① a,

$$\begin{pmatrix} -2-0 \\ 0+3 \\ -1-1 \end{pmatrix} \cdot (2,1,1) = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} \cdot (2\ 1\ 1) = \overset{-4}{-2\cdot 2} + \overset{3}{3\cdot 1} \overset{-2}{-2\cdot 1} =$$

$$= -3$$

b,

$$\begin{pmatrix} 0+2 \\ -3+1 \\ 1+1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = 2 \begin{pmatrix} -1\cdot(-1)-1\cdot 0 \\ 1\cdot(-2)-1\cdot(-1) \\ 1\cdot 0-(-1)(-2) \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

c,

$$(\underline{a},\underline{b},\underline{c}) = (\underline{a} \times \underline{b})\cdot \underline{c} = \left[\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \right] \cdot (2\ 1\ 1) = \begin{pmatrix} 0\cdot 1-(-1)\cdot 0 \\ -1\cdot 0-(-2)\cdot 1 \\ -2\cdot(-3)-0\cdot 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0+2+6 = \underline{8}$$

d,

$$(\underline{a},\underline{b})\sphericalangle = \frac{\cancel{a \times} \underline{a}\cdot\underline{b}}{|\underline{a}|\cdot|\underline{b}|} = \frac{(-2\ 0\ -1)\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}}{\sqrt{4+1}\cdot\sqrt{9+1}} =$$

$$= \frac{-1}{\sqrt{5}\cdot\sqrt{10}} = -\frac{1}{5\sqrt{2}}$$

e,

$(\underline{a},\underline{b},\underline{c}) = 0 \Rightarrow$ bármelyik egy síkban nem a determináns =0.

$\cancel{\neq}$ $\underline{b} = (0,-1,1)$ ~~pl:~~ $\underline{b}^{\perp} = (1,0,0)$

$$\underline{b} \cdot \underline{b}^{\perp} = 0+0+0 = 0$$

②

(A)| = 3·3 − (−1)(−1) = 8

|B| = 1·2·3 + (1)4(−2) + (−2)·(−1)4 − (−2)2(−1) − (1)(−1)·3 − 1·4·4 =
= 6 + 8 + 8 − 8 − 3 − 16 = −5

|C| = +1·$\begin{vmatrix} 2 & -1 & 4 \\ 4 & -1 & 2 \\ -2 & 2 & 2 \end{vmatrix}$ − 1·$\begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & 1 \\ -2 & 2 & 2 \end{vmatrix}$ =

= 2·2·2 + 2(−1)2 + (−1)·2·(−2) + 1·1·2 − 1·(−1)(2) − 1(−1)2 − 2·2·2
= $\Big($ 4·2·1 + 2·(−1)(−2) + (−1)2(−2) − (−1)1(−2) − 2·2·2 − 4(2)(1) $\Big)$

= −4 + 4 + 4 − 2 + 2 − 8 − (8 + 4 + 4 − 2 − 8 − 8) =
= 2 + 8 + 2 = $\underline{\underline{12}}$

⑤ a)
$\begin{cases} x_1 + x_2 + 2x_3 = 1 \\ 2x_1 − x_2 + x_3 = −2 \\ 3x_1 + 5x_2 + 7x_3 = 0 \end{cases}$
 → $-\dfrac{62}{20} − \dfrac{7}{20} + \dfrac{29}{20} = \dfrac{-40}{20}$ ✓

$-\dfrac{93}{20} + \dfrac{35}{20} + \dfrac{58}{20} = 0$ ✓

_____
$x_2 − 3x_3 = −4$ (II) − 2(I)

$3x_2 − 4x_3 = −3$ (III) − 3(I)
_____

$20x_3 = 29$ (II) − 3·(II)

$x_3 = \dfrac{29}{20}$
_____

$x_2 = −4 + 3x_3 = −\dfrac{80}{20} + \dfrac{87}{20} = \dfrac{7}{20}$ $x_1 = 1 + x_2 − 2x_3 = 1 + \dfrac{7}{20} − \dfrac{58}{20} = -\dfrac{31}{20}$

③ b,
$$\begin{cases} -x_1 + 3x_2 + x_3 = 1 \\ x_1 + 3x_2 + x_3 = 0 \\ 4x_1 + x_2 - 3x_3 = 1 \end{cases}$$

$6x_2 + 2x_3 = 1$ (II)+(I)

$13x_2 + x_3 = 5$ III+4(I)

$78x_2 + 26x_3 = 13$ 13·(I)

$78x_2 + 6x_3 = 30$ 6·(II)

$-20x_3 = 17$ (II)-(I)

$$x_3 = -\frac{17}{20}$$

$x_2 = \frac{1}{6}(1 - 2x_3) = \frac{1}{6}\left(\frac{20}{20} + \frac{34}{20}\right) =$
$= \frac{9}{20}$

