LACSKÓ GÁBOR ISTVÁN BQQQFX

(1) a) $F \cdot A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2+1 & 1+1 & 1+3 \\ 4 & 4+2 & 6 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ 4 & 6 & 6 \end{pmatrix}$

$2+2$

b) $\underset{2\times3}{C} \cdot \underset{3\times1}{B} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Leftarrow$ ∄ végezhető el a művelet.
The operation cannot be done.

(c) $\underset{2\times3}{A^T} + \underset{3\times2}{F} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} \Leftarrow$ ∄ végezhető el a művelet / The operation cannot be done.

(d) $\underset{2\times3}{C} \cdot \underset{2\times2}{E^T} = \begin{pmatrix} 2 & 4 \\ 4 & 2 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \\ 2+2 & 4+1 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 5 & 4 \\ 4 & 5 \end{pmatrix}$ ✓

(e) $\underset{2\times2}{E} \cdot \underset{3\times1}{B} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Leftarrow$ ∄ végezhető el a művelet.
The operation cannot be done.

(f) $A^{-1} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

$|A| = 1\cdot1\cdot3 + 2\cdot1\cdot1 + 0 - 0 - 2\cdot2\cdot3 - 1 =$
$= 3 + 2 - 12 - 1 = \underline{-8} \neq 0$
$\Downarrow$
Reguláris mátrix
Regular matrix

$\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 2$ $\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$ $\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$\begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix} = 6$ $\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = 3$ $\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1$

$\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$ $\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$ $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

$$\begin{pmatrix} 2 & 5 & 1 \\ 6 & 3 & -1 \\ 2 & 1 & -3 \end{pmatrix} \xrightarrow[\text{chess}]{\text{jann}} \begin{pmatrix} 2 & -5 & 1 \\ -6 & 3 & +1 \\ 2 & -1 & -3 \end{pmatrix} \xrightarrow{\text{transp.}} \begin{pmatrix} 2 & 6 & 2 \\ -5 & 3 & -1 \\ 1 & +1 & -3 \end{pmatrix} \to A^{-1} = -\frac{1}{8}\begin{pmatrix} 2 & 6 \\ -5 & 3 \\ 1 & 1 & -3 \end{pmatrix}$$

Ellenőrzés / check:

→ itt kész
It is ready here.

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix} \cdot -\frac{1}{8}\begin{pmatrix} 2 & -6 & 2 \\ -5 & 3 & -1 \\ 1 & 1 & -3 \end{pmatrix} = -\frac{1}{8}\begin{pmatrix} 2-10 & -6+6 & 2-2 \\ 4-5+1 & -12+3+1 & 4-1-3 \\ 2-5+3 & -6+3+3 & 2-1-9 \end{pmatrix} = -\frac{1}{8}\begin{pmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ✓$$

(g) $C^{-1}_{3\times 2} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}$ ← Csak n×n-es (négyzetes) mátrix inverz található

Only square matrix (n×n) can be inverted.

(2)

$$\lambda_1\begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \lambda_2\begin{pmatrix} 3 \\ 2 \\ 0 \\ 2 \end{pmatrix} + \lambda_3\begin{pmatrix} 4 \\ 1 \\ 1 \\ 6 \end{pmatrix} + \lambda_4\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 2 \\ 4 \end{pmatrix}$$

$$\begin{cases} 2\lambda_1 + 3\lambda_2 + 4\lambda_3 + 2\lambda_4 = 5 \\ 2\lambda_2 + \lambda_3 + \lambda_4 = 1 \\ \lambda_1 + \lambda_3 = 2 \quad \to \quad \lambda_3 = 2 - \lambda_1 \\ \lambda_1 + 2\lambda_2 = 4 \quad \to \quad \lambda_2 = \frac{1}{2}(4 - \lambda_1) \end{cases}$$

