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(MATNA1902) Lineáris algebra 3. zárthelyi dolgozat

1. Adottak a következő mátrixok:

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ -2 & 2 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ -1 & -1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix}$$

Végezze el az alábbiak közül az elvégezhető műveleteket! (a) $|\mathbf{A}|$; $|\mathbf{C}|$; $|\mathbf{D}|$ (b) $\mathbf{A}+\mathbf{B}$; $\mathbf{B}+\mathbf{C}$; $\mathbf{C}+\mathbf{D}$; $4\mathbf{A}-\mathbf{B}$ (c) $\mathbf{A}\cdot\mathbf{B}$; $\mathbf{B}\cdot\mathbf{C}$; $\mathbf{B}\cdot\mathbf{D}$ (d) $\mathbf{A}^T$; $\mathbf{D}^T$; $\mathbf{A}^T\cdot\mathbf{B}$; (e) $\rho(\mathbf{B})$; $\rho(\mathbf{D})$; (f) $\mathbf{A}^{-1}$; $\mathbf{D}^{-1}$ (10 pont)

2. Oldja meg az $\mathbf{A}\cdot\mathbf{X}=\mathbf{B}$ mátrixegyenletet, ha

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{pmatrix}$$

(10 pont)

3. Oldja meg az alábbi lineáris egyenletrendszert!

$$\begin{aligned} x_1+x_2+x_3 &= 3 \\ 2x_1+3x_2+2x_3 &= 7 \\ 2x_1+2x_2+3x_3 &= 6 \end{aligned}$$

(10 pont)

4. Lineárisan függetlenek-e az $\mathbf{a}=(-2,0,5)$, a $\mathbf{b}=(1,2,3)$ és a $\mathbf{c}=(-3,2,13)$ vektorok? (10 pont)
5. Lineáris altér-e az $\mathbb{R}^4$-on az $L=\{(x_1,x_2,x_1,2x_2)\,|\,x_1,x_2\in\mathbb{R}\}$? (10 pont)
6. Adja meg meg az $\mathbf{a}=(1,0,0)$ vektort az $(1,2,5)$; $(3,7,8)$; $(2,5,2)$ bázisban. (10 pont)
7. Adja meg a következő pontokon átmenő sík egyenletét: $A(0,0,1)$, $B(1,1,1)$, $C(1,1,0)$. (10 pont)
8. Adja meg az alábbi mátrix sajátértékeit és a saját altereket, majd diagonizálja a mátrixot!

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

(10 pont)

9. Az alábbi leképezések közül melyik lineáris? Adja meg a leképezés mátrixát is!

a.)

$$f(\mathbf{x}) = \begin{pmatrix} x_1+2x_2 \\ 2x_1+x_2 \\ 3x_3+3_2 \\ x_3 \end{pmatrix} \quad (\mathbf{x}\in\mathbb{R}^3)$$

b.)

$$g(\mathbf{x}) = \begin{pmatrix} 2x_1 \\ x_1x_2 \\ 3x_2 \\ x_1 \end{pmatrix} \quad (\mathbf{x}\in\mathbb{R}^2)$$

(10 pont)

10. Írja át az alábbi vektorokat ortogonális bázissá a Gram-Schmidt ortogonalizáció segítségével!

$$\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \begin{pmatrix}0\\0\\1\\1\end{pmatrix} \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

(10 pont)

A fenti feladatsor két részre oszlik. Az (1)-(5) feladatok megoldásával a első zárthelyit lehet javítani, illetve pótolni. A (6)-(10) feladatokkal pedig a másodikat. A zárthelyik osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). Mindkét témából zárthelyiből legalább elégségest (2) kell elérni a gyakorlati jegyhez.

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Pécs, 2025. május 15.

Alskó

TÓCSKÓ GÁBOR BFQQQX

1: $|A| = \cancel{\exists}$

$|C| : \cancel{\exists}$

$|B| = 0+0+2+1-0-0 = \underline{\underline{3}}$

b, $\underline{A+B} = \begin{pmatrix} 1 & -1 & 3 \\ -2 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 0 & 5 \\ -2 & 4 & 1 \end{pmatrix}}}$

$B+C : \cancel{\exists}$

$C+D : \cancel{\exists}$

b, $4A - B = 4 \cdot \begin{pmatrix} 1 & -1 & 3 \\ -2 & 2 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -4 & 12 \\ -8 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 0 & 2 & 0 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 2 & -5 & 10 \\ -8 & 6 & 4 \end{pmatrix}}}$

c, $A \cdot B : \nexists$ lehetséges

$B \cdot C = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 2+2-2 & 6+5-2 \\ 4 & 10 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 & 9 \\ 4 & 10 \end{pmatrix}}}$

