Név / Name: DR. FAZSI GÁBOR Neptun kód / code: B7Q0QY

(MATNA1902) Alkalmazott lineáris algebra 3. zárthelyi dolgozat
(ENKEMNA0302) Applied Linear Algebra Test 3

1. Adottak a következő vektorok és mátrixok / We have the following vectors and matrixes:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

Számolja ki a következő dyadikus és Kronecker szorzatokat / Calculate the following dyadic and Kronecker products: (a) $\mathbf{a} \otimes \mathbf{b}$; (b) $\mathbf{C} \otimes \mathbf{D}$. (10 pont)

2. Adott az $\mathbf{a}(1, 1, 1)$ vektor.
   (a) Írja fel a forgatási mátrixokat, amivel az $(0, 0, 1)$ irányba lehet forgatni az $\mathbf{a}$ vektort! / Give the rotational matrixes to rotate this vector to the direction of the $(0, 0, 1)$ vector.
   (b) Adja meg azt a $\mathbf{S}$ eltolási mátrixot, amivel az $\mathbf{a}$ vektort el lehet tolni a $(1, -1, 0)$ irányba! / Determine the $\mathbf{S}$ transformation matrix that shifts this $\mathbf{a}$ vector to the $(1, -1, 0)$ direction.

(10 pont)

3. Adottak a következő mátrixok / We have the following matrixes:

$$\mathbf{D_1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \mathbf{D_2} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 13 \end{pmatrix} \mathbf{T} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{S} = \begin{pmatrix} 0 & 0 & 3 \\ 3 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Végezze el a következő műveleteket! / Calculate the following expressions: (a) $\mathbf{D_1} + \mathbf{D_2}$; $\mathbf{D_1} \cdot \mathbf{D_2}$; (b) $|\mathbf{D_1}|$; $|\mathbf{S}|$; $|\mathbf{P}|$; (c) $\mathbf{D}_1^{-1}$; $\mathbf{D}_2^{-1}$; (d) $\mathbf{D}_1^2$; $\mathbf{D}_2^2$; (e) $\mathbf{D}_1^3$; $\mathbf{D}_2^3$. Csak ellenőrzésre használják a Sarrus-szabályt és az adjungálást. Használják az adott mátrixokról tanultakat. / Use the Sarrus rule and adjudication for check only. Use the learned features of these matrixes. (10 pont)

4. Adott a következő mátrix / We have the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 3 & 3 & 2 \\ 3 & 3 & 2 & 2 \end{pmatrix}$$

Bontsa fel ezt a mátrixokat egy szimmetrikus és egy ferdén szimmetrikus mátrix összegére! / Divide this matrix into a sum of symmetric and skew-symmetric matrixes. (10 pont)

5. Adottak a következő blokk mátrixok / We have the following block matrixes:

$$\mathbf{A} = \left(\begin{array}{ccc|ccc} 0 & 2 & 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 3 & 0 & 3 & 2 & 3 \end{array}\right) \mathbf{B} = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 2 & 1 \\ \hline 0 & 1 \\ 2 & 0 \\ 0 & 1 \end{array}\right)$$

Végezze el a következő műveleteket, ha lehetséges! / Calculate the following expressions, if it is possible: (a) $\mathbf{A} + \mathbf{B}$ (b) $2 \cdot \mathbf{A}$; $3 \cdot \mathbf{B}$; (c) $\mathbf{A} \cdot \mathbf{B}$. A műveleteket a blokokkal végezze, a rendes mátrix szorzást csak ellenőrzésre használja! / Calculate with the block. Use the normal matrix operations only for checking. (10 pont)

6. Mátrixok / Matrices

   a.) Adja meg az alábbi mátrixok nyomát! / What is the trace of the matrices below?

