Név / Name: DR. FACSKÓ GÁBOR ISTVÁN Neptun kód / code: BQQQ7Y

(MATNA1902) Alkalmazott lineáris algebra 1. zárthelyi dolgozat
(ENKEMNA0302) Applied Linear Algebra Test 1

1. Adottak a következő vektorok és mátrixok / We have the following vectors and matrixes:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 0 \end{pmatrix}$$

Számolja ki a következő dyadikus és Kronecker szorzatokat / Calculate the following dyadic and Kronecker products: (a) $\mathbf{a} \otimes \mathbf{b}$; (b) $\mathbf{C} \otimes \mathbf{D}$. (10 pont)

2. Adott az $\mathbf{a}\,(1,1,1)$ vektor.
   (a) Írja fel a forgatási mátrixokat, amivel az $(0,1,0)$ irányba lehet forgatni az $\mathbf{a}$ vektort! / Give the rotational matrixes to rotate this vector to the direction of the $(0,1,0)$ vector.
   (b) Adja meg azt a $\mathbf{S}$ eltolási mátrixot, amivel az $\mathbf{a}$ vektort el lehet tolni a $(1,1,0)$ irányba! / Determine the $\mathbf{S}$ transformation matrix that shifts this $\mathbf{a}$ vector to the $(1,1,0)$ direction.

(10 pont)

3. Adottak a következő mátrixok / We have the following matrixes:

$$\mathbf{D}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{D}_2 = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 11 \end{pmatrix} \mathbf{T} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{S} = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & 0 & 0 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Végezze el a következő müveleteket! / Calculate the following expressions: (a) $\mathbf{D}_1 + \mathbf{D}_2$; $\mathbf{D}_1 \cdot \mathbf{D}_2$; (b) $|\mathbf{D_1}|$; $|\mathbf{T}|$; $|\mathbf{S}|$; $|\mathbf{P}|$; (c) $\mathbf{D}_1^{-1}$; $\mathbf{D}_2^{-1}$; $\mathbf{T}^{-1}$; (d) $\mathbf{D}_1^2$; $\mathbf{D}_2^2$; (e) $\mathbf{D}_1^3$; $\mathbf{D}_2^3$. Csak ellenőrzésre használják a Sarrus-szabályt és az adjungálást. Használják az adott mátrixokról tanultakat. / Use the Sarrus rule and adjudication for check only. Use the learned features of these matrixes. (10 pont)

4. Adott a következő mátrix / We have the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 2 & 2 & 3 \\ 2 & 2 & 1 & 1 \end{pmatrix}$$

Bontsa fel ezt a mátrixokat egy szimmetrikus és egy ferdén szimmetrikus mátrix összegére! / Divide this matrix into a sum of symmetric and skew-symmetric matrixes. (10 pont)

5. Adottak a következő blokk mátrixok / We have the following block matrixes:

$$\mathbf{A} = \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ \hline 2 & 1 & 2 & 2 & 3 & 2 \\ 0 & 3 & 0 & 3 & 2 & 3 \end{array}\right) \mathbf{B} = \left(\begin{array}{cc} 1 & 0 \\ 0 & 2 \\ 2 & 1 \\ \hline 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{array}\right)$$

Végezze el a következő műveleteket, ha lehetséges! / Calculate the following expressions, if it is possible: (a) $\mathbf{A} + \mathbf{B}$ (b) $2 \cdot \mathbf{A}$; $3 \cdot \mathbf{B}$; (c) $\mathbf{A} \cdot \mathbf{B}$. A műveleteket a blokokkal végezze, a rendes mátrix szorzást csak ellenőrzésre használja! / Calculate with the block. Use the normal matrix operations only for checking. (10 pont)

A zárthelyi osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). / Grades: 0-20 points: Fail 81), 21-27 points: Pass (2), 28-35 points: Satisfactory (3), 36-42 points: Good (4) és 43-50 points: Excellent (5).

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Pécs, 2025. március 13. / March 13, 2025

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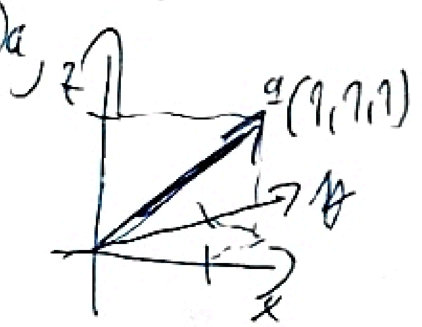
① a

$$a \otimes b = \begin{pmatrix}1\\2\\0\end{pmatrix}\begin{pmatrix}1 & 2 & 2\end{pmatrix} = \underline{\begin{pmatrix}1 & 2 & 2\\2 & 4 & 4\\0 & 0 & 0\end{pmatrix}}$$

$$\underline{C} \otimes \underline{D} = \begin{pmatrix}1 & 0 & 0\\2 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\0 & 3 & 0\end{pmatrix} = \underline{\begin{pmatrix}1&1&1&0&0&0&0&0&0\\0&3&0&0&0&0&0&0&0\\2&2&2&1&1&1&0&0&0\\0&6&0&0&3&0&0&0&0\end{pmatrix}}$$

