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(ONFIZ1-0401) Elemi lineáris algebra 5. zárthelyi dolgozat / Elementary Linear Algebra, Test 5

1. Adottak a következő vektorok: $\mathbf{a} = (1, 3, -2)$, $\mathbf{b} = (-1, 2, 4)$ és $\mathbf{c} = (4, 1, 3)$. Határozza meg a következő összefüggéseket / Calculate the following expressions:
   a.) $(\mathbf{a} - \mathbf{b})\,\mathbf{c}$
   b.) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a}$
   c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
   d.) Mennyi az **a** és **b** vektorok által közbezárt szög? / What is the angle of Vectors **a** and **b**?
   e.) Egy síkban vannak-e az **a**, **b**, **c** vektorok? / Are Vectors **a**, **b**, and **c** in the same plane?
   f.) Adjon meg egy vektort, mely merőleges az **b** vektorra. / Determine a perpendicular vector to Vector **b**.

   (10 po(i)nt)

2. Számítsa ki a következő mátrixok determinánsát! / Calculate the deteminant of the following matrixes:

$$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ -3 & -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -2 & 1 & 5 \\ -2 & -2 & 3 \\ 1 & 0 & 2 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 4 & 3 & 2 & -1 \\ 1 & -3 & 0 & 2 \\ -1 & 3 & 6 & 1 \\ -1 & 2 & 0 & 3 \end{pmatrix}$$

   (10 po(i)nt)

3. Oldja meg az alábbi lineáris egyenletrendszert! / Solve the following system of linear equations:

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 6 \\ 2x_1 - 3x_2 + x_3 &= -1 \\ 3x_1 + x_2 + x_3 &= 5 \end{aligned}$$

   (10 po(i)nt)

4. Lineárisan függetlenek-e az $\mathbf{a} = (6, 4, -1)$, a $\mathbf{b} = (2, 1, 6)$ és a $\mathbf{c} = (1, 0, 4)$ vektorok? / Are independent linear Vectors $\mathbf{a} = (6, 4, -1)$, $\mathbf{b} = (2, 1, 6)$, and $\mathbf{c} = (1, 0, 4)$? (10 po(i)nt)

5. Lineáris altér-e az $\mathbb{R}^4$-on az $L = \{(x_1, x_2, 2x_1, 3x_2)\,|\,x_1, x_2 \in \mathbb{R}\}$? / Is a linear subspace on $\mathbb{R}^4$ the $U = \{(x_1, x_2, 2x_1, 3x_2)\,|\,x_1, x_2 \in \mathbb{R}\}$ set? (10 po(i)nt)

6. Adja meg az $\mathbf{a} = (1, 0, 0)$ vektort az $(1, 2, 5)$; $(3, 7, 8)$; $(2, 5, 2)$ bázisban. / Give the Vector $\mathbf{a} = (1, 0, 0)$ in the $(1, 2, 5)$; $(3, 7, 8)$; $(2, 5, 2)$ basis. (10 po(i)nt)

7. Adottak a következő mátrixok:

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 2 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ -1 & 1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix}$$

   Végezze el az alábbiak közül az elvégezhető műveleteket! / Calculate the following terms if possible:
   (a) $\mathbf{A} + \mathbf{B}$; $\mathbf{B} + \mathbf{C}$; $\mathbf{C} + \mathbf{D}$; $4\mathbf{A} - \mathbf{B}$;(b) $\mathbf{A} \cdot \mathbf{B}$; $\mathbf{A} \cdot \mathbf{C}$; $\mathbf{B} \cdot \mathbf{C}$; $\mathbf{B} \cdot \mathbf{D}$ (c) $\mathbf{A}^T$; $\mathbf{D}^T$; $\mathbf{A}^T \cdot \mathbf{B}$; (d) $\rho(\mathbf{A})$; $\rho(\mathbf{D})$; (e) $\mathbf{A}^{-1}$ (g) $\mathbf{D}^{-1}$ (10 po(i)nt)

8. Oldja meg az $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ mátrixegyenletet, ha

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 7 \\ -3 & 2 & 2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 10 & 1 \\ 29 & 5 \\ 8 & 5 \end{pmatrix}$$

   Solve the $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ matrix equation above.

