Név / Name: DR. FACSKÓ GÁBOR Neptun kód / code: BFQQQY

(MATNA1902) Alkalmazott lineáris algebra 2. zárthelyi dolgozat
(ENKEMNA0302) Applied Linear Algebra Test 2

6. Mátrixok / Matrices

   a.) Adja meg az alábbi mátrixok nyomát! / What is the trace of the matrices below?

$$\mathbf{A} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \mathbf{B} = \begin{pmatrix} 8 & 1+i \\ 1-i & 5 \end{pmatrix} \mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1+i & -1+i \\ 1+i & 1-i \end{pmatrix}$$

$$\mathbf{D} = \frac{1}{3} \begin{pmatrix} 1 & 2 & -2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \mathbf{E} = \begin{pmatrix} 1 & 2+3i & 4i \\ 2-3i & 0 & 6-7i \\ -4i & 6+7i & 3 \end{pmatrix} \mathbf{F} = \frac{1}{2} \begin{pmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{pmatrix}$$

   b.) Melyik mátrix (szemi)ortogonális? Melyik mátrix önadjungált (Hermitikus), melyik unitér? / Which matrix is (semi)orthogonal? Which matrix is Hermitian or unitary?

   c.) Számolja ki az ortogonális (de nem hermitikus, vagy unitér) mátrixok inverzét! / Calculate the inverse of the orthogonal (but neither Hermitian nor unitary) matrices.

   d.) Mi az $(1, 1, 0)$ és $(0, i, i)$ pontok távolsága? / What is the distance between the points $(1, 1, 0)$ and $(0, i, i)$? (20 pont)

7. LU-felbontás / LU decomposition

   a.) Adja meg az alábbi mátrix LU-felbontását! / What is the LU decomposition of the matrix below?

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

   b.) Oldja meg az alábbi lineáris egyenletrendszert az **A** mátrix LU-felbontását használva! / Solve the following system of linear equations using LU decomposition.

$$\begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

   c.) Számítsa ki az **A** mátrix az inverzét LU felbontással! / Calculate the inverse of matrix **A** using LU decomposition. (20 pont)

8. Adja meg az alábbi mátrix sajátértékeit és a saját altereket, majd diagonizálja a mátrixot! / Calculate the eigenvalues and eigenspaces. Diagonalize the matrices.

$$\mathbf{A} = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$$

(10 pont)

A zárthelyi osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). / Grades: 0-20 points: Fail 81), 21-27 points: Pass (2), 28-35 points: Satisfactory (3), 36-42 points: Good (4) és 43-50 points: Excellent (5).

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Pécs, 2025. május 8. / May 8, 2025

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6) a) trace($\underline{A}$) = 2 cos α   trace($\underline{D}$) = 5

trace($\underline{B}$) = 13   trace($\underline{E}$) = 4

trace($\underline{C}$) = 2   trace($\underline{F}$) = 2

b) $\underline{A}$ is orthogonal / ortogonális ($\underline{A} = \underline{A}^T$)

$\underline{B}$, $\underline{E}$ are Hermitian ($\underline{A} = \underline{A}^H$)

unitary / unitér matrix $\Leftrightarrow$ $\underline{A}^H = \underline{A}^{-1}$

$$\underline{A}\cdot\underline{A}^H = \begin{pmatrix} \cancel{\cos\alpha} & \cancel{\sin\alpha} \\ \cancel{\sin\alpha} & \cancel{\cos\alpha} \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark \Rightarrow \underline{A} \text{ unitér / unitary matrix}$$

$$\underline{B}\cdot\underline{B}^H = \begin{pmatrix} \cancel{8} & \cancel{1+i} \\ \cancel{1-i} & \cancel{5} \end{pmatrix}\begin{pmatrix} 8 & 1+i \\ 1+i & 5 \end{pmatrix} = \begin{pmatrix} 64+(1+i)^2 & 8-8i+5+5i \\ 8-8i+5+5i & (1-i)^2+25 \end{pmatrix} =$$

$$= \begin{pmatrix} 64+\cancel{1^2}+2i-\cancel{1} & 13-3i \\ 13-3i & \cancel{1}-\cancel{1}-2i+25 \end{pmatrix} = \begin{pmatrix} 64+2i & 13-3i \\ 13-3i & 25-2i \end{pmatrix} \neq \underline{E}$$

$\Downarrow$ not unitary / unitér

$$\underline{C}\cdot\underline{C}^H = \frac{1}{2}\begin{pmatrix} \cancel{1+i} & \cancel{-1+i} \\ \cancel{1+i} & \cancel{1-i} \end{pmatrix}\begin{pmatrix} 1-i & 1+i \\ -1-i & 1+i \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} (1+i)(1-i)+(-1+i)(-1-i) & (1+i)(1-i)+(-1+i)(1+i) \\ (1+i)(1-i)+(1-i)(-1-i) & (1+i)(1-i)+(1-i)(1+i) \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 1^2+1+1^2+1 & 1^2+1-1-1 \\ 1+1-1-1 & 1+1+1+1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow \text{not unitary / unitér}$$

$$D \cdot D^H = \frac{1}{9}\begin{pmatrix} 1 & 2 & -2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -2 & 2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Downarrow$

