Solving using Cramer's rule:

x-2y-3z=6
2x-3y+z=-1
3x+y+z=5

$$A = \begin{vmatrix} 1 & -2 & -3 \\ 2 & -3 & 1 \\ 3 & 1 & 1 \end{vmatrix} \quad \text{beta} = (6, -1, 5)$$

$$\det(A) = -3-6-6-27+4-1 = -39 \neq 0$$

$$D_x = \begin{vmatrix} 6 & -2 & -3 \\ -1 & -3 & 1 \\ 5 & 1 & 1 \end{vmatrix} = -18-10+3-45-2-6 = -78$$

$$x = D_x/\det(A) = 2$$

$$D_y = \begin{vmatrix} 1 & 6 & -3 \\ 2 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = -1+18-30-9-12-5 = -39$$

$$y = D_y/\det(A) = 1$$

$$D_z = \begin{vmatrix} 1 & -2 & 6 \\ 2 & -3 & -1 \\ 3 & 1 & 5 \end{vmatrix} = -15+6+12+54+20+1 = 78$$

$$z = D_z/\det(A) = -2$$

Solving by Gauss elimination:

x+y+z=1
8x-y+2z=0
25x-2y+7z=1

-9y-6z=-8 (II)-8(I)
-27y-18z=-24 (III)-25(I)

9y+6z=8 -(I)
9y+6z=8 -1/3(II)

z=(8-9y)/6=4/3-3/2y
x=1-y-z=1-y-4/3+3/2y=-1/3+1/2y

Solving by Cramer's rules:

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 8 & -1 & 2 \\ 25 & -2 & 7 \end{vmatrix} = -7+50-16+25-56+4=0$$

$$D_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & -2 & 7 \end{vmatrix} = -7+2+0+1-0+4=0$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 8 & 0 & 2 \\ 25 & 1 & 7 \end{vmatrix} = 0+50+8-0-56-2=0$$

$$D_z = \begin{vmatrix} 1 & 1 & 1 \\ 8 & -1 & 0 \\ 25 & -2 & 1 \end{vmatrix} = -1+0-16+25-8-0=0$$

Are the following vectors independent linearly?

$\underset{\sim}{x}(6,4,-1)$
$\underset{\sim}{y}(2,1,6)$
$\underset{\sim}{z}(1,0,4)$

$a\underset{\sim}{x}+b\underset{\sim}{y}+c\underset{\sim}{z}=0$

$$a\begin{pmatrix}6\\4\\-1\end{pmatrix}+b\begin{pmatrix}2\\1\\6\end{pmatrix}+c\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$6a+2b+c=0$
$4a+b=0$
$-a+6b+4c=0$

$-a+6b+4c=0$
$6a+2b+c=0$
$4a+b=0$

$38b+25c=0$ (II)+6(I)
$25b+16c=0$ (III)+4(I)

$-16*38/25c=0$ (II)-38/25(I)

$c=0$

$a=0\ b=0\ c=0$

By Gauss elimination

x+2y+5z=-9
x-y+3z=2
3x-6y-z=25

-3y-2z=11 (II)-(I)
-12y-16z=52 (III)-3(I)

3y+2z=-11 -(I)
3y+4z=-13 -1/4(II)

2z=-2 (II)-(I)

x=-9-2y-5z=-9+6+5=2
y=-1/3(11+2z)=-3
z=-1
x=2 y=-3   z=-1

Cramer's rule

$$|A|=\begin{vmatrix}1 & 2 & 5\\ 1 & -1 & 3\\ 3 & -6 & -1\end{vmatrix}=1+18-30+15+2+18=24$$

$$Dx=\begin{vmatrix}-9 & 2 & 5\\ 2 & -1 & 3\\ 25 & -6 & -1\end{vmatrix}=-9+150-60+125+4-162=48$$

x=Dx/det(A)=48/24=2

$$Dy=\begin{vmatrix}1 & -9 & 5\\ 1 & 2 & 3\\ 3 & 25 & -1\end{vmatrix}=-2-81+125-30-9-75=-72$$

y=Dy/deat(A)=-3

$$
Dz=\begin{vmatrix}1 & 2 & -9\\ 1 & -1 & 2\\ 3 & -6 & 25\end{vmatrix}=-25+12+54-27-50+12=-24
$$

$$
z=Dz/\det(A)=-24/24=-1
$$

$$
x=2 \quad y=-3 \quad z=-1
$$