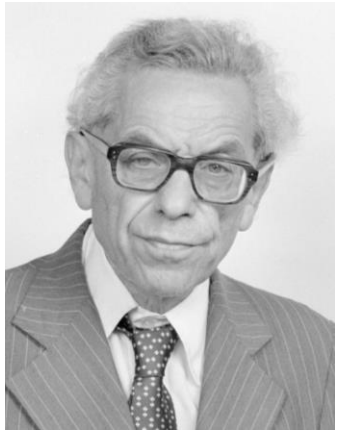


Erdős Pál
(1913-1996)

Alfréd Rényi
(1921-1970)

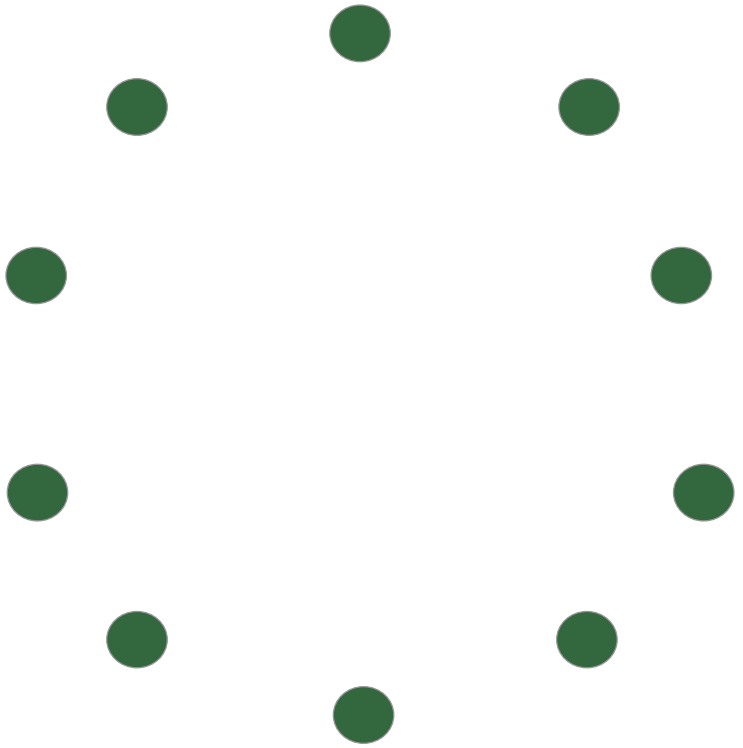
How does **nature** choose to connect nodes and connections into a network?

Random Networks (1959)



Erdős Pál
(1913-1996)

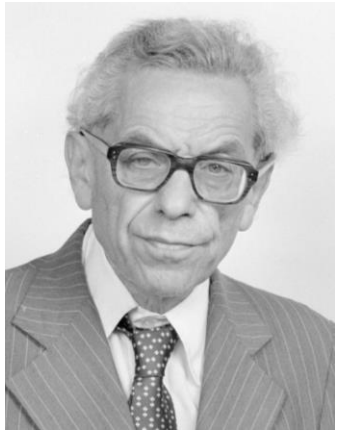
Alfréd Rényi
(1921-1970)



Connect with probability p

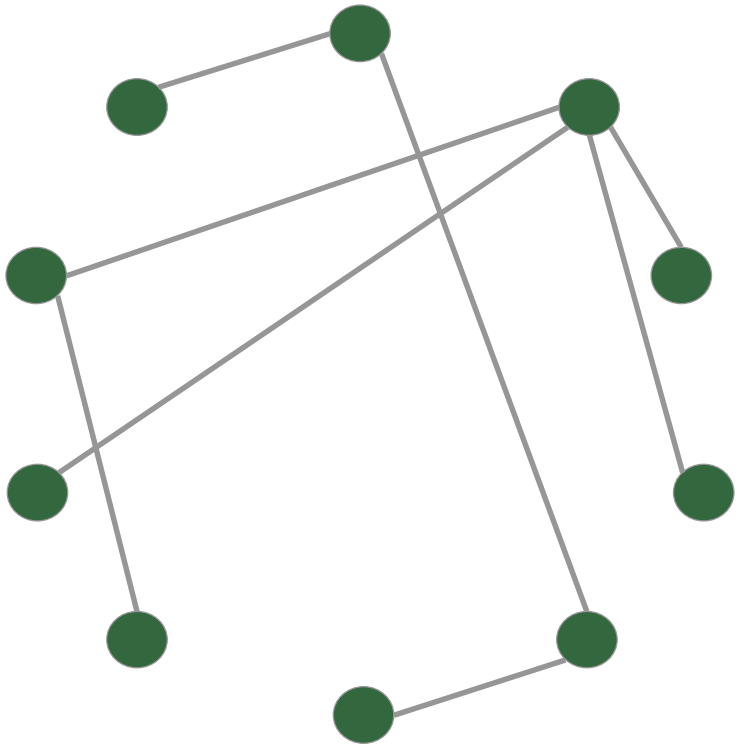
p=1/6 N=10

Random Networks (1959)



Erdős Pál
(1913-1996)

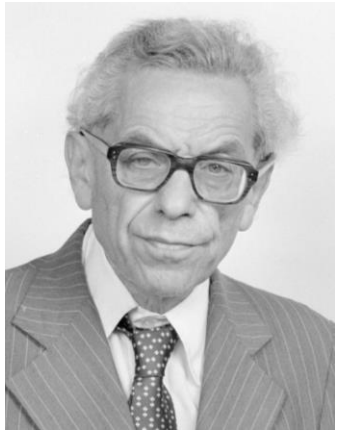
Alfréd Rényi
(1921-1970)



Connect with probability *p*

p=1/6 N=10
<k> ~ 1.5

Random Networks (1959)



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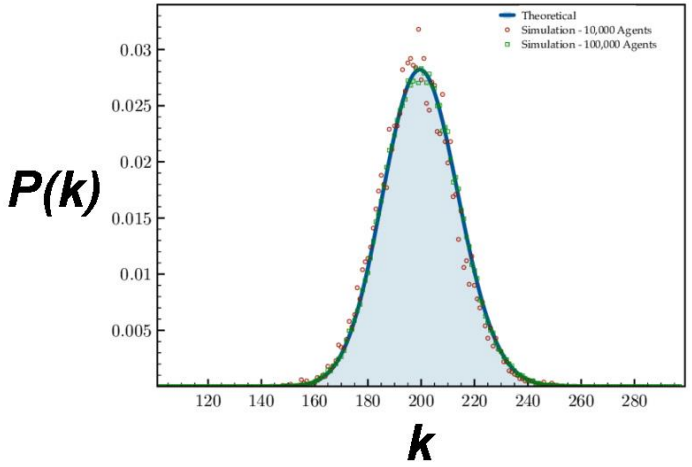


Degree distribution
(in very large networks)

Connect with probability *p*

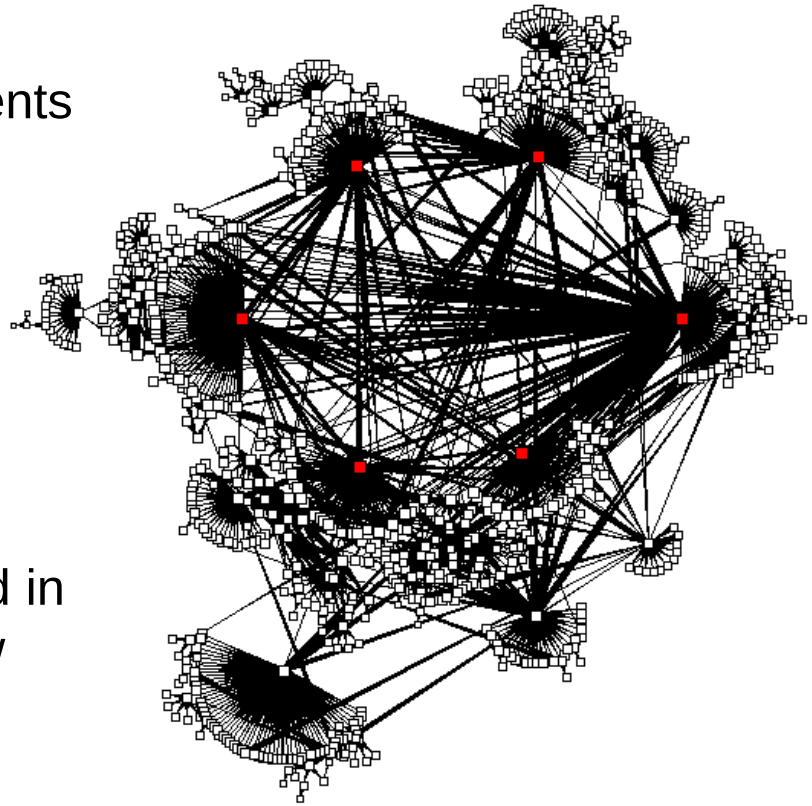
0 < p < 1/N disconnected trees *O(log n)*
p = 1/N large connected component
p > 1/N connected giant component

The emergence of a network is NOT a GRADUAL process !

