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(MATNA1902) Lineáris algebra 1. zárthelyi dolgozat

1. Adottak a következő mátrixok:

$$\mathbf{A}=\begin{pmatrix}1 & -2 & 3\\ -3 & 2 & 1\end{pmatrix}\ \mathbf{B}=\begin{pmatrix}3 & 1 & 2\\ 0 & 2 & 0\end{pmatrix}\ \mathbf{C}=\begin{pmatrix}1 & 4\\ 2 & 5\\ -1 & 1\end{pmatrix}\ \mathbf{D}=\begin{pmatrix}1 & 0 & -1\\ 3 & 1 & 2\\ -1 & 1 & 0\end{pmatrix}$$

Végezze el az alábbiak közül az elvégezhető műveleteket! (a) $|\mathbf{A}|$; $|\mathbf{C}|$; $|\mathbf{D}|$ (b) $\mathbf{A}+\mathbf{B}$; $\mathbf{B}+\mathbf{C}$; $\mathbf{C}+\mathbf{D}$; $4\mathbf{A}-\mathbf{B}$ (c) $\mathbf{A}\cdot\mathbf{B}$; $\mathbf{B}\cdot\mathbf{C}$; $\mathbf{B}\cdot\mathbf{D}$ (d) $\mathbf{A}^T$; $\mathbf{D}^T$; $\mathbf{A}^T\cdot\mathbf{B}$; (e) $\rho(\mathbf{B})$; $\rho(\mathbf{D})$; (f) $\mathbf{A}^{-1}$; $\mathbf{D}^{-1}$ (10 pont)

2. Oldja meg az $\mathbf{A}\cdot\mathbf{X}=\mathbf{B}$ mátrixegyenletet, ha

$$\mathbf{A}=\begin{pmatrix}1 & 0 & 2\\ 2 & -1 & 7\\ -3 & 2 & 2\end{pmatrix}\ \mathbf{B}=\begin{pmatrix}10 & 1\\ 29 & 5\\ 8 & 5\end{pmatrix}$$

(10 pont)

3. Oldja meg az alábbi lineáris egyenletrendszert!

$$\begin{aligned}x_1-2x_2-3x_3 &= 6\\ 2x_1-3x_2+x_3 &= -1\\ 3x_1+x_2+x_3 &= 5\end{aligned}$$

(10 pont)

4. Lineárisan függetlenek-e az $\mathbf{a}=(6,4,-1)$, a $\mathbf{b}=(2,1,6)$ és a $\mathbf{c}=(1,0,4)$ vektorok? (10 pont)
5. Lineáris altér-e az $\mathbb{R}^4$-on az $L=\{(x_1,x_2,2x_1,3x_2)\,|x_1,x_2\in\mathbb{R}\}$? (10 pont)

A zárthelyi osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5).

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① a) $|A|$: ∄ lehetséges, ∄ □-es mátrix

$|C|$: —"—

$$|D| = 1\cdot1\cdot0 + 0\cdot2\cdot(-1) + (-1)\cdot3\cdot1 - 1\cdot(-1)^2 - 3\cdot0\cdot0 - 1^2\cdot2 =$$

$$= -3 - 1 - 2 = \underline{\underline{-6}}$$

b) $$A+B = \begin{pmatrix} 1+3 & -2+1 & 3+2 \\ -3+0 & 2+2 & 1+0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 & -1 & 5 \\ -3 & 4 & 1 \end{pmatrix}}}$$

$\underline{B+C}$: ∄ lehetséges, mert ∄ azonos a mérete

$\underline{C+D}$: —"—

$$4A - B = 4\begin{pmatrix} 1 & -2 & 3 \\ -3 & 2 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 4\cdot1-3 & -8-1 & 12-2 \\ -12 & 8-2 & 4 \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 1 & -9 & 10 \\ -12 & 6 & 4 \end{pmatrix}}}$$

c) $\underline{A}\cdot\underline{B}$: ∄ lehetséges, mert ∄ megfelelő a mérete

$$\underline{B}\cdot\underline{C} = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3+2-2 & 12+5+2 \\ 4 & 10 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 19 \\ 4 & 10 \end{pmatrix}}}$$

$$\underline{B}\cdot\underline{D} = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3+3-2 & 0+1+2 & -3+2+0 \\ 6 & 2 & 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 & 3 & -1 \\ 6 & 2 & 4 \end{pmatrix}}}$$

d) $A^T = \begin{pmatrix}1 & -3\\ -2 & 2\\ 3 & 1\end{pmatrix}$ $\qquad D^T = \begin{pmatrix}1 & 3 & -1\\ 0 & 1 & 1\\ -1 & 2 & 0\end{pmatrix}$