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a) $(\underline{a}-\underline{b})\cdot\underline{c} = \left[\begin{pmatrix}1\\3\\-2\end{pmatrix} - \begin{pmatrix}-1\\2\\4\end{pmatrix}\right]\cdot\begin{pmatrix}4\\1\\3\end{pmatrix} = \begin{pmatrix}2\\1\\-6\end{pmatrix}\cdot\begin{pmatrix}4\\1\\3\end{pmatrix} = 8+1-18 = \underline{\underline{-9}}$

b) $\left[\begin{pmatrix}-1\\2\\4\end{pmatrix} + \begin{pmatrix}4\\1\\3\end{pmatrix}\right] \times \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}3\\3\\7\end{pmatrix} \times \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}3\cdot(-2) - 7\cdot 3\\ 7\cdot 1 - 3\cdot(-2)\\ 3\cdot 3 - 3\cdot 1\end{pmatrix} = \begin{pmatrix}-27\\13\\6\end{pmatrix} = 3\begin{pmatrix}-9\\5\\2\end{pmatrix}$

c) $\left[\begin{pmatrix}1\\3\\-2\end{pmatrix} \times \begin{pmatrix}-1\\2\\4\end{pmatrix}\right]\cdot\begin{pmatrix}4\\1\\3\end{pmatrix} = \begin{pmatrix}3\cdot 4 - (-2)\cdot 2\\ (-2)\cdot(-1) - 1\cdot 4\\ 1\cdot 2 - 3\cdot(-1)\end{pmatrix}\cdot\begin{pmatrix}4\\1\\3\end{pmatrix} = \begin{pmatrix}16\\-2\\5\end{pmatrix}\cdot\begin{pmatrix}4\\1\\3\end{pmatrix} =$

$= 64 - 2 + 15 = \underline{\underline{77}}$

d) $\cos\gamma = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}|\cdot|\underline{b}|} = \frac{\begin{pmatrix}1\\3\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\4\end{pmatrix}}{\sqrt{1^2+3^2+(-2)^2}\cdot\sqrt{(-1)^2+2^2+4^2}} = \frac{-1+6-8}{\sqrt{14}\cdot\sqrt{21}} =$

$= \frac{-3}{7\sqrt{6}} = -\frac{3\sqrt{6}}{42} = -\underline{\underline{\frac{\sqrt{6}}{14}}}$

e) Nem, mert $(\underline{a},\underline{b},\underline{c}) = 77 \neq 0$

No, because — " —

f) $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\4\end{pmatrix} = 0 \qquad -x+2y+4z = 0$

pl: $\boxed{\underline{b}^{\perp} = \begin{pmatrix}6\\1\\1\end{pmatrix}}$

pl:

$\begin{pmatrix}6\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\4\end{pmatrix} = -6+2+4 = 0$

② Jegyzet / Notes 22/2.5 feladat / exercise

$$A = \begin{pmatrix} 6 & 4 \\ -3 & -2 \end{pmatrix} = 6\cdot(-2) - 4\cdot(-3) = -12+12 = \underline{\underline{0}}$$

Sarrus szabály / rule

$$B = \begin{pmatrix} -2 & 1 & 5 \\ -2 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix} = (-2)\cdot(-2)\cdot 2 + 1\cdot 3\cdot 1 + 5(-2)\cdot 0 - 5(-2)1 -$$

$$- 1(-2)\cdot 2 - (-2)0\cdot 3 = 8+3+10+4 = \underline{\underline{25}}$$

$$C = \begin{pmatrix} 4 & 3 & 2 & -1 \\ 1 & -3 & 0 & 2 \\ -1 & 3 & 6 & 1 \\ -1 & 2 & 0 & 3 \end{pmatrix} = 2\cdot \begin{vmatrix} 1 & -3 & 2 \\ -1 & 3 & 1 \\ -1 & 2 & 3 \end{vmatrix} + 6\cdot \begin{vmatrix} 4 & 3 & -1 \\ 1 & -3 & 2 \\ -1 & 2 & 3 \end{vmatrix} =$$

$$= 2\cdot\left(1\cdot 3\cdot 3 + (-3)\cdot 1\cdot(-1) + 2(-1)\cdot 2 - 2\cdot 3(-1) - 1(3)\cdot 3 - 1\cdot 2\cdot 1\right) +$$

$$+ 6\left(4(-3)3 + 3\cdot 2(-1) + (-1)1\cdot 2 - (-1)(-3)(-1) - 1\cdot 3\cdot 3 - 4\cdot 2\cdot 2\right) =$$

$$= 2(9+3-4+6-9-2) + 6(-36-6-2+3-9-16) =$$

$$= 2(3) - 6\cdot(66) = 6 - 6\cdot 66 = -6\cdot 65 = \underline{\underline{-390}}$$

