VII. MATRIXES

17. $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 1 & 1 \end{pmatrix}$

$\mathbf{D} = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$ $\mathbf{E} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ $\mathbf{F} = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

Calculate the following operations:

a.) $\mathbf{F} \cdot \mathbf{A}$

b.) $\mathbf{C} \cdot \mathbf{B}$

c.) $\mathbf{A}^T + \mathbf{F}$

d.) $\mathbf{C} \cdot \mathbf{E}^T$

e.) $\mathbf{E} \cdot \mathbf{B}$

f.) $\mathbf{A}^{-1}$

g.) $\mathbf{C}^{-1}$

18. $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$

$\mathbf{D} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$ $\mathbf{E} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ $\mathbf{F} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

Calculate the following terms:

a.) $\mathbf{D} \cdot \mathbf{A}$

b.) $\mathbf{B} \cdot \mathbf{C}$

c.) $\mathbf{A}^T + \mathbf{F}$

d.) $\mathbf{C} \cdot \mathbf{E}^T$

e.) $\mathbf{E} \cdot \mathbf{B}$

f.) $\mathbf{E}^{-1}$

g.) $\mathbf{F}^{-1}$

VIII. BASISTRANSFORMATIONS

19. What is the $\mathbf{a} = (5, 1, 2, 4)^T$ vector in the following basis?

$$\begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \\ 5 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

20. What is the $\mathbf{a} = (2, 1, 2, 1)^T$ vector in the following basis?

$$\begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

IX. MATRIXEQUATIONS

21. Solve the following matrix equations:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 4 \\ 3 & 1 & 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 0 & 5 & 21 \\ 8 & 4 & 0 \\ 4 & 12 & 14 \end{pmatrix}$$

a.) $\mathbf{X} \cdot \mathbf{A} - 3\mathbf{B} = \mathbf{C}$

b.) $\mathbf{A} \cdot \mathbf{X}^{-1} + \mathbf{B}^{-1} = 2\mathbf{X}^{-1}$

22. Solve the following matrix equations:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

a.) $\mathbf{X} \cdot \mathbf{A} - 2\mathbf{B} = \mathbf{C}$

b.) $\mathbf{B} \cdot \mathbf{X}^{-1} + \mathbf{C}^{-1} = \mathbf{X}^{-1}$

X. EIGENVALUES

23. Calculate the eigenvalues and eigenvectors of the following matrix:

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$$

24. Calculate the eigenvalues and eigenvectors of the following matrix:

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

XI. LINEAR TRANSFORMATIONS

25. Which transformation is linear? Give the matrix of the linear transformation.

a.)

$$f(\mathbf{x}) = \begin{pmatrix} 2x_1 + x_2 \\ x_1 + x_2 \\ x_3 + 3x_2 \\ x_3 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^3)$$

b.)

$$f(\mathbf{x}) = \begin{pmatrix} 3x_1 \\ x_1 x_2 \\ 2x_2 \\ x_1 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^2)$$

26. Which transformation is linear? Give the matrix of the linear transformation.

a.)

$$f(\mathbf{x}) = \begin{pmatrix} x_1 + x_2 \\ x_1 + 2x_2 \\ x_3 + 2x_2 \\ 2x_3 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^3)$$

b.)

$$f(\mathbf{x}) = \begin{pmatrix} x_1 \\ 2x_1 x_2 \\ 3x_2 \\ 2x_1 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^2)$$

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