$B \cdot D = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 2+2-2 & 1-2 & -2+2 \\ 4 & 2 & 4 \end{pmatrix} =$

$= \underline{\underline{\begin{pmatrix} 2 & -1 & 0 \\ 4 & 2 & 4 \end{pmatrix}}}$

d) $A^T = \begin{pmatrix} 1 & -2 \\ -1 & 2 \\ 3 & 1 \end{pmatrix}$ $D^T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -1 & 2 & 0 \end{pmatrix}$

$A^T B \cdot 1$ Ersatz

e) $\rho(B) = 2$ — elvel torpiz dollban van

$\rho(D) = ?$

$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix} \xrightarrow{(II)-2(I)} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ -1 & -1 & 0 \end{pmatrix} \xrightarrow{(III)+(I)} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & -1 & -1 \end{pmatrix} \rightarrow$

$\xrightarrow{(III)+(II)} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{pmatrix}$ ← 3 sor van n torpiz dollban

$\underline{\rho(D) = 3}$

f) A^{-1}: $\nexists$

$|D| = 3 \neq 0 \Rightarrow$ invertálható

$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ -1 & -1 & 0 \end{pmatrix}$ aldet $\rightarrow$ $\begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} = 2$ $\begin{vmatrix} 2 & 2 \\ -1 & 0 \end{vmatrix} = 2$ $\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = -1$

$\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = -1$ $\begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -1$ $\begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} = -1$

$\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1$ $\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = 4$ $\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$

$\begin{pmatrix} 2 & 2 & -1 \\ -1 & -1 & -1 \\ 1 & 4 & 1 \end{pmatrix}$

⇓ rakkban

$\begin{pmatrix} 2 & -2 & -1 \\ +1 & -1 & 1 \\ 1 & -4 & 1 \end{pmatrix}$

FOLSIKÓ GÁBOR BTQQQY

(1/f) feladat

$\rightarrow$ transzponált $\begin{matrix} 2 & -2 & -1 \\ -1 & 1 & 1 \\ 1 & -4 & 1 \end{matrix}$ $\begin{pmatrix} 2 & +1 & 1 \\ -2 & -1 & -4 \\ -1 & 1 & 1 \end{pmatrix} \rightarrow \underline{D}^{-1} = \frac{1}{3}\begin{pmatrix} 2 & +1 & 1 \\ -2 & -1 & -4 \\ -1 & 1 & 1 \end{pmatrix}$

E: $\underline{D} \cdot \underline{D}^{-1}$ $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \frac{1}{3}\begin{pmatrix} 2 & +1 & 1 \\ -2 & -1 & -4 \\ -1 & 1 & 1 \end{pmatrix} =$

$= \frac{1}{3}\begin{pmatrix} 2+1 & 1-1 & 1-1 \\ 4-2-2 & +2-1+2 & 2-4+2 \\ -2+2 & 1-1 & -1+4 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓

(2) $\underline{A} \cdot \underline{X} = \underline{B}$ $\quad (\cdot \underline{A}^{-1} \cdot)$

$\underline{X} = \underline{A}^{-1} \cdot \underline{B}$

$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \rightarrow$

$\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$

$\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & +1 \\ 1 & -1 \end{vmatrix} = -2$

$\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 2 & -2 \\ 1 & 2 & -1 \end{pmatrix} \xrightarrow[\text{Adj SR}]{\text{Joahn}} \begin{pmatrix} 1 & -2 & -3 \\ 0 & 2 & 2 \\ 1 & -2 & -1 \end{pmatrix} \xrightarrow[\text{pndo}]{\text{transp.}} \begin{pmatrix} 1 & 0 & 1 \\ -2 & 2 & -2 \\ -3 & 2 & -1 \end{pmatrix} \rightarrow A^{-1} = \frac{1}{2} \begin{pmatrix} \\ -2 \\ -3 \; 2 \end{pmatrix}$$

FALSCH

$$|A| = 1 + 0 + 2 + 1 - 2 - 0 = 2 \neq 0$$

$$X = A^{-1} \cdot B = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ -2 & 2 & -2 \\ -3 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & +2 & 5 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1+1 & -1-2 & 3+ \\ -2+8-2 & 2+6+4 & -6+4 \\ -3+8-1 & 3+6+2 & -9+4- \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 2 & -3 & 8 \\ 4 & 12 & -12 \\ 4 & 11 & -10 \end{pmatrix}$$

