Név / Name: DR. FALSKÓ GÁBOR Neptun kód / code: BQQQTY

(ONFIZ1-0401) Elemi lineáris algebra 3. zárthelyi dolgozat / Elementary Linear Algebra, Test 3

1. Adottak a következő vektorok: $\mathbf{a} = (2,0,1)$, $\mathbf{b} = (0,3,1)$ és $\mathbf{c} = (2,1,1)$. Határozza meg a következő összefüggéseket / Calculate the following expressions:
   a.) $(\mathbf{a} - \mathbf{b})\,\mathbf{c}$
   b.) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a}$
   c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$
   d.) Mennyi az **a** és **b** vektorok által közbezárt szög? / What is the angle of Vectors **a** and **b**?
   e.) Egy síkban vannak-e az **a**, **b**, **c** vektorok? / Are Vectors **a**, **b**, and **c** in the same plane?
   f.) Adjon meg egy vektort, mely merőleges az **b** vektorra. / Determine a perpendicular vector to Vector **b**.

(10 po(i)nt)

2. Számítsa ki a következő mátrixok determinánsát! / Calculate the deteminant of the following matrixes:

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 4 & 2 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

(10 po(i)nt)

3. Oldja meg az alábbi lineáris egyenletrendszert! / Solve the following system of linear equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= -2 \\ 3x_1 + 5x_2 + 2x_3 &= 0 \end{aligned}$$

(10 po(i)nt)

4. Lineárisan függetlenek-e az $\mathbf{a} = (1,2,1,3)$, a $\mathbf{b} = (0,5,2,2)$ és a $\mathbf{c} = (1,1,3,1)$ vektorok? / Are independent linear Vectors $\mathbf{a} = (1,2,1,3)$, $\mathbf{b} = (0,5,2,2)$, and $\mathbf{c} = (1,1,3,1)$? (10 po(i)nt)

5. Altér -e az $\mathbb{R}^3$-on az $U = \{(x_1 + x_2, x_1 - x_2, 5x_2)\,|x_1, x_2 \in \mathbb{R}\}$? / Is subspace on $\mathbb{R}^3$ the $U = \{(x_1 + x_2, x_1 - x_2, 5x_2)\,|x_1, x_2 \in \mathbb{R}\}$ set? (10 po(i)nt)

6. Adja meg az $\mathbf{a} = (1,0,0,0)$ vektort az $(1,0,0,0)$; $(0,1,0,0)$; $(0,0,2,0)$; $(0,0,0,1)$ bázisban. / Give the Vector $\mathbf{a} = (1,0,0,0)$ in the $(1,0,0,0)$; $(0,1,0,0)$; $(0,0,2,0)$; $(0,0,0,1)$ basis. (10 po(i)nt)

7. Adottak a következő mátrixok:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \mathbf{E} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{F} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Végezze el az alábbiak közül az elvégezhető műveleteket! / Calculate the following terms if possible:
(a) $\mathbf{F} \cdot \mathbf{A}$ (b) $\mathbf{C} \cdot \mathbf{B}$ (c) $\mathbf{A}^T + \mathbf{F}$ (d) $\mathbf{C} \cdot \mathbf{E}^T$ (e) $\mathbf{E} \cdot \mathbf{B}$ (f) $\mathbf{A}^{-1}$ (g) $\mathbf{C}^{-1}$ (10 po(i)nt)

8. Oldja meg az $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ mátrixegyenletet, ha

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$

Solve the $\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$ matrix equation above.

9. Adja meg az alábbi mátrix sajátértékeit és egy-egy, a sajátértékhez tartozó sajátvektort! / Calculate the eigenvalues of Matrix **A** and give an eigenvector for each eigenvalues:

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

(10 po(i)nt)

10. Az alábbi leképezés lineáris? Adja meg a leképezés mátrixát is, ha létezik! / Is this transformation a linear transformation? Give the matrix of the linear transformation if it exists.

$$f(\mathbf{x}) = \begin{pmatrix} 3x_1 + x_2 \\ -x_1 + x_2 \\ x_3 + 3x_2 \end{pmatrix} \quad (\mathbf{x} \in \mathbb{R}^3)$$

(10 po(i)nt)

A fenti feladatsor két részre oszlik. Az (1)-(5) feladatok megoldásával a első zárthelyit lehet javítani, illetve pótolni. A (6)-(10) feladatokkal pedig a másodikat. A zárthelyik osztályzása: 0-20 pont: elégtelen (1), 21-27 pont: elégséges (2), 28-35 pont: közepes (3), 36-42 pont: jó (4) és 43-50 pont: jeles (5). Mindkét témából zárthelyiből legalább elégségest (2) kell elérni a gyakorlati jegyhez. Ha mindkét zárthelyi legalább közepes (3), akkor megajánlott vizsgajegyet kapnak. A megajánlott jegyet nem szerzőknek, vagy a jegyet nem elfogadóknak vizsgáznia kell a kiírt időpo(i)ntokban.

