(PTIA0301) Elementary Linear Algebra

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November 28, 2024

Gram-Schmidt Orthogonalization I

- Orthogonalization procedure:
  1. Set $\mathbf{e}_1^{'} = \mathbf{b}_1$ and $\mathbf{e}_1 = \frac{\mathbf{e}_1^{'}}{\left\|\mathbf{e}_1^{'}\right\|}$.
  2. Compute the vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_k$.
  3. Finally,

$$\mathbf{e}_{k+1}^{'} = \mathbf{b}_{k+1} - (\mathbf{b}_{k+1} \cdot \mathbf{e}_1)\,\mathbf{e}_1 - (\mathbf{b}_{k+1} \cdot \mathbf{e}_2)\,\mathbf{e}_2 - \cdots - (\mathbf{b}_{k+1} \cdot \mathbf{e}_k)\,\mathbf{e}_k,$$

and

$$\mathbf{e}_{k+1} = \frac{\mathbf{e}_{k+1}^{'}}{\left\|\mathbf{e}_{k+1}^{'}\right\|}.$$

Eigenvalue, Eigenvector I

- Definition: Let V be a vector space over $\mathbb{R}$. Let $\varphi : V \to V$ be a linear mapping. If for a nonzero vector $\mathbf{a} \in V$ and a scalar $\lambda \in \mathbb{R}$, the equation $\varphi(\mathbf{a}) = \lambda\mathbf{a}$ holds, we say that $\mathbf{a}$ is an eigenvector of φ, and λ is the eigenvalue of φ corresponding to $\mathbf{a}$.
- Definition: Let $L_\lambda = \{\mathbf{a} \in V : \varphi(\mathbf{a}) = \lambda\mathbf{a}\}$ be the set of eigenvectors corresponding to λ, along with the zero vector. The set L_λ forms a subspace, and it is called the eigenspace corresponding to λ.
- Definition: (Determination of Eigenvalues) The characteristic polynomial of a matrix $A \in \mathcal{M}_{n\times n}$ is defined as the n^{th}-degree polynomial

$$f(x) = |A - xE_n| = \begin{vmatrix} a_{11}-x & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-x & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}-x \end{vmatrix}.$$

The End

Thank you for your attention!