(ENKEMNA0302) Applied Linear Algebra

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Schedule I

- **Classes: April 9-10, 16-17, 30, and May 7, 2025.**
- Merőleges vetítés és a legjobb közelítés, ortonormált bázis, ortogonális mátrixok, komplex és végest test feletti terek.
- ~~Kvadratikus formák.~~
- Szinguláris érték, szinguláris vektor, SVD, PCA.
- ~~Mátrixok összehasonlítása, pozitív mátrixok, nemnegatív mátrixok, irreducibilis mátrixok, SMRC, NMF.~~
- Reakcióegyenletek sztöchiometrikus rendezése.
- ~~Lineáris programozási feladatok mátrixaritmetikai megoldhatósága. (MLF?)~~
- ~~Power of matrices. Applications: linear recursions, power of incidence matrixes.~~
- ~~Gram-Schmidt ortogonalization. LU and~~ QR decomposition. Fourier-series.

Schedule II

- ~~Further applications. (MLF?)~~

Orthogonal projection and best approximation I

- Sum and direct sum of subspaces: Wettl book.
- Properties of complementary subspaces: Wettl notes
- Direct sum: Wettl notes
- Properties of the orthogonal complement subspace: Wettl notes
- Orthogonal projection onto a subspace of $\mathbb{R}^n$: Wettl notes
- Matrix of projection onto a subspace: Wettl notes
- Calculation of the orthogonal projection: Wettl notes
- Matrices of orthogonal projections: Wettl notes
- The best approximation theorem: Wettl notes
- ~~Decomposition of a vector into components~~
- ~~Optimal solution of a system of equations~~
- ~~Linear and polynomial regression~~
- ~~Linearizable regression models~~

Orthogonal projection and best approximation II

- ~~Projection onto a subspace~~
- ~~Properties of the projection~~
- ~~Equivalent definition of the projection~~
- ~~When is a projection orthogonal?~~

Orthonormal basis - orthogonal matrix I

- The analysis of subspaces is facilitated when the basis vectors are pairwise orthogonal, since in that case the scalar product of different basis vectors is 0. It becomes even simpler if the basis vectors are unit vectors, because then the scalar product of a vector with them gives the length of the orthogonal projection.
- A set of pairwise orthogonal vectors is called an orthogonal system. An orthogonal system may contain zero vectors. A set of pairwise orthogonal unit vectors is called an orthonormal system. Orthonormal systems do not contain zero vectors. From the following theorem, it immediately follows that an orthogonal system without zero vectors or any orthonormal system is always a basis of the subspace it spans. This is called an orthogonal basis (OB), or orthonormal basis (ONB), respectively. From an orthogonal basis, an orthonormal basis can always be obtained by normalizing each basis vector (i.e., dividing each by its length).

Orthonormal basis - orthogonal matrix II

- Independence of orthogonal vectors: Wettl notes
- Best approximation in the case of an ONB: Wettl notes
- Definition: (Orthogonal and semi-orthogonal matrix). A real square matrix is called orthogonal if its column vectors or row vectors form an orthonormal system. If we do not require the matrix to be square, we speak of a semi-orthogonal matrix.
- Theorem: (Equivalent definitions of semi-orthogonal matrices) Let $m \geq n$ and $\mathbf{Q} \in \mathbb{R}^{m \times n}$. The following statements are equivalent:
  1. $\mathbf{Q}$ is semi-orthogonal,
  2. $\mathbf{Q}^T\mathbf{Q} = \mathbf{E}_n$.

  Similarly, for $m \leq n$, $\mathbf{Q}$ is semi-orthogonal if and only if $\mathbf{Q}\mathbf{Q}^T = \mathbf{E}_m$. Statement 2 expresses in algebraic terms that for $m \leq n$, $\mathbf{Q}$ is semi-orthogonal if and only if its transpose is its left inverse.

  Proof: Wettl notes.

The End

Thank you for your attention!