CSIKÓ GÁBOR ISTVÁN BQQQTY

3) folyt. / Cont'd

$$2\lambda_1 + 3\cdot\frac{1}{2}(4-\lambda_1) + 4(2-\lambda_1) + 2\lambda_4 = 5$$

$$2\cdot\frac{1}{2}(4-\lambda_1) + (2-\lambda_1) + \lambda_4 = 1$$

$$2\lambda_1 + \frac{3}{2}(4-\lambda_1) + 8 - 4\lambda_1 + 2\lambda_4 = 5$$

$$4 - \lambda_1 + 2 - \lambda_1 + \lambda_4 = 1$$

$$2\lambda_1 + 6 - \frac{3}{2}\lambda_1 + 8 - 4\lambda_1 + 2\lambda_4 = 5$$

$$6 - 2\lambda_1 + \lambda_4 = 1 \rightarrow \lambda_4 = 2\lambda_1 - 5$$

$$-\frac{7}{2}\lambda_1 + 14 + 2\lambda_4 = 5$$

$$-\frac{7}{2}\lambda_1 + 2\lambda_4 = -9$$

$$-\frac{7}{2}\lambda_1 + 4\lambda_1 - 10 = -9$$

$$\frac{1}{2}\lambda_1 = 1$$

$\lambda_1 = 2$ $\lambda_2 = 1$ $\lambda_3 = 0$

$\lambda_4 = -1$

Ellenőrzés / Check:

$$2\begin{pmatrix}2\\0\\1\\1\end{pmatrix} + \begin{pmatrix}3\\2\\0\\2\end{pmatrix} + 0\begin{pmatrix}\ \\ \ \\ \ \\ \ \end{pmatrix} - \begin{pmatrix}2\\1\\0\\0\end{pmatrix} = \begin{pmatrix}4+3-2\\0+2-1\\2+0-0\\2+2-0\end{pmatrix} = \begin{pmatrix}5\\1\\2\\4\end{pmatrix} \checkmark$$

③ a, $X \cdot \underline{A} - 3\underline{B} = \underline{C} \quad / +3 \cdot \underline{B}$

$$X \cdot \underline{A} = \underline{C} + 3\underline{B} \quad / \cdot \underline{A}^{-1}$$

$$X = (\underline{C} + 3\underline{B})\underline{A}^{-1} = \left[\begin{pmatrix} 0 & 5 & 4 \\ 5 & 4 & 0 \\ 4 & 0 & 1 \end{pmatrix} + 3\begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}\right] \cdot \underline{A}^{-1} =$$

$$= \left[\begin{pmatrix} 0 & 5 & 4 \\ 5 & 4 & 0 \\ 4 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 9 \\ 0 & 3 & 9 \\ 9 & 9 & 3 \end{pmatrix}\right] \cdot \underline{A}^{-1} = \begin{pmatrix} 6 & 5 & 13 \\ 5 & 7 & 9 \\ 13 & 9 & 4 \end{pmatrix} \cdot \underline{A}^{-1} \; \textcircled{=}$$

folytatva cont'd

$$\underline{A}^{-1} \rightarrow \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -1 \quad \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} = 2$$

$$\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1 \quad \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -8 \quad \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = 2 \quad \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5 \quad \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$\rightarrow \begin{pmatrix} -1 & -1 & 2 \\ -1 & -8 & -5 \\ 2 & -5 & -4 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 0 + 6 + 6 - 0 - 4 - 1 = \cancel{7} \; \neq 0$$

szinguláris
$\in$ nem szinguláris

sakk
$\rightarrow$
chess

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & -8 & 5 \\ 2 & 5 & -4 \end{pmatrix} \xrightarrow[\text{transpose}]{\text{transzponált}} \begin{pmatrix} -1 & 1 & 2 \\ 1 & -8 & 5 \\ 2 & 5 & -4 \end{pmatrix}$$

