Gy. f. 8

$$\boxed{\lambda_1 \underline{a} + \lambda_2 \underline{b} + \lambda_3 \underline{c} = 0 \iff \lambda_1 = \lambda_2 = \lambda_3 = 0}$$

$$\lambda_1 \begin{pmatrix} -1 \\ 2 \\ 1 \\ 4 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 5 \\ -1 \\ 1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 1 \\ 5 \\ 2 \end{pmatrix} = 0$$

$$\left\{\begin{aligned} -\lambda_1 + \lambda_3 &= 0 \\ 2\lambda_1 + 5\lambda_2 + \lambda_3 &= 0 \\ \lambda_1 - \lambda_2 + 5\lambda_3 &= 0 \\ 4\lambda_1 + \lambda_2 + 2\lambda_3 &= 0 \end{aligned}\right.$$

$$\to \lambda_1 = \boxed{\lambda_3 = 0}$$

$$2\lambda_1 + 5\lambda_2 + \lambda_1 = 0$$

$$\lambda_1 - \lambda_2 + 5\lambda_1 = 0$$

$$4\lambda_1 + \lambda_2 + 2\lambda_1 = 0$$

$$3\lambda_1 + 5\lambda_2 = 0$$

$$6\lambda_1 - \lambda_2 = 0 \to \boxed{\lambda_2} = 6\lambda_1$$

$$6\lambda_1 + \lambda_2 = 0$$

$$12\lambda_1 = 0 \to \boxed{\lambda_1 = 0}$$

Gy. $f^{\text{II}}/8$

$$\boxed{\lambda_1 \underline{a} + \lambda_2 \underline{b} + \lambda_3 \underline{c} = 0 \iff \lambda_1 = \lambda_2 = \lambda_3 = 0}$$

$$\lambda_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$\underline{E}: \; 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$\lambda_1 + \lambda_3 = 0 \rightarrow \lambda_1 = -\lambda_3$$

$$\lambda_2 + \lambda_3 = 0 \rightarrow \lambda_2 = -\lambda_3$$

$$\lambda_1 = \lambda_2 = -\lambda_3$$

$$(\lambda_1, \lambda_2, \lambda_3) = (1, 1, -1)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$(\lambda_1, \lambda_1, -\lambda_1)$$

Gy. $f^{\underline{0}}/_{0}$

$$\boxed{\lambda_1 \underline{a} + \lambda_2 \underline{b} + \lambda_3 \underline{c} = 0 \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0}$$

$$\lambda_1 \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \cdot \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} = 0$$

1E:
$$7 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} - 3 \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} = 0$$

$$\begin{cases} +\lambda_1 + 2\lambda_2 + 3\lambda_3 = 0 \\ \lambda_1 - \lambda_2 + 2\lambda_3 = 0 \end{cases}$$

$$-3\lambda_2 - \lambda_3 = 0$$

$$3\lambda_2 + \lambda_3 = 0$$

(II) − (I)

$$\lambda_3 = -3\lambda_2$$

$\lambda_1 \cdot (1)$
$$\rightarrow \lambda_1 + 2\lambda_2 - 9\lambda_2 = 0$$

$$\lambda_1 = 7\lambda_2$$

$$(7\lambda_2 \mid \lambda_2 \mid -3\lambda_2)$$

$$\begin{pmatrix} -7 - 2 + 9 \\ 1 - 1 + 2 \\ 0 + 0 + 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Gy. f. VI/11

$\underline{a}+\underline{b} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{pmatrix} = a_1+b_1$

(I) $\forall\, \underline{a}, \underline{b} \in U$ $\quad \underline{a}+\underline{b} \in U$

(II) $\lambda \in \mathbb{R}, \forall\, \underline{a} \in U$ $\quad \lambda \underline{a} \in U$

$$U = \begin{pmatrix} x_1 + 2x_2 \\ x_1 + x_2 \\ x_2 \end{pmatrix}, \; x_1, x_2 \in \mathbb{R}$$

$(x_1, x_2, x_3) \in \mathbb{R}^3$

(I) $$\begin{pmatrix} a_1+2a_2 \\ a_1+a_2 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1+2b_2 \\ b_1+b_2 \\ b_2 \end{pmatrix} = \begin{bmatrix} (a_1+b_1)+2(a_2+b_2) \\ (a_1+b_1)+(a_2+b_2) \\ (a_2+b_2) \end{bmatrix} = \begin{pmatrix} c_1+2c_2 \\ c_1+c_2 \\ c_2 \end{pmatrix} \in U$$

