Név / Name: DR. FACSÓ GÁBOR Neptun kód / code: BQQQ7Y

(ONFIZ1-0401) Elemi lineáris algebra 4 zárthelyi dolgozat / Elementary Linear Algebra, Test 4

1. Adottak a következő vektorok: $\mathbf{a} = (1, 0, 1)$, $\mathbf{b} = (1, 3, 0)$ és $\mathbf{c} = (1, 2, 2)$. Határozza meg a következő összefüggéseket / Calculate the following expressions:
   a.) $(\mathbf{a} - \mathbf{b})\mathbf{c}$
   b.) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a}$
   c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
   d.) Mennyi az **a** és **b** vektorok által közbezárt szög? / What is the angle of Vectors **a** and **b**?
   e.) Egy síkban vannak-e az **a**, **b**, **c** vektorok? / Are Vectors **a**, **b**, and **c** in the same plane?
   f.) Adjon meg egy vektort, mely merőleges az **b** vektorra. / Determine a perpendicular vector to Vector **b**.

   (10 po(i)nt)

2. Számítsa ki a következő mátrixok determinánsát! / Calculate the deteminant of the following matrixes:

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

   (10 po(i)nt)

3. Oldja meg az alábbi lineáris egyenletrendszert! / Solve the following system of linear equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 5x_1 - x_2 + x_3 &= -2 \\ 2x_1 + x_2 + 2x_3 &= 0 \end{aligned}$$

   (10 po(i)nt)

4. Lineárisan függetlenek-e az $\mathbf{a} = (1, 2, 1, 2)$, a $\mathbf{b} = (0, 2, 2, 2)$ és a $\mathbf{c} = (1, 1, 2, 1)$ vektorok? / Are independent linear Vectors $\mathbf{a} = (1, 2, 1, 2)$, $\mathbf{b} = (0, 2, 2, 2)$, and $\mathbf{c} = (1, 1, 2, 1)$? (10 po(i)nt)

5. Altér -e az $\mathbb{R}^3$-on az $U = \{(x_1 + x_2, x_1 - x_2, 3x_2) \,|\, x_1, x_2 \in \mathbb{R}\}$? / Is subspace on $\mathbb{R}^3$ the $U = \{(x_1 + x_2, x_1 - x_2, 3x_2) \,|\, x_1, x_2 \in \mathbb{R}\}$ set? (10 po(i)nt)

6. Adja meg az $\mathbf{a} = (1, 0, 0, 1)$ vektort az $(1, 1, 0, 0)$; ~~$(0, 1, 1, 0)$; $(0, 0, 1, 1)$; $(1, 0, 0, 1)$ bázisban. / Give~~ the Vector $\mathbf{a} = (1, 0, 0, 1)$ in the $(1, 1, 0, 0)$; $(0, 1, 1, 0)$; $(0, 0, 1, 1)$; $(1, 0, 0, 1)$ basis. (10 po(i)nt)

7. Adottak a következő mátrixok:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \\ 2 & 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 0 & 2 & 1 \end{pmatrix} \mathbf{E} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{F} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

   Végezze el az alábbiak közül az elvégezhető müveleteket! / Calculate the following terms if possible:
   (a) $\mathbf{F} \cdot \mathbf{A}$ (b) $\mathbf{C} \cdot \mathbf{B}$ (c) $\mathbf{A}^T + \mathbf{F}$ (d) $\mathbf{C} \cdot \mathbf{E}^T$ (e) $\mathbf{E} \cdot \mathbf{B}$ (f) $\mathbf{A}^{-1}$ (g) $\mathbf{C}^{-1}$ (10 po(i)nt)

8. Oldja meg az $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ mátrixegyenletet, ha

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

   Solve the $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ matrix equation above.

9. Adja meg az alábbi mátrix sajátértékeit és egy-egy, a sajátértékhez tartozó sajátvektort! / Calc... the eigenvalues of Matrix **A** and give an eigenvector for each eigenvalues:

$$\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$$

(10 po(i)nt)

10. Az alábbi leképezés lineáris? Adja meg a leképezés mátrixát is, ha létezik! / Is this transformation a linear transformation? Give the matrix of the linear transformation if it exists.

$$f(\mathbf{x}) = \begin{pmatrix} x_1 + x_2 \\ -2x_1 + x_2 \\ x_3 + 2x_2 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^3)$$

(10 po(i)nt)

A fenti feladatsor két részre oszlik. Az (1)-(5) feladatok megoldásával a első zárthelyit lehet javítani, illetve pótolni. A (6)-(10) feladatokkal pedig a másodikat. A zárthelyik osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). Mindkét témából zárthelyiből legalább elégségest (2) kell elérni a gyakorlati jegyhez. Ha mindkét zárthelyi legalább közepes (3), akkor megajánlott vizsgajegyet kapnak. A megajánlott jegyet nem szerzőknek, vagy a jegyet nem elfogadóknak vizsgázniuk kell a kiírt időpo(i)ntokban.

