EXERCIZE #1

ⓐ $1 + z = \frac{\lambda_{obs}}{\lambda_{rest}}$, $V_{rot} = c \cdot \frac{\Delta\lambda}{\lambda}$

- Lyman α rest wavelength $= 1216 \mathring{A} = \lambda_{rest}$
- From spectrum : $\lambda_{obs} = 1214 \mathring{A}$

$z = \frac{\lambda_{obs}}{\lambda_{rest}} - 1 = \left(\frac{1214}{1216}\right)\mathring{A} - 1 = -0.00164$ $\Rightarrow$ M31 is blueshifted

$V_{rot} = c \cdot \frac{\Delta\lambda}{\lambda} = c \cdot \left(\frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}\right) = -490 \frac{Km}{S}$

ⓑ $d = 780$ Kpc , $H_0 = 71 \frac{Km}{S\,Mpc}$

$V_r = H_0 d = 71 \frac{Km}{S\,Mpc} \cdot (780 \times 10^{-3}\ Mpc) = 55 \frac{Km}{S}$??

ⓒ $z = 0.05$

$V_r = cz = 2.98 \times 10^5 \frac{Km}{S} \times 0.05 = 14.9 \times 10^3 \frac{Km}{S}$

$d = \frac{V_r}{H_0} = 14.9 \times 10^3 \left[\frac{Km}{S}\right] \times \frac{1}{71} \left[\frac{Mpc\,S}{Km}\right] = 210$ Mpc

EXERCIZE #2

ⓐ $\gamma + \gamma \rightarrow e^+ + e^-$

$E = m_e c^2 = K_B T = 0.51$ MeV

$\Rightarrow T_{50} = \frac{m_e c^2}{K_B} = 5.94 \times 10^9$ K

$\Rightarrow$ at $T \lesssim 6$ billion kelvin the electron-positron pairs can no longer be produced efficiently $\Rightarrow$ n° density of photons with energies above pair production threshold is too small

(b) $T[K] \sim 1.5 \times 10^{10} (t\,[s])^{-1/2}$

$\eta = \frac{n_\gamma}{n_b} = 10^9 \Rightarrow 10^9$ photons for each stable baryon

$T \sim 1.5 \times 10^{10} (1\,[s])^{-1/2} = 1.5 \times 10^{10}\ K$

$E = k_B T = 1.3\ MeV$

(c) $p + n \rightleftharpoons D + \gamma \qquad m_p = m_n = 1.67 \times 10^{-27}\ kg$

$\rightarrow m_D = 3.34 \times 10^{-27}\ kg = 1875.61\ MeV/c^2$

- Energy needed to disintegrate D: $E > 2.23\ MeV$

$$\Delta m = Z m_p + (A - Z) m_n - m_{nuc} \Rightarrow A = 2\ \&\ Z = 1 \text{ for } D$$
$$= m_p + m_n - m_{nuc}$$

Deuteron binding energy:

- $E_B = \Delta m c^2 = (m_p + m_n - m_{nuc}) c^2 = \underline{2.23\ MeV}$

$\Rightarrow$ Energy required to photo-disintegrate D must be greater than $2.23\ MeV \Rightarrow E_{PD} > 2.23\ MeV$

- $E = \hbar \omega \rightarrow \omega = \frac{E}{\hbar} = \frac{2.23 \times 10^6\ eV}{6.58 \times 10^{-16}\ eV/s} = 3.39 \times 10^{21}\ Hz$

- Deuterons are present in such small amount since it is the element with lowest binding energy and thus it is easy to break up