$$\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\underline{A} \cdot \varphi = \overset{\in \mathbb{R}}{\lambda}\, \varphi$$

$$\underline{A}\,\underline{x} - \lambda \underline{x} = 0$$

$$\underline{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\underline{A}\,\underline{x} - \lambda \underline{x} = 0$$

$$(\underline{A} - \lambda E_2)\,\underline{x} = 0$$

$\lambda \in \mathbb{R}$

$x \in \mathbb{R}^3$

$A: \mathbb{R}^3 \to \mathbb{R}^3$

$$\underline{A} - \lambda E_2 = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

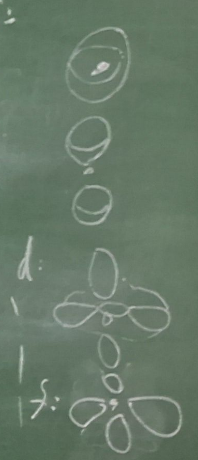
$$(2-\lambda)(2-\lambda) - 1 = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$D = 16 - 4 \cdot 3 = 4$$

$$\lambda_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} 1 \\ \underline{\underline{3}} \end{cases}$$



λ_1: $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = 0$

$x_1 + x_2 = 0$

$x_1 + x_2 = 0$

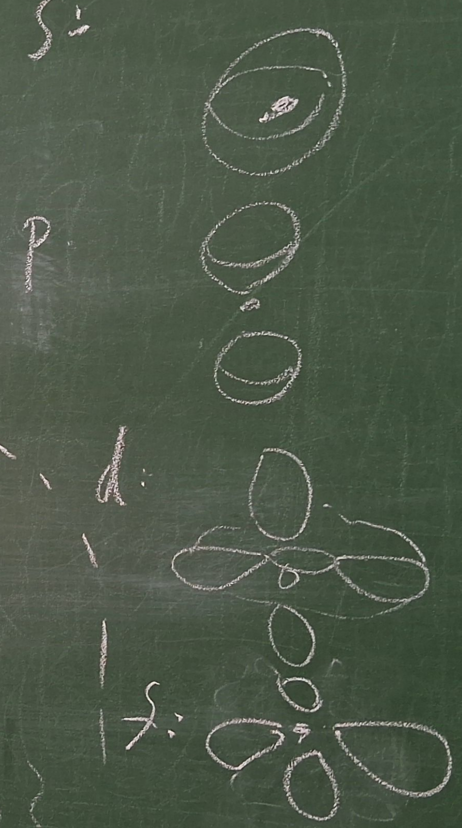
$x_2 = -x_1$

λ_2: $\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = 0$

$x_1 = x_2$

$\mathcal{L}_{\lambda_1} = \left\{ \begin{pmatrix}t\\ -t\end{pmatrix} \middle| t \in \mathbb{R} \right\}$

$\mathcal{L}_{\lambda_2} = \left\{ \begin{pmatrix}t\\ t\end{pmatrix} \middle| t \in \mathbb{R} \right\}$



$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix} \rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\lambda_{1,2,3} = \begin{cases} 1 \\ 2 \\ 3 \end{cases}$$