(PTIA0301) Elementary Linear Algebra thematics

Scalars and vectors. Equality of two vectors. Additive vectors. Planar and spatial vectors. Coordinates of vectors. Length/magnitude and equality of vectors given coordinates. Null vector. Inverse vector. Unit vector. Normal vector. Multiplication of a vector with a scalar, sum of vectors, and difference of vectors. The properties of vector addition. The properties of vector multiplication with scalar. Distance of points, equation of a sphere.

Scalar and vector products. Scalar (inner) product of vectors. The properties of scalar multiplication. The scalar product of vectors is given in coordinates. The angle of two non-zero vectors using the two definitions of scalar product. Orthogonal vectors. Perpendicular projection of a vector to the direction of another vector. Vector Product. Right-hand system. Properties of the vector product. Parallel vectors and vector product. Vector product of the same vector. Vector product with components. Triple product. The volume of the parallelepiped is constructed by three vectors.

Operators and matrixes. Sets, pairs, ordered pairs, and relations. Injections, surjections, bijections, and functions. Linear relations and their properties. Definition of operator, identical, null, mirror and projection operators. Matrixes. The spur of the matrix. Row index, column index.

Determinant of square matrixes. Permutation. Determinant of a square matrix. Calculation of a determinant using Laplace (or cofactor) expansion. Sarrus rule. Triple product in determinant form. The volume of a parallelepiped using a determinant. Basic properties determinants. Gaussian elimination. Upper triangular matrix form. The determinant of an upper triangular matrix.

Linear combination, system of linear equations. Linear combination of vectors and equations. System of linear equations. Coefficient matrix of a system of linear equations. Gauss Elimination Method for Solving System of Linear Equations. Equivalence of two systems of linear equations. Cramer's rule. Linear Independence of vectors.

Vector Space. Vector space definition. Cartesian product, polynomial. Subspace, linear subspace. Generator system (GS), finite GS and linear independent GS, and dimension of a vector space. The base of a vector space. Plane and affine subspace. A (Minkowski-)sum of set. The sum and intersection of subspaces. Direct sum of subspace.

Matrix operations, inverse matrix, the rank of matrix. Transpose. The sum of matrixes. Multiplication of a scaler with a matrix. Multiplication of matrixes. Identity matrix. Regular and singular matrixes. The inverse of a square matrix. Inverse matrix calculation by elemental transformations. Calculation of inverse matrix by subdeterminant. Exponentiation of matrixes and their equations. Trapeziod form of matrixes. The rank of matrixes and its calculation by transforming them to trapezoid form. The common rank of the maximal ranked non-disappearing subdeterminants and the matrix rank.

Linear Transformations. Linear mapping. Matrix representation. Linear transformation. Linear and bilinear forms. Symmetric bilinear form. Quadratic form. Positive and negative (semi) definite quadratic forms. Inner product, mapping, isomorphism. Isomorphism of two vector spaces and their dimensions.

Gram-Schmidt Orthogonalization. Euclidean vector space. Orthogonal and orthonormal vector systems. All Euclidean space has an orthonormal basis. Orthogonalization.

Eigenvalue, Eigenvector. Definition of eigenvalue and eigenvector. Subspace of eigenvectors. Determination of eigenvalues. The characteristic polynomial of a matrix, a linear transformation, and their relationship. Characteristic solution and eigenvalue of a linear transformation. A number $\lambda \in \mathbb{R}$ is a characteristic solution of the linear transformation φ if λ is a solution of the characteristic polynomial of φ. Eigenvalues and characteristic solution of linear transformations.

Dr. Gabor FACSKO
facskog@gamma.ttk.pte.hu

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