$A^T \cdot B = \begin{pmatrix}1 & -3\\ -2 & 2\\ 3 & 1\end{pmatrix} \cdot \begin{pmatrix}[illegible]\end{pmatrix} = \begin{pmatrix}3 & 1-6 & 2\\ -6 & -2+2^2 & -2^2\\ 9 & 3+2 & 6\end{pmatrix} = \underline{\begin{pmatrix}3 & -5 & 2\\ -6 & 2 & -4\\ 9 & 5 & 6\end{pmatrix}}$

e) $\rho(B) = \underline{\underline{2}}$, mert (felső) trapéz alakú a mátrix és 2 sora van.

$\rho(D) = ?$

$\begin{pmatrix}1 & 0 & -1\\ 3 & 1 & 2\\ -1 & 1 & 0\end{pmatrix} \xrightarrow[(III)+(I)]{(II)-3(I)} \begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 5\\ 0 & 1 & -1\end{pmatrix} \xrightarrow{(III)-(II)} \begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 5\\ 0 & 0 & -6\end{pmatrix} \Rightarrow \rho(D) = \underline{\underline{3}}$

f) A^{-1}: $\nexists$ ~~nat~~ mert 1O-s mátrix

$D^{-1} = ?$ $\qquad |D| = \begin{vmatrix}1 & 0 & -1\\ 3 & 1 & 2\\ -1 & 1 & 0\end{vmatrix} = 0 + 0 - 3 - 1 - 0 - 2 = \underline{-6}$

$\begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix} = -2 \quad \begin{vmatrix}3 & 2\\ -1 & 0\end{vmatrix} = 2 \quad \begin{vmatrix}3 & 1\\ -1 & 1\end{vmatrix} = 4$

$\begin{vmatrix}0 & -1\\ 1 & 0\end{vmatrix} = 1 \quad \begin{vmatrix}1 & -1\\ -1 & 0\end{vmatrix} = -1 \quad \begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix} = +1$

$\begin{vmatrix}0 & -1\\ 1 & 2\end{vmatrix} = 1 \quad \begin{vmatrix}1 & -1\\ 3 & 2\end{vmatrix} = 5 \quad \begin{vmatrix}1 & 0\\ 3 & 1\end{vmatrix} = 1$

$\begin{pmatrix}-2 & 2 & 4\\ 1 & -1 & +1\\ 1 & 5 & 1\end{pmatrix} \xrightarrow{\text{sakk t.-sz.}} \begin{pmatrix}-2 & -2 & 4\\ -1 & -1 & +1\\ 1 & -5 & 1\end{pmatrix} \xrightarrow{\text{transp.}} \begin{pmatrix}-2 & -1 & 1\\ -2 & -1 & -5\\ 4 & 1 & 1\end{pmatrix} \rightarrow \underline{D^{-1} = \frac{1}{6}\begin{pmatrix}2 & 1 & -1\\ 2 & 1 & 5\\ -4 & -1 & -1\end{pmatrix}}$

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② $AX = B \quad / A^{-1}\cdot$

$X = A^{-1}\cdot B$?

$|A| = -2+0+8-6-0-14 = -14$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 7 \\ -3 & 2 & 2 \end{pmatrix} \to \begin{vmatrix} -1 & 7 \\ 2 & 2 \end{vmatrix} = -16 \quad \begin{vmatrix} 2 & 7 \\ -3 & 2 \end{vmatrix} = 25 \quad \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} = 1$$

$$\begin{vmatrix} 0 & 2 \\ 2 & 2 \end{vmatrix} = -4 \quad \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} = 8 \quad \begin{vmatrix} 1 & 0 \\ -3 & 2 \end{vmatrix} = 2$$

$$\begin{vmatrix} 0 & 2 \\ -1 & 7 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 2 \\ 2 & 7 \end{vmatrix} = 3 \quad \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1$$

$$\begin{pmatrix} -16 & 25 & 1 \\ -4 & 8 & 2 \\ 2 & 3 & -1 \end{pmatrix}$$

↓ sakkt.sz

$$\begin{pmatrix} -16 & -25 & 1 \\ 4 & 8 & -2 \\ 2 & -3 & -1 \end{pmatrix}$$

transz. ↓

$$\begin{pmatrix} -16 & 4 & 2 \\ -25 & 8 & -3 \\ 1 & -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{14}\begin{pmatrix} 16 & -4 & -2 \\ 25 & -8 & 3 \\ -1 & 2 & 1 \end{pmatrix}$$