FACSKÓ GÁBOR ISTVÁN BQQQ4 7X

$$\underline{A}^{-1} = \frac{1}{7}\begin{pmatrix} -1 & 1 & 2 \\ 1 & -8 & 5 \\ 2 & 5 & -4 \end{pmatrix}$$

ellenőrzés / check

$$\frac{1}{7}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 & 2 \\ 1 & -8 & 5 \\ 2 & 5 & -4 \end{pmatrix} =$$

$$\frac{1}{7}\begin{pmatrix} -1+2+6 & 1-16+15 & 2+10-12 \\ -2+2 & 2+5 & 4-4 \\ -3+1+2 & 3-8+5 & 6+5-4 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

feladat

Ⓑ $\frac{1}{7}\begin{pmatrix} 6 & 5 & 13 \\ 5 & 7 & 9 \\ 13 & 9 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 & 2 \\ 1 & -8 & 5 \\ 2 & 5 & -4 \end{pmatrix} =$

Cont'd

$$= \frac{1}{7}\begin{pmatrix} -6+5+26 & 6-40+65 & 12+25-52 \\ -5+7+18 & 5-56+45 & 10+35-36 \\ -13+9+8 & 13-72+20 & 26+45-16 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 25 & 31 & -15 \\ 20 & -6 & 9 \\ 4 & -39 & 55 \end{pmatrix}$$

Ⓒ

$$\underline{A}\cdot\underline{X}^{-1} + \underline{B}^{-1} = 2\underline{X}^{-1} \quad / \cdot \underline{X}$$

$$\underline{A}\underbrace{(\underline{X}^{-1}\cdot\underline{X})}_{\underline{E}} + \underline{B}^{-1}\cdot\underline{X} = 2\cdot\underbrace{(\underline{X}^{-1}\cdot\underline{X})}_{\underline{E}}$$

$$\underline{A} + \underline{B}^{-1}\cdot\underline{X} = 2\cdot\underline{E}$$

$$\underline{B}^{-1}\cdot\underline{X} = 2\underline{E} - \underline{A} \quad / \underline{B}\cdot$$

$$\underbrace{(\underline{B}\cdot\underline{B}^{-1})}_{\underline{E}}\underline{X} = \underline{B}(2\underline{E} - \underline{A})$$

$$\underline{X} = \underline{B}(2\underline{E} - \underline{A})$$

$$X = B(2E - A) = B\left[\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix} - \begin{pmatrix}1&2&3\\2&0&1\\3&1&1\end{pmatrix}\right] = B\begin{pmatrix}1&-2&-3\\-2&2&-1\\-3&-1&1\end{pmatrix}$$

$$= \begin{pmatrix}2&0&3\\0&1&3\\3&3&1\end{pmatrix}\begin{pmatrix}1&-2&-3\\-2&2&-1\\-3&-1&1\end{pmatrix} = \begin{pmatrix}2-9&-4-3&-6+3\\-2-9&2-3&-1+3\\3-6-3&-6+6-1&-9-3+1\end{pmatrix}$$

$$= \begin{pmatrix}-7&-7&-3\\-11&-1&2\\-6&-1&-11\end{pmatrix}$$

---

4. $Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda E)x = 0$

$\Updownarrow$

$x = 0 \quad \wedge \quad |A - \lambda E| = 0$

Sajátértékek / Eigenvalues

$$\begin{vmatrix}3-\lambda & 2\\ 2 & 1-\lambda\end{vmatrix} = (3-\lambda)(1-\lambda) - 4 =$$

$$= 3 - 3\lambda - \lambda + \lambda^2 - 4 = \lambda^2 - 4\lambda - 1 =$$

$$= 0$$

$$\lambda_{1,2} = \frac{\cancel{+4} \pm 2\sqrt{5}}{2} = \frac{+2 \pm \sqrt{5}}{} \qquad D = 16 + 4 = (2\sqrt{5})^2$$

(1) $\lambda_1 = +2 - \sqrt{5}$ sfm

$$\cancel{\begin{vmatrix}3+2+\sqrt{5} & 2\\ 2 & 1+2+\sqrt{5}\end{vmatrix} = \begin{vmatrix}5+\sqrt{5} & 2\\ 2 & 3+\sqrt{5}\end{vmatrix}}$$

