



# (ENKEMNA0302) Applied Linear Algebra

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# Questions

- ▶ Teams, YouTube.com page
- ▶ TDK / Scientific Student thesis
- ▶ Stipendium Hungaricum Scholarship
- ▶ Purpose of your life

# Requirements

- ▶ You will write tests based on the exercises of the practical courses. You can use everything during the test
- ▶ The minimum requirement is 41 % of both tests.
- ▶ Failed tests must be corrected
- ▶ You must take an oral written exam. You cannot use anything
- ▶ Grades: Insufficient/Fail (1): 0-40 %, Sufficient/Pass (2): 41-55 %, Average (3): 56-70 %, Good (4): 71-85 %, Excellent (5): 86-100 %.
- ▶ Mid-term test 1: March 13, mid-term test 2: May 8, retake tests: May 15, 2025.

## Bibliography

*Bernard Kolman and David Hill: Elementary Linear Algebra with Applications, 9th ed., Person, 2007*

*Philip N. Klein: Coding the Matrix: Linear Algebra through Applications to Computer Science, Newtonian Press 2013*

*K. F. Riley, M. P. Hobson, S. J. Bence: Mathematical Methods for Physics and Engineering: A Comprehensive Guide, Cambridge University Press, 3rd. ed. (2006)*

# Practice I

- ▶ Kronecker-delta:  $\delta_{ij} = \begin{cases} 0, & \text{if } i \neq j. \\ 1, & \text{if } i = j. \end{cases}$
- ▶ Levi-Civita symbol:  $\epsilon_{ijk} = \begin{cases} 1, & \text{if } (i, j, k) \text{ is } (1, 2, 3), (2, 3, 1), \text{ or } (3, 1, 2). \\ -1, & \text{if } (i, j, k) \text{ is } (3, 2, 1), (2, 1, 3), \text{ or } (1, 3, 2). \\ 0, & \text{if } i = j, j = k, \text{ or } i = k. \end{cases}$

Vector product etc

- ▶ Dyadic product:  $\mathbf{a} \otimes \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} (b_1, b_2, b_3) = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$
- ▶ Coordinate systems
  - ▶ Cartesian
  - ▶ Spherical
  - ▶ Cylindric

## Practice II

- ▶ Astronomical and Geophysical Coordinate Systems.
  - ▶ Geocentric solar ecliptic (GSE): This system has its X axis towards the Sun and its Z axis perpendicular to the plane of the Earth's orbit around the Sun (positive North). This system is fixed with respect to the Earth-Sun line. It is convenient for specifying magnetospheric boundaries. It has also been widely adopted as the system for representing vector quantities in space physics databases.
  - ▶ Geocentric solar magnetospheric (GSM): This system has its X axis towards the Sun and its Z axis is the projection of the Earth's magnetic dipole axis (positive North) on to the plane perpendicular to the X axis. The direction of the geomagnetic field near the nose of the magnetosphere is well-ordered by this system. Thus it is considered the best system to use when studying the effects of interplanetary magnetic field components (e.g.  $B_z$ ) on magnetospheric and ionospheric phenomena.
  - ▶ Heliocentric Earth ecliptic (HEE): This system has its X axis towards the Earth and its Z axis perpendicular to the plane of the Earth's orbit around the Sun (positive North). This system is fixed with respect to the Earth-Sun line.

## Practice III

- ▶ NASA SPICE kernels: <https://naif.jpl.nasa.gov/naif/data.html>
- ▶ TREPS (Transformations de Repères En Physique Spatiale) enables to transform vector time series (magnetic fields, velocities, positions, ...) in a choice of heliospheric reference frames. Input data may be a local file (ASCII, CDF, netCDF, VOTable), a URL or come from manual edition or via the SAMP protocol. The tool also handles different common time formats. Transformations are based on Spice Kernels from NAIF enabled in 3DView (access via web services). CNRS AMDA: <https://cdpp.irap.omp.eu/index.php/services/treps>

# The End

Thank you for your attention!