0

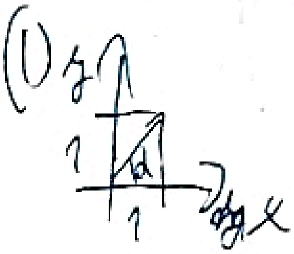
② a



→ (I) Rotation around Z / forgatás Z körül

(II) Rotation around x / forgatás x körül

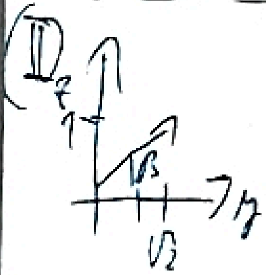
(I)



$\alpha = 45°$

$$\rightarrow R_{(I)} = \begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix} = \underline{\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0\\ 0 & 0 & 1\end{pmatrix}} \leftarrow R(45°)$$

(II)



$$R_{(I)}\,\underline{a} = \begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\ \sqrt{2}\\ 1\end{pmatrix}$$

$$\cos\beta = \frac{(0,1,0)\cdot(0,\sqrt{2},1)}{1\cdot\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3} \rightarrow \underline{\beta = -54{,}74}$$

$$R_{(II)} = \begin{pmatrix}1 & 0 & 0\\ 0 & \frac{\sqrt{6}}{3} & \cancel{+0{,}577}\ 0{,}82\\ 0 & \cancel{+0{,}57}\ -0{,}82 & \frac{\sqrt{6}}{3}\end{pmatrix}$$

0,82

b) $$\underline{S} = \underline{\begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}}$$

$$\rightarrow E/Ch: \begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\2\\1\\1\end{pmatrix} \checkmark$$

$\beta = 54{,}74°$

$\Downarrow$

$(\cancel{\beta = 35{,}26°})$

③ a) $\underline{D}_1+\underline{D}_2 = \begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}+\begin{pmatrix}5&0&0\\0&7&0\\0&0&11\end{pmatrix}=\underline{\underline{\begin{pmatrix}6&0&0\\0&9&0\\0&0&14\end{pmatrix}}}$

$\underline{D}_1\cdot\underline{D}_2 = \begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}\cdot\begin{pmatrix}5&0&0\\0&7&0\\0&0&11\end{pmatrix}=\underline{\underline{\begin{pmatrix}5&0&0\\0&14&0\\0&0&33\end{pmatrix}}}$

b) $|\underline{D}_1| = \underline{\underline{6}}$

$|\underline{T}| = \underline{\underline{8}}$

$|\underline{S}| = \underline{\underline{8}}$

$|\underline{P}| = \underline{\underline{1}}$

c) $\underline{D}_1^{-1} = \begin{pmatrix}1&0&0\\0&1/2&0\\0&0&1/3\end{pmatrix}$ $\quad \underline{D}_2^{-1} = \begin{pmatrix}1/5&0&0\\0&1/7&0\\0&0&1/11\end{pmatrix}$

$\text{adj}(\underline{T}) = \begin{pmatrix}4&0&0\\2&4&0\\-1&2&4\end{pmatrix} \xrightarrow{\text{class sohn}} \begin{pmatrix}4&0&0\\-2&4&0\\-1&-2&4\end{pmatrix} \xrightarrow{\text{Transp}} \begin{pmatrix}4&-2&-1\\0&4&-2\\0&0&4\end{pmatrix} \rightarrow \underline{T}^{-1} = \frac{1}{8}\begin{pmatrix}4&-2&-1\\0&4&-2\\0&0&4\end{pmatrix} = \underline{\underline{\begin{pmatrix}\frac{1}{2}&-\frac{1}{4}&-\frac{1}{8}\\0&\frac{1}{2}&-\frac{1}{4}\\0&0&\frac{1}{2}\end{pmatrix}}}$

d) $\underline{D}_1^2 = \begin{pmatrix}1&0&0\\0&4&0\\0&0&9\end{pmatrix}$ $\quad \underline{D}_2^2 = \begin{pmatrix}25&0&0\\0&49&0\\0&0&121\end{pmatrix}$

e) $\underline{D}_1^3 = \begin{pmatrix}1&0&0\\0&8&0\\0&0&27\end{pmatrix}$ $\quad \underline{D}_2^3 = \begin{pmatrix}125&0&0\\0&343&0\\0&0&1331\end{pmatrix}$