$\cancel{x_1(5+\sqrt{5}) + 2x_2 = 0}$

$2x_1 + x_2(3+\sqrt{5}) = 0$

$\lambda_1 = 2 - \sqrt{5}$

$$\begin{pmatrix} 3-2+\sqrt{5} & 2 \\ 2 & 1-2+\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1+\sqrt{5} & 2 \\ 2 & -1+\sqrt{5} \end{pmatrix}$$

$x_1(1+\sqrt{5}) + 2x_2 = 0 \quad \rightarrow \quad \cancel{x_2 = \frac{1}{2}(1+\sqrt{5})x_1}$

$2x_1 - x_2(1-\sqrt{5}) = 0 \qquad x_1 = + \frac{1-\sqrt{5}}{2} x_2$

$\cancel{2x_1 + (1-\sqrt{5}) \cdot \frac{1}{2}(1+\sqrt{5})x_1 = 0}$

$\cancel{2x_1 + \frac{1}{2}(1-5)x_1 = 0}$

$\cancel{2x_1 + \frac{1}{2} \cdot 4 \cdot x_1 = 0}$

$\cancel{2x_1 + 2x_1 = 0}$

$\cancel{x_1 = 0 \Leftrightarrow x_1}$

$+ \frac{1-\sqrt{5}}{2} \cdot (1+\sqrt{5}) x_2 + 2x_2 = 0$

$+ x_2 \left(\frac{1-5}{2}\right) + 2x_2 = 0$

$4x_2 = 0$ ✓

$x_2 = 0$

$\mathcal{L}_{\lambda_1} = \left\{ t \begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{pmatrix} ; t \in \mathbb{R} \right\} \Rightarrow \underline{V} = \left\{ \begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{pmatrix} \right\}$ ✓

$\lambda_2 = 2 + \sqrt{5}$

$$\begin{pmatrix} 3-2-\sqrt{5} & 2 \\ 2 & 1-2-\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1-\sqrt{5} & 2 \\ 2 & -1-\sqrt{5} \end{pmatrix}$$

1. F OCSNB G Albour István BQQQ7X

7(b) folytatás / cont'd

$x_1(1-\sqrt{5}) + 2x_2 = 0 \quad \to x_2 = \frac{1-\sqrt{5}}{2} x_1$

$2x_1 - x_2(1+\sqrt{5}) = 0$

$2x_1 - \frac{1-\sqrt{5}}{2}(1-\sqrt{5})x_1 = 0$

$2x_1 - \frac{1-5}{2}x_1 = 0$

$2x_1 - 2x_1 = 0$

$0 = 0$

$\alpha_{c_2} = \left\{ t \begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \end{pmatrix} ; t \in \mathbb{R} \right\}$

$\underline{\underline{u = \left\{ \begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \end{pmatrix} \right\}}}$

⑤ g $f(\underline{x}) = \begin{pmatrix} 2x_1 + x_2 \\ -x_1 + x_2 \\ x_3 + 7x_2 \\ -x_3 \end{pmatrix} \quad x \in \mathbb{R}^3$

(I) $f(\underline{a} + \underline{b}) = f(\underline{a}) + f(\underline{b})$ additivitás / additivity

$$f(\underline{a}+\underline{b}) = \begin{pmatrix} 2a_1 + 2b_1 + a_2 + b_2 \\ -a_1 - b_1 + a_2 + b_2 \\ a_3 + 7a_2 + b_3 + 7b_2 \\ -a_3 - b_3 \end{pmatrix} = \begin{pmatrix} 2a_1 + a_2 \\ -a_1 + a_2 \\ a_3 + 7a_2 \\ -a_3 \end{pmatrix} + \begin{pmatrix} 2b_1 + b_2 \\ -b_1 + b_2 \\ b_3 + 7b_2 \\ -b_3 \end{pmatrix} =$$

