(KTXFI2EBNF) Physics II. Lecture

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Status Report I

- No communication with Mr. Szilárd ZSÓKA. :(
- You need to do your homework and take a test – no practice.
- How many tests should you take?
- I give my lectures and do the exercises with you.

Where are we? I

- ~~Motion of charged particles in electromagnetic fields.~~
- ~~Elements of quantum mechanics. Heisenberg's uncertainty principle. The stationary Schrödinger equation and its applications.~~
- ~~Limits of the classical conceptual framework. Thermal radiation. Photoelectric effect. Compton effect. The dual nature of electromagnetic radiation. The dual nature of particles.~~
- **Moving reference frames. Inertial forces in accelerating reference frames. Elements of special relativity. Dirac-equation, antimatter.**
- *The classical theory of atomic structure (Rutherford, Franck-Hertz experiment, Bohr model, quantum numbers, Pauli exclusion principle).*
- Physics of condensed matter. Metallic bonding. Electrical conduction in metals based on the free electron model and the wave model. Hall effect. Band theory of solids.

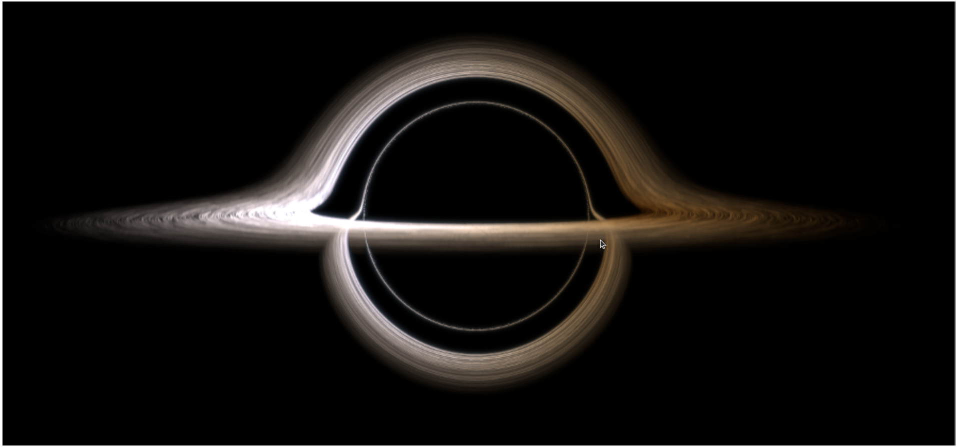
Where are we? II

- Semiconductors. Elements of Fermi-Dirac statistics. Thermoelectric phenomena. Magnetic properties.
- Ferroelectricity. Piezoelectricity and electrostriction. Liquid crystals. Superconductivity.
- Luminescence. Lasers. Basic knowledge of nuclear physics. Basic knowledge of particle physics.

Elements of the Special Relativity Theory – Clarification I

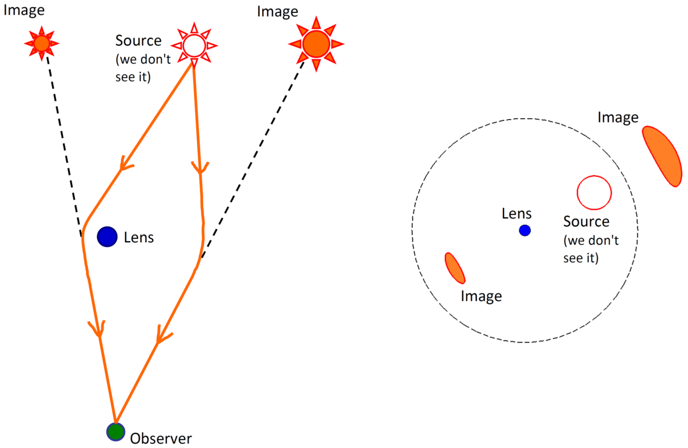
- Special Relativity Theory vs. General Relativity Theory: both are classical theories.
- Special Relativity Theory: reference frames moving uniformly in a straight line
  - Lorentz, Minkowski, Einstein, and others
  - Lorentz transformation, time dilation, mass increase, addition of velocities, electrodynamics, Maxwell's equations
  - Quantum Mechanics + Special Relativity Theory = Dirac equation, positron, antimatter, Relativistic Quantum Electrodynamics, Quantum Chromodynamics, Quarks, Higgs boson
- General Relativity Theory: any reference frame
  - Gravitational theory: a geometrical theory
  - Time dilation caused by gravitation (GPS!)
  - Gravitational lenses
  - Developed by Albert Einstein (with help from his wife and a circle of mathematician friends)
  - **ASSIGNMENT: Watch (again) the Interstellar movie.**

Elements of the Special Relativity Theory – Clarification II



The spacetime is twisted, therefore, we can the accretion disc behind the black hole.