$$\mathbf{A} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 3 & 2+i \\ 2-i & 3 \end{pmatrix} \quad \mathbf{D} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

   b.) Melyik mátrix (szemi)ortogonális? Melyik mátrix önadjungált (Hermitikus), melyik unitér? / Which matrix is (semi)orthogonal? Which matrix is Hermitian or unitary?

   c.) Számolja ki az ortogonális és az unitér mátrixok inverzét! / Calculate the inverse of the orthogonal and unitary matrices.

   d.) Mi az $(1,1,0)$ és $(0,i,i)$ pontok távolsága? / What is the distance between the points $(1,1,0)$ and $(0,i,i)$? (20 pont)

7. LU-felbontás / LU decomposition

   a.) Adja meg az alábbi mátrix LU-felbontását! / What is the LU decomposition of the matrix below?

$$\mathbf{A} = \begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix}$$

   b.) Oldja meg az alábbi lineáris egyenletrendszert az $\mathbf{A}$ mátrix LU-felbontását használva! / Solve the following system of linear equations using LU decomposition.

$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$$

   c.) Számítsa ki az $\mathbf{A}$ mátrix az inverzét LU felbontással! / Calculate the inverse of matrix $\mathbf{A}$ using LU decomposition. (20 pont)

8. Adja meg az alábbi mátrix sajátértékeit és a saját altereket, majd diagonizálja a mátrixot! / Calculate the eigenvalues and eigenspaces. Diagonalize the matrices.

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

(10 pont)

A fenti feladatsor két részre oszlik. Az (1)-(5) feladatok megoldásával a első zárthelyit lehet javítani, illetve pótolni. A (6)-(10) feladatokkal pedig a másodikat. A zárthelyik osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). Mindkét témából zárthelyiből legalább elégségest (2) kell elérni a gyakorlati jegyhez.

You can improve the results of the first mid-term test by solving exercises (1)-(5). If you want to replace your second mid-term exam you must solace exercises (6)-(10). Grades: 0-20 points: Fail 81), 21-27 points: Pass (2), 28-35 points: Satisfactory (3), 36-42 points: Good (4) és 43-50 points: Excellent (5). You must pass both mid-term tests to get a grade for the practice.

Facskó Gábor / Gabor FACSKO
facskog@gamma.ttk.pte.hu

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ZASHÓ GÁBOR ISTVÁN BFQRQY

6 a) $\mathrm{trace}(A) = \underline{2\cos\alpha}$ $\mathrm{trace}(C) = \underline{6}$

$\mathrm{trace}(B) = \underline{0}$ $\mathrm{trace}(D) = \frac{1}{\sqrt{2}}(1-i) = \underline{\frac{\sqrt{2}}{2}(1-i)}$

b) $A^{-1} = A^{T} \Leftrightarrow$ ortogonális / orthogonal matrix

$A = A^{H} \Leftrightarrow$ Hermitikus / Hermitian matrix

$A^{-1} = A^{H} \Leftrightarrow$ Unitér mátrix / unitary matrix

$B = B^{T} \Rightarrow B$ ortogonál / orthogonal

	orth	Herm	U
A	+	−	+
B	+	+	+
C	~~−~~	~~+~~	−
D	−	+	~~+~~

$$\underline{A}\cdot\underline{A}^{T} = \begin{pmatrix} \cancel{\cos\alpha} & \cancel{-\sin\alpha} \\ \cancel{\sin\alpha} & \cancel{\cos\alpha} \end{pmatrix} \cdot \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\cdot\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cdot\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix} =$$

$$= \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \Rightarrow \underline{A} \text{ ortogonális}$$

$$\underline{B}\cdot\underline{B}^{T} = \begin{pmatrix} \cancel{1} & \cancel{0} \\ \cancel{0} & \cancel{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \Rightarrow \underline{B} \text{ ortogonális}$$

$$\underline{C}\cdot\underline{C}^{T} = \begin{pmatrix} \cancel{3} & \cancel{2+i} \\ \cancel{2-i} & \cancel{3} \end{pmatrix} \begin{pmatrix} 3 & 2+i \\ 2+i & 3 \end{pmatrix} = \begin{pmatrix} 9+\overset{4+4i-1}{(2+i)^2} & \overset{12}{6-3i+6+3i} \\ \overset{12}{6-3i+6+3i} & \overset{4-4i-1}{(2-i)^2}+9 \end{pmatrix} =$$

$$= \begin{pmatrix} 12+4i & 12 \\ 12 & 12-4i \end{pmatrix} \neq \underline{E} \Rightarrow \text{NEM ortogonális}$$