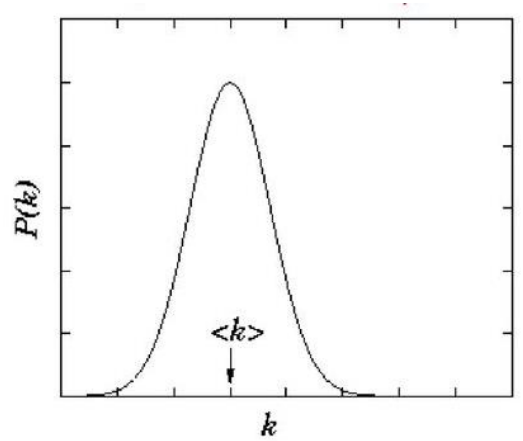
World Wide Web

Nodes: WWW documents
Links: URL links

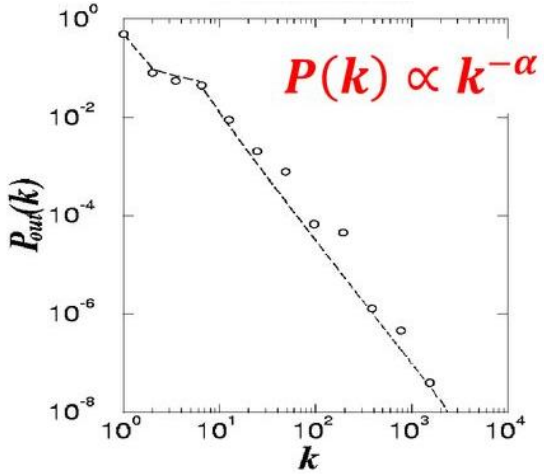
Over 1 Trillion documents



Collect all URL‘s found in a document and follow them recursively



Expected



Found in data

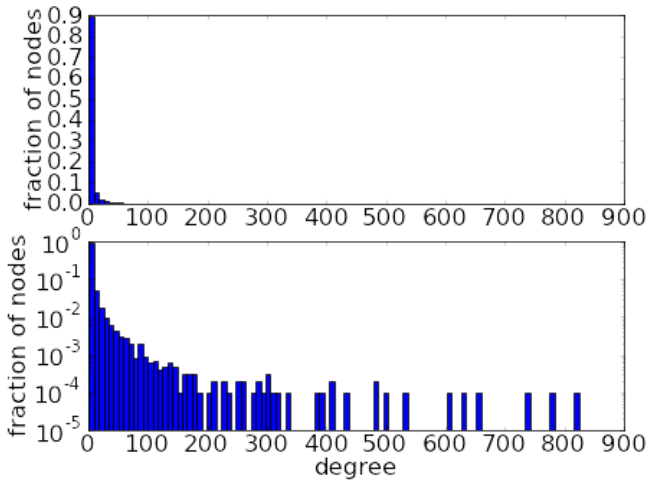
Albert, Jeong & Barabási, *Nature*, 401 130 (1999).

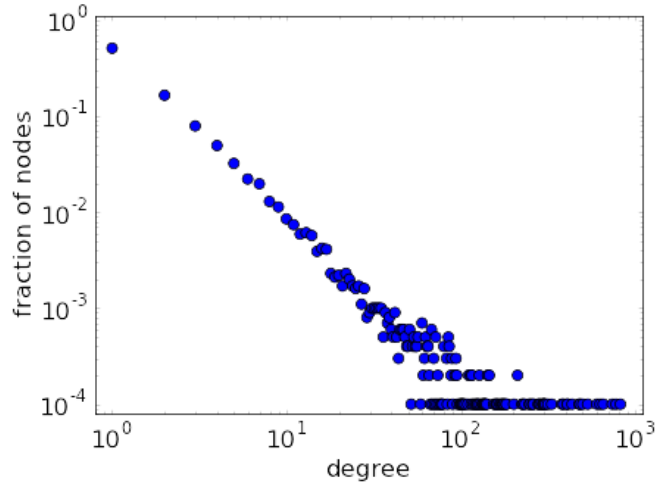
Scale-Free Networks (1999)

Albert-László Barabási
(1967-)

Scale free networks
(large hubs network)

$$P_{\deg}(k) \propto k^{-\gamma}$$





$N = 10,000 \qquad \gamma = 2$

average degree is about 7
3/4 of the nodes have a degree of 3 or less

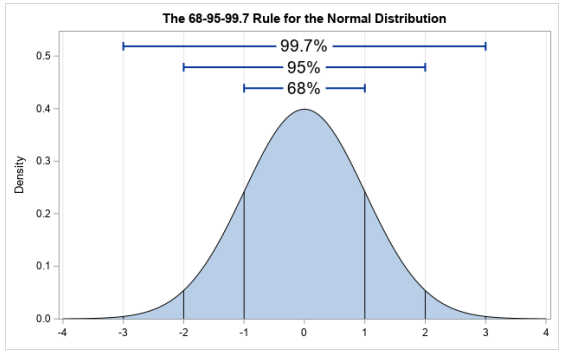
Albert, Jeong & Barabási, *Nature*, 401 130 (1999)

Erdős-Rényi (ER) Networks :: Barabasi-Albert (BA) Networks

Random Network

Gaussian

Degree distribution



Albert, Jeong & Barabási, *Nature*, 401 130 (1999).