Elements of the Special Relativity Theory – Clarification III



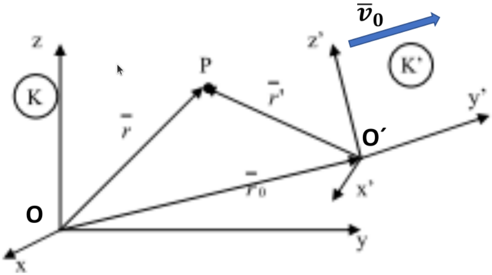
The massive object bends the light curve and behaves as a lens.

Galilean Principle of Relativity, Galilean Transformations I

- Description of motion in reference frames moving uniformly in a straight line
- Reference frames performing uniform linear motion: Galilean principle of relativity
- On a straight railway track, a passenger train is either at rest or moving with a constant (magnitude and direction) velocity. Sitting in a completely curtained passenger car, can one decide — without looking out of the windows — whether the train is at rest or in motion? Can we feel, through our body, what kind of motion state the train is in?

Galilean Principle of Relativity, Galilean Transformations II

- What forces act on a body resting at a point P in space when observed from a uniformly moving reference frame?



K: reference frame at rest

K': reference frame moving with velocity $\mathbf{v_0}$ relative to K

Galilean Principle of Relativity, Galilean Transformations III

- From the vector triangle OO'P:

$$\mathbf{r_o} + \mathbf{r}' = \mathbf{r}.$$

- Differentiating both sides with respect to time:

$$\mathbf{v_o} + \mathbf{v}' = \mathbf{v}.$$

- Differentiating again with respect to time:

$$\mathbf{a_o} + \mathbf{a}' = \mathbf{a},$$

  then multiplying by the mass m:

$$m\mathbf{a}' = m\mathbf{a}.$$

Galilean Principle of Relativity, Galilean Transformations IV

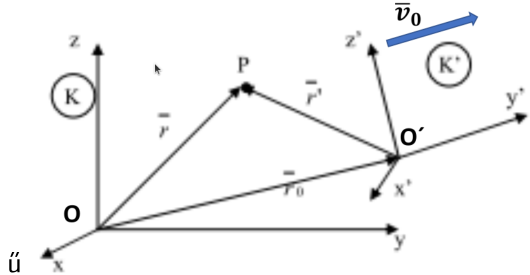
- Therefore,
$$\mathbf{F}' = \mathbf{F}.$$
- Galilean Principle of Relativity: Reference frames that are at rest or moving uniformly and linearly relative to each other are completely equivalent with respect to mechanical phenomena. If one of them is an inertial frame, then all of them are.
- Consequence of the Galilean Principle of Relativity: If there exists one inertial frame, then infinitely many inertial frames exist.

  Newton's First Law: There exists a reference frame in which ... This is an inertial frame. $\rightarrow$ Therefore, an inertial frame exists, and infinitely many exist.

Galilean Principle of Relativity, Galilean Transformations V

- Galilean Transformations:

$$
\begin{aligned}
\mathbf{r_o} + \mathbf{r}' &= \mathbf{r}, \\
\mathbf{v_o} + \mathbf{v}' &= \mathbf{v}, \\
\mathbf{a}' &= \mathbf{a}, \\
\mathbf{t}' &= \mathbf{t}.
\end{aligned}
$$



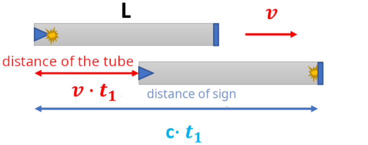
Galilean Principle of Relativity, Galilean Transformations VI

- What about other physical phenomena when we apply Galilean transformations to their descriptive equations? Do the equations remain equivalent or not? For example: in electromagnetism — are Maxwell's equations invariant under Galilean transformations?
- What about light? Does the speed of light add to the velocity of moving objects? For example: does the velocity of light emitted on a moving train add to (or subtract from) the train's velocity?
- Light clock: A light clock is a tube with a plane mirror at one end and a flashing light source installed on the tube's central axis at the other end.

Galilean Principle of Relativity, Galilean Transformations VII

- Light Clock Experiment

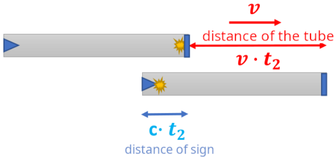
a.)



distance of light impuse: $c \cdot t_1 = L + v \cdot t_1$

$$\boxed{t_1 = \frac{L}{c - v}}$$

b.)



distance of light impuse: $\mathrm{L} = c \cdot t_2 + v \cdot t_2$

$$\boxed{t_2 = \frac{L}{c + v}}$$

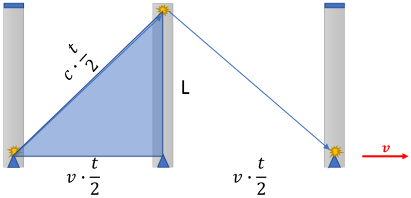
$$\boxed{t_1 \neq t_2}$$

$$\boxed{t_{\text{total}} = t_1 + t_2 = \frac{2L}{c \cdot (1 - \frac{v^2}{c^2})}}$$

Galilean Principle of Relativity, Galilean Transformations VIII

- Light Clock Experiment - cont'd

c.)



