Wettl 196/5.30: Számítsuk ki az alábbi mátrixok inverzeit, négyzetét és köbét!

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 4 \\ 3 & 0 & 0 \end{pmatrix} \quad |A| = 24 \quad A^{-1} = \begin{pmatrix} 0 & 0 & 1/3 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{pmatrix} \quad A^2 = \begin{pmatrix} 0 & 0 & 8 \\ 12 & 0 & 0 \\ 0 & 6 & 0 \end{pmatrix} \quad A^3 = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 4 & 0 \\ 3 & 0 & 0 \end{pmatrix} \quad |B| = -24 \quad B^{-1} = \begin{pmatrix} 0 & 0 & 1/3 \\ 0 & 1/4 & 0 \\ 1/2 & 0 & 0 \end{pmatrix} \quad B^2 = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 6 \end{pmatrix} \quad B^3 = \begin{pmatrix} 0 & 0 & 12 \\ 0 & 64 & 0 \\ 18 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 3 & 0 \\ 4 & 0 & 0 & 0 \end{pmatrix} \quad |C| = -2 \begin{vmatrix} 0 & 0 & 5 \\ 0 & 3 & 0 \\ 4 & 0 & 0 \end{vmatrix} = 120 \quad C^{-1} = \begin{pmatrix} 0 & 0 & 0 & 1/4 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 1/5 & 0 & 0 \end{pmatrix} \quad C^2 = \begin{pmatrix} 0 & 0 & 0 & 10 \\ 20 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 8 & 0 & 0 \end{pmatrix}$$

$$C^3 = \begin{pmatrix} 40 & 0 & 0 & 0 \\ 0 & 40 & 0 & 0 \\ 0 & 0 & 27 & 0 \\ 0 & 0 & 0 & 40 \end{pmatrix}$$

Bontsuk fel az A mátrixot egy szimmetrikus és egy ferdén szimmetrikus mátrixra!

A=1/2 (A+A^T)+1/2(A-A^T)

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix} \quad A^T=\begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

szimmetrikus tag: $1/2\begin{pmatrix} 2 & 5 & 5 \\ 5 & 2 & 5 \\ 5 & 5 & 2 \end{pmatrix}$

ferdén szimmetrikus tag: $1/2\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

Wettl 152/4.12: Számolják ki A+3C-t és AB-t!

$$A=\left(\begin{array}{cc|c|c}1&0&1&0\\0&1&1&2\\\hline 0&0&3&0\end{array}\right)\qquad C=\left(\begin{array}{cc|c|c}0&3&0&1\\2&0&0&0\\\hline 1&1&1&1\end{array}\right)\qquad B=\left(\begin{array}{cc}2&4\\1&5\\\hline 2&2\\\hline 0&1\end{array}\right)$$

$$A+3C=\begin{pmatrix}1&9&1&3\\6&1&1&2\\3&3&6&3\end{pmatrix}$$

$$A+3C=\left(\begin{array}{c|c|c}\begin{pmatrix}1&0\\0&1\end{pmatrix}+3*\begin{pmatrix}0&3\\2&0\end{pmatrix}&\begin{pmatrix}1\\1\end{pmatrix}+3*\begin{pmatrix}0\\0\end{pmatrix}&\begin{pmatrix}0+3*1\\2\quad 0\end{pmatrix}\\\hline (0\ 0)+3*(1\ 1)&3+3*1&0+3*1\end{array}\right)=\left(\begin{array}{c|c|c}\begin{pmatrix}1&9\\6&1\end{pmatrix}&\begin{pmatrix}1\\1\end{pmatrix}&\begin{pmatrix}3\\2\end{pmatrix}\\\hline (3\ 3)&6&3\end{array}\right)=\left(\begin{array}{cc|c|c}1&9&1&3\\6&1&1&2\\\hline 3&3&6&3\end{array}\right)$$

$$AB=\begin{pmatrix}4&6\\3&9\\6&6\end{pmatrix}$$

$$AB=\left(\begin{array}{c}\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}2&4\\1&5\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}(2\ 2)+\begin{pmatrix}0\\2\end{pmatrix}(0\ 1)\\\hline (0\ 0)\begin{pmatrix}2&4\\1&5\end{pmatrix}+3\,(2\ 2)+0\,(0\ 1)\end{array}\right)=\left(\begin{array}{c}\begin{pmatrix}2&4\\1&5\end{pmatrix}+\begin{pmatrix}2&2\\2&2\end{pmatrix}+\begin{pmatrix}0&0\\0&2\end{pmatrix}\\\hline (0\ 0)+(6\ 6)+(0\ 0)\end{array}\right)=\begin{pmatrix}4&6\\3&9\\6&6\end{pmatrix}$$

Kronecker-szorzat

$$A=\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \qquad B=\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad A\ B=\begin{pmatrix} 2 & 4 & 6 & 0 & 0 & 0 \\ 6 & 4 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 6 & 9 \\ 0 & 0 & 0 & 9 & 6 & 3 \end{pmatrix}$$