Erdős-Rényi (ER) Networks :: Barabasi-Albert (BA) Networks

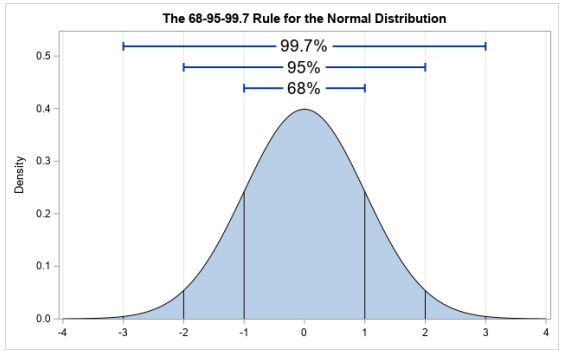
Random Network

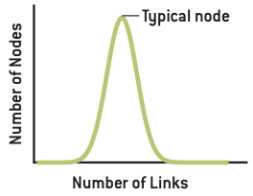
Scale-free Network

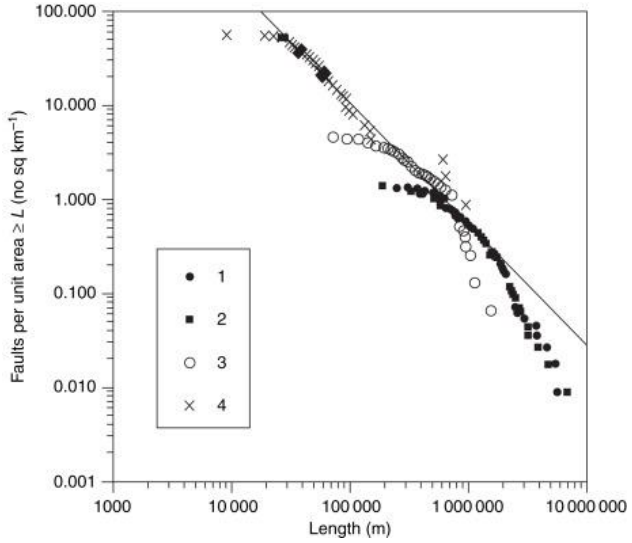
Gaussian

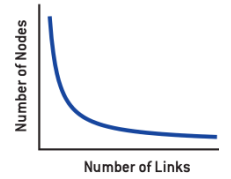
Power-Law

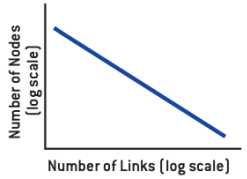
Degree distribution



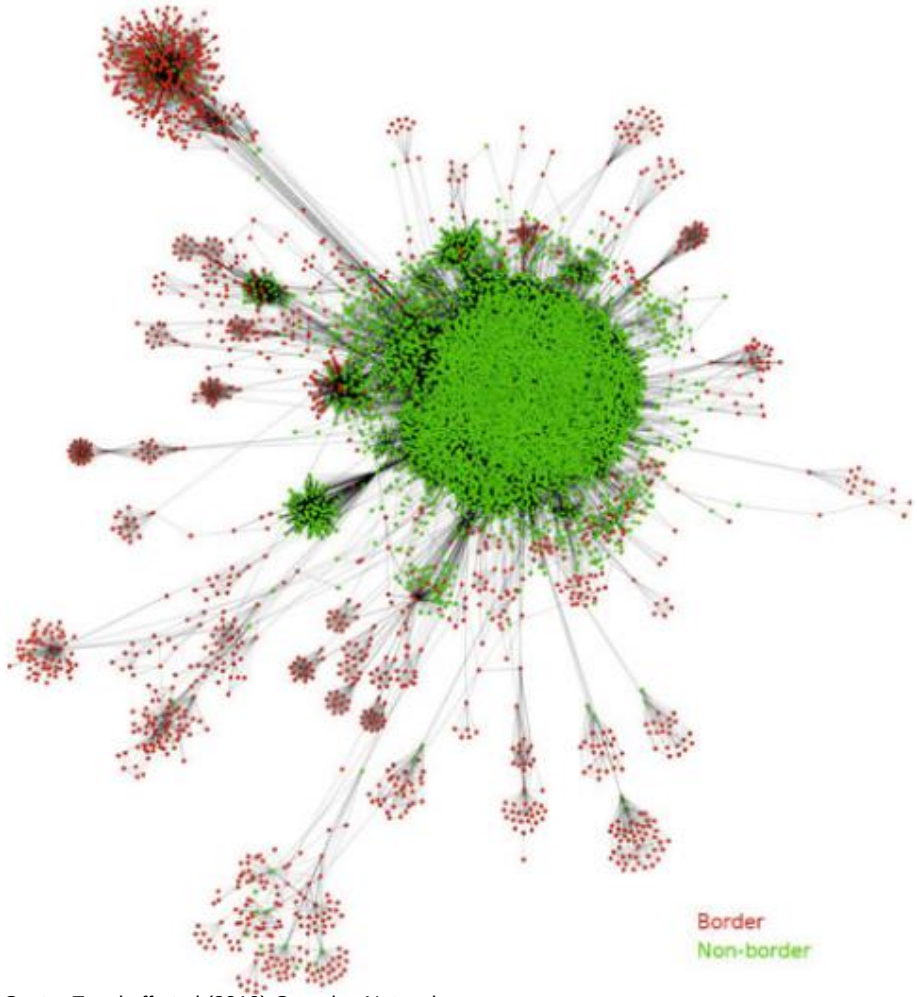






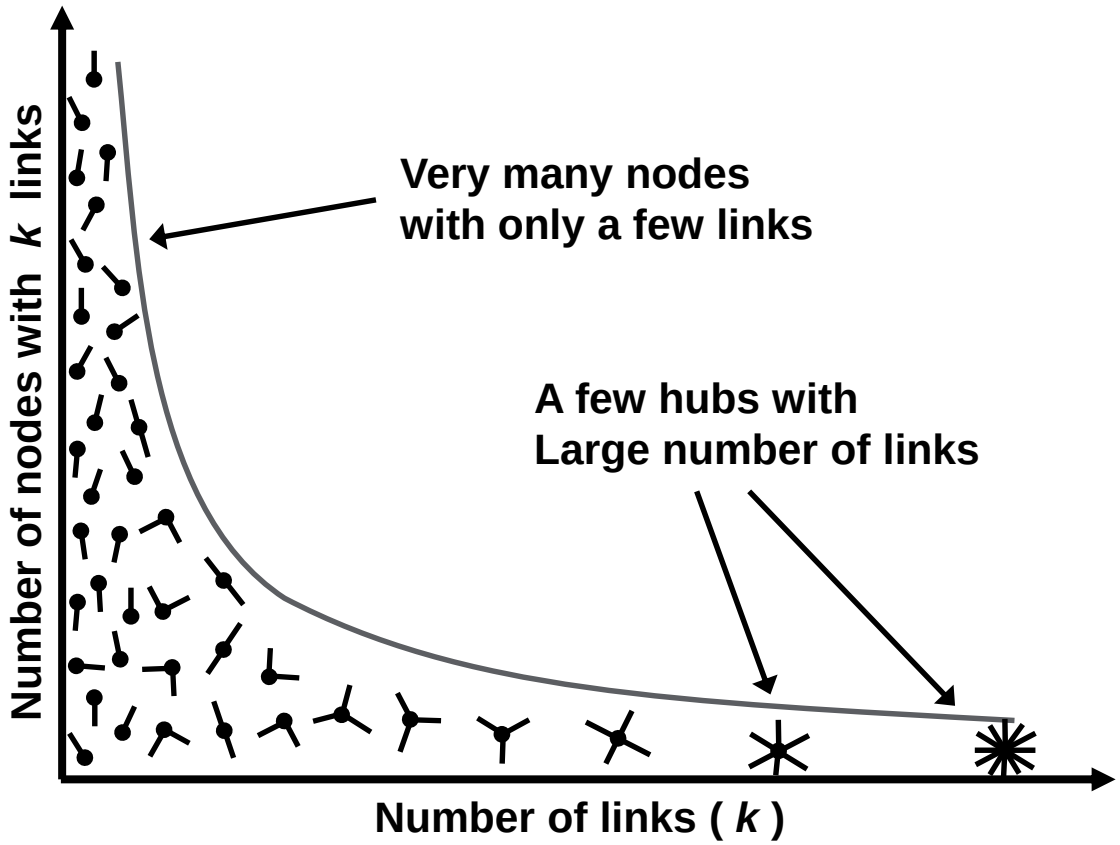


Networks held together by Hubs

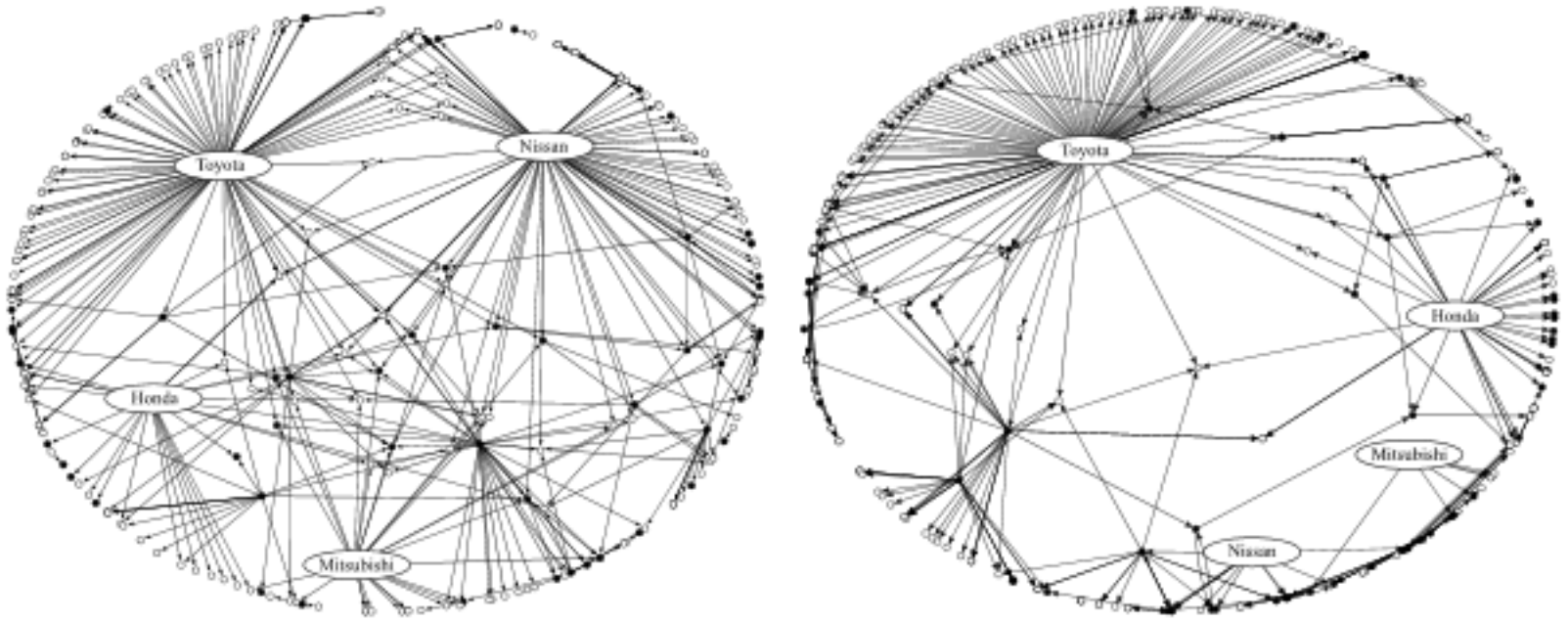


Costa, Evsukoff et al (2010) Complex Networks

Power Law Distribution



Barabasi Networks :: Examples



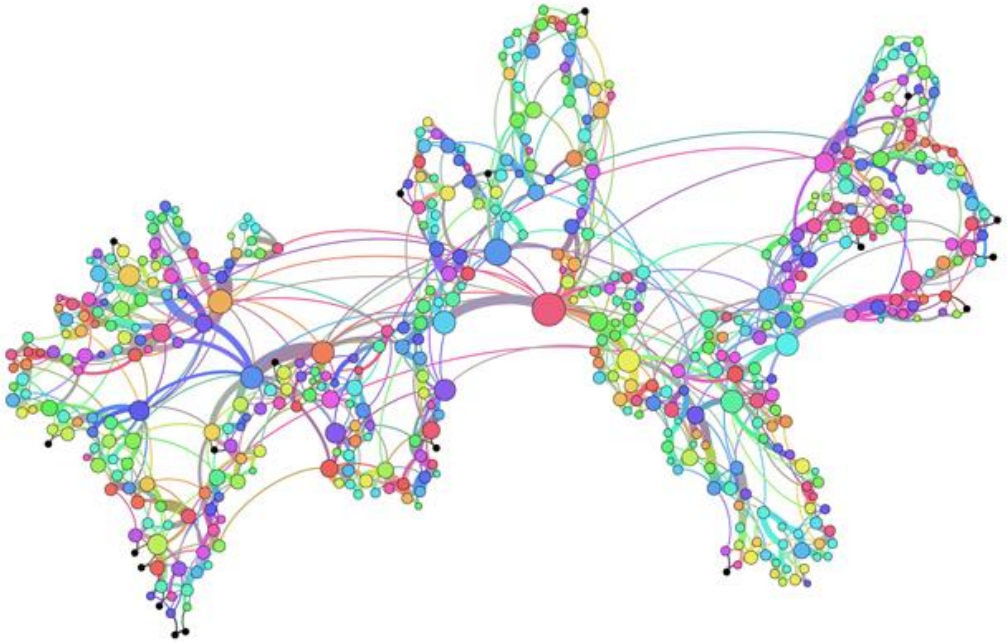
1985 **2003**

The Network Behind an Organisation :: Shareholder-Network of the Japanese Automotive Industry