You can improve the results of the first mid-term test by solving exercises (1)-(5). If you want to replace your second mid-term exam you must solace exercises (6)-(10). Grades: 0-20 points: Fail 81), 21-27 points: Pass (2), 28-35 points: Satisfactory (3), 36-42 points: Good (4) és 43-50 points: Excellent (5). You must pass both mid-term tests to get a grade for the practice. If you get at least an Average (3) grade for both mid-term tests I will offer you an exam grade based on the test results. If neither exam grade was offered nor you wish for a better grade you must pass a written exam.

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a) $(\underline{a}-\underline{b})\cdot\underline{c} = \left[\begin{pmatrix}2\\0\\1\end{pmatrix}-\begin{pmatrix}0\\3\\1\end{pmatrix}\right]\cdot(2,1,1) = \begin{pmatrix}2\\-3\\0\end{pmatrix}\cdot(2,1,1) =$

$= 4-3 = \underline{\underline{1}}$

b) $(\underline{b}+\underline{c})\times\underline{a} = \left[\begin{pmatrix}0\\3\\1\end{pmatrix}+\begin{pmatrix}2\\1\\1\end{pmatrix}\right]\times\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}2\\4\\2\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix} =$

$= \begin{pmatrix}4\cdot1-2\cdot0\\2\cdot2-2\cdot1\\2\cdot0-4\cdot2\end{pmatrix} = \underline{\underline{\begin{pmatrix}4\\2\\-8\end{pmatrix}}}$

c) $\cos\alpha = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}|\cdot|\underline{b}|} = \dfrac{\begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\3\\1\end{pmatrix}}{\sqrt{2^2+1^2}\cdot\sqrt{3^2+1^2}} = \dfrac{0+0+1}{\sqrt{5}\cdot\sqrt{10}} =$

$= \dfrac{1}{5\sqrt{2}} = \underline{\underline{\dfrac{\sqrt{2}}{10}}}$

23 32

d) $(\underline{a},\underline{b},\underline{c}) = (\underline{a}\times\underline{b})\cdot\underline{c} = \left[\begin{pmatrix}2\\0\\1\end{pmatrix}\times\begin{pmatrix}0\\3\\1\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\1\end{pmatrix} =$

$= \begin{pmatrix}0\cdot1-1\cdot3\\1\cdot0-2\cdot1\\2\cdot3-0\cdot0\end{pmatrix}\cdot\begin{pmatrix}2\\1\\1\end{pmatrix} = \begin{pmatrix}-3\\-2\\6\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix} = -6+6 = 0$ -2

e) ~~Igen~~ Nem, mert az $(\underline{a},\underline{b},\underline{c}) \neq 0$ ($\in$ c feladat).

~~Yes~~ No, because $(\underline{a},\underline{b},\underline{c}) \neq 0$ ($\in$ Exercise c).

f) Pl. $(1,0,0)$ hisz: $\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}0\\3\\1\end{pmatrix} = \underline{0}$

(2) $|A| = 3\cdot 3 - 1\cdot 1 = \underline{8}$

$|B| = 1\cdot 2\cdot 3 + 1\cdot 4\cdot 2 + 2\cdot 1\cdot 4 - 2^3 - 1\cdot 1\cdot 3 - 1\cdot 4\cdot 4 =$

$= 12 + 8 + \underline{8} - \underline{8} - 3 - 16 = \underline{\underline{-5}}$

$|C| = 1\cdot \begin{vmatrix} 4 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 2 \end{vmatrix} - 1\cdot \begin{vmatrix} 4 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} =$

$= (4\cdot 1\cdot 2 + 1\cdot 2\cdot 2 + 1\cdot 2\cdot 2 - 2\cdot 1\cdot 1 - 2\cdot 1\cdot 2 - 4\cdot 2\cdot 2) -$

