(ONFIZ1-0401) Elementary Linear Algebra, Test 1

1. $\mathbf{a} = (-2, 0, -1)$, $\mathbf{b} = (0, -3, 1)$ és $\mathbf{c} = (2, 1, 1)$. Calculate the following expressions:

   a.) $(\mathbf{a} - \mathbf{b})\,\mathbf{c}$

   b.) $(\mathbf{b} + \mathbf{c}) \times \mathbf{a}$

   c.) $(\mathbf{a}, \mathbf{b}, \mathbf{c})$

   d.) What is the angle of Vectors $\mathbf{a}$ and $\mathbf{b}$?

   e.) Are Vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in the same plane?

   f.) Determine a perpendicular vector to Vector $\mathbf{b}$.

   (8 point)

2. Calculate the deteminant of the following matrixes:

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 2 & 4 \\ -2 & 4 & 3 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 4 & 2 & -1 & 1 \\ 2 & 1 & -1 & 2 \\ 1 & 0 & 0 & 1 \\ -2 & -2 & 2 & 2 \end{pmatrix}$$

   (10 point)

3. Solve the following systems of linear equations:

   a.)

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= -2 \\ 3x_1 + 5x_2 + 2x_3 &= 0 \end{aligned}$$

   b.)

$$\begin{aligned} -x_1 + 3x_2 + x_3 &= 1 \\ x_1 + 3x_2 + x_3 &= 0 \\ 4x_1 + x_2 - 3x_3 &= 1 \end{aligned}$$

   (12 point)

4. Are independent linear Vectors $\mathbf{a} = (-1, 2, 1, 3)$, $\mathbf{b} = (0, 5, -2, 2)$, and $\mathbf{c} = (1, 1, 3, 1)$? (8 point)

5. Is subspace on $\mathbb{R}^3$ the $U = \{(x_1 + x_2, -x_1 - x_2, 4x_2)\,|x_1, x_2 \in \mathbb{R}\}$ set? (4 point)

6. Give the Vector $\mathbf{a} = (-1, 0, 0)$ in the $(-1, 1, 1); (0, 1, 0); (1, 2, 1)$ basis. (8 point)

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