D unitary / unitar

$$C \cdot C^H = \begin{pmatrix} 1 & 2+3i & 4i \\ 2-3i & 0 & 6+7i \\ -4i & 6+7i & 3 \end{pmatrix}\begin{pmatrix} 1 & 2+3i & 4i \\ 2-3i & 0 & 6+7i \\ -4i & 6+7i & 3 \end{pmatrix} =$$

$4+9$

$$= \begin{pmatrix} 1+(2+3i)(2-3i)-16(i^2) & 2+3i+24i-28 & 4i+(2+3i)(6+7i)+12i \\ 2-3i-24i-28 & 4+9+36+49 & 8i+12+18-21i \\ -4i+(6+7i)(2-3i)-12i & -8i+12+18+21i & 16+36+49+9 \end{pmatrix}$$

$$= \begin{pmatrix} 30 & -26+27i & \\ \neq & & \end{pmatrix}$$

7 unitar / ~~unit~~ nicht unitary

$$E \cdot E^H = \frac{1}{4}\begin{pmatrix} 1 & -1 & -1+i \\ i & 1 & 1+i \\ -1+i & -1+i & 0 \end{pmatrix}\begin{pmatrix} 1 & -i & 1+i \\ i & 1 & 1-i \\ -1+i & -1+i & 0 \end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix} 1+1+1+1 & -i-1+1+1 & 1-i-i+1+1+1-i+1 \\ i+i+2 & +1+1+2 & i+1-1-i \\ 1+i+i-1+2 & \cancel{1+1}+i+1+2 \;\; -i+1-1+i & 2+2 \end{pmatrix}$$

$\Rightarrow$ 7 unitar

7 unitär

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6.b folyt./Cont'd

$$= \frac{1}{4}\begin{pmatrix} 4 & 2-2i & 2-2i \\ 2+2i & 4 & 0 \\ 2+2i & 0 & 4 \end{pmatrix}$$

A is unitary / unitér mátrix

c, $$A^{-1} = A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

d, $P_1(1,1,0)$, $P_2(0,i,i)$ $\Rightarrow \overrightarrow{P_1P_2}(1, 1-i, -i)$

$$d = |\overrightarrow{P_1P_2}| = \left(\bar{1}\cdot 1 + \overline{(1-i)}(1-i) + \overline{(-i)}(-i)\right)^{\frac{1}{2}} =$$

$$= \left(1 + (1+i)(1-i) + i\cdot(-i)\right)^{\frac{1}{2}} = (1+2+1)^{\frac{1}{2}} = \underline{\underline{2}}$$

7. a $$\begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix} \qquad E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(II) − (I)

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \quad \cancel{\downarrow (III) + 2(I)}$$

$$E_2 = \cancel{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_2 = \cancel{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}$$

$$\underbrace{\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}}_{\underline{U}}$$

$$\underline{E}_2 \, \underline{E}_1 \cdot \underline{A} = U \quad / \, \underline{E}_1^{-1} \underline{E}_2^{-1}$$

$$\boxed{\underline{A} = \left(\underline{E}_1^{-1} \underline{E}_2^{-1}\right) U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}}_{\underline{U}}}$$

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$$\begin{pmatrix}3&-1&0\\3&-1&1\\0&2&1\end{pmatrix}\underline{x}=\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\underline{A}\cdot\underline{x}=\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\underline{L}\cdot\underbrace{\underline{U}\cdot\underline{x}}_{\underline{y}}=\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\underline{L}\cdot\underline{y}=\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}2\\2\\2\end{pmatrix}$$

$y_1=2$

$y_2=2$

$y_1+y_3=2 \Rightarrow y_3=0$

$$\boxed{\underline{y}=\begin{pmatrix}2\\2\\0\end{pmatrix}}$$

$$\underline{U}\cdot\underline{x}=\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$$\begin{pmatrix}3&-1&0\\0&2&1\\0&0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$$\begin{cases}3x_1-x_2=2\\2x_2+x_3=2\\\boxed{x_3=0}\end{cases}$$

$3x_1-x_2=2$

$2x_2=2 \rightarrow \boxed{x_2=1}$

$3x_1-1=2$

$3x_1=3$

$\boxed{x_1=1}$

$$\underline{x}=\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$C:\begin{pmatrix}3&-1&0\\3&-1&1\\0&2&1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}2\\2\\2\end{pmatrix}\checkmark$$