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(3)

$$\begin{cases} x_1 + x_2 + x_3 = 3 \\ 2 \cdot x_1 + 3 x_2 + 2 x_3 = 7 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 = 6 \end{cases}$$

P: $2 + 1 + 0 = 3$ ✓

$4 + 3 + 0 = 7$ ✓

$4 + 2 + 0 = 6$ ✓

$x_1 + x_2 + x_3 = 3$

$\boxed{x_2 = 1}$ (II) − 2(I)

(III) − 2(I)

$\boxed{x_3 = 0}$

$\boxed{x_1 = 2}$

FALUSKÓ GÁBOR BTQQQY

c) lineáris függetlenség:

$\lambda_1 \cdot a + \lambda_2 \cdot b + \lambda_3 \cdot c = 0 \iff \lambda_1 = \lambda_2 = \lambda_3 = 0$

$$\lambda_1 \begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda_3 \cdot \begin{pmatrix} -3 \\ 2 \\ 13 \end{pmatrix} = 0$$

$$\begin{cases} -2\lambda_1 + \lambda_2 - 3\lambda_3 = 0 \\ 2\lambda_2 + 2\lambda_3 = 0 \quad /:2 \\ 5\lambda_1 + 3\lambda_2 + 13\lambda_3 = 0 \end{cases}$$

(II) ⟷ (III)

$-2\lambda_1 + \lambda_2 - 3\lambda_3 = 0$

$5\lambda_1 + 3\lambda_2 + 13\lambda_3 = 0$

$\lambda_2 + \lambda_3 = 0 \to \lambda_3 = -\lambda_2$

$-2\lambda_1 + \lambda_2 + 3\lambda_2 = 0$

$5\lambda_1 + 3\lambda_2 - 13\lambda_2 = 0$

$-2\cdot\lambda_1 + 4\lambda_2 = 0$

$5\lambda_1 - 10\lambda_2 = 0$

$\lambda_1 - 2\lambda_2 = 0 \to \lambda_1 = 2\lambda_2$

$\lambda_1 - 2\lambda_2 = 0$

$$\begin{pmatrix} 2\lambda_2 \\ \lambda_2 \\ -\lambda_2 \end{pmatrix} = \lambda_2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \lambda_2 \in \mathbb{R}$$

LINEÁRISAN FÜGGŐ VEKTOROK

(5) $U = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_1 \\ 2x_2 \end{pmatrix} \middle| \; x_1, x_2 \in \mathbb{R} \right\}$

(I) additív

$$\cancel{\begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ a_1+b_1 \\ 2a_2+2b_2 \end{pmatrix}} = \begin{pmatrix} a_1 \\ a_2 \\ a_1 \\ 2a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_1 \\ 2b_2 \end{pmatrix} = \begin{pmatrix} (a_1+b_1) \\ (a_2+b_2) \\ (a_1+b_1) \\ 2(a_2+b_2) \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_1 \\ 2c_2 \end{pmatrix} \in U$$

(II) homogén

$c_1 = a_1 + b_1$

$$\lambda \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_1 \\ 2a_2 \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda a_2 \\ \lambda a_1 \\ 2\lambda a_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_1 \\ 2d_2 \end{pmatrix} \in U \quad ✓$$

$d_i = \lambda a_i$

U tehát lineáris altér $\mathbb{R} \times \mathbb{R}$-en

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(6) $\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 7 \\ 8 \end{pmatrix} + \lambda_3 \cdot \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda_1 + 3\lambda_2 + 2\lambda_3 = 1$

$2\lambda_1 + 7\lambda_2 + 5\lambda_3 = 0$

$5\lambda_1 + 8\lambda_2 + 2\lambda_3 = 0$

$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 7 & 5 \\ 5 & 8 & 2 \end{pmatrix}$