FALCHÓ GÁBOR BI2QQPY

5a) feladat

$\underline{D} \cdot \underline{D}^T = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ i & i \end{pmatrix}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & -1+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \underline{D}$ ∄ ortogonális

$\underline{A} \neq \underline{A}^H \Rightarrow$ ∄ Hermitikus / Hermitikus

$\underline{B} = \underline{B}^H \Rightarrow$ Hermitikus / −"−

$\cancel{C \neq C^H}$ Hermitikus / Hermitikus $\Leftarrow$ $\underline{C} = \underline{C}^H$

$\underline{D} = \underline{D}^H =$ Hermitikus / Hermitikus

$\underline{A} \cdot \underline{A}^H = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \underline{A}$ unitér / unitér

$\underline{B} \cdot \underline{B}^H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \underline{B}$ unitér / unitér

$\underline{C} \cdot \underline{C}^H = \begin{pmatrix} 3 & 2+i \\ 2-i & 3 \end{pmatrix}\begin{pmatrix} 3 & 2+i \\ 2-i & 3 \end{pmatrix} = \begin{pmatrix} 9+4+1 & 6+3i+6+3i \\ 6-3i+6-3i & 4+1+9 \end{pmatrix} =$

$= \begin{pmatrix} 14 & 12+6i \\ 12-6i & 14 \end{pmatrix} \Rightarrow$ ∄ unitér / unitér

$\underline{D} \cdot \underline{D}^H = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} 1 & -i \\ 1 & +i \end{pmatrix} = \frac{2}{2}\begin{pmatrix} 2 & 0 \\ 0 & +1+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ D unitér

FAZEKAS GÁBOR   BTQQDY

6c)
$$\underline{A}^{-1} = \underline{A}^T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$D^{-1} = D^H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & i \end{pmatrix}$$

$$\underline{B}^{-1} = \underline{B}^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

d) $A(1,1,0)$
$B(0,i,1)$

$\overrightarrow{AB}(1, 1-i, -i)$

$$d_{AB} \; |\overrightarrow{AB}| = \sqrt{1^2 + \overline{(1-i)}(1-i) + \overline{(-i)} \cdot i} = \sqrt{1 + (1+i)(1-i) + i^2} =$$
$$= \sqrt{1 + 1 + 1 - 1} = \underline{\underline{\sqrt{2}}}$$

7a)
$$\underline{A} = \begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix}$$

$\downarrow (III) + \frac{1}{2}(I)$

$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 4 & 1 \end{pmatrix} \qquad E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

T. FOLCSKÓ GÁBOR BTQQQX

(1.) felbontás

$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 4 & 1 \end{pmatrix} \qquad \underline{E}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\downarrow (III)+2(II)$$

$$\underline{U} = \begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

$$\underline{E}_2 \cdot \underline{E}_1 \cdot \underline{A} = \underline{U} \quad / \underline{E}_2^{-1} \cdot$$

$$\underline{E}_1 \cdot \underline{A} = \underline{E}_2^{-1} \cdot \underline{U} \quad / \underline{E}_1^{-1} \cdot$$

$$\underline{A} = \underbrace{\left(\underline{E}_1^{-1} \cdot \underline{E}_2^{-1}\right)}_{\underline{L}} \underline{U}$$

$$\underline{L} = \left(\underline{E}_1^{-1} \cdot \underline{E}_2^{-1}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -2 & 1 \end{pmatrix}$$

$$\underline{A} \begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix} = \underline{L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -2 & 1 \end{pmatrix} \cdot \underline{U} \begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{pmatrix} \checkmark$$