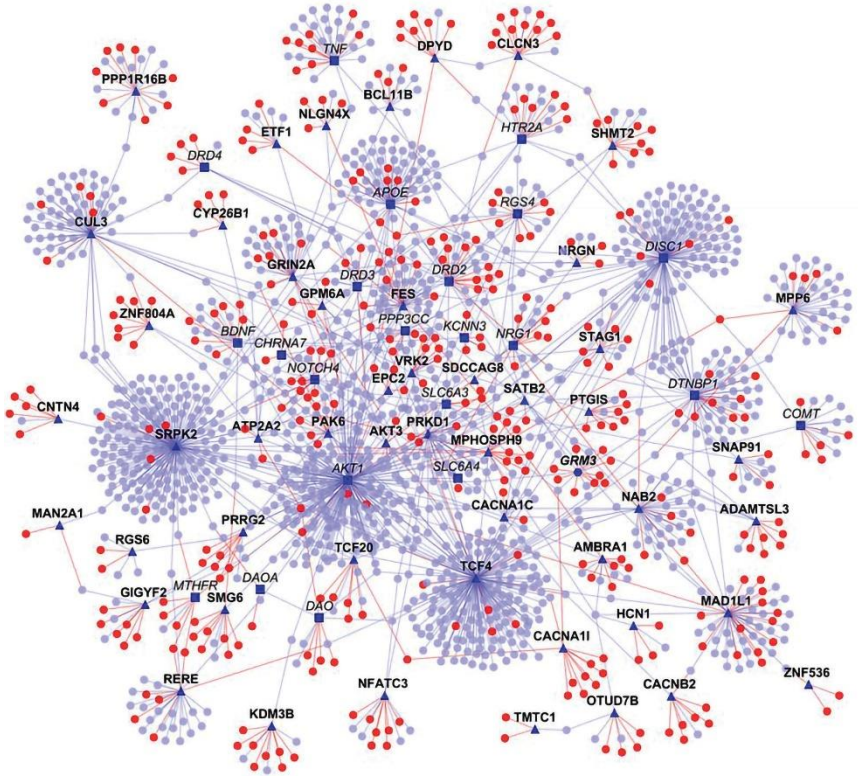
https://www.quantamagazine.org/20130904-evolution-as-opportunist/

Krischke & Röpcke (2014) Graphen und Netzwerktheorie

Barabasi Networks :: Examples



Metabolic Network

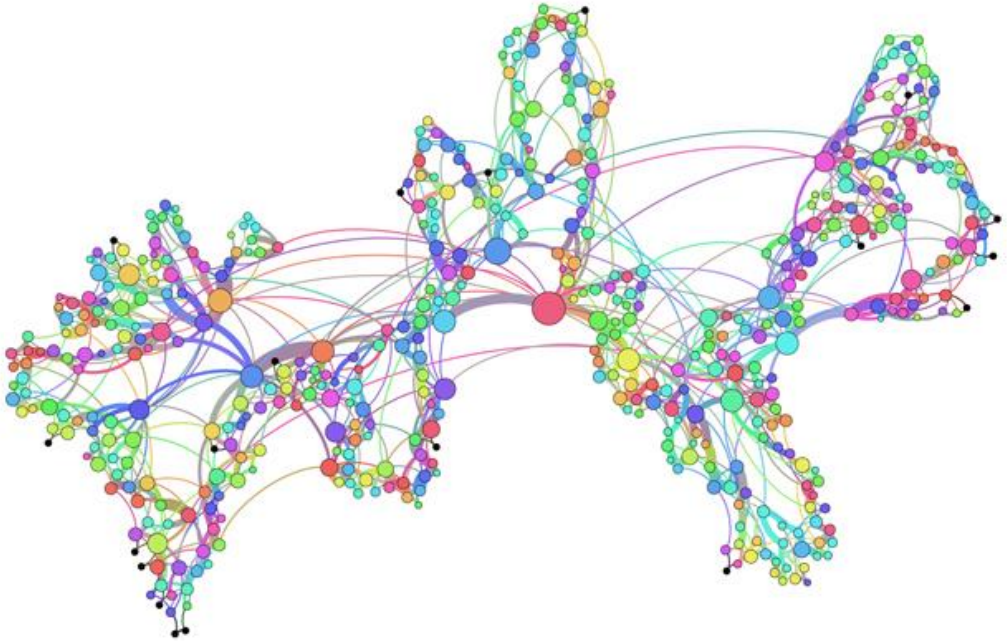


Protein Interactions

https://www.quantamagazine.org/20130904-evolution-as-opportunist/

Krischke & Röpcke (2014) Graphen und Netzwerktheorie

Barabasi Networks :: The Matter of Life



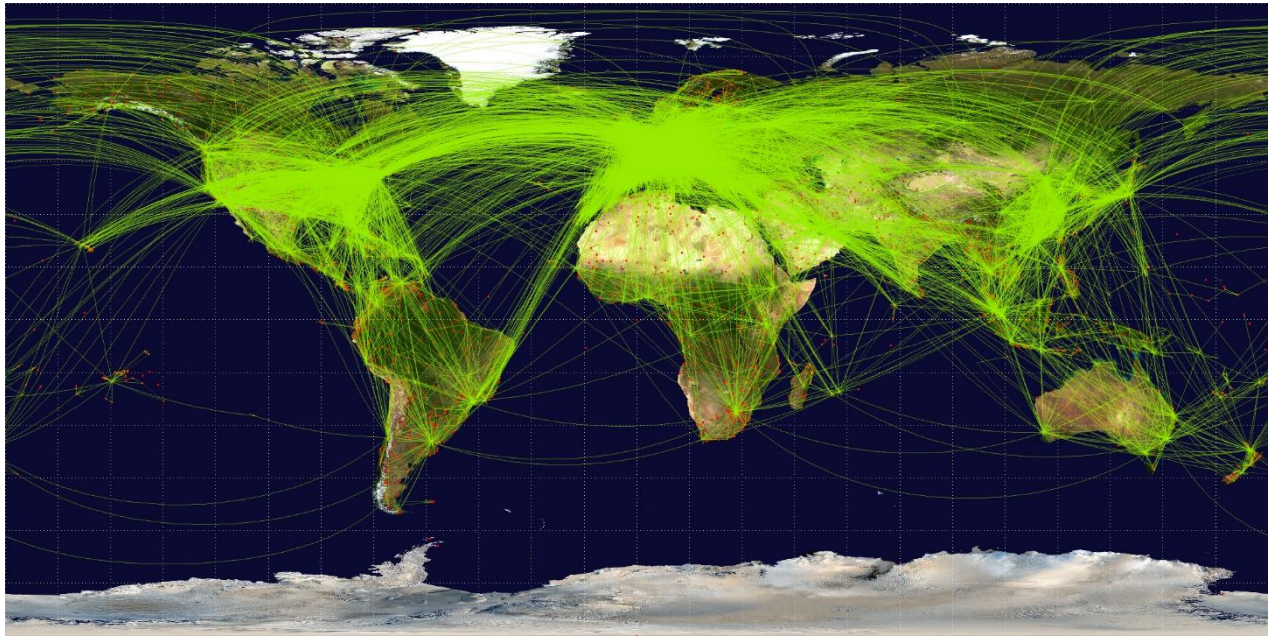
Metabolic Network

Social Network

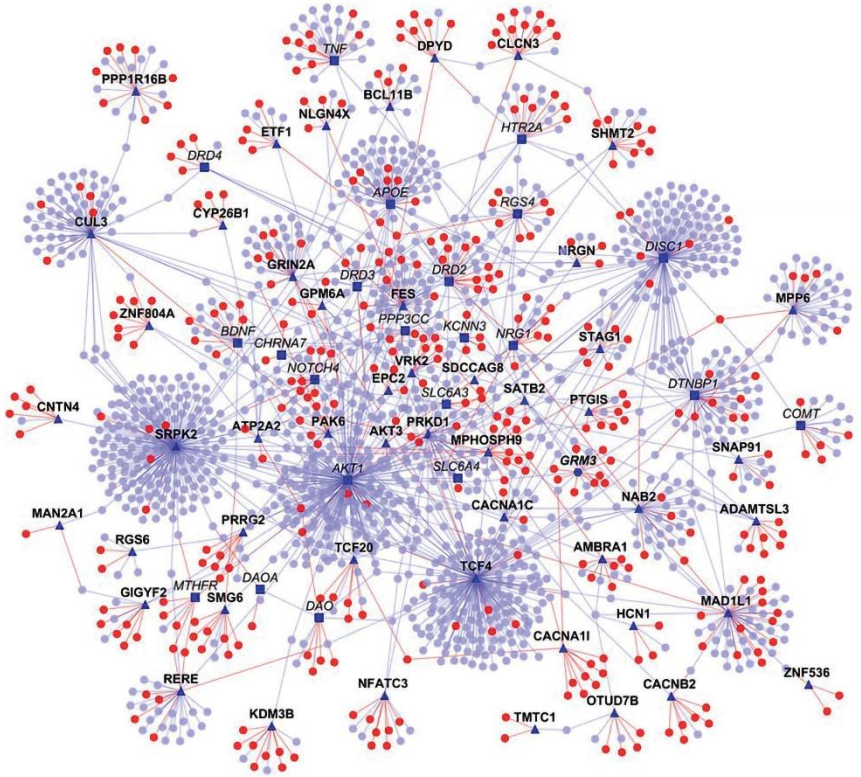
https://www.quantamagazine.org/20130904-evolution-as-opportunist/

