(PTIA0301) Elementary Linear Algebra

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Multiply vector with a scalar and sum of vectors

- Definition: Sum of vectors. If $\mathbf{a}\,(a_1, a_2, a_3)$ and $\mathbf{b}\,(b_1, b_2, b_3)$, then

$$\mathbf{a} + \mathbf{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)\,,$$

  where $a_1,\ a_2,\ a_3,,\ b_1,\ b_2,\ b_3 \in \mathbb{R}$.

- Definition: Subtracktion of vectors. If $\mathbf{a}\,(a_1, a_2, a_3)$ and $\mathbf{b}\,(b_1, b_2, b_3)$, then

$$\mathbf{a} - \mathbf{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)\,,$$

  where $a_1,\ a_2,\ a_3,\ b_1,\ b_2,\ b_3 \in \mathbb{R}$.

Multiply vector with a scalar, difference of vectors

- Definition: Multiplication of vectors with a scalar. If $\lambda \in \mathbb{R}$ and $\mathbf{a}\,(a_1, a_2, a_3)$, where a_1, a_2, $a_3 \in \mathbb{R}$, then

$$\lambda \mathbf{a} = (\lambda a_1, \lambda a_2, \lambda a_3)\,.$$

  Mind if λ is 0, 1, -1, <1, or >1.
- The sum of vectors is
  - Commutative: $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$, where $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$.
  - Associative: $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$, where $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$.
  - The null vector exists: $\exists \mathbf{0} \in \mathbb{R}^3$, where $\mathbf{a} + \mathbf{0} = \mathbf{a}$, where $\mathbf{a} \in \mathbb{R}^3$.
  - All vectors have an inverse vector: $\forall \mathbf{a} \in \mathbb{R}^3\ \exists\,(-\mathbf{a}) \in \mathbb{R}^3$, where $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$.
- The vector multiplication with scalar is
  - Associative: $\lambda\,(\mu \mathbf{a}) = (\lambda \mu)\,\mathbf{a}$, where $\mathbf{a} \in \mathbb{R}^3, \lambda, \mu \in \mathbb{R}$.
  - Distributive: $\lambda\,(\mathbf{a} + \mathbf{b}) = \lambda \mathbf{a} + \lambda \mathbf{b}$, where $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3, \lambda \in \mathbb{R}$.
  - Distributive: $(\lambda + \mu)\,\mathbf{a} = \lambda \mathbf{a} + \mu \mathbf{a}$, where $\mathbf{a} \in \mathbb{R}^3, \lambda, \mu \in \mathbb{R}$.
  - $\forall \in \mathbf{a} \cdot 1 = \mathbf{a}$, where $\mathbf{a} \in \mathbb{R}^3$.

The End

Thank you for your attention!