(ENKEMNA0302) Applied Linear Algebra

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March 20, 2025

Eigenvalue, Eigenvector, Eigenspace I

- Definition: (Determination of Eigenvalues) The characteristic polynomial of a matrix $A \in \mathcal{M}_{n \times n}$ is defined as the n^{th}-degree polynomial

$$f(x) = |A - xE_n| = \begin{vmatrix} a_{11} - x & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - x & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} - x \end{vmatrix}.$$

- Statement: (Eigenvalues of Triangular Matrices). The eigenvalues of triangular matrices, and thus also of diagonal matrices, are equal to the elements of their main diagonal.

Eigenvalue, Eigenvector, Eigenspace II

- Statement: (Determinant, Trace, and Eigenvalues). If the eigenvalues of an $n \times n$ matrix $\mathbf{A}$ are $\lambda_1, \ldots, \lambda_n$, then

$$\begin{aligned} \det(\mathbf{A}) &= \lambda_1 \lambda_2 \ldots \lambda_n \\ trace(\mathbf{A}) &= \lambda_1 + \lambda_2 + \cdots + \lambda_n \end{aligned}$$

  These values appear in the characteristic polynomial: the determinant corresponds to the constant term, while the trace is the coefficient of $(-\lambda)^{n-1}$.

Eigenvalue, Eigenvector, Eigenspace III

- Theorem: (Eigenspaces of 2×2 Symmetric Matrices). Let $\mathbf{A} \in \mathbb{R}^{2\times 2}$ be a symmetric matrix. Then:
  1. Every eigenvalue of $\mathbf{A}$ is real.
  2. $\mathbf{A}$ has two identical eigenvalues if and only if it is of the form $a\mathbf{I}$, in which case every vector in the plane is an eigenvector.
  3. If $\mathbf{A}$ has two distinct eigenvalues, then its eigenspaces are orthogonal to each other.
- Theorem: (Matrix Invertibility and the Eigenvalue 0). A matrix $\mathbf{A}$ is invertible if and only if 0 is not an eigenvalue.
- Theorem: (Eigenvalues of Special Matrices). Let $\mathbf{A}$ be an $n \times n$ real matrix. Then:
  1. If $\mathbf{A}$ is symmetric, all of its eigenvalues are real.
  2. If $\mathbf{A}$ is skew-symmetric, all of its eigenvalues are imaginary.

The End

Thank you for your attention!