Krischke & Röpcke (2014) Graphen und Netzwerktheorie

Barabasi Networks :: Examples



Airline Network



Protein Interactions

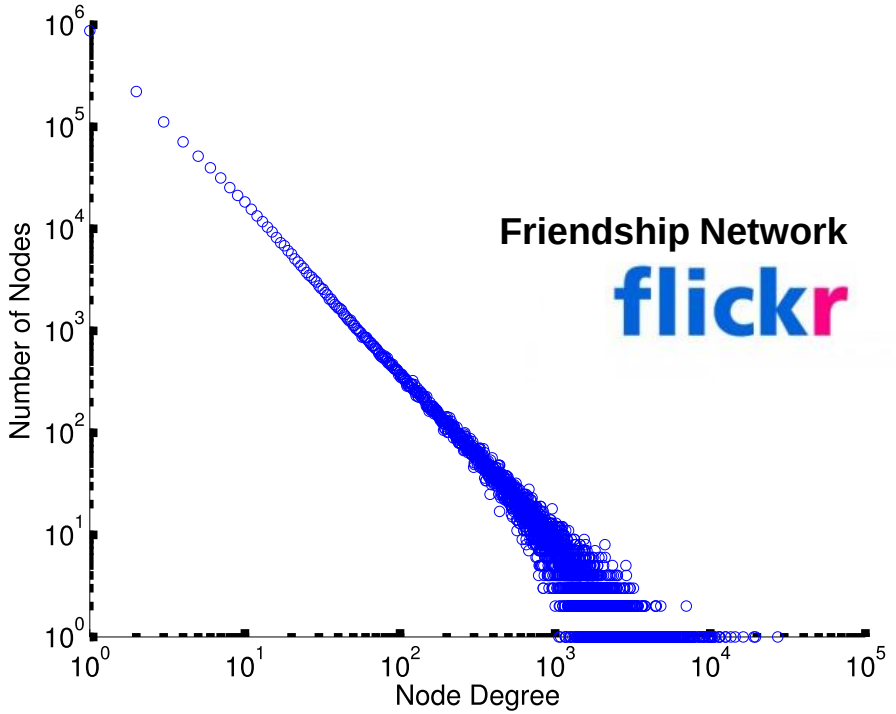
https://www.quantamagazine.org/20130904-evolution-as-opportunist/

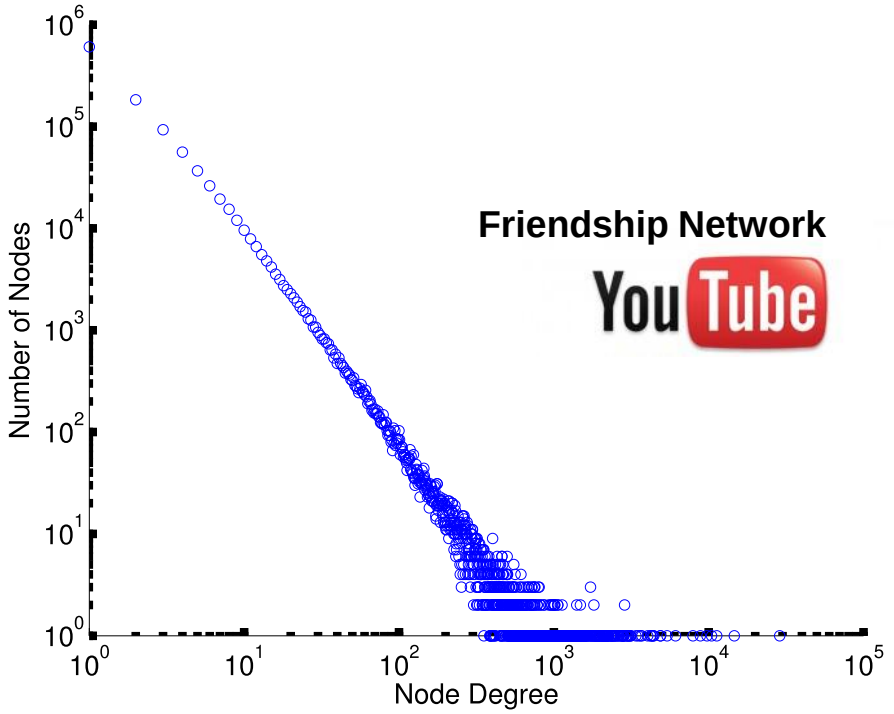
Krischke & Röpcke (2014) Graphen und Netzwerktheorie

Barabasi Networks :: Examples

Log-Log plot

Power law distribution becomes a straight line if plot is in a log-log scale

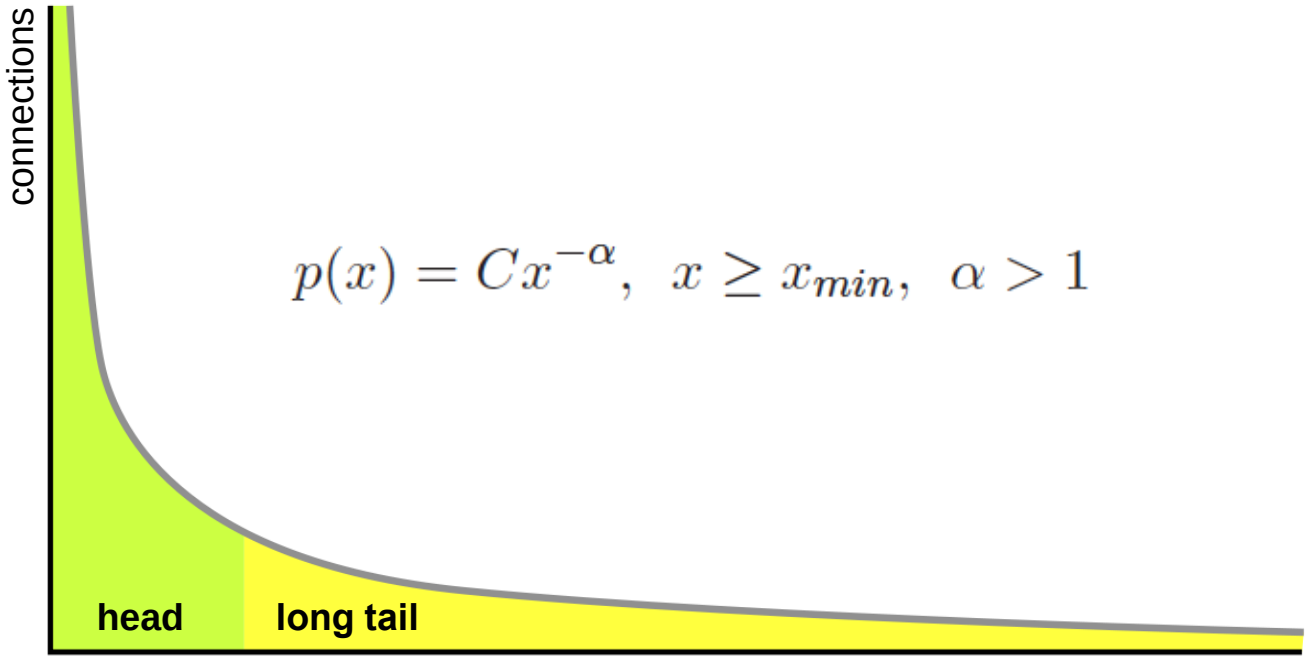




Scale-free Networks

Scale-free Distributions

Degree distribution in large-scale networks often follows a power law.



e.g., long tail distribution, scale-free distribution

An example **power law graph**, being used to demonstrate ranking of popularity.