$-(4\cdot 1\cdot 2 + 2\cdot 1\cdot 2 + 1\cdot 2\cdot 2 - 2\cdot 1\cdot 1 - 2^3 - 4\cdot 2\cdot 1) =$

$= (8 + 4 + 4 - 2 - 4 - 16) - (8 + 4 + 4 - 2 - 8 - 8) =$

$= \underline{16} - 22 - \underline{16} + 18 = \underline{\underline{-4}}$ $\underline{\underline{0}}$

$16 \quad -14 \quad - \quad 16 + 18$

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(3) $\begin{cases} x_1 - x_2 + 2x_3 = 1 \\ 2x_1 - x_2 + x_3 = -2 \\ 3x_1 + 5x_2 + 2x_3 = 0 \end{cases}$

II-2I: $x_2 - 3x_3 = -5$

III-3I: $8x_2 - 4x_3 = -3$

III-8(II): $+20x_3 = 37$

$x_3 = \underline{\underline{\frac{37}{20}}}$

$x_2 = -5 + 3\frac{37}{20} =$

$= -5 + \frac{97}{20} = -\frac{100}{20} + \frac{97}{20} =$

$= -\frac{3}{20}$

$x_1 = 1 - \frac{3}{20} - 2\cdot\frac{37}{20} =$

$= \frac{20}{20} - \frac{3}{20} - \frac{74}{20} = -\frac{57}{20}$

④

$$\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 5 \\ 2 \\ 2 \end{pmatrix} + \lambda_3 \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \lambda_1 + \lambda_3 = 0 & \rightarrow \lambda_3 = -\lambda_1 \\ 2\lambda_1 + 5\lambda_2 + \lambda_3 = 0 \\ \lambda_1 + 2\lambda_2 + 3\lambda_3 = 0 \\ 3\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \end{cases}$$

$$\begin{cases} 2\lambda_1 + 5\lambda_2 - \lambda_1 = 0 \\ \lambda_1 + 2\lambda_2 - 3\lambda_1 = 0 \\ 3\lambda_1 + 2\lambda_2 - \lambda_3 = 0 \end{cases}$$

$$\lambda_1 + 5\lambda_2 = 0 \rightarrow \lambda_1 = -5\lambda_2$$

$$-2\lambda_1 + 2\lambda_2 = 0 \rightarrow \lambda_1 = \lambda_2 \quad \Rightarrow \quad \lambda_1 = \lambda_2 = \lambda_3 = 0$$

$$2\lambda_1 + 2\lambda_2 = 0 \rightarrow \lambda_1 = -\lambda_2$$

$$\Downarrow$$

~~Nem~~ függetlenek.

[illegible]

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⑤

$$\left(\begin{pmatrix} a_1 + a_2 + b_1 + b_2 \\ a_1 \cancel{\ldots} b_1 - b_2 \; -(a_2+b_2) \\ 5a_2 + 5b_2 \end{pmatrix}\right) = \left(\begin{pmatrix} c_1 + c_2 \\ c_1 - c_2 \\ 5c_2 \end{pmatrix}\right) \checkmark$$

$$\left.\begin{matrix} c_1 = a_1 + b_1 \\ c_2 = a_2 + b_2 \\ c_3 = a_3 + b_3 \end{matrix}\right\} \quad c_i = a_i + b_i \quad i \in \{1,2,3\}$$

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(5) feladat / Cont'd

$$\left\{\begin{pmatrix} \lambda a_1 + \lambda a_2 \\ \lambda a_1 - \lambda a_2 \\ 5\lambda a_2 \end{pmatrix}\right\} = \left\{\begin{pmatrix} d_1 + d_2 \\ d_1 - d_2 \\ 5 d_2 \end{pmatrix}\right\}$$

$d_i = \lambda a_i \quad i \in \{1,2,3\}$

Mindkét feltétel teljesül, tehát az U altér $\mathbb{R}^3$-an.

Both conditions are fullfilled, therefore U is a subspace of $\mathbb{R}^3$.