$= f(\underline{a}) + f(\underline{b})$ ✓

homogenitás / homogenity

(II) $$f(\lambda \underline{a}) = \begin{pmatrix} 2\lambda a_1 + \lambda a_2 \\ -\lambda a_1 + \lambda a_2 \\ \lambda a_3 + \cancel{\lambda a_2} 7\lambda a_2 \\ -\lambda a_3 \end{pmatrix} = \lambda \begin{pmatrix} 2a_1 + a_2 \\ -a_1 + a_2 \\ a_3 + 7a_2 \\ -a_3 \end{pmatrix} = \lambda f(\underline{a})$$ ✓

A transzformáció lineáris / The transformation is linear. A transzformációs mátrix / The transformation matrix is:

FACSKÓ GÁBOR ISTVÁN BQQQ7Y

5a, folyt / Cont'd

$$\begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

5b $g(\underline{x}) = \begin{pmatrix} x_1 \\ -3x_1x_2 \\ 5x_2 \\ -x_1 \end{pmatrix}, \; \underline{x} \in \mathbb{R}^2$

1 (lineáris)
1 lineár

(1) additivitás

$$\begin{pmatrix} a_1+b_1 \\ -3(a_1+b_1)(a_2+b_2) \\ 5(a_2+b_2) \\ -(a_1+b_1) \end{pmatrix} = \underbrace{\begin{pmatrix} a_1 \\ -3a_1a_2 \\ 5a_2 \\ -a_1 \end{pmatrix}}_{A(\underline{a})} + \underbrace{\begin{pmatrix} b_1 \\ -3b_1b_2 \\ 5b_2 \\ -b_1 \end{pmatrix}}_{\cancel{A(\underline{b})}} \; -3a_1b_2-3b_1a_2$$

Nincs transzformáció mátrixa / ∄ transformation ~~matrix~~ matrix

④ Cllarsrail Check

$$\begin{pmatrix} 3 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{pmatrix} = \begin{pmatrix} 3+1+\sqrt{5} \\ 2+\frac{1+\sqrt{5}}{2} \end{pmatrix} = \begin{pmatrix} 4+\sqrt{5} \\ \frac{5+\sqrt{5}}{2} \end{pmatrix} ?$$

$$\left(4+\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}\right) = \frac{1}{2}\left(4+4\sqrt{5}+\sqrt{5}+5\right) =$$

$$= \frac{1}{2}\left(9+5\sqrt{5}\right)$$

(3a) Cllausrail Check

$$\frac{1}{7}\begin{pmatrix} 25 & 31 & -15 \\ 20 & -6 & 9 \\ 4 & -39 & 55 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 1 \\ 3 & 1 & 1 \end{pmatrix} - 3 \cdot \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix} =$$

$$= \frac{1}{7}\begin{pmatrix} 25+62-45 & 50-15 & 75+31-15 \\ 20-12+27 & 40+9 & 60-6+9 \\ 4-78+165 & 8+55 & 12-39+55 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 9 \\ 0 & 3 & 9 \\ 9 & 9 & 3 \end{pmatrix} =$$

$$= \frac{1}{7}\begin{pmatrix} 42 & 35 & 91 \\ 35 & 49 & 63 \\ 91 & 63 & 28 \end{pmatrix} = \begin{pmatrix} 6 & 5 & 13 \\ 5 & 7 & 9 \\ 13 & 9 & 4 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 9 \\ 0 & 3 & 9 \\ 9 & 9 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 5 & 4 \\ 5 & 4 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