$|A| = 14 + \cancel{\quad}^{75} + 32 - 70 - 12 - 40 = \cancel{46} -1 \neq 0$

FALSKÓ GÁBOR BFQQRY

6) feladat

$$\begin{vmatrix} 7 & 5 \\ 8 & 2 \end{vmatrix} = -26 \quad \begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix} = -21 \quad \begin{vmatrix} 2 & 7 \\ 5 & 8 \end{vmatrix} = -19$$

$$\begin{vmatrix} 3 & 2 \\ 8 & 2 \end{vmatrix} = -10 \quad \begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} = -8 \quad \begin{vmatrix} 1 & 3 \\ 5 & 8 \end{vmatrix} = -7$$

$$\begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = +1 \quad \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 1$$

$$\begin{pmatrix} -26 & -21 & -19 \\ -10 & -8 & -7 \\ 1 & +1 & 1 \end{pmatrix}$$

$\downarrow$ előjelek

$$\begin{pmatrix} -26 & +21 & -19 \\ 10 & -8 & 7 \\ 1 & -1 & 1 \end{pmatrix}$$

$\downarrow$ transzponálás

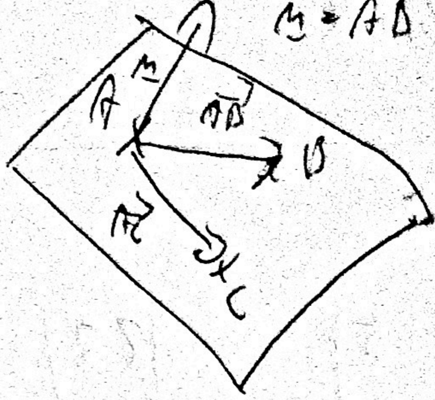
$$A^{-1} = \begin{pmatrix} 26 & -10 & -1 \\ -21 & 8 & +1 \\ 19 & -7 & -1 \end{pmatrix} \Leftarrow \begin{pmatrix} -26 & 10 & 1 \\ 21 & -8 & -1 \\ -19 & 7 & 1 \end{pmatrix}$$

$$x = A^{-1} \cdot b = \begin{pmatrix} \cancel{-26} & \cancel{10} & \cancel{1} \\ 21 & -8 & -1 \\ -19 & 7 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -26 \\ 21 \\ -19 \end{pmatrix}$$

ε: ✓

⑦ $A(0,0,1)$
$B(1,1,1)$
$C(1,1,0)$

S: ?



$\underline{n} = \vec{AB} \times \vec{AC}$

$\vec{AB}\,(1,1,0)$
$\vec{AC}\,(1,1,-1)$ $\Rightarrow$ $\underline{n} = \vec{AB} \times \vec{AC} = \begin{pmatrix} 1(-1) - 0\cdot 1 \\ 0\cdot 1 - 1\cdot(-1) \\ 1\cdot 1 - 1\cdot 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\underline{n}(\underline{x} - \underline{x}_0) = 0$

$-x + y = 0$

$\underline{y = x}$

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⑧ $\underline{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

S.e., S.v., D = ?

$\underline{A} \cdot x = \lambda \underline{x}$

$(\underline{A} - \lambda \underline{E})\underline{x} = \underline{0}$

$\det(\underline{A} - \lambda \underline{E}) = 0$

$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$

$9 - 6\lambda + \lambda^2 - 1 = 0$

$\Rightarrow \lambda^2 - 6\lambda + 8 = 0$

$\cancel{(\lambda - 1)(\lambda - 5) = 0}$

$\cancel{\lambda_{1,2} = \frac{6 \pm \sqrt{36 - 40}}{2} = 3 \pm 2 = \begin{matrix} 1 \\ 5 \end{matrix}}$

$(\lambda - 2)(\lambda - 4) = 0$

$\underline{\lambda_1 = 2}$; $\underline{\underline{\lambda_2 = 4}}$

$\lambda_1 = 2$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = x_2$ $\Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\boxed{x_{\lambda_1 = 2} = \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \middle| t \in \mathbb{R} \right\}}$

Falskó Gábor B7QQQY

$\lambda_2 = 4$

$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad L_{\lambda=4} = \left\{ \begin{pmatrix} t \\ t \end{pmatrix} \middle| t \in \mathbb{R} \right\} \to \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x_1 = x_2$

$D = \underline{C}^{-1} \cdot \underline{A} \cdot \underline{C}$, ahol $\underline{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$|\underline{C}| = 1 + 1 = 2 \neq 0$

$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \xrightarrow[\text{tükr.}]{} \begin{pmatrix} 1 & +1 \\ -1 & 1 \end{pmatrix} \xrightarrow[\text{ném}]{\text{transp}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \to \underline{C}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$\underline{C} \cdot \underline{C}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$D = \underline{C}^{-1} \cdot \underline{A} \cdot \underline{C} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\underline{C} = \frac{1}{2}\begin{pmatrix} 3-1 & 1-3 \\ 3+1 & 1+3 \end{pmatrix}\underline{C}$

