DR. FACSKÓ GÁBOR   BQQQ7Y

1. a) $\begin{pmatrix} -2 & - & 0 \\ 0 & + & 3 \\ -1 & - & 1 \end{pmatrix} \cdot (2,1,1) = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} \cdot (2\ 1\ 1) = \overset{-4}{-2\cdot 2} + \overset{3}{3\cdot 1} \overset{-2}{-2\cdot 1} =$

$= \underline{\underline{-3}}$

b) $\begin{pmatrix} 0 & +2 \\ -3 & +1 \\ 1 & +1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = 2 \begin{pmatrix} -1\cdot(-1) - 1\cdot 0 \\ 1\cdot(-2) - 1\cdot(-1) \\ 1\cdot 0 - (-1)(-2) \end{pmatrix} = 2 \underline{\underline{\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}}}$

c) $(\underline{a}, \underline{b}, \underline{c}) = (\underline{a} \times \underline{b}) \cdot \underline{c} = \left[\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}\right] \cdot (2\ 1\ 1) = \begin{pmatrix} 0\cdot 1 - (-1)\cdot 0 \\ -1\cdot 0 - (2)\cdot 1 \\ -2(-3) - 0\cdot 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} =$

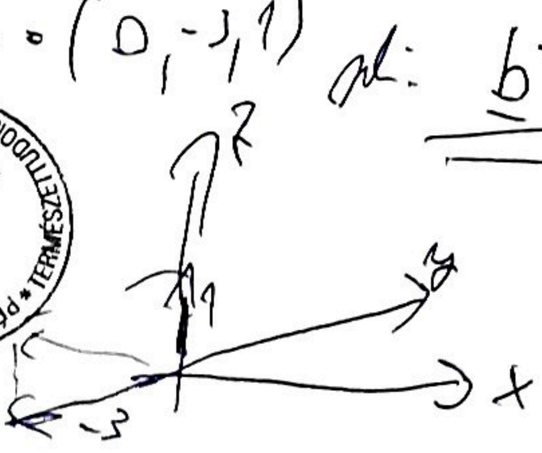
$= \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 + 2 + 6 = \underline{\underline{8}}$

d) $(\underline{a}, \underline{b})\sphericalangle = \dfrac{\cos\ \underline{a}\cdot\underline{b}}{|\underline{a}| \cdot |\underline{b}|} = \dfrac{(-2\ 0\ -1)\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}}{\sqrt{4+1} \cdot \sqrt{9+1}} =$

$= \dfrac{-1}{\sqrt{5} \cdot \sqrt{10}} = -\underline{\underline{\dfrac{1}{5\sqrt{2}}}}$

e) $(\underline{a}, \underline{b}, \underline{c}) = 8 \Rightarrow$ Nincsenek egy síkban mert a determináns $\neq 0$.

f) $\underline{b} = (0, -3, 1)$ pl: $\underline{\underline{b^{\perp} = (1, 0, 0)}}$



$\underline{b} \cdot \underline{b}^{\perp} = 0 + 0 + 0 = 0$

2

$|A| = 3\cdot 3 - (-1)(-1) = \underline{\underline{8}}$

$|B| = 1\cdot 2\cdot 3 + (-1)4(-2) + (-2)\cdot(-1)4 - (-2)2(-1) - (-1)(-1)\cdot 3 - 1\cdot 4\cdot 4 =$

$= 6 + 8 + \cancel{8} - \cancel{8} - 3 - 16 = \underline{\underline{-5}}$

$|C| = +1\cdot \begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & 2 \\ -2 & 2 & 2 \end{vmatrix} - 1\cdot \begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & -1 \\ -2 & -2 & 2 \end{vmatrix} =$

$= \cancel{1\cdot 2\cdot 2 +}\ 2(-1)2 + (-1)\cdot 2(-2) + 1\cdot 1\cdot 2 - 1\cdot(1)(2) - 1(-1)2 - 2\cdot 2\cdot 2 -$

$\big(4\cdot 1\cdot 1 + 2\cdot(-1)(-2) + (-1)2(-2) - (-1)1(-2) - 2\cdot 2\cdot 2 - 4(-1)(-1)\big)$

$= \cancel{-4} + \cancel{4} + \cancel{2} - \cancel{2} + 2 - 8 - (\cancel{8} + \cancel{4} + \cancel{4} - 2 - \cancel{8} - \cancel{8}) =$

$= 2 + 8 + 2 = \underline{\underline{12}}$

3 a)

$x_1 - x_2 + 2x_3 = 1$

$2x_1 - x_2 + x_3 = -2$ $\rightarrow -\frac{62}{20} - \frac{7}{20} + \frac{29}{20} = \underline{\underline{-\frac{40}{20}}}$ ✓

$3x_1 + 5x_2 + 2x_3 = 0$

$-\frac{93}{20} + \frac{35}{20} + \frac{58}{20} = \underline{\underline{0}}$ ✓

$x_2 - 3x_3 = -4$ (II) − 2(I)

$8x_2 - 4x_3 = -3$ (III) − 3(I)

$20x_3 = 29$ (III) − 8(II)

$\underline{x_3 = \frac{29}{20}}$

$x_2 = -4 + 3x_3 = -\frac{80}{20} + \frac{87}{20} = \underline{\underline{\frac{7}{20}}}$ $x_1 = 1 + x_2 - 2x_3 = \frac{20}{20} + \frac{7}{20} - \frac{58}{20} = \underline{\underline{-\frac{31}{20}}}$

