$A=\begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 1 & 2 \\ -2 & 2 & 0 \end{pmatrix}$ $C=\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}$ $D=\begin{pmatrix} -1 & 0 & 1 \\ 5 & 1 & 2 \end{pmatrix}$

Calculate the following terms if it is possible:

(a) A+B, B+C, C+D, 2A-B
(b) AB, AC, BC, BD
(c) A^T, D^T
(d) $g(A)$, $g(C)$
(e) A^{-1}, D^{-1}

(a) A+B You cannot do the operation because the size of the matrixes are different.

B+C You cannot do the operation because the size of the matrixes are different.

C+D You cannot do the operation because the size of the matrixes are different.

2A-B You cannot do the operation because the size of the matrixes are different.

(b) AB You cannot do this operation because the number of columns of A and the number of rows of B are different.

$$AC=\begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}=\begin{pmatrix} -1\cdot 0+0\cdot(-2)+2\cdot 3 & -1\cdot 1+0\cdot(-5)+2\cdot 7 \\ 0\cdot 0+2\cdot(-2)+1\cdot 3 & 0\cdot 1+2\cdot(-5)+1\cdot 7 \\ -2\cdot 0+2(-2)+5\cdot 3 & -2\cdot 1+2\cdot(-5)+5\cdot 7 \end{pmatrix}=$$

$$=\begin{pmatrix} 6 & 13 \\ -1 & -3 \\ 11 & 23 \end{pmatrix}$$

$$BC=\begin{pmatrix} 1 & 1 & 2 \\ -2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -5 \\ 3 & 7 \end{pmatrix}=\begin{pmatrix} 1*0+1(-2)+2*3 & 1*1+1(-5)+2*7 \\ -2*0+2(-2)+0*3 & -2*1+2(-5)+0*7 \end{pmatrix}=\begin{pmatrix} 4 & 10 \\ -4 & -12 \end{pmatrix}$$

BD You cannot do this operation because the number of columns of A and the number of rows of B are different.

(c) $A=\begin{pmatrix}-1 & 0 & 2\\ 0 & 2 & 1\\ -2 & 2 & 5\end{pmatrix}$ $A^T=\begin{pmatrix}-1 & 0 & -2\\ 0 & 2 & 2\\ 2 & 1 & 5\end{pmatrix}$

$D=\begin{pmatrix}-1 & 0 & 1\\ 5 & 1 & 2\end{pmatrix}$ $D^T=\begin{pmatrix}-1 & 5\\ 0 & 1\\ 1 & 2\end{pmatrix}$

$\rho(C) = ?$ $\rho(C) = \underline{\underline{2}}$

$$\begin{pmatrix}0 & 1\\ -2 & -5\\ 3 & 7\end{pmatrix} \longrightarrow \begin{pmatrix}3 & 7\\ -2 & -5\\ 0 & 1\end{pmatrix} \xrightarrow[3(II)]{2(I)} \begin{pmatrix}6 & 14\\ -6 & -15\\ 0 & 1\end{pmatrix}$$

$$\xrightarrow{(II)+(I)} \begin{pmatrix}6 & 14\\ 0 & -1\\ 0 & 1\end{pmatrix} \xrightarrow{(III)+(II)} \begin{pmatrix}6 & 14\\ 0 & -1\\ 0 & 0\end{pmatrix}$$

(d) $\rho(A) = ?$

$$\begin{pmatrix}-1 & 0 & 2\\ 0 & 2 & 1\\ -2 & 2 & 5\end{pmatrix} \xrightarrow{(III)-2(I)} \begin{pmatrix}-1 & 0 & 2\\ 0 & 2 & 1\\ 0 & 2 & 1\end{pmatrix} \xrightarrow{(III)-(II)} \begin{pmatrix}-1 & 0 & 2\\ 0 & 2 & 1\\ 0 & 0 & 0\end{pmatrix}$$

$\rho(A) = 2$

(e)

$$A = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ -2 & 2 & 5 \end{pmatrix} \quad A^{-1} = ? \quad det(A)=-1*2*5+0*1(-2)+0*1(-2)-2*2(-2)-0*0*5-(-1)1*2=-10+8+2=0$$

The det(A)=0, therefore the inversion of A is not possible.

The inverse of D matrix cannot be calculated because the D is not a square matrix.