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③ Jegyzet / Notes 27/1 feladat / exercise

$\underline{\underline{x_1 = 2}}$; $\underline{\underline{x_2 = 1}}$; $\underline{\underline{x_3 - 2}}$

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Jegyzet / Notes 33/12 feladat / exercise

Igen / Yes, - ld. ott / See there.

⑤ Jegyzet / Notes 43/15

(I) additivitás / additivity

$$\begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ 2a_1+2b_1 \\ 3a_2+3b_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ 2c_1 \\ 3c_2 \end{pmatrix} \in L$$

$c_i = a_i + b_i$; ahol / where $i \in \{1, 2, 3\}$

(II) homogenitás / homogenity

$$\begin{pmatrix} \lambda a_1 \\ \lambda a_2 \\ 2\lambda a_1 \\ 3\lambda a_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ 2d_1 \\ 3d_2 \end{pmatrix} \in L$$

$d_i = \lambda a_i$, $i \in \{1, 2, 3\}$

Mindkettő feltétel teljesül, tehát altér. / Both conditions are fulfilled, therefore, it is a subspace.

⑤ Jezyczek / Wróbel 44/17 Przykład, rozwiąż

$$\lambda_1 \cdot \begin{pmatrix}1\\2\\5\end{pmatrix} + \lambda_2 \cdot \begin{pmatrix}3\\7\\8\end{pmatrix} + \lambda_3 \cdot \begin{pmatrix}2\\5\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{cases} \lambda_1 + 3\lambda_2 + 2\lambda_3 = 1 \\ 2\lambda_1 + 7\lambda_2 + 5\lambda_3 = 0 \\ 5\lambda_1 + 8\lambda_2 + 2\lambda_3 = 0 \end{cases}$$

$$\underbrace{\begin{pmatrix}1 & 3 & 2\\2 & 7 & 5\\5 & 8 & 2\end{pmatrix}}_{A} \underbrace{\begin{pmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{pmatrix}}_{\lambda} = \underbrace{\begin{pmatrix}1\\0\\0\end{pmatrix}}_{\underline{b}}$$

$$A \quad \lambda = \underline{b} \rightarrow \lambda = A^{-1} \cdot \underline{b} = ?$$

$$\det A \quad |A| = 1\cdot7\cdot2 + 3\cdot5\cdot5 + 2\cdot2\cdot8 - 5\cdot7\cdot2 - 2\cdot3\cdot2 - 40 =$$

$$= 14 + 75 + 32 - 70 - 12 - 40 = \underline{-1} \neq 0 \quad \neg \text{ singular}$$

Singular

$$\begin{vmatrix}7 & 5\\8 & 2\end{vmatrix} = 14-40 = -26 \quad \begin{vmatrix}2 & 5\\5 & 2\end{vmatrix} = 4-15 = -21 \quad \begin{vmatrix}2 & 7\\5 & 8\end{vmatrix} = 16-35 = -19$$

$$\begin{vmatrix}3 & 2\\8 & 2\end{vmatrix} = 6-16 = -10 \quad \begin{vmatrix}1 & 2\\5 & 2\end{vmatrix} = 2-10 = -8 \quad \begin{vmatrix}1 & 3\\5 & 8\end{vmatrix} = 8-15 = -7$$

$$\begin{vmatrix}3 & 2\\7 & 5\end{vmatrix} = 15-14 = 1 \quad \begin{vmatrix}1 & 2\\2 & 5\end{vmatrix} = 5-4 = 1 \quad \begin{vmatrix}1 & 3\\2 & 7\end{vmatrix} = 7-6 = 1$$

