

N 68/1

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

a) d. Vai d. ve.

b) $|A|=?$, $\text{tr}(A) = ?$

c) diagonal form

$$A \cdot x = \lambda x \quad x \in \mathbb{R}^2 \setminus \{0\}$$

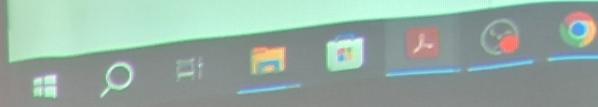
$$A \cdot x - \lambda x = 0$$

$$(A - \lambda E)x = 0 \quad (\Leftrightarrow) |A - \lambda E| = 0$$

2-
3

12-

8-
13



$$\begin{vmatrix} 2-\lambda & 1 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - 3 = 0$$

$$8 - 6\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda - 5)(\lambda - 1) \geq 0 \Rightarrow \lambda \in \left\{ \begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 5 \end{array} \right\}$$

$$\lambda_{12} =$$

$$\lambda_1 = 1$$

$$\begin{pmatrix} 2 & 1 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 = -x_2$$

$$\mathcal{L}_{\lambda=1} \supset \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 &= -x_1 \\ 3x_1 + 3x_2 &= 0 \quad /:3 \end{aligned}$$

$$x_2 = -x_1 - x_1$$

$$\begin{aligned} 5 \\ x_2 &= 0 \quad \text{+} \\ -x_1 & \\ x_2 &= 0 \quad \text{/ .5} \\ \Rightarrow -x_1 &= -x_1 \\ & \end{aligned}$$

$$\begin{pmatrix} -3 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-3x_1 + x_2 = 0$$

$$3x_1 - x_2 = 0 \quad \left\{ \begin{array}{l} x_1 = \begin{cases} t \\ 3t \end{cases} \\ x_2 = 5 \end{array} \right| t \in \mathbb{R} \quad \text{/ .5}$$

$$x_2 = 3x_1$$

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$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

a) d. Vai l. ve.

b) $|A|=?$, $\text{tr}(A)=?$
c) diagonal form

$$A \cdot X = J \cdot X \quad X \in \mathbb{R}^2 \setminus \{0\}$$

$$A \cdot X - J \cdot X = 0$$

$$(A - J \cdot E)X = 0 \quad (\Leftrightarrow) \quad |A - J \cdot E| \approx 0$$

$$\lambda_1 = \{5\}$$

$$\mathcal{L}_{\lambda_1} = \left\{ \begin{pmatrix} t \\ -t \end{pmatrix} \mid t \in \mathbb{R} \right\} \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\mathcal{L}_{\lambda_2} = \left\{ \begin{pmatrix} t \\ t \end{pmatrix} \mid t \in \mathbb{R} \right\} \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$b) |A| =$$

$$-3(A)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$8 -$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Ha egy $n \times n$ -es mátrixnak van

MEGNEZEM AZ ERRÓL A KÉPLETRŐL SZÓLÓ TANANYAGOT

1.

Az oldali cookie-kat használ.

ELFOGADOK

$$b) |A| = -3 + 5 = 2 \quad \lambda_1 = 1.5 = 5$$

$$-tr(A) = -2 + 4 = 6 \quad \lambda_1 + \lambda_2 = 1 + 5 = 6 = -tr(A)$$

$$X = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$$

$$|X| = 3 + 1 = 4 \neq 0$$

$$X^{-1} \rightarrow \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \xrightarrow{\text{cholesky}} \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \xrightarrow{\text{trans}}$$

b) \rightarrow $\text{det} S$

$$\rightarrow X^{-1} = \frac{1}{4} \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{4}$$

$$= \frac{1}{4}$$

=

$$\underline{x}^T \cdot A \cdot \underline{x} = \frac{1}{4} \begin{pmatrix} 3-1 \\ 1+1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \underline{x} =$$

$$= \frac{1}{4} \begin{pmatrix} 6-3 & 3-4 \\ 2+3 & 1+4 \end{pmatrix} \cdot \underline{x} = \frac{1}{4} \begin{pmatrix} 3 & -1 \\ 5 & 5 \end{pmatrix} \cdot \underline{x}$$

$$= \frac{1}{4} \begin{pmatrix} 3-1 \\ 5-5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3+1 & 3-3 \\ 5-5 & 5+15 \end{pmatrix} =$$

$$= \frac{1}{4} \begin{pmatrix} 4 & 0 \\ 0 & 20 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$$

N 77/5

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 5 & -2 & 14 \\ 9 & 1 & 3 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} -5 \\ 29 \\ 8 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 0 \\ -7 \\ 7 \end{pmatrix}, \quad x_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\lambda_{123} = \begin{cases} 1 \\ -4 \\ 5 \end{cases}$$

$$X^{-1} \cdot A \cdot X = ?$$
$$X = \begin{pmatrix} 5 & 0 & 0 \\ 29 & -7 & 2 \\ 8 & 1 & 1 \end{pmatrix}$$

$$X^{-1} A X$$



$$X = \begin{pmatrix} -9 & 0 & 0 \\ -13 & -5 & 10 \\ 85 & 5 & 35 \end{pmatrix}$$

$$X^T A X = \begin{pmatrix} -\frac{9}{45} & 0 & 0 \\ \frac{52}{45} & \frac{29}{45} & -\frac{40}{45} \\ \frac{225}{45} & \frac{25}{45} & \frac{165}{45} \end{pmatrix} \cdot X =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$