$= \frac{1}{2}\begin{pmatrix} 2 & -2 \\ 4 & 4 \end{pmatrix}\underline{C} = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$

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1) a) $f(a+b) = \begin{pmatrix} (a_1+b_1) + 2(a_2+b_2) \\ 2(a_1+b_1) + (a_2+b_2) \\ 3(a_3+b_3) + 3(a_2+b_2) \end{pmatrix} = \begin{pmatrix} a_1 + 2a_2 \\ 2a_1 + a_2 \\ 3a_3 + 3a_2 \end{pmatrix} + \begin{pmatrix} b_1 + 2b_2 \\ 2b_1 + b_2 \\ 3b_3 + 3b_2 \end{pmatrix} = f(a) + f(b)$ ✓

$$f(\lambda a)=\begin{pmatrix}\lambda a_1+2\lambda a_2\\ 2\lambda a_1+\lambda a_2\\ 3\lambda a_3+3\lambda a_2\\ \lambda a_3\end{pmatrix}=\lambda\begin{pmatrix}a_1+2a_2\\ 2a_1+a_2\\ 3a_3+3a_2\\ a_3\end{pmatrix}=\underline{\lambda f(a)}$$

Ez egy lineáris transzformáció, a mátrixa pedig:
$$\begin{pmatrix}1&2&0\\2&1&0\\0&3&3\\0&0&1\end{pmatrix}$$

e)
$$g(a+b)=\begin{pmatrix}2a_1+2b_1\\ (a_1+b_1)(a_2+b_2)\\ 3a_2+3b_2\\ a_1+b_1\end{pmatrix}=\begin{pmatrix}2a_1\\ a_1a_2\\ 3a_2\\ a\end{pmatrix}+$$

$$+\begin{pmatrix}2b_1\\ b_1b_2\\ 3b_2\\ b_1\end{pmatrix}+\begin{pmatrix}0\\ a_1b_2+a_2b_1\\ 0\\ 0\end{pmatrix}\Leftarrow \neg$$ lineáris fgv.

$\downarrow$

$\neg$ lineáris transzformáció

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(11)
$$e_1'=\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\qquad e_1=\frac{e_1'}{|e_1'|}=\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$e_2'=\begin{pmatrix}0\\0\\1\\1\end{pmatrix}-\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\left[\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\right]=\begin{pmatrix}0\\0\\1\\1\end{pmatrix}-\frac{1}{2}\cdot 1\begin{pmatrix}1\\0\\0\\1\end{pmatrix}=$$

$$=\begin{pmatrix}-\frac{1}{2}\\0\\1\\\frac{1}{2}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix}\qquad e_2=\frac{e_2'}{|e_2'|}=\frac{\frac{1}{2}}{\frac{1}{2}\sqrt{6}}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix}=\frac{\sqrt{6}}{6}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix}$$

1. PAKSIKÓ GÁBOR BFQQQX

(10) folytatás

$$e_3' = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \left[\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\cdot\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\right]\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} - \left[\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\cdot\frac{\sqrt{6}}{6}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix}\right]\frac{\sqrt{6}}{6}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \frac{1}{2}\cdot 1\cdot\begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \frac{1}{6}\cdot 1\begin{pmatrix}-1\\0\\2\\1\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} - \cancel{\frac{3}{6}}\,\frac{1}{2}\begin{pmatrix}3\\0\\1\end{pmatrix} +$$

$$+\frac{1}{6}\begin{pmatrix}-1\\0\\2\\1\end{pmatrix} = \begin{pmatrix}\frac{6}{6}-\frac{3}{6}-\frac{1}{6}\\ \frac{6}{6}-0+\frac{0}{2}\\ 0-0+\frac{2}{6}\\ 0-\frac{3}{6}+\frac{1}{6}\end{pmatrix} = \begin{pmatrix}\frac{2}{6}\\1\\\frac{2}{6}\\-\frac{2}{6}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1\\3\\1\\-1\end{pmatrix}$$

$$e_3 = \frac{e_3'}{|e_3'|} = \frac{1}{3}\begin{pmatrix}1\\3\\1\\-1\end{pmatrix}\frac{1}{\frac{1}{3}\sqrt{12}} = \frac{1}{2\sqrt{3}}\begin{pmatrix}1\\3\\1\\-1\end{pmatrix} = \frac{\sqrt{3}}{6}\begin{pmatrix}1\\3\\1\\-1\end{pmatrix}$$