To the right is the long tail, and to the left is the “head”, the few that dominate (also known as the 80–20 rule)

Scale-free Networks

Comparing network models

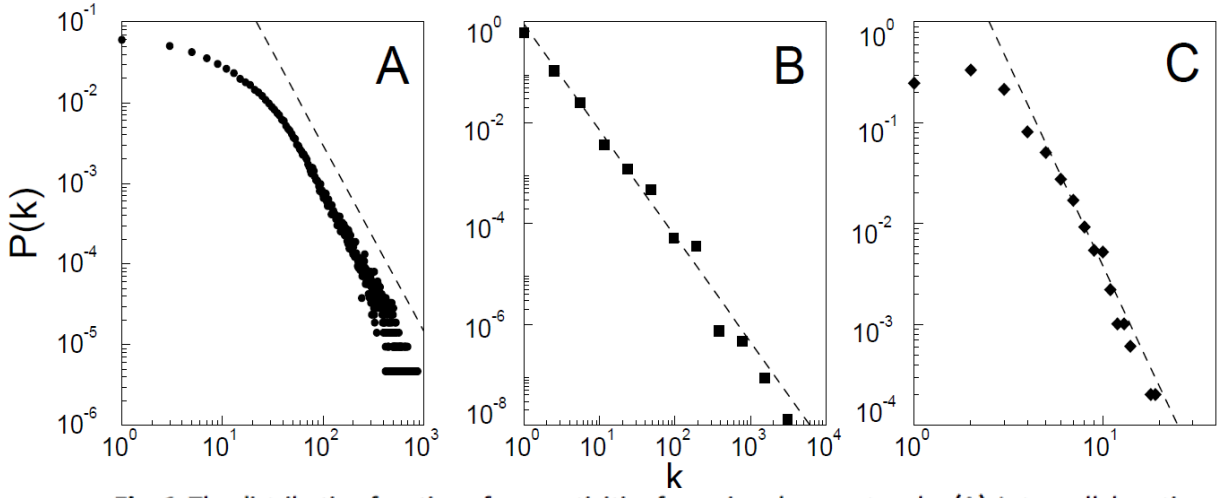
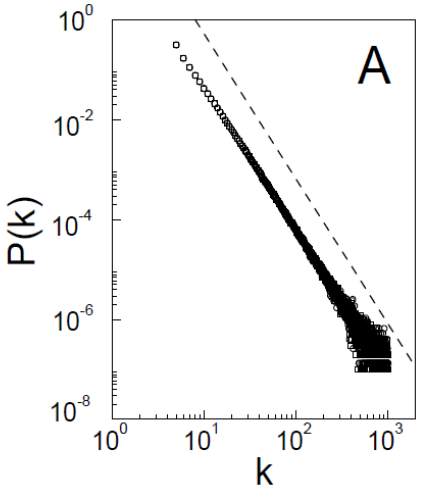


Fig. 1. The distribution function of connectivities for various large networks. **(A)** Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. **(B)** WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (6). **(C)** Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{actor} = 2.3$, (B) $\gamma_{www} = 2.1$ and (C) $\gamma_{power} = 4$.

observations over various real-word large-scale networks



The power-law connectivity distribution as obtained from the model

outcome of a network model

NETWORK	NODES	LINKS
Cellular metabolism	Molecules involved in burning food for energy	Participation in the same biochemical reaction
Hollywood	Actors	Appearance in the same movie
Internet	Routers	Optical and other physical connections
Protein regulatory network	Proteins that help to regulate a cell's activities	Interactions among proteins
Research collaborations	Scientists	Co-authorship of papers
Sexual relationships	People	Sexual contact
World Wide Web	Web pages	URLs

A.-L. Barab´asi and R. Albert. Emergence of Scaling in Random Networks. Science, 286(5439):509–512, Oct. 1999.

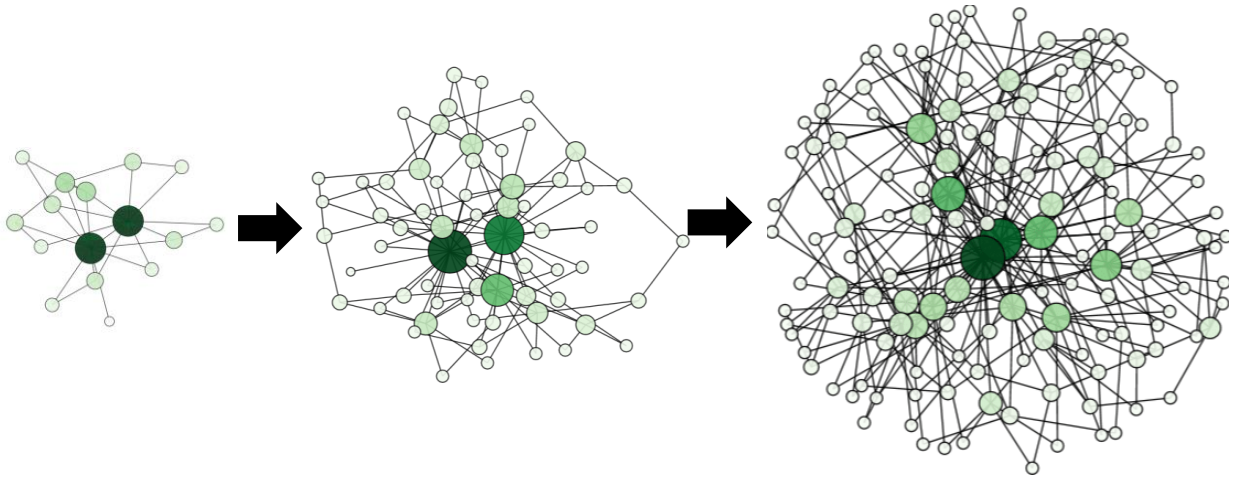
SF Networks :: Growth and Preferential Attachment

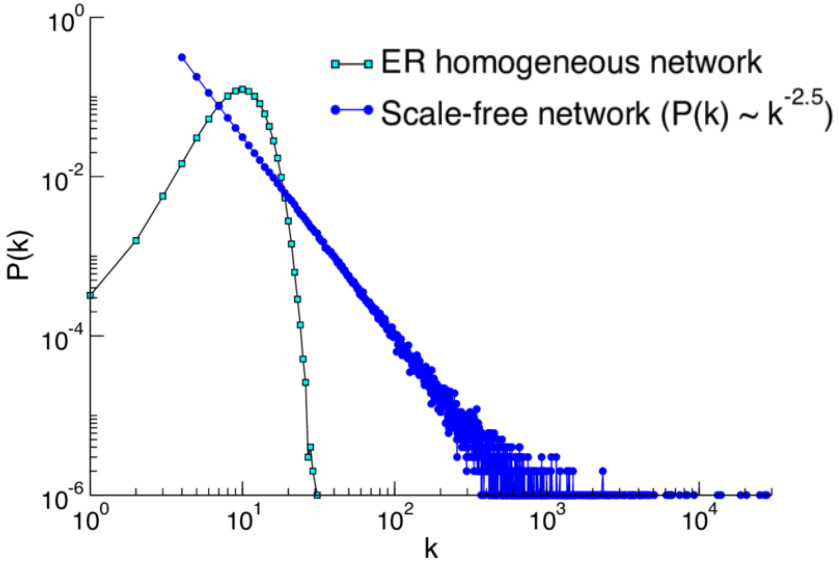
(1) Networks continuously expand by the addition of new nodes
www: addition of new documents

(2) New nodes prefer to link to highly connected nodes.
www: linking to well known sites

Growth:
Add a new node with *m* links

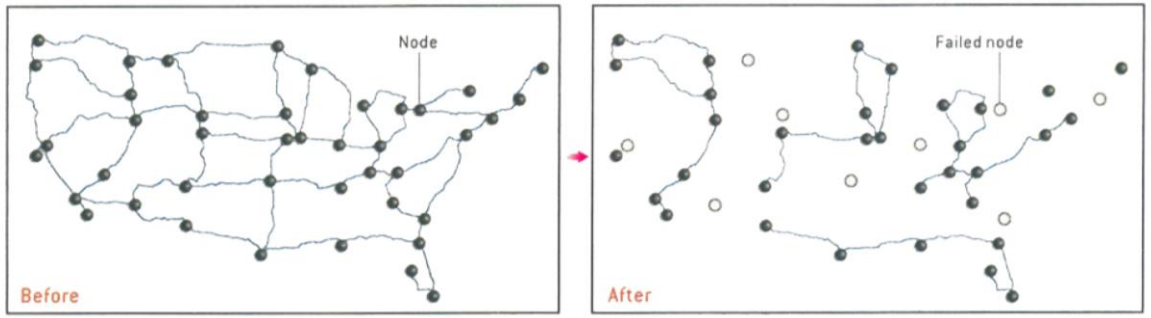
Preferential Attachment:
The probability that a node connects to a node with *k* links is proportional to *k*.