Give the element of the Matrix X if the following equation is satified:

$$\underbrace{\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}}_{\underline{A}} \underline{X} = \underbrace{\begin{pmatrix}4 & -6\\ 2 & 1\end{pmatrix}}_{\underline{B}} \qquad \underline{X} = ?$$

$$\underline{A}\,\underline{X} = \underline{B} \quad / \underline{A}^{-1}\cdot$$

$$\underbrace{\underline{A}^{-1}\underline{A}}_{\underline{E}}\,\underline{X} = \underline{A}^{-1}\cdot\underline{B}$$

$$\underline{X} = \underline{A}^{-1}\cdot\underline{B}$$

$$\left(\begin{array}{cc|cc}2 & 5 & 1 & 0\\ 1 & 3 & 0 & 1\end{array}\right) \xrightarrow{\frac{1}{2}(I)} \left(\begin{array}{cc|cc}1 & \frac{5}{2} & \frac{1}{2} & 0\\ 1 & 3 & 0 & 1\end{array}\right) \xrightarrow{(II)-(I)} \left(\begin{array}{cc|cc}1 & \frac{5}{2} & \frac{1}{2} & 0\\ 0 & \frac{1}{2} & -\frac{1}{2} & 1\end{array}\right)$$

$$\xrightarrow{(I)-5(II)} \left(\begin{array}{cc|cc}1 & 0 & 3 & -5\\ 0 & \frac{1}{2} & -\frac{1}{2} & 1\end{array}\right) \xrightarrow{2\cdot(II)} \left(\begin{array}{cc|cc}1 & 0 & 3 & -5\\ 0 & 1 & -1 & 2\end{array}\right)$$

$$\underline{A}^{-1} = \begin{pmatrix}3 & -5\\ -1 & 1\end{pmatrix}$$

$$\underline{X} = \begin{pmatrix}3 & -5\\ -1 & 2\end{pmatrix}\cdot\begin{pmatrix}4 & -6\\ 2 & 1\end{pmatrix} = \begin{pmatrix}12-10 & -18-5\\ -4+4 & 6+2\end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix}2 & -23\\ 0 & 8\end{pmatrix}}}$$

$$\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}\begin{pmatrix}2 & -23\\ 0 & 8\end{pmatrix} = \begin{pmatrix}4+0 & -46+40\\ 2+0 & -23+24\end{pmatrix} = \underline{\underline{\begin{pmatrix}4 & -6\\ 2 & 1\end{pmatrix}}}$$

$$\underbrace{\begin{pmatrix} 2 & 1 & -1 \\ 4 & -1 & 2 \\ 3 & 0 & 1 \end{pmatrix}}_{A} X = \underbrace{\begin{pmatrix} 5 & 1 & 6 \\ 7 & -8 & 7 \\ 6 & -4 & 7 \end{pmatrix}}_{B}$$

$$A X = B \quad / A^{-1} *$$

$$A^{-1} A X = A^{-1} B$$

$$X = A^{-1} B = ?$$

$$D_{11} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1 \qquad D_{12} = \begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = 4-6=2 \qquad D_{13} = \begin{vmatrix} 4 & -1 \\ 3 & 0 \end{vmatrix} = 3$$

$$D_{21} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 \qquad D_{22} = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 5 \qquad D_{23} = \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} = -3$$

$$D_{31} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 1 \qquad D_{32} = \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} = 8 \qquad D_{33} = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$$

$$\begin{pmatrix} -1 & -2 & 3 \\ +1 & 5 & -3 \\ 1 & 8 & -6 \end{pmatrix} \to \begin{pmatrix} -1 & -1 & 1 \\ -2 & 5 & 8 \\ 3 & -3 & -6 \end{pmatrix} \to \begin{pmatrix} -1 & -1 & 1 \\ +2 & 5 & -8 \\ 3 & 3 & -6 \end{pmatrix}$$

det(A)=2(-1)1+1*2*3+(-1)4*0-(-1)(-1)3-1*4*1-2*2*0=
-2+6-3-4=-3

$$X = \frac{1}{3}\begin{pmatrix} 1 & 1 & -1 \\ -2 & -5 & 8 \\ -3 & -3 & 6 \end{pmatrix}\begin{pmatrix} 5 & 1 & 6 \\ 7 & -8 & 7 \\ 6 & -4 & 7 \end{pmatrix} = \qquad A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 & -1 \\ -2 & -5 & 8 \\ -3 & -3 & 6 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 5+7-6 & 1-8+4 & 6+7-7 \\ -10-35+48 & -2+40-32 & -12-35+56 \\ -15-21+36 & -3+24-24 & -18-21+42 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 & -3 & 6 \\ 3 & 6 & 9 \\ 0 & -3 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}$$