

I. VEKTORS

1. $\mathbf{a} = (-1, 0, 2)$, $\mathbf{b} = (0, -1, 3)$, and $\mathbf{c} = (-2, 1, 1)$. Calculate the following expressions:
 - a.) $\mathbf{c}(\mathbf{a} - \mathbf{b})$
 - b.) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a}$
 - c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
 - d.) What is the angle of Vectors \mathbf{a} and \mathbf{b} ?
 - e.) What is the angle of Vectors \mathbf{b} and \mathbf{c} ?
 - f.) What is the angle of Vectors \mathbf{a} and \mathbf{c} ?
 - g.) Are Vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} in the same plane?
 - h.) Determine a perpendicular vector to Vector \mathbf{a} .
2. $\mathbf{a} = (1, 1, 0)$, $\mathbf{b} = (0, -1, 2)$ és $\mathbf{c} = (-1, 2, 1)$. Calculate the following expressions:
 - a.) $(\mathbf{a} - \mathbf{b})\mathbf{c}$
 - b.) $(\mathbf{a} + \mathbf{b}) \times \mathbf{c}$
 - c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
 - d.) What is the angle of Vectors \mathbf{a} and \mathbf{b} ?
 - e.) What is the angle of Vectors \mathbf{b} and \mathbf{c} ?
 - f.) What is the angle of Vectors \mathbf{a} and \mathbf{c} ?
 - g.) Are Vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} in the same plane?
 - h.) Determine a perpendicular vector to Vector \mathbf{b} .
3. $\mathbf{a} = (2, 1, 1)$, $\mathbf{b} = (1, 3, 0)$, and $\mathbf{c} = (0, 1, 3)$. Calculate the following expressions:
 - a.) $(\mathbf{a} - \mathbf{b})\mathbf{c}$
 - b.) $(\mathbf{a} + \mathbf{b}) \times \mathbf{c}$
 - c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
 - d.) What is the angle of Vectors \mathbf{a} and \mathbf{b} ?
 - e.) What is the angle of Vectors \mathbf{b} and \mathbf{c} ?
 - f.) What is the angle of Vectors \mathbf{a} and \mathbf{c} ?
 - g.) Are Vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} in the same plane?
 - h.) Determine a perpendicular vector to Vector \mathbf{c} .

II. DETERMINANTS

4. Calculate the determinant of the following matrixes using Laplace expansion.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 & -2 \\ -2 & -2 & 1 \\ -1 & 4 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \\ 3 & 0 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{pmatrix}$$

5. Calculate the determinant of the following matrixes using Sarrus rules.

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 & 1 & 1 \\ 3 & -2 & -1 \\ -1 & 3 & -1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 3 & 0 & 0 \end{pmatrix}$$

6. Calculate the determinant of the following matrixes using Gauss elimination.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & -2 \\ 1 & 0 & 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 & -3 \\ -1 & -2 & 1 \\ -4 & 4 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 3 & 3 & 2 & 0 \\ 2 & -2 & 1 & 3 \\ 2 & 3 & 0 & 1 \\ -2 & 1 & 2 & 1 \end{pmatrix}$$

III. SYSTEM OF LINEAR EQUATIONS

7.) Solve the following systems of linear equations using Gauss eliminaton and Cramer's rules.

a.)

$$\begin{aligned}x_1 - 2x_2 + x_3 &= 1 \\2x_1 - x_2 + 3x_3 &= -2 \\3x_1 + 7x_2 + 2x_3 &= 0\end{aligned}$$

b.)

$$\begin{aligned}x_1 - 3x_2 + 4x_3 &= -2 \\x_1 - 3x_2 + x_3 &= 2 \\2x_1 + x_2 - 5x_3 &= -1\end{aligned}$$

c.)

$$\begin{aligned}2x_1 + 4x_2 - x_3 &= -2 \\3x_1 - x_2 - x_3 &= 1 \\-2x_1 - x_2 - 3x_3 &= -2\end{aligned}$$

IV. LINEAR INDEPENDENCE

8. Are independent linearly Vectors $\mathbf{a} = (-1, 2, 1, 4)$, $\mathbf{b} = (0, 5, -1, 1)$, and $\mathbf{c} = (1, 1, 5, 2)$?
9. Are independent linearly Vectors $\mathbf{a} = (-1, 1, 0)$, $\mathbf{b} = (-2, -1, 0)$, and $\mathbf{c} = (-3, 2, 0)$?
10. Are independent linearly Vectors $\mathbf{a} = (-1, 2, -1)$, $\mathbf{b} = (-1, -2, 1)$, and $\mathbf{c} = (-1, 3, 1)$?

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