You can improve the results of the first mid-term test by solving exercises (1)-(5). If you want to replace your second mid-term exam you must solace exercises (6)-(10). Grades: 0-20 points: Fail 81), 21-27 points: Pass (2), 28-35 points: Satisfactory (3), 36-42 points: Good (4) és 43-50 points: Excellent (5). You must pass both mid-term tests to get a grade for the practice. If you get at least an Average (3) grade for both mid-term tests I will offer you an exam grade based on the test results. If neither exam grade was offered nor you wish for a better grade you must pass a written exam.

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a) $(\underline{a}-\underline{b})\cdot\underline{c} = \left[\begin{pmatrix}1\\0\\1\end{pmatrix}-\begin{pmatrix}1\\3\\0\end{pmatrix}\right]\cdot(1,2,2) = \begin{pmatrix}0\\-3\\1\end{pmatrix}\begin{pmatrix}1\\2\\2\end{pmatrix} =$

$= 0 - 6 + 2 = \underline{-4}$

b) $(\underline{b}+\underline{c})\times\underline{a} = \left[\begin{pmatrix}1\\3\\0\end{pmatrix}+\begin{pmatrix}1\\2\\2\end{pmatrix}\right]\times\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}2\\5\\2\end{pmatrix}\times\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}5\cdot1-2\cdot0\\2\cdot1-2\cdot1\\2\cdot0-5\cdot1\end{pmatrix} =$

$= \underline{\underline{\begin{pmatrix}5\\0\\-5\end{pmatrix}}} = 5\begin{pmatrix}1\\0\\-1\end{pmatrix}$

c) $(\underline{a},\underline{b},\underline{c}) = (\underline{a}\times\underline{b})\,\underline{c} = \left[\begin{pmatrix}1\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\3\\0\end{pmatrix}\right]\cdot\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}0\cdot0-1\cdot3\\1\cdot1-1\cdot0\\1\cdot3-0\cdot1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix} =$

$= \begin{pmatrix}-3\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix} = -3+2+6 = \underline{\underline{5}}$

d) $\cos\gamma = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}|\cdot|\underline{b}|} = \dfrac{\begin{pmatrix}1\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix}}{\sqrt{1^2+0^2+1^2}\cdot\sqrt{1^2+3^2+0^2}} = \dfrac{1}{\sqrt{2}\cdot\sqrt{10}} = \dfrac{1}{2\sqrt{5}} =$

$= \underline{\underline{\dfrac{\sqrt{5}}{10}}}$

e) Nem, mert $(\underline{a},\underline{b},\underline{c}) \neq 0$. / No, because $(\underline{a},\underline{b},\underline{c}) \neq 0$.

f) $\underline{\underline{\underline{b}_\perp = (0,0,1)}}$ $\quad \underline{b}_\perp\cdot\underline{b} = \begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix} = \underline{0}$ ✓

② $\underline{A} = 3^2 - 2^2 = \underline{\underline{5}}$

$\underline{B} = 1 \cdot 2 \cdot 1 + 1 \cdot 2 \cdot 2 + 2 \cdot 1 \cdot 2 - 2^3 - 1^3 - 1 \cdot 2^2 = 2 + 4 + 4 - 8 - 1 - 4 =$

$= \underline{\underline{-3}}$

$$\underline{C} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \overset{2 + 2 + 1 - 1 - 1 - 4}{\left(2 \cdot 1^2 + 1^2 \cdot 2 + 1^3 - 1^3 - 1^3 - 1 \cdot 2^2\right)} -$$

$$- \overset{1 + 2 + 2 - 1 - 4 - 1}{\left(1^3 + 2 \cdot 1^2 + 1 \cdot 2 \cdot 1 - 1^3 - 2^2 \cdot 1 - 1^3\right)} = -1 - (-1) = \underline{\underline{0}}$$

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③
$$\begin{cases} x_1 - x_2 + 2x_3 = 1 \\ 5x_1 - x_2 + x_3 = -2 \\ 2x_1 + x_2 + 2x_3 = 0 \end{cases}$$

$\underline{A}\,\underline{x} = \underline{b} \qquad \rightarrow \underline{x} = \underline{A}^{-1} \cdot \underline{b}$

$$\underline{A} = \begin{pmatrix} 1 & -1 & 2 \\ 5 & -1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$\det(\underline{A}) = -2 - 2 + 10 + 4 + 10 - 1 = 19 \neq 0 \Rightarrow \exists\, \underline{A}^{-1}$