$$X = A^{-1}\cdot B = \frac{1}{14}\begin{pmatrix} 16 & -4 & -2 \\ 25 & -8 & 3 \\ -1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 10 & 1 \\ 29 & 5 \\ 8 & 4 \end{pmatrix} = \frac{1}{14}\cdot\begin{pmatrix} 160-116-16 & 16-20-10 \\ 250-232+24 & 25-40+15 \\ -10+58+8 & -1+10+5 \end{pmatrix} =$$

$$= \frac{1}{14}\begin{pmatrix} 28 & -14 \\ 42 & 0 \\ 56 & 14 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 7 & 0 \\ 4 & 1 \end{pmatrix} = X$$

③ $\begin{cases} x_1 - 2x_2 - 3x_3 = 6 \\ 2x_1 - 3x_2 + x_3 = -1 \\ 3x_1 + x_2 + x_3 = 5 \end{cases}$

$\begin{cases} x_1 - 2x_2 - 3x_3 = 6 & (II) - 2(I) \\ \cancel{x_2 + 7x_3 = -13} & (III) - 3(I) \\ \cancel{7x_2 + 10x_3 = -13} \\ x_2 + 7x_3 = -13 \\ 7x_2 + 10x_3 = -13 \end{cases}$

$x_1 - 2x_2 - 3x_3 = 6$ $\quad (III) - 7(II)$ $\quad \boxed{x_1 = 6 + 2x_2 + 3x_3 = 6 + 2 - 6 = 2}$

$x_2 + 7x_3 = -13 \rightarrow \boxed{x_2 = -13 - 7x_3 = 1}$

$-39x_3 = 78 \rightarrow \boxed{x_3 = -2}$

P:

$2 - 2 + 6 = 6$ ✓

$4 - 3 - 2 = -1$ ✓

$6 + 1 - 2 = 5$ ✓

④ $\boxed{\lambda_1 \underline{a} + \lambda_2 \underline{b} + \lambda_3 \underline{c} = 0 \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0}$

$$\lambda_1 \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = 0$$

$\begin{cases} 6\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \\ 4\lambda_1 + \lambda_2 = 0 \\ -\lambda_1 + 6\lambda_2 + 4\lambda_3 = 0 \end{cases}$

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4. feladat

$$\begin{cases} 6\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \\ 4\lambda_1 + \lambda_2 = 0 \rightarrow \lambda_2 = -4\lambda_1 = 0 \\ -\lambda_1 + 6\lambda_2 + 4\lambda_3 = 0 \end{cases}$$

$$\begin{cases} 6\lambda_1 - 8\lambda_1 + \lambda_3 = 0 \\ -\lambda_1 - 24\lambda_1 + 4\lambda_3 = 0 \end{cases}$$

$$-2\lambda_1 + \lambda_3 = 0 \rightarrow \lambda_3 = 2\lambda_1 = 0$$

$$-25\lambda_1 + 4\lambda_3 = 0$$

$$-25\lambda_1 + 8\lambda_1 = 0$$

$$-17\lambda_1 = 0$$

$$\lambda_1 = 0$$

$\lambda_1 = \lambda_2 = \lambda_3 = 0$, tehát a 3 vektor lineárisan független

5. $U = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ 2x_1 \\ 3x_2 \end{pmatrix} \in \middle| \, x_1, x_2 \in \mathbb{R} \right\}$

(I): additív

(II) homogén

I.

$$\begin{pmatrix} a_1 \\ a_2 \\ 2a_1 \\ 3a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ 2b_1 \\ 3b_2 \end{pmatrix} = \begin{bmatrix} (a_1+b_1) \\ (a_2+b_2) \\ 2(a_1+b_1) \\ 3(a_2+b_2) \end{bmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ 2c_1 \\ 3c_2 \end{pmatrix} \in U \checkmark$$

$c_1 = a_1 + b_1$

II konzeptes

$$\lambda\begin{pmatrix} a_1 \\ a_2 \\ 2a_1 \\ 3a_2 \end{pmatrix} = \begin{pmatrix} (\lambda a_1) \\ (\lambda a_2) \\ 2(\lambda a_1) \\ 3(\lambda a_2) \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ 2d_1 \\ 3d_2 \end{pmatrix} \in U \checkmark$$

$d_i = \lambda a_i$

Tehát az U-ra teljesül az addíció és a konzgatás, így U alt. $\mathbb{R}^4$-en.