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(6)

$$d_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + d_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + d_3 \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} + d_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\underline{\underline{d_1 = 1}}$

$\underline{\underline{d_2 = 0}}$

$2d_3 = 0 \Rightarrow \underline{\underline{d_3 = 0}}$

$\underline{\underline{d_4 = 0}}$

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(7) a

$$\underline{\underline{B}} \cdot \underline{\underline{A}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0+1+0 & 0+1+1 & 0+1+1 \\ 1+0+0 & 1+0+1 & 0+0+1 \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \end{pmatrix}}}$$

b) $\underset{2\times3}{\underline{C}} \cdot \underset{1\times3}{\underline{B}}$ → Nem lehetséges, It is not possible.

c) $\underset{3\times3}{\underline{A}}^T + \underset{3\times2}{\underline{I}}$ → — // —

d) $\underline{C} \cdot \underline{C}^T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 2\cdot1+1\cdot3 & 2\cdot3+1\cdot1 \\ 1\cdot1+2\cdot3 & 1\cdot3+2\cdot1 \\ 2\cdot1+1\cdot3 & 2\cdot3+1\cdot1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 & 7 \\ 7 & 5 \\ 5 & 7 \end{pmatrix}}}$

e) $\underset{2\times2}{\underline{C}} \cdot \underset{2\times3}{\underline{B}}$ → Nem lehetséges, It is not possible.

f) ~~Adott~~ $\underline{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow[\text{(lusz)}]{\text{sakk}} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

$|\underline{A}| = 1^3 + 0 + 0 - 0 - 1 - 1 = -1$

transzp. \ trans

$\underline{A}^{-1} = \underline{\underline{\begin{pmatrix} 0 & +1 & -1 \\ +1 & -1 & +1 \\ -1 & +1 & 0 \end{pmatrix}}} \xleftarrow{-\frac{1}{1}\text{-el}} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

3) $\underline{C}_{2\times3}$ → 1 négyzetes mátrix, 1 lehetséges

1 square matrix, 1 possible.

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(8) $\underline{A}\,\underline{X}=\underline{B} \quad /\underline{A}^{-1}\cdot$

$(\underline{A}^{-1}\underline{A})\,\underline{X}=\underline{A}^{-1}\cdot\underline{B}$

$\underline{E}$

$\underline{X}=\underline{A}^{-1}\cdot\underline{B}=?$

$$\begin{vmatrix}1&2&3\\2&0&1\\3&1&1\end{vmatrix}=0+6+3\cdot2\cdot1-0-4-1=$$

$$=\underline{7}$$

$$\underline{A}=\begin{pmatrix}1&2&3\\2&0&1\\3&1&1\end{pmatrix}$$

$$\begin{vmatrix}0&1\\1&1\end{vmatrix}=-1\quad\begin{vmatrix}2&1\\3&1\end{vmatrix}=-1\quad\begin{vmatrix}2&0\\3&1\end{vmatrix}=2$$

$$\begin{vmatrix}2&3\\1&1\end{vmatrix}=-1\quad\begin{vmatrix}1&3\\3&1\end{vmatrix}=-8\quad\begin{vmatrix}1&2\\3&1\end{vmatrix}=-5$$

$$\begin{vmatrix}2&3\\0&1\end{vmatrix}=2\quad\begin{vmatrix}1&3\\2&1\end{vmatrix}=-5\quad\begin{vmatrix}1&2\\2&0\end{vmatrix}=-4$$

$$\begin{pmatrix}-1&-1&2\\-1&-8&-5\\2&-5&-4\end{pmatrix}\xrightarrow[\text{chess}]{\text{sakk}}\begin{pmatrix}-1&+1&2\\+1&-8&+5\\2&+5&-4\end{pmatrix}\xrightarrow{\text{transzp}}\begin{pmatrix}-1&1&2\\1&-8&5\\2&5&-4\end{pmatrix}\Rightarrow$$

$$\xrightarrow{\frac{1}{\det}}\underline{A}^{-1}=\frac{1}{7}\begin{pmatrix}-1&1&2\\1&-8&5\\2&5&-4\end{pmatrix}$$

$$X=\underline{A}^{-1}\cdot\underline{B}=\frac{1}{7}\begin{pmatrix}\cancel{-1}&\cancel{1}&\cancel{2}\\\cancel{1}&\cancel{-8}&\cancel{5}\\\cancel{2}&\cancel{5}&\cancel{-4}\end{pmatrix}\cdot\begin{pmatrix}2&0&3\\0&1&3\\3&3&1\end{pmatrix}=\frac{1}{7}\begin{pmatrix}-2+6&1+6&-3+3+2\\2+15&-8+15&3-24+5\\4-12&5-12&6+15-4\end{pmatrix}=$$

$$=\frac{1}{7}\begin{pmatrix}4&7&2\\17&7&-16\\-8&-7&17\end{pmatrix}$$

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$$\begin{pmatrix} 2-\lambda & 0 \\ 0 & \cancel{3+\lambda} \; 3-\lambda \end{pmatrix} = (2-\lambda)(3-\lambda) = \quad \underline{\underline{\lambda_1 = 2}} \quad \underline{\underline{\lambda_2 = 3}}$$

