$$A=\begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix} \quad B=\begin{pmatrix} 1 & 1 & 2 \\ -2 & 2 & 0 \end{pmatrix} \quad C=\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix} \quad D=\begin{pmatrix} -1 & 0 & 1 \\ 5 & 1 & 2 \end{pmatrix}$$

Calculate the following terms if it is possible:

(a) A+B, B+C, C+D, 2A-B
(b) AB, AC, BC, BD
(c) $A^{T}$, $D^{T}$
(d) (A), (C)
(e) $A^{-1}$, $D^{-1}$

(a) A+B You cannot do the operation because the size of the matrixes are different.
B+C You cannot do the operation because the size of the matrixes are different.
C+D You cannot do the operation because the size of the matrixes are different.
2A-B You cannot do the operation because the size of the matrixes are different.

(b) AB You cannot do this operation because the number of columns of A and the number of rows of B are different.

$$AC=\begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}=\begin{pmatrix} -1\cdot 0+0\cdot(-2)+2\cdot 3 & -1\cdot 1+0\cdot(-5)+2\cdot 7 \\ 0\cdot 0+2\cdot(-2)+1\cdot 3 & 0\cdot 1+2\cdot(-5)+1\cdot 7 \\ -2\cdot 0+2(-2)+5\cdot 3 & -2\cdot 1+2\cdot(-5)+5\cdot 7 \end{pmatrix}=$$

$$=\begin{pmatrix} 6 & 13 \\ -1 & -3 \\ 11 & 23 \end{pmatrix}$$

$$BC=\begin{pmatrix} 1 & 1 & 2 \\ -2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}=\begin{pmatrix} 1*0+1(-2)+2*3 & 1*1+1(-5)+2*7 \\ -2*0+2(-2)+0*3 & -2*1+2(-5)+0*7 \end{pmatrix}=\begin{pmatrix} 4 & 10 \\ -4 & -12 \end{pmatrix}$$

BD You cannot do this operation because the number of columns of A and the number of rows of B are different.

(c) $A=\begin{pmatrix}-1&0&2\\0&2&1\\-2&2&5\end{pmatrix}$ $A^T=\begin{pmatrix}-1&0&-2\\0&2&2\\2&1&5\end{pmatrix}$

$D=\begin{pmatrix}-1&0&1\\5&1&2\end{pmatrix}$ $D^T=\begin{pmatrix}-1&5\\0&1\\1&2\end{pmatrix}$

(d) $\rho(\underline{A}) = ?$

$$\begin{pmatrix}-1&0&2\\0&2&1\\-2&2&5\end{pmatrix} \xrightarrow{(III)-2(I)} \begin{pmatrix}-1&0&2\\0&2&1\\0&2&1\end{pmatrix} \xrightarrow{(III)-(II)} \begin{pmatrix}-1&0&2\\0&2&1\\0&0&0\end{pmatrix}$$

$\rho(\underline{A}) = 2$

$\rho(\underline{C}) = ?$

$$\begin{pmatrix}0&1\\-2&-5\\3&7\end{pmatrix} \longrightarrow \begin{pmatrix}3&7\\-2&-5\\0&1\end{pmatrix} \xrightarrow[3(II)]{2(I)} \begin{pmatrix}6&14\\-6&-15\\0&1\end{pmatrix}$$

$$\xrightarrow{(II)+(I)} \begin{pmatrix}6&14\\0&-1\\0&1\end{pmatrix} \xrightarrow{(III)+(II)} \begin{pmatrix}6&14\\0&-1\\0&0\end{pmatrix}$$

$\rho(\underline{C}) = \underline{\underline{2}}$

(e)

$$A=\begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix} \quad A^{-1} = ? \quad det(A)=-1*2*5+0*1(-2)+0*1(-2)-2*2(-2)-0*0*5-(-1)1*2=-10+8+2=0$$

The det(A)=0, therefore the inversion of A is not possible.

The inverse of D matrix cannot be calculated because the D is not a square matrix.