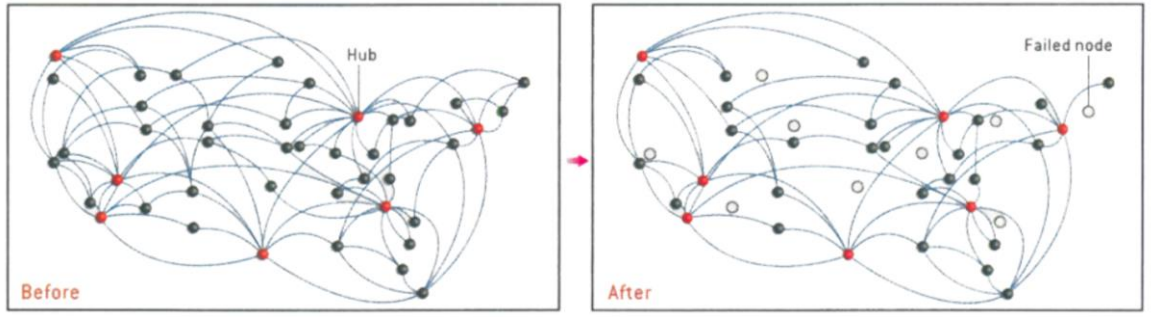


Robustness of Scale-free Networks

Random Network, Accidental Node Failure



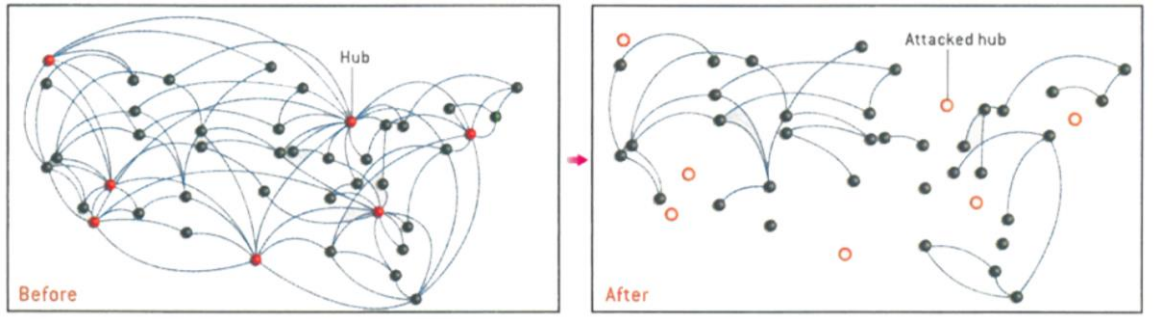
Scale-free Network, Accidental Node Failure



Scale-free Networks are

- **Ultra-resilient against failures**
- **Suffer from direct attacks on hubs**

Scale-free Network, Attack on Hubs



https://zaguan.unizar.es/record/15295/files/TAZ-TFG-2014-1036.pdf

Robustness of Scale-free Networks

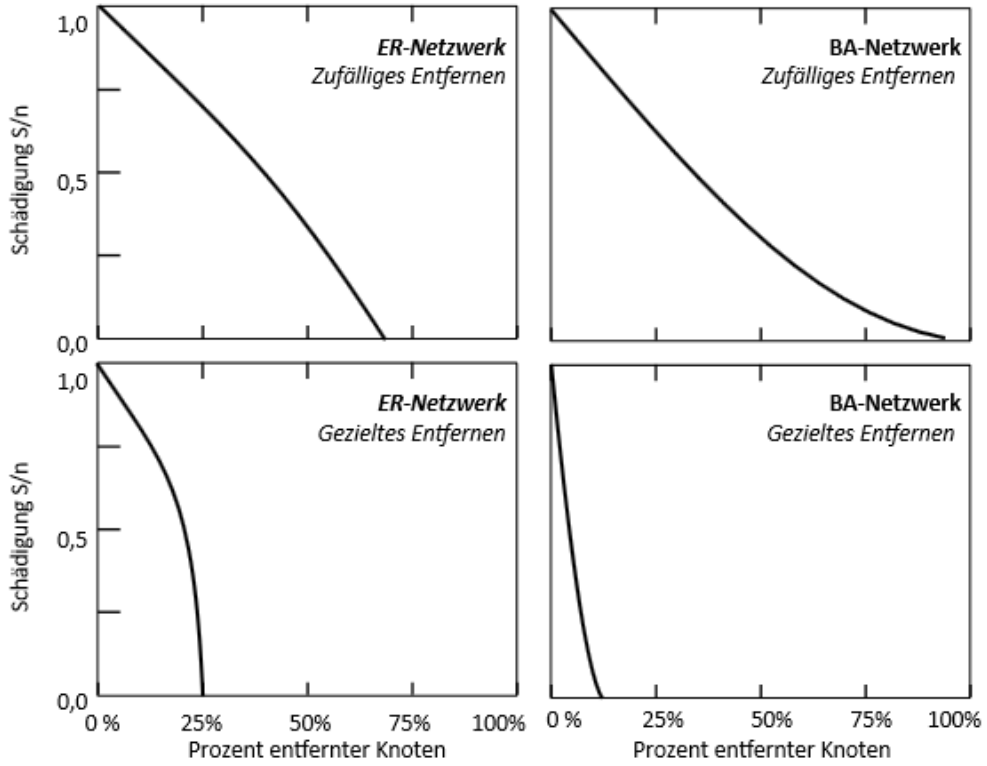


Bild 12.2 Schädigung durch *zufälliges Entfernen* (oben) und *gezieltes Entfernen* (unten) von Knoten: Größte zusammenhängende Komponente im Verhältnis zur ursprünglichen Größe des Netzwerks, aufgetragen über den Anteil f der entfernten Knoten. Verlauf für einen ER-Zufallsgraphen (links) und für ein skalenfreies Netzwerk nach dem BA-Modell gleicher Größe (rechts) [5]. – Nachdruck mit Genehmigung

Krischke & Röpcke (2014) Graphen und Netzwerktheorie

Failures

Attacks

Scale-free Networks are

(i) ultra-resilient against failures

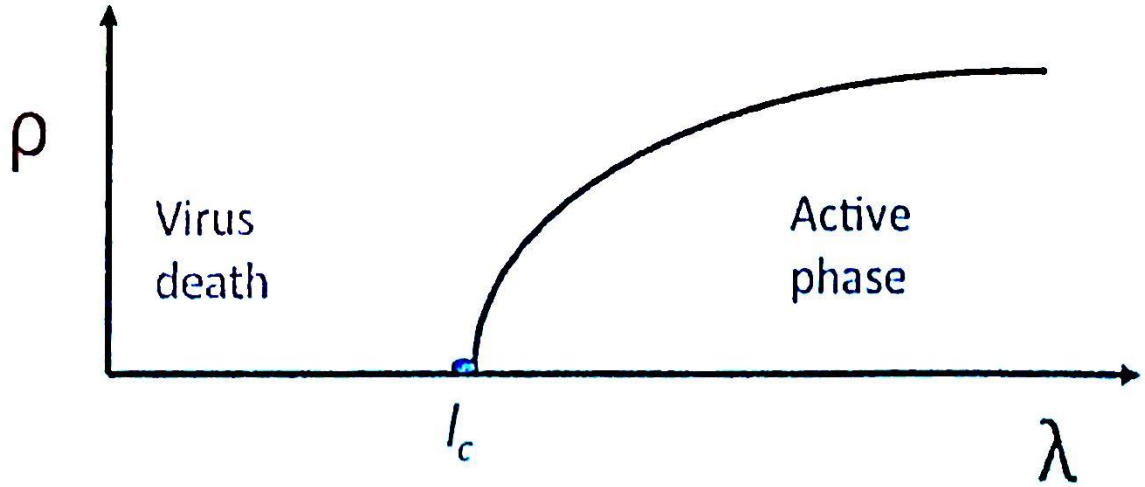
(ii) but suffer from direct attacks on hubs

Contagion :: Erdős-Rényi Networks

Biology:
If a virus is not too infectious
> it dies out.

Economics:
If a product/idea is not too ‚sticky'
> it does not succeed.

spreading rate
of a virus



density of infected individuals

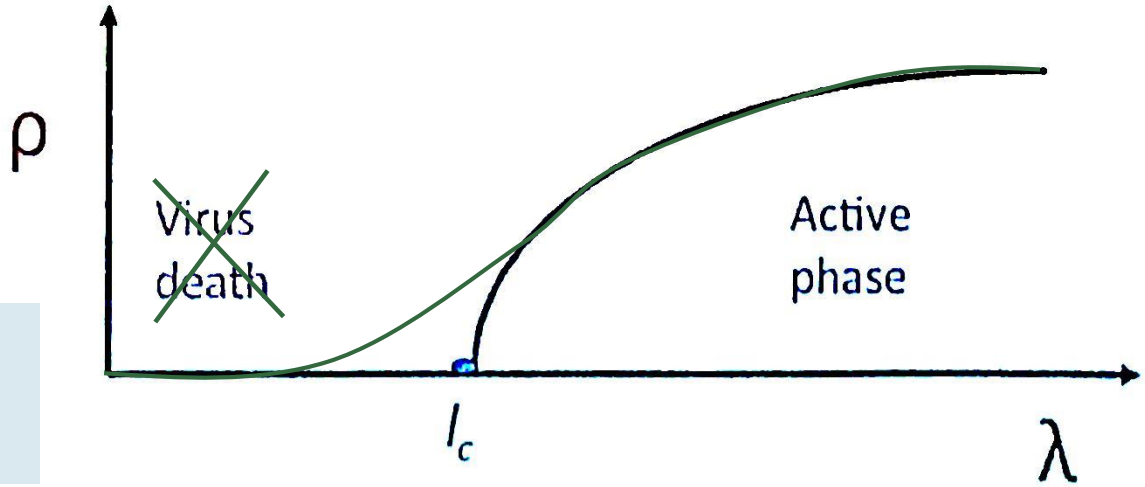
Pastor-Satorras & Vespignani, Physical Review Letters (2001) DOI: 10.1103/PhysRevLett.86.3200

Contagion :: Erdős-Rényi Networks

Biology:
If a virus is not too infectious
> it dies out.