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$$\underline{A}\cdot\underline{X}=\underline{E}$$

$$\underline{L}\cdot\underbrace{(\underline{U}\cdot\underline{X})}_{\underline{Y}}=\underline{E}$$

$$\underline{L}\cdot\underline{Y}=\underline{E}$$

$$\begin{pmatrix} \cancel{1\ 0\ 0} \\ \cancel{0\ 1\ 0} \\ 1\ 0\ 1 \end{pmatrix}\begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cancel{y_{11}=}\begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{11}+y_{31} & y_{12}+y_{32} & y_{13}+y_{33} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{array}{lll} y_{11}=1 & y_{12}=0 & y_{13}=0 \\ y_{21}=0 & y_{22}=1 & y_{23}=0 \\ y_{11}+y_{31}=0 & y_{12}+y_{32}=0 & y_{13}+y_{33}=1 \\ \Downarrow & \Downarrow & \\ y_{31}=-1 & y_{32}=0 & y_{33}=1 \end{array}\right\} \Rightarrow \underline{Y}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

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$$\underline{U} \cdot \underline{X} = \underline{Y}$$

$$X = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Determinált permutáció

$$X = \begin{pmatrix} \frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -1 & 1 & 0 \end{pmatrix}$$

$3x_{11} - x_{21} = 1$ $\quad$ $3x_{12} - x_{22} = 0$ $\quad$ $3x_{13} - x_{23} = 0$

$2x_{21} + x_{31} = 0$ $\quad$ $2x_{22} + x_{32} = 1$ $\quad$ $2x_{23} + x_{33} = 0$

$\boxed{x_{31} = -1}$ $\quad$ $\boxed{x_{32} = 0}$ $\quad$ $\boxed{x_{33} = 1}$

$3x_{11} - x_{21} = 1$ $\quad$ $x_{22} = 3x_{12}$ $\quad$ $x_{23} = 3x_{13}$

$2x_{21} - 1 = 0$ $\quad$ $2x_{22} = 1$ $\quad$ $2x_{23} + 1 = 0$

$3x_{11} - x_{21} = 1$ $\quad$ $x_{22} = 3x_{12}$ $\quad$ $x_{23} = 3x_{13}$

$\boxed{x_{21} = \frac{1}{2}}$ $\quad$ $\boxed{x_{22} = \frac{1}{2}}$ $\quad$ $\boxed{x_{23} = -\frac{1}{2}}$

$3x_{11} - \frac{1}{2} = 1$ $\quad$ $\frac{1}{2} = 3x_{12}$ $\quad$ $-\frac{1}{2} = 3x_{13}$

$\boxed{x_{11} = \frac{1}{2}}$ $\quad$ $\boxed{x_{12} = \frac{1}{6}}$ $\quad$ $\boxed{x_{13} = -\frac{1}{6}}$

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$$A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$$

$$\det(A - \lambda E) = 0$$

$$\begin{vmatrix} 5-\lambda & 1 \\ 1 & 5-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)^2 - 1 = 0$$

$$25 - 10\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 10\lambda + 24 = 0$$

$$(\lambda - 4)(\lambda - 6) = 0$$

$$\boxed{\lambda_1 = 4} \qquad \boxed{\lambda_2 = 6}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 + x_2 = 0 \qquad x_1 - x_2 = 0$$

$$x_2 = -x_1 \qquad x_2 = x_1$$

$$\boxed{L_{\lambda_1 = 4} = \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}} \to \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \boxed{L_{\lambda_2 = 6} = \left\{ \begin{pmatrix} t \\ t \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \to \det(C) = 2 \neq 0 \qquad C^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & +1 \\ -1 & 1 \end{pmatrix} \xrightarrow{\text{tausch}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \xrightarrow{\text{trans}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \to C^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

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① folyt / Cont'd $|C| = 2 \neq 0$ ✓

$$\underline{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \xrightarrow[\text{sakk chess}]{} \begin{pmatrix} 1 & +1 \\ -1 & 1 \end{pmatrix} \xrightarrow[\text{transp}]{} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \to \underline{C}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\underline{E} =: \underline{C} \cdot \underline{C}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} ✓$$

$$\underline{C}^{-1} \cdot \underline{A} \cdot \underline{C} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}\underline{C} = \frac{1}{2}\begin{pmatrix} 4 & -4 \\ 6 & 6 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 8 & 0 \\ 0 & 12 \end{pmatrix} = \underline{\begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}}$$

6a ÚJRA / RE CALCULATION

$$\underline{F} \cdot \underline{F}^{*} = \frac{1}{4}\begin{pmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{pmatrix}\begin{pmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{pmatrix} =$$

$\boxed{\underline{F} \text{ unitér / unitar}}$ ⇑

$$= \frac{1}{4}\begin{pmatrix} 1+1+2 & \begin{matrix}-i+i-1+i+i+1 \\ \cancel{2+2i}\end{matrix} & 1-i-i+1 \\ \begin{matrix} 2i \cancel{\ldots} -1-i \\ -i+1 \end{matrix} & 2+2 & i+1 \cancel{+1} -1 \\ 1+i+i-1+\ldots & -i+1-1+i & 2+2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 4 & \cancel{2+2i}\ 0 & 0 \\ \cancel{2+2i}\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$