$\lambda_1 = 2$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \cancel{2\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$0 = 0$ $\cancel{0 = 2x_1 \rightarrow x_1 = 0}$

$\cancel{x_2 = 2x_2 \rightarrow x_2 = 0}$

$x_2 = 0$

$\begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}$ $\quad \alpha_{\lambda_1} = \left\{ t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mid t \in \mathbb{R} \right\}$

$\underline{\underline{v_1 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}}}$

$\lambda_2 = 3$

$$\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\alpha_{\lambda_2} = \left\{ t \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}$

$x_1 = 0$ $\rightarrow$ $\underline{\underline{v_1 = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}}}$

x_2 a. beliebig

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$$f(x) = \begin{pmatrix} 3x_1 + x_2 \\ -x_1 + x_2 \\ x_3 + 3x_2 \end{pmatrix}$$

(1) $f(a+b) = \begin{pmatrix} 3a_1 + a_2 + 3b_1 + b_2 \\ -a_1 + a_2 - b_1 + b_2 \\ a_3 + 3a_2 + b_3 + 3b_2 \end{pmatrix} = \begin{pmatrix} 3a_1 + a_2 \\ -a_1 + a_2 \\ a_3 + 3a_2 \end{pmatrix} + \begin{pmatrix} 3b_1 + b_2 \\ -b_1 + b_2 \\ b_3 + 3b_2 \end{pmatrix} =$

$= f(a) + f(b)$ ✓

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(10) folyt. / Cont'd

(II) $f(\lambda a) = \begin{pmatrix} 3\lambda a_1 + \lambda a_2 \\ -\lambda a_1 + \lambda a_2 \\ \lambda a_3 + 3\lambda a_2 \end{pmatrix} = \lambda \begin{pmatrix} 3a_1 + a_2 \\ -a_1 + a_2 \\ a_3 + 3a_2 \end{pmatrix} = \lambda f(a)$ ✓

Tehát a leképezés lineáris / Therefore the transformation is linear.

A transzformáció mátrixa / Its transformation matrix is:

$$\begin{pmatrix} 3 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$$

(3)
$$\begin{cases} x_1 - x_2 + 2x_3 = 1 \\ 2x_1 - x_2 + x_3 = -2 \\ 3x_1 + 5x_2 + 2x_3 = 0 \end{cases}$$

$$\underline{\begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 5 & 2 \end{pmatrix}}_{\underline{A}} \underline{x} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}_{\underline{b}}$$

$\begin{vmatrix} -1 & 1 \\ 5 & 2 \end{vmatrix} = -7 \quad \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1 \quad \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$

$\begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} = -12 \quad \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4 \quad \begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = 8$

$\begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = 1 \quad \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \quad \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = +1$

$|A| = -2 - 3 + 20 + 6 + 4 - 5 = \underline{20}$

$\underline{x} = \underline{A}^{-1} \cdot \underline{b}$

$$\begin{pmatrix} -7 & 1 & 18 \\ -12 & -4 & 8 \\ 1 & -3 & 1 \end{pmatrix} \xrightarrow[\text{Chyss}]{\text{Sahn}} \begin{pmatrix} -7 & -1 & 13 \\ +12 & -4 & -8 \\ 1 & +3 & 1 \end{pmatrix} \xrightarrow{\text{transp}} \begin{pmatrix} -7 & 12 & 1 \\ -1 & -4 & 3 \\ 13 & -8 & 1 \end{pmatrix}$$

$$\underline{A}^{-1} = \frac{1}{20} \begin{pmatrix} -7 & 12 & 1 \\ -1 & -4 & 3 \\ 13 & -8 & 1 \end{pmatrix}$$

$$\underline{A}^{-1} \underline{b} = \frac{1}{20} \begin{pmatrix} -7 - 24 \\ -1 + 8 \\ 13 + 16 \end{pmatrix} = \frac{1}{20} \begin{pmatrix} -31 \\ 7 \\ 29 \end{pmatrix}$$