Tk 68/1

$$\underline{A} = \begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}$$

a) s.é. ; s.v.

b) $|\underline{A}|$; $tr(\underline{A})$

c) diagonalizálás

$$\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}1\\ 3\end{pmatrix} = \begin{pmatrix}5\\ 15\end{pmatrix} = 5\begin{pmatrix}1\\ 3\end{pmatrix}$$

$$\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

$$\underline{A}\cdot\underline{x} = \lambda\underline{x} \quad ; \underline{x} \in \mathbb{R}^2 \quad \lambda \in \mathbb{R}$$

$$\underline{A}\cdot\underline{x} - \lambda\underline{x} = 0$$

$$(\underline{A} - \lambda\cdot\underline{E})\underline{x} = 0$$

$$\Updownarrow$$

$$|\underline{A} - \lambda\cdot\underline{E}| = 0$$

$$\begin{vmatrix}2-\lambda & 1\\ 3 & 4-\lambda\end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - 3 = 0$$

$$8 - 6\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda-1)(\lambda-5) = 0$$

$$\underline{\underline{\lambda_1 = 1}} \qquad \underline{\underline{\lambda_2 = 5}}$$

$\lambda_1 = 1$

$$\begin{pmatrix}1 & 1\\ 3 & 3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$x_1 + x_2 = 0 \rightarrow x_2 = -x_1$$

$$3x_1 + 3x_2 = 0 \rightarrow x_2 = -x_1$$

$$\mathcal{L}_{\lambda_1=1} = \left\{\begin{pmatrix}t\\ -t\end{pmatrix} \middle| t \in \mathbb{R}\right\}$$

$\lambda_2 = 5$

$$\begin{pmatrix}-3 & 1\\ 3 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$-3x_1 + x_2 = 0$$

$$3x_1 - x_2 = 0$$

$$x_2 = 3x_1$$

$$\mathcal{L}_{\lambda_2=5} = \left\{\begin{pmatrix}t\\ 3t\end{pmatrix} \middle| t \in \mathbb{R}\right\}$$

Tk 68/1

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

a) s.é. ; s.v.

b) $|A|$; $tr(A)$

c) diagonalizálás

b) $|A| = 8 - 3 = 5$

$|A| = \lambda_1 \cdot \lambda_2 = 1 \cdot 5 = 5$

$tr(A) = 6$

$tr(A) = \lambda_1 + \lambda_2 = 1 + 5 = 6$

$\underline{\underline{\lambda_1 = 1}}$ $\underline{\underline{\lambda_2 = 5}}$

$\mathcal{L}_{\lambda_1 = 1} = \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \middle| t \in \mathbb{R} \right\}$

$\mathcal{L}_{\lambda_2 = 5} = \left\{ \begin{pmatrix} t \\ 3t \end{pmatrix} \middle| t \in \mathbb{R} \right\}$

Tk 68/1

$$\underline{A} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

a) s.é. ; s.v.

b) $|\underline{A}|$; $tr(\underline{A})$

c) diagonalizálás

$$\underline{X}^{-1}\,\underline{A}\cdot\underline{X}$$

$$\underline{X} = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \to |\underline{X}| = \underline{4} \neq 0$$

$$\underline{X} = (x_1, x_2)$$

$$\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \xrightarrow{s.t.} \begin{pmatrix} 3 & +1 \\ -1 & 1 \end{pmatrix} \xrightarrow{tr.} \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \to \underline{X}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\underline{X}^{-1}\cdot\underline{A}\cdot\underline{X} = \frac{1}{4}\begin{pmatrix} \cancel{3} & \cancel{-1} \\ \cancel{1} & \cancel{1} \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \underline{X} =$$

$$= \frac{1}{4}\begin{pmatrix} 3 & -1 \\ 5 & 5 \end{pmatrix} \underline{X} =$$

$$= \frac{1}{4}\begin{pmatrix} 3 & 1 \\ 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix} 3+1 & 3-3 \\ 5-5 & 5+15 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 4 & 0 \\ 0 & 20 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$$

$$\lambda_1 = 1 \qquad \lambda_2 = 5$$

$$\boxed{\mathcal{L}_{\lambda_1 = 1} = \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \middle| t \in \mathbb{R} \right\}}$$

$$\boxed{\mathcal{L}_{\lambda_2 = 5} = \left\{ \begin{pmatrix} t \\ 3t \end{pmatrix} \middle| t \in \mathbb{R} \right\}}$$

Tk 77/5

$$\underline{A} = \begin{pmatrix} 1 & 0 & 0 \\ 5 & -2 & 14 \\ 9 & 1 & 3 \end{pmatrix}$$

$\lambda_1 = 1 \ ; \ \lambda_2 = -4 \ ; \ \lambda_3 = 5$

$$\underline{x}_1 = \begin{pmatrix} -5 \\ 29 \\ 8 \end{pmatrix} \quad \underline{x}_2 = \begin{pmatrix} 0 \\ -7 \\ 1 \end{pmatrix} \quad \underline{x}_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

diagonizálás

$$\underline{X} = \begin{pmatrix} -5 & 0 & 0 \\ 29 & -7 & 2 \\ 8 & 1 & 1 \end{pmatrix}$$

$$\underline{X}^{-1} = \frac{1}{45} \cdot \begin{pmatrix} -9 & 0 & 0 \\ -13 & -5 & 10 \\ 85 & 5 & 35 \end{pmatrix}$$

$$\underline{X}^{-1} \cdot \underline{A} \cdot \underline{X} = \frac{1}{45} \begin{pmatrix} 45 & 0 & 0 \\ 52 & 20 & -40 \\ 225 & 25 & 165 \end{pmatrix} \underline{X} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$\underline{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad \underline{B} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \longrightarrow \underline{C} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \lambda_1 = 2$$

$$\underline{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \lambda_2 = 3$$

$$\underline{x}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \lambda_3 = 5$$

$$\underline{v}_1 = \begin{pmatrix} i \\ 1 \end{pmatrix} \qquad \lambda_1 = i$$

$$\underline{v}_2 = \begin{pmatrix} -i \\ 1 \end{pmatrix} \qquad \lambda_2 = -i.$$