$\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = -3 \quad \begin{vmatrix} 5 & 1 \\ 2 & 2 \end{vmatrix} = 8 \quad \begin{vmatrix} 5 & -1 \\ 2 & 1 \end{vmatrix} = 7 \qquad \begin{pmatrix} -3 & 8 & 7 \\ 4 & -2 & 3 \\ 1 & -9 & 4 \end{pmatrix}$

$\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} = -4 \quad \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = -2 \quad \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$

$\begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9 \quad \begin{vmatrix} 1 & -1 \\ 5 & -1 \end{vmatrix} = 4$

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③ Feladat (Cont'd)

transzp.
$$\begin{pmatrix} -3 & -4 & 1 \\ 8 & -2 & -9 \\ 7 & 3 & 4 \end{pmatrix} \xrightarrow[\text{előj.}]{\text{Chess}} \begin{pmatrix} -3 & 4 & 1 \\ -8 & -2 & 9 \\ 7 & -3 & 4 \end{pmatrix} \to A^{-1} = \frac{1}{19} \begin{pmatrix} -3 & 4 & 1 \\ -8 & 2 & 9 \\ 7 & -3 & 4 \end{pmatrix}$$

$$x = \frac{1}{19} \begin{pmatrix} -3 & 4 & 1 \\ -8 & -2 & 9 \\ 7 & -3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \frac{1}{19} \begin{pmatrix} -3-8 \\ -8+4 \\ 7+6 \end{pmatrix} = \frac{1}{19} \begin{pmatrix} -11 \\ -4 \\ 13 \end{pmatrix}$$

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④
$$\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 2 \\ 2 \\ 2 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \;\&\; \lambda_1 = \lambda_2 = \lambda_3 = 0$$

⇓

Lineárisan független
Linearly independent.

$$\lambda_1 + \lambda_3 = 0 \to \lambda_3 = -\lambda_1$$

$$\begin{cases} 2\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \\ \lambda_1 + 2\lambda_2 + \lambda_3 = 0 \\ 2\lambda_1 + \lambda_2 + \lambda_3 = 0 \end{cases}$$

$$\begin{cases} 2\lambda_1 + 2\lambda_2 - \lambda_1 = 0 \\ \lambda_1 + 2\lambda_2 - \lambda_1 = 0 \\ 2\lambda_1 + \lambda_2 - \lambda_1 = 0 \end{cases}$$

$$\Rightarrow \lambda_1 + 2\lambda_2 = 0$$
$$2\lambda_2 = 0 \to \lambda_2 = 0 ✓$$
$$\lambda_1 + \lambda_2 = 0$$

$\lambda_1 = 0$ ✓

$\lambda_3 = 0$ ✓

Lineárisan független
Linearly independent.

(5) $U = \left\{ \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 3x_2 \end{pmatrix} \in \mathbb{R}^3 ; \ x_1, x_2 \in \mathbb{R} \right\}$

(I) addicio: $\begin{pmatrix} a_1 + b_1 + a_2 + b_2 \\ a_1 + b_1 - a_2 - b_2 \\ 3a_2 + 3b_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 \\ a_1 - a_2 \\ 3a_2 \end{pmatrix} + \begin{pmatrix} b_1 + b_2 \\ b_1 - b_2 \\ 3b_2 \end{pmatrix} =$

$\underset{c_i = a_i + b_i}{=} \begin{pmatrix} c_1 + c_2 \\ c_1 - c_2 \\ 3c_2 \end{pmatrix} \in U$

(II) Pregunta $\begin{pmatrix} \lambda a_1 + \lambda a_2 \\ \lambda a_1 - \lambda a_2 \\ \lambda a_2 \end{pmatrix} \underset{d_i = \lambda a_i}{=} \begin{pmatrix} d_1 + d_2 \\ d_1 - d_2 \\ d_2 \end{pmatrix} \in U$

logen, dir / Yes, it is a subspace

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(6) $\lambda_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \lambda_4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\lambda_1 + \lambda_4 = 1$
$\lambda_1 + \lambda_2 = 0 \to \lambda_2 = -\lambda_1$
$\lambda_2 + \lambda_3 = 0 \to \lambda_3 = -\lambda_2 = \lambda_1$
$\lambda_3 + \lambda_4 = 1$

$\lambda_1 + \lambda_4 = 1$

$\lambda_1 + \lambda_4 = 1$

p.ej. $\lambda_1 = \frac{1}{2} = \lambda_4 = \lambda_3 \quad \lambda_2 = -\frac{1}{2}$