FALSKÓ GÁBOR ISTVÁN BFQQQY

(1b)
$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 1 & 5 & 2 \end{pmatrix} \underline{x} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$$

$$\underline{A} \cdot x = \underline{b}$$

$$\underline{L} \cdot \underbrace{(\underline{U} \cdot x)}_{y} = 6$$

$$\underline{L} \cdot \underline{y} = \underline{b}$$

$$\cancel{\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{pmatrix} \underline{y} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}}$$

$$\cancel{2y_1 - 2y_2 - 2y_3 = -4}$$

$$\cancel{-2y_2 + 2y_3 = -2}$$

$$\cancel{5y_3 = 6}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -2 & 1 \end{pmatrix} \underline{y} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$$

$$\boxed{y_1 = -4}$$

$$\boxed{y_2 = -2}$$

$$\frac{y_1}{2} - 2 \cdot y_2 + y_3 = 6$$

$$+2 + 4 + y_3 = 6 \rightarrow \boxed{y_3 = 0}$$

$$\underline{y} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix}$$

Jacsikó Gábor István BFQQQX

b) feladat

~~[illegible]~~ $\underline{U}\,\underline{x} = \underline{y}$

$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix}$$

$x_1 - x_2 - x_3 = -2$

$-x_2 + x_3 = -1 \rightarrow \boxed{x_2 = 1}$

$5x_3 = 0 \rightarrow \boxed{x_3 = 0}$

$x_1 - 1 - 0 = -2$

$\boxed{x_1 = -1}$

$$\boxed{\underline{x} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}$$

E:
$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} \checkmark$$

E!
$$\begin{pmatrix} 2 & -2 & -2 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix} \checkmark$$

70 $\underline{A} \cdot \underline{x} = \underline{b}$

PALKÓ GÁBOR BTQQQX

1) feladat

$A \cdot X = E$

$$X = A^{-1} = \begin{pmatrix} \frac{7}{10} & \frac{3}{10} & \frac{2}{5} \\ \frac{1}{10} & -\frac{1}{10} & \frac{1}{5} \\ \frac{1}{10} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

$$\begin{pmatrix} \cancel{2 \quad -2 \quad -2} \\ \cancel{0 \quad -2 \quad 2} \\ \cancel{0 \quad 0 \quad 5} \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{7}{10} & \frac{3}{10} & \frac{2}{5} \\ \frac{1}{10} & -\frac{1}{10} & \frac{1}{5} \\ \frac{1}{10} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

$2x_{11} - 2x_{21} - 2x_{31} = 1$

$-2x_{21} + 2x_{31} = 0$

$5x_{31} = \frac{1}{2}$

$x_{31} = \frac{1}{10}$

$x_{21} = x_{31} = \frac{1}{10}$

$\cancel{(2x_{11} - 0 - 0 = 1)}$

$\cancel{x_{11} = \frac{1}{2}}$

$2x_{11} - \frac{1}{5} - \frac{1}{5} = 1$

$2x_{11} = \frac{7}{5}$

$\boxed{x_{11} = \frac{7}{10}}$

$2x_{12} - 2x_{22} - 2x_{32} = 0$

$-2x_{22} + 2x_{32} = 1$

$5x_{32} = 2$

$x_{32} = \frac{2}{5}$

$-2x_{22} + \frac{4}{5} = 1$

$-2x_{22} = \frac{1}{5}$

$x_{22} = -\frac{1}{10}$

$2x_{12} + \frac{1}{5} - \frac{4}{5} = 0$

$x_{12} = +\frac{3}{10}$

$2x_{13} - 2x_{23} - 2x_{33} = 0$

$-2x_{23} + 2x_{33} = 0$

$5x_{33} = 1$

$x_{33} = \frac{1}{5}$

$x_{33} = x_{23} = \frac{1}{5}$

$2x_{13} - \frac{2}{5} - \frac{2}{5} = 0$

$x_{13} = \frac{2}{5}$

FACSKÓ GÁBOR BFQQQY

-at, uta (30 pont van 55

(10) (felyt.)