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...st. Cont'd

$$\begin{pmatrix} -26 & -21 & -19 \\ -10 & -8 & -7 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[\text{Class}]{\text{Sahh}} \begin{pmatrix} -26 & 21 & -19 \\ 10 & -8 & 7 \\ 1 & -1 & 1 \end{pmatrix} \xrightarrow[\text{tm}]{\text{tmt}} \begin{pmatrix} -26 & 10 & 1 \\ 21 & -8 & -1 \\ -19 & 7 & 1 \end{pmatrix}$$

$$\rightarrow A^{-1} = -\begin{pmatrix} -26 & 10 & 1 \\ 21 & -8 & -1 \\ -19 & 7 & 1 \end{pmatrix} = \begin{pmatrix} 26 & -10 & -1 \\ -21 & 8 & +1 \\ 19 & -7 & -1 \end{pmatrix}$$

$$\lambda = \begin{pmatrix} \cancel{26} & \cancel{-10} & \cancel{-1} \\ \cancel{-21} & 8 & 1 \\ 19 & -7 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 26 \\ -21 \\ 19 \end{pmatrix}$$

$\lambda_1 = 26 \quad ; \lambda_2 = -21 \quad ; \lambda_3 = 19$

$$\stackrel{?}{=} \; 26 \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 21 \begin{pmatrix} 3 \\ 7 \\ 8 \end{pmatrix} + 19 \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 26-63+38 \\ 52-147+95 \\ 130-168+38 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

② geograph / Notes 50/1 blodad / resist

dd. att / See this.

(8) Jegyzet / Note 52/6c feladat / levezetése

dd. off / See here

$$X = \frac{1}{14}\begin{pmatrix} 16 & -4 & -2 \\ 25 & -8 & 3 \\ -1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 5 \\ 1 & 5 \end{pmatrix} =$$

$$= \frac{1}{14}\begin{pmatrix} 16 - 16 - 16 & 16 - 20 - 10 \\ 25 - 232 + 24 & 25 - 40 + 15 \\ -1 + 4 + 8 & -1 + 10 - 15 \end{pmatrix}$$

(9) Jegyzet / Note 69/1 feladat / levezetése

dd. off / See here

$$\frac{1}{14}\begin{pmatrix} 28 & -14 \\ 42 & 0 \\ 56 & 14 \end{pmatrix} =$$

(10) Jegyzet / Note 61/1 feladat / levezetése

dd. off / See here

$$= \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 4 & 1 \end{pmatrix}$$

(8) Jegyzet / Note 52/6c feladat levezetése

$\underline{A}\underline{X} = \underline{B} \rightarrow$

$\underline{X} = \underline{A}^{-1} \cdot \underline{B} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 4 & 1 \end{pmatrix}$

$$\underline{A} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 7 \\ -3 & 2 & 2 \end{pmatrix} \rightarrow$$

$$\begin{vmatrix} 1 & 7 \\ 2 & 2 \end{vmatrix} = -16 \quad \begin{vmatrix} 2 & 7 \\ -3 & 2 \end{vmatrix} = 25 \quad \begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = 7$$

$$\begin{vmatrix} 0 & 2 \\ 2 & 2 \end{vmatrix} = -4 \quad \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} = 8 \quad \begin{vmatrix} 1 & 0 \\ -3 & 2 \end{vmatrix} = 2$$

$$\begin{vmatrix} 0 & 2 \\ 1 & 7 \end{vmatrix} = -2 \quad \begin{vmatrix} 1 & 2 \\ 2 & 7 \end{vmatrix} = 3 \quad \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$$

$$\rightarrow \begin{pmatrix} -16 & 25 & 7 \\ -4 & 8 & 2 \\ 2 & 3 & 1 \end{pmatrix} \xrightarrow{\text{chess}} \begin{pmatrix} -16 & -25 & 7 \\ 4 & 8 & -2 \\ -2 & -3 & 1 \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} -16 & 4 & 2 \\ -25 & 8 & -3 \\ 7 & -2 & 1 \end{pmatrix} \rightarrow \underline{A}^{-1} = \frac{1}{-14}\begin{pmatrix} -16 & 4 & 2 \\ -25 & 8 & -3 \\ 7 & -2 & 1 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 16 & -4 & -2 \\ 25 & -8 & 3 \\ -1 & 2 & 1 \end{pmatrix}$$

$|A| = -2 + 0 + 8 - 6 - 0 - 14 = -14$