BQQ Q4FY

4)

$$\underline{A} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 2 & 2 & 3 \\ 2 & 2 & 1 & 1 \end{pmatrix} \rightarrow \underline{A}^T = \begin{pmatrix} 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix}$$

$$\underline{A} = \frac{1}{2}(\underline{A} + \underline{A}^T) + \frac{1}{2}(\underline{A} - \underline{A}^T) =$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 2 & 5 & 3 \\ 2 & 4 & 5 & 4 \\ 5 & 5 & 4 & 4 \\ 3 & 4 & 4 & 2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & +2 \\ 1 & 0 & -2 & 0 \end{pmatrix} =$$

$$= \underbrace{\begin{pmatrix} 1 & 1 & \frac{5}{2} & \frac{3}{2} \\ 1 & 2 & \frac{5}{2} & 2 \\ \frac{5}{2} & \frac{5}{2} & 2 & 2 \\ \frac{3}{2} & 2 & 2 & 1 \end{pmatrix}}_{\text{Szim / Sym}} + \underbrace{\begin{pmatrix} 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & +\frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & +1 \\ \frac{1}{2} & 0 & -1 & 0 \end{pmatrix}}_{\text{ferdén szim / skew sym.}}$$

5)

a) $\underline{A} + \underline{B}$: ¬ possible / ¬ lehetséges

b) $$2 \cdot \underline{A} = \left(\begin{array}{c|c} 2 \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} & 2 \cdot \begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \\ \hline 2 \cdot \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & 0 \end{pmatrix} & 2 \cdot \begin{pmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \end{pmatrix} \end{array}\right) = \left(\begin{array}{c|c} \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix} & \begin{pmatrix} 4 & 0 & 4 \\ 2 & 4 & 2 \end{pmatrix} \\ \hline \begin{pmatrix} 4 & 2 & 4 \\ 0 & 6 & 0 \end{pmatrix} & \begin{pmatrix} 4 & 6 & 4 \\ 6 & 4 & 6 \end{pmatrix} \end{array}\right) \neq \left(\begin{array}{ccc|ccc} 0 & 2 & 0 & 4 & 0 & 4 \\ 2 & 0 & 2 & 2 & 4 & 2 \\ \hline 4 & 2 & 4 & 4 & 6 & 4 \\ 0 & 6 & 0 & 6 & 4 & 6 \end{array}\right)$$

$$3\cdot B = \begin{pmatrix} 3\cdot\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{pmatrix} \\ \hline 3\cdot\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 6 \\ 6 & 3 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 3 \\ 3 & 0 \\ 0 & 3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 6 \\ 6 & 3 \\ \hline 0 & 3 \\ 3 & 0 \\ 0 & 3 \end{pmatrix}$$

c)

$$\underline{A}\cdot\underline{B} = \left(\begin{array}{ccc|ccc} 0 & 1 & 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ \hline 2 & 1 & 2 & 2 & 3 & 2 \\ 0 & 3 & 0 & 3 & 2 & 3 \end{array}\right) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \\ \hline 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \hline \begin{pmatrix} 2 & 1 & 2 \\ 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix} \\ \hline \begin{pmatrix} 6 & 4 \\ 0 & 6 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ 2 & 6 \end{pmatrix} \end{pmatrix} =$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & 6 \\ 5 & 3 \end{pmatrix} \\ \hline \begin{pmatrix} 9 & 8 \\ 2 & 12 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 5 & 3 \\ \hline 9 & 8 \\ 2 & 12 \end{pmatrix}$$

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② a) $\underline{C}/C$h:

$$\cancel{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0{,}82 & -0{,}57 \\ 0 & 0{,}57 & 0{,}82 \end{pmatrix}\begin{pmatrix} 0 \\ 1{,}41 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}8319 \\ 1{,}6179 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0{,}82 & +0{,}82 \\ 0 & -0{,}82 & 0{,}82 \end{pmatrix}\begin{pmatrix} 0 \\ 1{,}41 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2{,}30 \\ 0 \end{pmatrix} \checkmark$$

$\beta = \underline{-54{,}74}$

$$R_{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{6}}{3} & +0{,}82 \\ 1 & -0{,}82 & \frac{\sqrt{6}}{3} \end{pmatrix}$$

← RESULT EREDMÉNY

② a) 2nd version / ~~2. megold.~~

$$\cancel{\underline{P} = \underline{E} - \underline{n} \otimes \underline{n}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$\cancel{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}} \to \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\cancel{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}$$

$$\underline{P} = \frac{1}{\underline{b}\underline{b}^T}\, \underline{b} \otimes \underline{b}^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$