Name: _____ Neptun code _____

f(PTIA0301) Elementary Linear Algebra thematics

Scalars and vectors. Equality of two vectors. Additive vectors. Planar and spatial vectors. Coordinates of vectors. Length/magnitude and equality of vectors given coordinates. Null vector. Inverse vector. Unitvector. Normal vector. Multiplication of a vector with a scalar, sum of vectors, and difference of vectors. The properties of vector addition. The properties of vector multiplication with scalar. Distance of points, equation of sphere.

Scalar and vector products. Scalar (inner) product of vectors. The properties of scalar multiplication. The scalar product of vectors given in coordinates. The angle of two non-zero vectors using the two definitions of scalar product. Orthogonal vectors. Perpendicular projection of a vector to the dictection of another vector. Vector Product. Right-hand system. Properties of the vector product. Parallel vectors and vector product. Vector product of the same vector. Vector product with components. Triple product. The volume of the paralepipedon constructed by three vectors.

Operators

- <u>Definition:</u> The set is the sum of things. It is a fundamental term. You need a statement that collects the element. It means that you can decide whether an element is part of the set or not.

- <u>Definition:</u> The pair are sets consisting of two elements.

- <u>Definition:</u> Elements e_1 and e_2 consist of ordered pair if $\{e_1, \{e_2\}\}$. It sign is (e_1, e_2).

- <u>Definition:</u> Relation is the set of ordered pairs. [c] 0.25
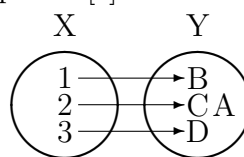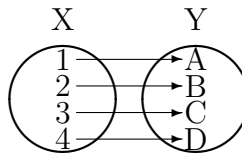

  0.75

- <u>Definition:</u> The injection orders different elements (X) to different elements (Y).   [c] 0.25
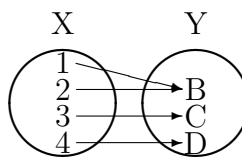
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- <u>Definition:</u> Surjections are those relations, that the values of the relation agree to the values of the set to order.   [c] 0.25

0.75

- <u>Definition:</u> Bijection is an injection and a surjection. All elements are related to all elements of the other set.

- <u>Definition:</u> The functions are such a set of ordered pairs in that one element shows up only once:

$$(\forall x)\,(\forall y_1)\,(\forall y_2)\,[(x, y_1) \in f \wedge (x, y_2) \in f \Rightarrow y_1 = y_2]$$

- <u>Definition:</u> V are U vectorspaces above $\mathbb{T}$ body. The $f : V \to U$ relation is linear if it is
    1. Additive, for all $v_1, v_2 \in V$ vectors $f\,(v_1 + v_2) = f\,(v_1) + f\,(v_2)$.
    2. Homogen, for all $v \in V$ vectors and $\lambda \in \mathbb{T}$ elements $f\,(\lambda v) = \lambda f\,(v)$.
- <u>Definition:</u> Operators are the linear vector-vector functions.
- Például:
    - Identical operator: $\mathbf{A} \cdot \mathbf{1} = \mathbf{A}$, for all $\mathbf{A}$ operators.
    - Null operator: $\mathbf{A} \cdot \mathbf{0} = \mathbf{0}$, for all $\mathbf{A}$ operators.

- Mirror operators: $(\mathbf{A} \cdot \mathbf{T}) \cdot \mathbf{T} = \mathbf{A}$, for all $\mathbf{A}$ operators.
- Projection operator: $\mathbf{A} \cdot \mathbf{P} = \mathbf{P}$, for all $\mathbf{A}$ operator.
- Rotational operator: later.

- Operators could be multiplied on both sides.

- The representation of operators is the matrixes. See $\alpha_{ij} \in \mathbb{R}$ for all $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$, where $m, n \in \mathbb{N}^+$. The

$$
A = \begin{pmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn}
\end{pmatrix}
$$

table is called $m \times n$ type matrix. The set of the $m \times n$ type matrixes is $M_{m \times n}$.

- The spur of the matrix is the set of $\{\alpha_{11}, \alpha_{22}, \ldots, \alpha_{nn}\}$.

- The first index of the elements α_{ij} is the rowindex (i), the 2nd index is the column index (j).

- The Row i of the Matrix is A_i , and the Column $j$of the matrix is A_jj.

- Determinant!!!

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