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⑥ fogl.kar / cont'd

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \checkmark$$

⑦

a) $\underline{I} \cdot \underline{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2+2 & 0+0+1 \end{pmatrix} = \underline{\begin{pmatrix} 2 & 1 & 2 \\ 1 & 4 & 1 \end{pmatrix}}$

b) $\underset{2\times3}{\underline{C}} \cdot \underset{1\times3}{\underline{B}} \rightarrow \nexists$ létezik / it is not possible.

c) $\underset{3\times3}{\underline{A}^T} + \underset{3\times2}{\underline{I}} \rightarrow$ — // —

d) $\underset{3\times2}{\underline{C}} \cdot \underset{2\times2}{\underline{C}^T} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \underline{\begin{pmatrix} 2 & 4 \\ 4 & 2 \\ 2 & 4 \end{pmatrix}}$

e) $\underset{2\times2}{\underline{E}} \cdot \underset{1\times3}{\underline{B}} = \longrightarrow \nexists$ létezik / it is not possible

f) $\underline{A}^{-1} \Rightarrow \det(\underline{A}) = 1^3 + 0 + 0 - 0 - 2^2 \cdot 1 - 1 \cdot 2^2 = 1 - 8 = \underline{-7}$

0/0

$\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \quad \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4$

$\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2$

$\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = \cancel{2}\,4 \quad \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

$$\begin{pmatrix} -3 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & -3 \end{pmatrix} \xrightarrow{\text{transpose}} \begin{pmatrix} -3 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & -3 \end{pmatrix} \xrightarrow{\text{chess}} \begin{pmatrix} -3 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{7} \begin{pmatrix} 3 & 2 & -4 \\ +2 & -1 & 2 \\ -4 & 2 & 3 \end{pmatrix}$$

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9) $\underline{C}_{2\times 3}$: $\neg$ adjugate matrix, $\nexists$ inverse.

$\neg$ square matrix $\Rightarrow$ $\nexists$ [illegible].

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8) $\underline{A}\,\underline{x} = \underline{b}$ / ~~[illegible]~~ $\underline{A}^{-1}$.

$\underline{x} = \underline{A}^{-1} \cdot \underline{b}$

$|A| = 0 + 2 + 2 - 0 - 4 - 1 = -1 \neq 0$

$\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$

$\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1$

$\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \quad \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$

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(8) felsr./conf'a

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & -4 \end{pmatrix} \xrightarrow{trsp.} \begin{pmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & -4 \end{pmatrix} \xrightarrow{A_{ij}} \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & +1 \\ 2 & +1 & -4 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} +1 & +1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & +4 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} \cancel{1} & \cancel{1} & \cancel{2} \\ \cancel{1} & \cancel{0} & \cancel{-1} \\ -2 & -1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-2 & 1 & 1-2 \\ 1-1 & 0 & 0 \\ -2+4 & -1 & -2+4 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 2 & -1 & 2 \end{pmatrix}$$

(9) $(3-\lambda)(5-\lambda) = 0 \Rightarrow \lambda_1 = 3 \quad \lambda_2 = 5$

λ_1: $\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow$ $\alpha_{\lambda_1} = \{ t \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; t \in \mathbb{R} \} \rightarrow \cancel{\ldots} t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\cancel{\ldots 2x_2 \Rightarrow x}$

$2x_2 = 0 \rightarrow x_2 = 0$

d_2: $\begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2x_1 = 0 \rightarrow x_1 = 0$

$0 = 0$

$\alpha_{d_2} = \left\{ t \begin{pmatrix} 0 \\ 1 \end{pmatrix} ; t \in \mathbb{R} \right\} \rightarrow \underline{\underline{B_1 = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}}}$

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(I) $f(\underline{a}+\underline{b}) = \begin{pmatrix} \cancel{a_1+b_1+t3} \cdot a_1+b_1+a_2+b_2 \\ -2a_1-2b_1+a_2+b_2 \\ a_3+b_3+2a_2+2b_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 \\ -2a_1+a_2 \\ a_3+2a_2 \end{pmatrix} + \begin{pmatrix} b_1+b_2 \\ -2b_1+b_2 \\ b_3+2b_2 \end{pmatrix} =$

homogeneitás

$= f(\underline{a}) + f(\underline{b})$ ✓

(II) multiplikatív

$f(\lambda a) = \begin{pmatrix} \lambda a_1 + \lambda a_2 \\ -2\lambda a_1 + \lambda a_2 \\ \lambda a_3 + 2\lambda a_2 \end{pmatrix} = \lambda \begin{pmatrix} a_1+a_2 \\ -2a_1+a_2 \\ a_3+2a_2 \end{pmatrix} = \lambda f(a)$ ✓

Ez egy lineáris transzformáció / This is a linear transformation.

$\begin{pmatrix} 1 & 1 & 0 \\ -2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$ ← transzformációs mátrix
transformation matrix