

$$(1, 2, 3), (2, 3, 1), (3, 1, 2)$$

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \rightarrow \det(A) = |A| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot 2 = \underline{\underline{-3}}$$

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} =$$

$$= (1 \cdot 1 - 2 \cdot 2) - 2(2 \cdot 1 - 2 \cdot 3) + 3(2 \cdot 2 - 1 \cdot 3) = \\ = -3 - 2(-4) + 3(1) = \underline{\underline{8}}$$

$$-\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 4 \end{vmatrix} = -1 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 2 & 4 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} =$$

$$= (2 \cdot 4 - 4 \cdot 3) + (1 \cdot 4 - 4 \cdot 2) - (1 \cdot 3 - 2 \cdot 2) =$$

$$= +4 - 4 + 1 = +1$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 2 \end{vmatrix} = 1 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 3 + 3 \cdot 2 \cdot 2 - 3 \cdot 1 \cdot 3 - 2 \cdot 2 \cdot 1 - 1 \cdot 2 \cdot 2 =$$

$$= 1 + 12 + 12 - 9 - 4 - 4 = 8$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - 1 \cdot 1 = 3$$

$$\stackrel{def}{=} (\alpha_1 \mathbf{a}_1 + \alpha_2 \mathbf{a}_2 + \alpha_3 \mathbf{a}_3) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} ; \quad \text{Def} + \boxed{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}} \quad \text{read}$$

$$\stackrel{bize}{=} (\alpha_1 \mathbf{a}_1 + \alpha_2 \mathbf{a}_2 + \alpha_3 \mathbf{a}_3) = (\mathbf{a}_1 \times \mathbf{b}) \cdot \underline{c} = \left[ (a_2 b_3 - a_3 b_2) e_1 + \right. \\ \left. + (a_3 b_1 - a_1 b_3) e_2 + (a_1 b_2 - a_2 b_1) e_3 \right] \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \\ = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} c_1 + \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} c_2 + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} c_3 = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \boxed{\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - 1 \cdot 1 = \underline{\underline{3}}$$

$$|B| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot 2 = \underline{\underline{-3}}$$

$$|C| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 2 - 2 \cdot 1 = \underline{\underline{0}}$$

$$|D| = \begin{vmatrix} 2 & 1 \\ 3 & 6 \end{vmatrix} = 2 \cdot 6 - 1 \cdot 3 = \underline{\underline{9}}$$

$$|E| = \begin{vmatrix} 2 & 1 \\ 4 & 8 \end{vmatrix} = 16 - 4 = \underline{\underline{12}}$$

A : E

$$|G| = \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} = 2 \cdot 4 - 1 \cdot 5 = \underline{\underline{3}}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 0 & \frac{3}{2} \end{vmatrix} = 3 - \frac{1}{2}$$

$$\begin{vmatrix} 1 & 3 & 2 & -1 \\ -2 & -3 & 0 & 2 \\ -1 & 3 & 6 & 1 \\ -1 & 2 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 & -1 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 5 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 & -1 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{44}{3} & -1 \end{vmatrix} = 0$$

$$\begin{aligned} & (II) + 2(IV) \\ & (III) - 2(II) \\ & (IV) - \frac{5}{3}(II) \end{aligned}$$

A:  $\mathbb{R}$