Feladat Gábor [illegible] BQQQ7Y

36) Ellenőrzés / Check:

$$A \cdot X^{-1} + B^{-1} = 2X^{-1}$$

$$X = \begin{pmatrix} -7 & -1 & -3 \\ -11 & -1 & 2 \\ -6 & -1 & -11 \end{pmatrix}$$

$$X^{-1} = \frac{1}{125} \begin{pmatrix} 13 & -74 & -17 \\ -133 & 59 & 47 \\ 5 & 35 & -10 \end{pmatrix}$$

$$\to \cancel{X^{-1} = \frac{1}{123} \begin{pmatrix} 13 & -74 & -17 \\ -127 & 72 & 47 \\ 11 & -7 & -10 \end{pmatrix}}$$

$$B^{-1} = \frac{1}{25} \begin{pmatrix} 8 & -9 & 3 \\ -9 & 7 & 6 \\ 3 & 6 & -2 \end{pmatrix}$$

?

$$\cancel{A \cdot X^{-1} = \frac{1}{123} \begin{pmatrix} -196 & 59 & 71 \\ 37 & -135 & -12 \\ -71 & -132 & -2 \end{pmatrix}}$$

$$A \cdot X^{-1} = \frac{1}{125} \begin{pmatrix} -238 & 149 & -133 \\ 31 & -113 & -104 \\ -89 & -128 & -74 \end{pmatrix}$$

$$A \cdot X^{-1} - B^{-1} = \to \frac{1}{125} \begin{pmatrix} 26 & -148 & 34 \\ -266 & 118 & 94 \\ 10 & 70 & 140 \end{pmatrix}$$

$$4 \quad 2X^{-1}$$

(4)

$$\frac{1}{\frac{1+\sqrt{5}}{2}} = \frac{2}{1+\sqrt{5}} = \frac{2(1-\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{2}{-4}(1-\sqrt{5}) = -\frac{1-\sqrt{5}}{2}$$

$\lambda_1 = 2-\sqrt{5}$

$$\begin{pmatrix} 3-2+\sqrt{5} & 2 \\ 2 & 1-2+\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1+\sqrt{5} & 2 \\ 2 & -1+\sqrt{5} \end{pmatrix}$$

$x_1(1+\sqrt{5}) + 2x_2 = 0 \Rightarrow$

$2x_1 - x_2(1-\sqrt{5}) = 0 \qquad x_1 = \frac{1-\sqrt{5}}{2} x_2 \qquad \left(\begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \end{pmatrix} \right)$

$\frac{1-\sqrt{5}}{2}(1+\sqrt{5}) x_2 + 2x_2 = 0$

$-2x_2 + 2x_2 = 0$

$0 = 0$

$$\mathcal{L}_{\lambda_1} = \left\{ t \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix} ; t \in \mathbb{R} \right\} \rightarrow v_1 = \left(\begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix} \right)$$

$\lambda_2 = 2+\sqrt{5}$

$$\begin{pmatrix} 3-2-\sqrt{5} & 2 \\ 2 & 1-2-\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1-\sqrt{5} & 2 \\ 2 & -1-\sqrt{5} \end{pmatrix}$$

2 | 7# | Fogarasi Gábor István | KQQQFX

$x_1(1-\sqrt{5})+2x_2=0$ $\cancel{\Rightarrow}$ $\to$ $\cancel{x_1}$ $\to$

$2x_1-x_2(1+\sqrt{5})=0$ $\cancel{\Rightarrow x_1 = \frac{1+\sqrt{5}}{2}x_2}$ $x_1=\frac{1+\sqrt{5}}{2}x_2$

$\frac{1+\sqrt{5}}{2}(1-\sqrt{5})x_2+2x_2=0$

$-2x_2+2x_2=0$

$0=0$

$\alpha \left\{\begin{pmatrix}1\\ \frac{1-\sqrt{5}}{2}\end{pmatrix}\right\}$

$\alpha_{\lambda_2}=\left\{t\cdot\begin{pmatrix}\frac{1+\sqrt{5}}{2}\\ 1\end{pmatrix} ; t\in\mathbb{R}\right\}\to V=\left\{\begin{pmatrix}\frac{1+\sqrt{5}}{2}\\ 1\end{pmatrix}\right\}$