(ENKEMNA0302) Applied Linear Algebra thematics

Scalars and vectors. Coordinates of vectors. Length and equality of vectors. Null and unit vectors. Multiply vector with a scalar, sum of vectors, and difference of vectors. Properties of vector operations. Distance of points, equation of sphere. Scalar product of vectors and its properties. Vector product of vectors and its properties. Triple product.

Operators Set, pair, oredered pair, relation, injection, surjection, bijection, and function. Linear vector-vector functions. Matrixes and their operations. Transpose of matrixes. Matrix exponentation and its equations. Rank of matrixes and its properties. Determinations of rank. Example for image processing. Moment of inertia.

Determinant of Square Matrixes Two definitions of determinants. Calculation of a determinant using Laplace (or cofactor) expansion. Sarrus rule. Triple product in determinat form. Volume of a paralepipedon. Gaussian elimination. Cramer's rule.

Matrix Inversion. Regular and singular matrixes. Inverse matrix calculation by elemental transformations. Calculation of inverse matrix by subdeterminant. ~~Sherman-Morrison-Woodbury Theorem.~~

Special matrixes-I. Transformation matrixes: rotational matrix in 2D and 3D. Mirror of the vectors for a line or a plan. Perpendicular projection to a line or a plan. Diagonal matrixes. Operations with diagonal matrixes. Permutation matrixes, snakes, their generation and operations. Triangular matrixes, their determinant and operations. Symmetric and skew-symmetric matrixes. Decomposition into the sum of a symmetric and a skew-symmetric matrix.

Spacial matrixes-II. Operations with Block Matrices. Kronecker Product and the Vec Function. Properties of the Kronecker Product. Hypermatrixes. Transpose of a hypermatrix. Trace of a Matrix. Trace as a linear mapping. LU Decomposition. Solving systems of equations using LU decomposition. Matrix inversion using LU decomposition.

Eigenvalue, Eigenvector, Eigenspace. Determination of Eigenvalues. Characteristic polynom. Determinant, Trace, and Eigenvalues. Eigenspaces of 2×2 Symmetric Matrices. Determining All Eigenvalues and Eigenvectors of a Matrix. Matrix Invertibility and the Eigenvalue 0. Eigenvalues of Special Matrices. ~~Le Verrier-Souriau algorithm.~~ Eigenvalue and eigenspace of a linear trasformation.

Diagonalization. Similarity. Similarity-invariant properties. Quadratic form. Eigenvalue-related invariants. Diagonalizability. Necessary and sufficient condition for diagonalizability. Diagonalization of Linear Transformations: mirroring of plane vectors across a line, orthogonal projection of plane vectors onto a line, rotation of space vectors around a line by an angle different from an integer multiple of 180^o, orthogonal projection of space vectors onto a plane, and mirroring of space vectors across a plane.

Orthonormal basis - orthogonal matrix. Orthogonal and orthonormal bases. Orthogonal and semi-orthogonal matrix. Equivalent definitions of orthogonal matrices. Matrix transformation associated with an orthogonal matrix. Properties of orthogonal matrices. ~~Gram–Schmidt orthogonalization.~~ QR-decomposition, description.

Spaces over the Complex Field and Finite Fields. Adjungate of a complex matrix (Hermitian transpose). Scalar product of complex vectors. Properties of the adjungate. Properties of complex scalar product. Unitary matrix. Cauchy–Bunyakovsky–Schwarz inequality. ~~Maldenbrot-, and Julia sets.~~

Discrete Fourier Transformation. Substitution values of the Fourier sum. Properties of Fourier matrices. Discrete Fourier Transform (DFT). Properties of the DFT. Computing the DFT. Filtering periodic components. Convolution of vectors. Convolution matrix.

Singular Value, Singular Vector, SVD. Singular value. Singular decomposition. Reduced singular value decomposition and dyadic form. Singular value decomposition (SVD). Principal Component Analysis (PCA) description. Covariance of vectors, covariance matrix.

~~**Stoichiometric analysis of reaction equations.** Molecular matrix. Augmented Molecular Matrix. Application of the Molecular Matrix to Element Balances. Stoichiometric Matrix. Application of the Stoichiometric Matrix to Reactions. Augmented Stoichiometric Matrix. Horiuti Numbers.~~

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