$c_i = a_i + b_i$

(II) $$\lambda \cdot \begin{pmatrix} a_1+2a_2 \\ a_1+a_2 \\ a_2 \end{pmatrix} = \begin{pmatrix} \lambda a_1 + 2(\lambda a_2) \\ (\lambda a_1)+(\lambda a_2) \\ (\lambda a_2) \end{pmatrix} = \begin{pmatrix} d_1+2d_2 \\ d_1+d_2 \\ d_2 \end{pmatrix} \in U$$

$d_i = \lambda a_i$

q.e.d

Gy. f. VII/12

$$\underline{U} = \begin{pmatrix} x_1 + x_3 \\ x_1 - x_3 \\ 4x_3 \end{pmatrix} \quad x_1, x_2, x_3 \in \mathbb{R}$$

(I) $\forall \underline{a}, \underline{b} \in \underline{U} \quad \underline{a} + \underline{b} \in \underline{U}$

(II) $\lambda \in \mathbb{R}, \forall \underline{a} \in \underline{U} \quad \lambda \underline{a} \in \underline{U}$

(I)
$$\begin{pmatrix} a_1 + a_3 \\ a_1 - a_3 \\ 4a_3 \end{pmatrix} + \begin{pmatrix} b_1 + b_3 \\ b_1 - b_3 \\ 4b_3 \end{pmatrix} = \begin{bmatrix} (a_1 + b_1) + (a_3 + b_3) \\ (a_1 + b_1) - (a_3 + b_3) \\ 4(a_3 + b_3) \end{bmatrix} = \begin{pmatrix} c_1 + c_3 \\ c_1 - c_3 \\ 4c_3 \end{pmatrix} \in \underline{U} \checkmark$$

$$c_i = a_i + b_i$$

(II)
$$\lambda \begin{pmatrix} a_1 + a_3 \\ a_1 - a_3 \\ 4a_3 \end{pmatrix} = \begin{bmatrix} (\lambda a_1) + (\lambda a_3) \\ (\lambda a_1) - (\lambda a_3) \\ 4(\lambda a_3) \end{bmatrix} = \begin{pmatrix} d_1 + d_3 \\ d_1 - d_3 \\ 4d_3 \end{pmatrix} \in \underline{U} \checkmark$$

$$d_i = \lambda a_i$$

q.e.d.

Gy. 5. VI/1.

$$\underline{C} = \begin{pmatrix} 2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \\ 3 & 0 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{pmatrix} = 3 \cdot \begin{vmatrix} 4 & -2 & 2 \\ -2 & 1 & 3 \\ 0 & 1 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ -1 & 0 & 1 \end{vmatrix} =$$

$$= 3(4+0-4-0-4-12) - (-4-4+0+4-4+0) =$$

$$= 3 \cdot (-16) - (-8) = \underline{\underline{-40}}$$

Gy. 5. $\overline{VI}$/

$$\underline{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$|\underline{A}| = 0+1+0-0-2-0 = \underline{\underline{-1}} \neq 0$$

$$\left.\begin{matrix}
\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \\
\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = 2 & \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} = 0 & \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \\
\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 & \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0 & \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1
\end{matrix}\right\} \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix} \xrightarrow[\text{sakk}]{} \begin{pmatrix} 1 & -1 & 0 \\ -2 & 0 & +1 \\ 1 & 0 & -1 \end{pmatrix} \xrightarrow[\text{Trasz}]{} \begin{pmatrix} 1 & -2 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} =$$

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} \begin{pmatrix} 1 & -2 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -1 & 2 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}}}$$

Gy. F III/66

$$\underline{A}\,\underline{X}^{-1} + \underline{B}^{-1} = 2\underline{X}^{-1} \quad /\cdot \underline{X}$$

$$\underline{A}(\underbrace{\underline{X}^{-1}\underline{X}}_{E}) + \underline{B}^{-1}\cdot X = 2(\underbrace{X^{-1}\cdot \underline{X}}_{E})$$

$$\underline{A} + B^{-1}\cdot X = 2\cdot \underline{E} \quad / -\underline{A}$$

$$B^{-1}\underline{X} = 2\underline{E} - A \quad / \underline{B}\cdot$$

$$\boxed{\underline{X} = \underline{B}(2\cdot \underline{E} - \underline{A})}$$

$$\begin{pmatrix}0&0&2\\0&2&0\\2&0&0\end{pmatrix}\cdot\left[\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix} - \begin{pmatrix}1&0&1\\0&1&0\\1&0&1\end{pmatrix}\right]$$

$$= \begin{pmatrix}0&0&2\\0&2&0\\2&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&-1\\0&1&0\\-1&0&1\end{pmatrix} =$$

$$= \begin{pmatrix}-2&0&2\\0&2&0\\2&0&-2\end{pmatrix}$$