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③ b,
$$\begin{cases} -x_1+3x_2+x_3=1 \\ x_1+3x_2+x_3=0 \\ 4x_1+x_2-3x_3=1 \end{cases}$$

$6x_2+2x_3=1$ (II)+(I)

$13x_2+x_3=5$ III+4(I)

$78x_2+26x_3=13$ $13\cdot$(I)

$78x_2+6x_3=30$ $6\cdot$(II)

$-20x_3=17$ (II)−(I)

$x_3=-\frac{17}{20}$

$x_2=\frac{1}{6}(1-2x_3)=\frac{1}{6}\left(\frac{20}{20}+\frac{34}{20}\right)=$
$=\frac{9}{20}$

$x_1=0-3x_2-x_3=-\frac{27}{20}+\frac{17}{20}=$
$=-\frac{10}{20}=-\frac{1}{2}$

$+\frac{10}{20}+\frac{27}{20}-\frac{17}{20}=\frac{20}{20}=1$ ✓

$-\frac{10}{20}+\frac{27}{20}-\frac{17}{20}=0$ ✓

$-\frac{40}{20}+\frac{9}{20}+\frac{51}{20}=\frac{20}{20}=1$ ✓

④
$$\lambda_1\begin{pmatrix}-1\\2\\1\\3\end{pmatrix}+\lambda_2\begin{pmatrix}0\\5\\-2\\2\end{pmatrix}+\lambda_3\begin{pmatrix}1\\1\\3\\1\end{pmatrix}=0$$

$-\lambda_1+\lambda_3=0 \rightarrow \lambda_1=\lambda_3$

$2\lambda_1+5\lambda_2+\lambda_3=0$

$\lambda_1-2\lambda_2+3\lambda_3=0$

$3\lambda_1+2\lambda_2+\lambda_3=0$

$2\lambda_1+5\lambda_2+\lambda_1=0$

$\lambda_1-2\lambda_2+3\lambda_1=0$

$3\lambda_1+2\lambda_2+\lambda_1=0$

$3\lambda_1+5\lambda_2=0$

$4\lambda_1-2\lambda_2=0$

$4\lambda_1+2\lambda_2=0$

$3\lambda_1+5\lambda_2=0$

$2\lambda_1-\lambda_2=0 \rightarrow \lambda_2=2\lambda_1$

$2\lambda_1+\lambda_2=0$

$$\begin{cases} 3\lambda_1 + 10\lambda_1 = 0 \\ 2\lambda_1 - 2\lambda_1 = 0 \\ 2\lambda_1 + 2\lambda_1 = 0 \end{cases}$$

$13\lambda_1 = 0$

$0 = 0$

$4\lambda_1 = 0$

$\underline{\underline{\lambda_1 = \lambda_2 = \lambda_3 = 0}}$ ⇐ lineárisan függetlenek

K 6 folytatás

$$-\frac{1}{2}\begin{pmatrix}-1\\1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}0\\1\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\2\\1\end{pmatrix} =$$

$$= \begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\end{pmatrix} + \begin{pmatrix}0\\-\frac{1}{2}\\0\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\1\\\frac{1}{2}\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} \checkmark$$

5

$$U = \left\{ \begin{pmatrix} x_1 + x_2 \\ -x_1 - x_2 \\ 4x_2 \end{pmatrix} \middle| \; x_1, x_2 \in \mathbb{R} \right\}$$

$c_1 = a_1 + b_1$

$c_2 = a_2 + b_2$

$$\underline{a} + \underline{b} = \begin{pmatrix} a_1 + a_2 + b_1 + b_2 \\ -a_1 - a_2 - b_1 - b_2 \\ 4a_2 + 4b_2 \end{pmatrix} = \begin{pmatrix} (a_1 + b_1) + (a_2 + b_2) \\ -(a_1 + b_1) - (a_2 + b_2) \\ 4(a_2 + b_2) \end{pmatrix} = \begin{pmatrix} c_1 + c_2 \\ -c_1 - c_2 \\ 4c_2 \end{pmatrix} \in U$$

$$\lambda \cdot \underline{a} = \lambda \begin{pmatrix} a_1 + a_2 \\ -a_1 - a_2 \\ 4a_2 \end{pmatrix} = \begin{pmatrix} \lambda a_1 + \lambda a_2 \\ -\lambda a_1 - \lambda a_2 \\ 4\lambda a_2 \end{pmatrix} = \begin{pmatrix} \cancel{\lambda a_1 + \lambda a_2}\; d_1 + d_2 \\ -d_1 - d_2 \\ 4d_2 \end{pmatrix} \in U \checkmark$$

$d_1 = \lambda a_1$

$d_2 = \lambda a_2$

6

$$\lambda_1 \begin{pmatrix}-1\\1\\1\end{pmatrix} + \lambda_2 \begin{pmatrix}0\\1\\0\end{pmatrix} + \lambda_3 \begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

K

$$\begin{cases} -\lambda_1 + \lambda_3 = 1 \\ \lambda_1 + \lambda_2 + 2\lambda_3 = 0 \\ \lambda_1 + \lambda_3 = 0 \end{cases} \quad \lambda_3 = -\lambda_1$$

$$\begin{cases} -\lambda_1 - \lambda_2 = 1 \\ \lambda_1 + \lambda_2 - 2\lambda_1 = 0 \end{cases}$$

$\Rightarrow \lambda_1 + \lambda_2 = -1$

$-\lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_2 = \lambda_1$

$2\lambda_1 = -1$

$\lambda_1 = -\frac{1}{2}$

$\underline{\underline{\lambda_2 = -\frac{1}{2}}}$

$\lambda_3 = \frac{1}{2}$