Name: _____ Neptun code _____

(ONFIZ1-0401) Elementary Linear Algebra, Test 1

1. Look at the following matrixes:

$$\mathbf{A}=\begin{pmatrix}1&2&0\\2&1&1\\1&1&3\end{pmatrix}\mathbf{B}=\begin{pmatrix}0\\1\\0\end{pmatrix}\mathbf{C}=\begin{pmatrix}2&1\\1&2\\2&1\end{pmatrix}\mathbf{D}=\begin{pmatrix}1&1&1\end{pmatrix}\mathbf{E}=\begin{pmatrix}1&2\\2&1\end{pmatrix}\mathbf{F}=\begin{pmatrix}0&1&1\\2&0&2\end{pmatrix}$$

Calculate the following terms if possible.
(a) $\mathbf{F}\cdot\mathbf{A}$ (b) $\mathbf{C}\cdot\mathbf{B}$ (c) $\mathbf{A}^T+\mathbf{F}$ (d) $\mathbf{C}\cdot\mathbf{E}^T$ (e) $\mathbf{E}\cdot\mathbf{B}$ (f) $\mathbf{A}^{-1}$ (g) $\mathbf{C}^{-1}$ (6 point)

2. Convert the $\mathbf{a}=(5,1,2,4)^T$ vector to the

$$\begin{pmatrix}2\\0\\1\\1\end{pmatrix}\begin{pmatrix}3\\2\\0\\2\end{pmatrix}\begin{pmatrix}4\\1\\1\\0\end{pmatrix}\begin{pmatrix}2\\1\\0\\0\end{pmatrix}$$

basis.. (6 point)

3. Solve the following matrix equations:

$$\mathbf{A}=\begin{pmatrix}1&2&3\\2&0&1\\3&1&1\end{pmatrix}\mathbf{B}=\begin{pmatrix}2&0&3\\0&1&3\\3&3&1\end{pmatrix}\mathbf{C}=\begin{pmatrix}0&5&4\\5&4&0\\4&0&1\end{pmatrix}$$

a.) $\mathbf{X}\cdot\mathbf{A}-3\mathbf{B}=\mathbf{C}$

b.) $\mathbf{A}\cdot\mathbf{X}^{-1}+\mathbf{B}^{-1}=2\mathbf{X}^{-1}$

(10 point)

4. Calculate the eigenvalues of Matrix $\mathbf{A}$ and give an eigenvector for each eigenvalues:

$$\mathbf{A}=\begin{pmatrix}3&2\\2&1\end{pmatrix}$$

(8 point)

5. Which transformation is a linear transformation? Give the matrix of the linear transformation too.

a.)

$$f(\mathbf{x})=\begin{pmatrix}2x_1+x_2\\-x_1+x_2\\x_3+7x_2\\-x_3\end{pmatrix}\left(\mathbf{x}\in\mathbb{R}^3\right)$$

b.)

$$f(\mathbf{x})=\begin{pmatrix}x_1\\-3x_1x_2\\5x_2\\-x_1\end{pmatrix}\left(\mathbf{x}\in\mathbb{R}^2\right)$$

(10 point)

Grades: 21-27 points: Pass (2), 28-35 points: Averages (3), 36-42 points: Good (4) és 43-50 points: Excellent (5). You must pass both mid-term tests to get a grade for the practice. If you get at least an Average (3) grade for both mid-term tests I will offer you an exam grade based on the test results. If you cannot write a test, you failed, or you desire a better grade, you can write a 3rd test after December 8, 2024. If neither exam grade was offered nor you wish for a better grade you must pass a written exam.

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Pécs, December 6, 2024