$\underline{A} \cdot \underline{X} = \underline{E}$

$\underline{L} \underbrace{(\underline{U} \cdot \underline{X})}_{\underline{Y}} = \underline{E}$

$\underline{L} \cdot \underline{Y} = \underline{E}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & -2 & 1 \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$y_{11} = 1$ $y_{12} = 0$ $y_{13} = 0$

$y_{21} = 0$ $y_{22} = 1$ $y_{23} = 0$

$-\frac{y_{11}}{2} - 2y_{21} + y_{31} = 0$

$-\frac{y_{12}}{2} - 2y_{22} + y_{32} = 0$

$-\frac{y_{13}}{2} - 2y_{23} + y_{33} = 1$

$-\frac{1}{2} - 0 + y_{31} = 0 \to \boxed{y_{31} = \frac{1}{2}}$

$0 - 2 + y_{32} = 0 \to \boxed{y_{32} = +2}$

$\cancel{-\frac{y_{12}}{2} - 2y_{22} + y_{32}} = 0$

$0 - 0 + y_{33} = 1$

$y_{33} = 1$

$$\underline{Y} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 2 & 1 \end{pmatrix}$$

FACSKÓ GÁBOR BTQQQX

1)

$$A = \begin{pmatrix}3&1\\1&3\end{pmatrix}$$

Saját. D=:

$$\begin{vmatrix}3-\lambda&1\\1&3-\lambda\end{vmatrix}=0$$

$$9-6\lambda+\lambda^2-1=0$$

$$\lambda^2-6\lambda+8=0$$

$$(\lambda-2)(\lambda-4)=0$$

$\underline{\underline{\lambda_1=2}}$ $\underline{\underline{\lambda_2=4}}$

$\lambda_1=2$

$$\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$x_1=-x_2$$

$$\mathcal{L}_{\lambda_1=2}=\left\{\begin{pmatrix}t\\-t\end{pmatrix} \middle| t\in\mathbb{R}\right\} \to \begin{pmatrix}1\\-1\end{pmatrix}$$

$\lambda_2=4$

$$\begin{pmatrix}-1&1\\1&-1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$-x_1+x_2=0$$

$$x_2=x_1$$

$$\mathcal{L}_{\lambda_2=4}=\left\{\begin{pmatrix}t\\t\end{pmatrix} \middle| t\in\mathbb{R}\right\} \to \begin{pmatrix}1\\1\end{pmatrix}$$

$$D = C^{-1}\cdot A\cdot C$$

$$C=\begin{pmatrix}1&1\\-1&1\end{pmatrix} \to |C|=2\neq0$$

$$\begin{pmatrix}1&-1\\1&1\end{pmatrix}\to\begin{pmatrix}1&+1\\-1&1\end{pmatrix}\to\begin{pmatrix}1&-1\\1&1\end{pmatrix}$$

$$C^{-1}=\frac{1}{2}\begin{pmatrix}1&-1\\1&1\end{pmatrix}$$

E! $C^{-1}C=\frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$$C^{-1}\cdot A\cdot C=\frac{1}{2}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}3&1\\1&3\end{pmatrix}C=\frac{1}{2}\begin{pmatrix}2&-2\\4&4\end{pmatrix}C=\begin{pmatrix}1&-1\\2&2\end{pmatrix}\begin{pmatrix}1&1\\-1&1\end{pmatrix}=\begin{pmatrix}2&0\\0&4\end{pmatrix}$$