Economics:
If a product/idea is not too ‚sticky'
> it does not succeed.

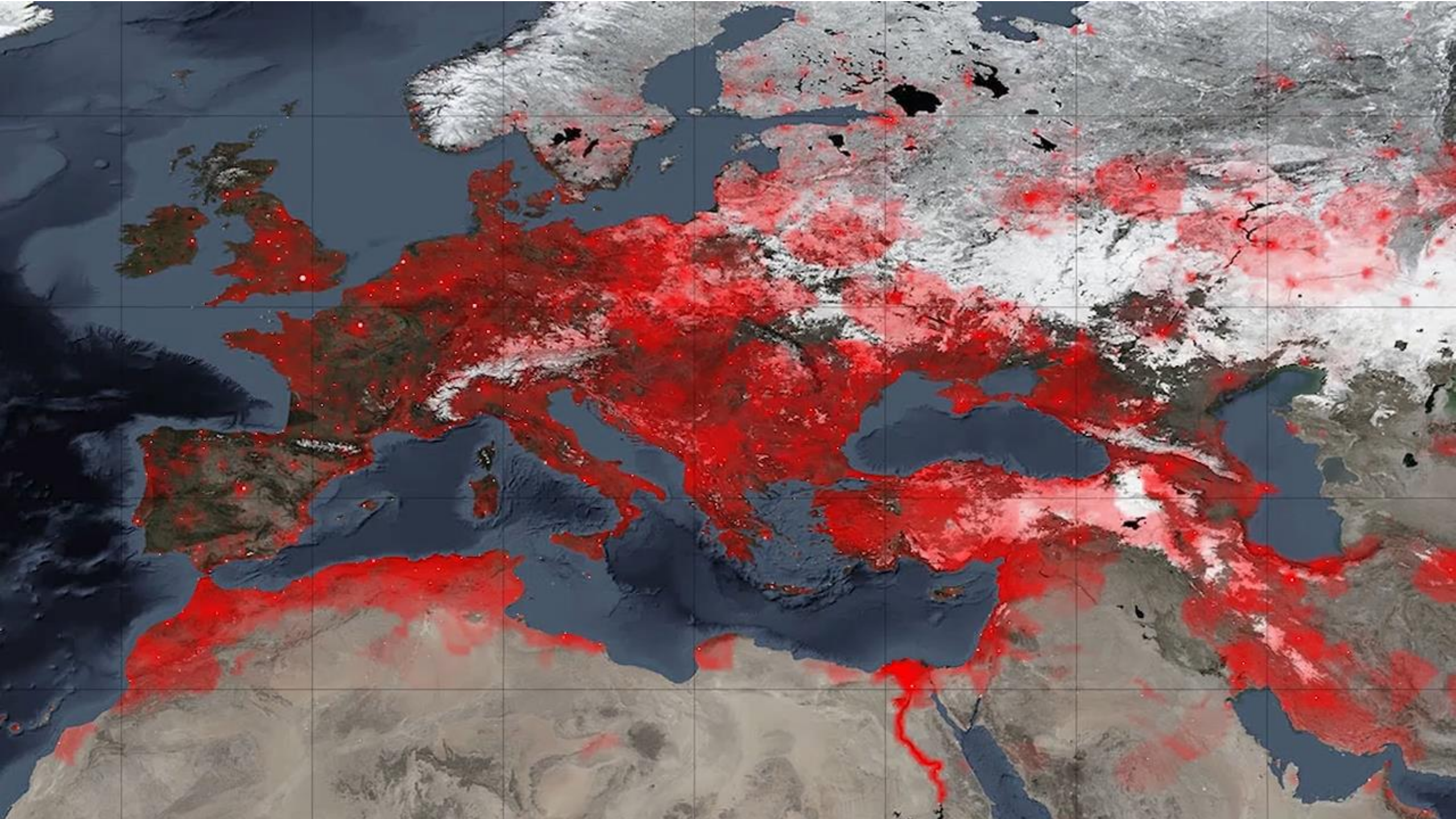
spreading rate
of a virus

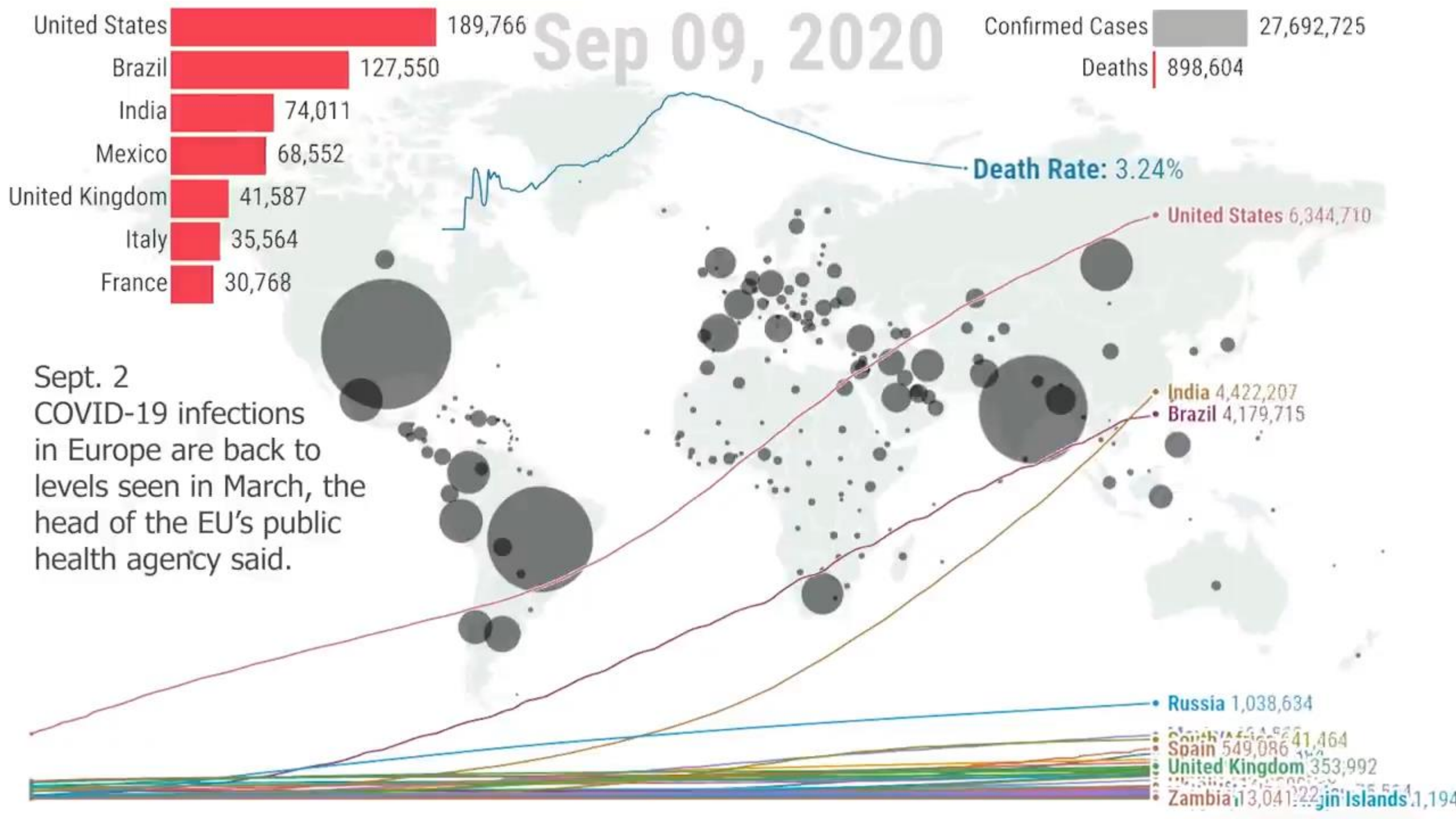


density of infected individuals

In the presence of **hubs**, weakly infectious viruses persist in the population.

Pastor-Satorras & Vespignani, Physical Review Letters (2001) DOI: 10.1103/PhysRevLett.86.3200




United States 189,766
Brazil 127,550
India 74,011
Mexico 68,552
United Kingdom 41,587
Italy 35,564
France 30,768
Sep 09, 2020
Confirmed Cases 27,692,725
Deaths 898,604
Death Rate: 3.24%
United States 6,344,710
India 4,422,207
Brazil 4,179,715
Russia 1,038,634
Spain 549,086
United Kingdom 353,992
Zambia 13,041
Sept. 2
COVID-19 infections
in Europe are back to
levels seen in March, the
head of the EU's public
health agency said.

Institute of
Pervasive Computing

Social Graphs

Algorithms and Data Structures 2, 340300
Lecture – 2023W
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