$x_1 = 0 - 3x_2 - x_3 = -\frac{27}{20} + \frac{17}{20} =$
$= -\frac{10}{20} = -\frac{1}{2}$

$+\frac{10}{20} + \frac{27}{20} * -\frac{17}{20} = \frac{20}{20} = 1$ ✓

$-\frac{10}{20} + \frac{27}{20} * -\frac{17}{20} = 0$ ✓

$-\frac{40}{20} + \frac{9}{20} + \frac{51}{20} = \frac{20}{20} = 1$ ✓

④
$$\lambda_1 \begin{pmatrix} -1 \\ 2 \\ 1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 5 \\ -2 \\ 2 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \end{pmatrix} = 0$$

$-\lambda_1 + \lambda_3 = 0$ → $\lambda_1 = \lambda_3$

$2\lambda_1 + 5\lambda_2 + \lambda_3 = 0$

$\lambda_1 - 2\lambda_2 + 3\lambda_3 = 0$

$3\lambda_1 + 2\lambda_2 + \lambda_3 = 0$

$2\lambda_1 + 5\lambda_2 + \lambda_1 = 0$

$\lambda_1 - 2\lambda_2 + 3\lambda_1 = 0$

$3\lambda_1 + 2\lambda_2 + \lambda_1 = 0$

$3\lambda_1 + 5\lambda_2 = 0$

$4\lambda_1 - 2\lambda_2 = 0$

$4\lambda_1 + 2\lambda_2 = 0$

$3\lambda_1 + 5\lambda_2 = 0$
$2\lambda_1 - \lambda_2 = 0$ → $\lambda_2 = 2\lambda_1$
$2\lambda_1 + \lambda_2 = 0$

$$\begin{cases} 3t_1 + 10t_1 = 0 \\ 2t_1 - 2t_1 = 0 \\ 2t_1 + 2t_1 = 0 \end{cases}$$

$$\underline{\qquad\qquad\qquad\qquad}$$

$$13\,t_1 = 0$$
$$0 = 0$$
$$4\,t_1 = 0$$

$$\underline{\underline{t_1 = t_2 = t_3 = 0}} \qquad \in \text{ lineáris függetlenek}$$

$$k(\delta) \text{ felírható}$$

$$-\frac{1}{2}\begin{pmatrix}-1\\1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}0\\1\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\2\\1\end{pmatrix} =$$

$$= \begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} + \begin{pmatrix}0\\-\frac{1}{2}\\6\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\1\\\frac{1}{2}\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}\ \checkmark$$

$$0$$

⑤
$$u = \left\{\begin{pmatrix} x_1 + x_2 \\ -x_1 - x_2 \\ 4x_2 \end{pmatrix}\ \Big|\ x_1, x_2 \in \mathbb{R}\right\}$$

$$c_1 = a_1 + b_1$$
$$c_2 = a_2 + b_2$$

$$\underline{a} + \underline{b} = \begin{pmatrix} a_1 + a_2 + b_1 + b_2 \\ -a_1 - a_2 - b_1 - b_2 \\ 4a_2 + 4b_2 \end{pmatrix} = \begin{pmatrix} (a_1+b_1) + (a_2+b_2) \\ -(a_1+b_1) - (a_2+b_2) \\ 4(a_2+b_2) \end{pmatrix} = \begin{pmatrix} c_1 + c_2 \\ -c_1 - c_2 \\ 4c_2 \end{pmatrix}$$

$$t \cdot \underline{a} = A\begin{pmatrix} a_1 + a_2 \\ -a_1 - a_2 \\ 4a_2 \end{pmatrix} = \begin{pmatrix} t a_1 + t a_2 \\ -t a_1 - t a_2 \\ 4t a_2 \end{pmatrix} = \begin{pmatrix} d_1 + d_2 \\ -d_1 - d_2 \\ 4 d_2 \end{pmatrix} \in u$$

$$d_1 = t a_1$$
$$d_2 = t a_2$$

⑥
$$t_1\begin{pmatrix}-1\\1\\1\end{pmatrix} + t_2\begin{pmatrix}0\\1\\0\end{pmatrix} + d_3\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{cases} -t_1 + d_3 = 1 \\ t_1 + t_2 + 2d_3 = 0 \\ t_1 + d_3 = 0 \end{cases}$$

$$\begin{cases} t_1 - d_2 = 1 \\ t_1 + t_2 - 2t_1 = 0 \end{cases} \qquad d_3 = t_1$$

$$t_1 + d_2 = -1$$
$$-t_1 + t_2 = 0 \Rightarrow t_2 = t_1$$

$$2t_1 = -1$$
$$t_1 = -\frac{1}{2}$$
$$d_3 = \frac{1}{2} \qquad d_2 = -\frac{1}{2}$$