(PTIA0301) Elementary Linear Algebra

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Unitvector

- Definition: The unit vectors are vectors with unit (1) length. The canonic basis of $\mathbb{R}^3$ is
$$\mathbf{i} = \mathbf{e}_1 = (1, 0, 0)\,, \mathbf{j} = \mathbf{e}_2 = (0, 1, 0)\,, \mathbf{k} = \mathbf{e}_3 = (0, 0, 1)\,.$$
- Thesis: For all $\mathbf{v}\,(v_1, v_2, v_3)$ 3D vector:
$$\mathbf{v} = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3.$$

  Definition: The normal of the $|\mathbf{v}| \neq \mathbf{0}$ vector is $\frac{\mathbf{v}}{|\mathbf{v}|}$.

Distance of points, equation of sphere I

- The M bisecting point of the segment between the $P_1(x_1, y_1, z_1)$ and the $P_2(x_2, y_2, z_2)$ points is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

- The distance of the $P_1(x_1, y_1, z_1)$ and the $P_2(x_2, y_2, z_2)$ points is the length of the **a**, **b** vectors with ending point of P_1 and P_2: $|\mathbf{a} - \mathbf{b}|$.
- <u>Definition:</u> The distance of the $P_1(x_1, y_1, z_1)$ and the $P_2(x_2, y_2, z_2)$ points is

$$|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

Distance of points, equation of sphere II

- Definition: The equation of the sphere with a radius and (x_0, y_0, z_0) centre is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2.$$

Scalar multiplication of vectors I

- Definition. The scaler (or inner) multiplication of two vectors is

$$\mathbf{ab} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta$$

  where $\theta = (\mathbf{a}, \mathbf{b}) \angle$ és $\left(\mathbf{a}, \mathbf{b} \in V^3\right)$.
- Note that $\mathbf{aa} = |\mathbf{a}|^2$.
- <u>Thesis:</u> (the features of scalar multiplication):
  commutative: $\mathbf{ab} = \mathbf{ba}$ és $\left(\mathbf{a}, \mathbf{b} \in V^3\right)$.
  additive: $(\mathbf{a} + \mathbf{b})\,\mathbf{c} = \mathbf{ac} + \mathbf{bc}$ és $\left(\mathbf{a}, \mathbf{b}, \mathbf{c} \in V^3\right)$.
  homogenous: $(\lambda\mathbf{a})\,\mathbf{b} = \lambda\,(\mathbf{ab})$, where $\lambda \in \mathbb{R}$ és$\left(\mathbf{ab} \in V^3\right)$.
  pozitive definit: $\mathbf{aa} \geq 0$, ahol $\left(\mathbf{a} \in V^3\right)$ és $\mathbf{aa} = 0 \Leftrightarrow \mathbf{a} = \mathbf{0}$.

Scalar multiplication of vectors II

- Thesis: The scalar multiplication of $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ vectors is

$$\mathbf{ab} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

- Thesis: The angle of two non-zero vectors ($\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$) is

$$\cos\theta = \frac{\mathbf{ab}}{|\mathbf{a}| \cdot |\mathbf{b}|}$$

- Definition: The $\mathbf{a}$ and $\mathbf{b}$ vectors are orthogonal (perpendicular) if $\mathbf{ab} = 0$.
- Definition: The perpendicular projection ($proj_{\mathbf{b}}\mathbf{a}$) of $\mathbf{a}$ vector to $\mathbf{b}$ vector is that $\mathbf{b}$ directed vector that ends in the point that is determined by a perpendicular line to $\mathbf{b}$ vector.

Scalar multiplication of vectors III

- Thesis: If $\left(\mathbf{ab} \in V^3\right)$, then

$$proj_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{ab}}{|\mathbf{b}|^2} \cdot \mathbf{b}.$$

- If $\mathbf{b}$ unit vector has unit length, then the formula is simple:

$$proj_{\mathbf{b}}\mathbf{a} = (\mathbf{ab})\,\mathbf{b}.$$

The End

Thank you for your attention!