Pythagoras Theorem

$$(c \cdot \frac{t}{2})^2 = L^2 + (v \cdot \frac{t}{2})^2$$

$$\boldsymbol{t = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c \cdot \sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boldsymbol{t_{\text{total}} = t_1 + t_2 = \frac{2L}{c \cdot (1 - \frac{v^2}{c^2})}}$$

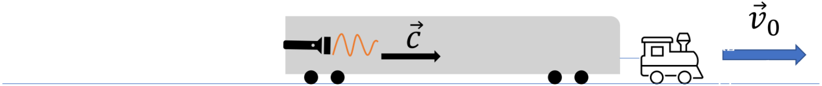
Summary: $\boldsymbol{t}_{\text{total}} \neq \boldsymbol{t}$???

Galilean Principle of Relativity, Galilean Transformations IX

- Train Experiment

How much is the speed of the train from the ground?

$$v = c + v_0$$



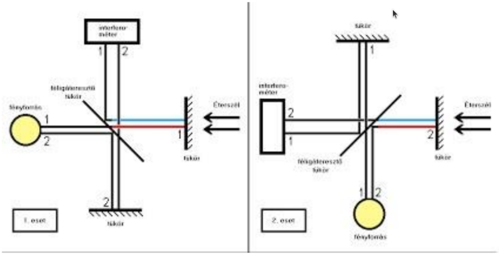
How much is the speed of the light from the ground?

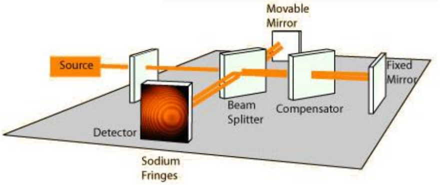
$$v = c + v_0$$

?

Galilean Principle of Relativity, Galilean Transformations X

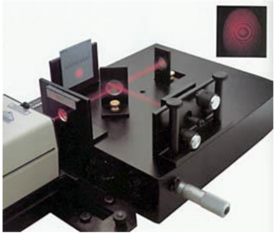
- Michelson-Morley Experiment: No interference





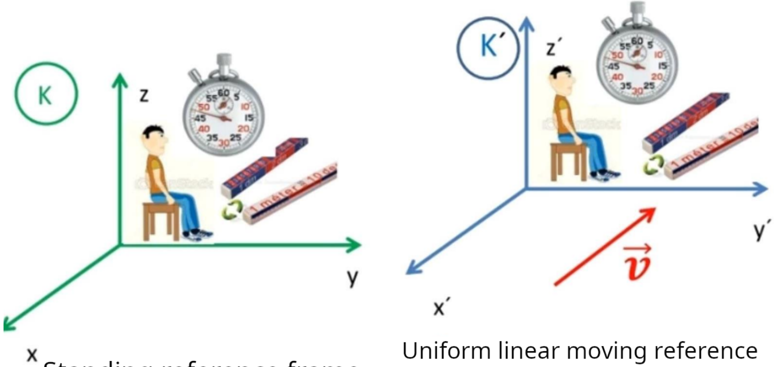
$$t_{II} \neq t_{\llcorner}$$

???



Lorentz Transformations I

- Reference frames for Lorentz transformations



Standing reference frame

Uniform linear moving reference frame (**v**)

Lorentz Transformations II

Lorentz Transformations

$$\begin{aligned} t' &= \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \\ x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \end{aligned}$$

Inverse Transformations

$$\begin{aligned} t &= \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}} \\ x &= \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y &= y' \\ z &= z' \end{aligned}$$

t, t': time; x, x', y, y', z, z': coordinates; v: velocity of the K' frame; c: speed of light

t, x, y, z belong to the K reference frame, t', x', y', z' belong to the K' reference frame

Lorentz Transformations III

- Consequences of the Lorentz transformation
  - For $v > c$ there is no real solution (thus $v > c$ is impossible)
  - For $v \ll c$ it reduces to the previously known Galilean principle of relativity
  - Convention: if $v \leq c/6$, a non-relativistic description should be used
  - The concept of simultaneity is relative
  - Length contraction ("Einstein's garage"):

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}.$$

  - Time dilation ("Twin Paradox"):

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Einstein's Postulates

- Principle of Equivalence: All inertial reference frames are equivalent for describing any physical phenomenon. In other words, reference frames that are at rest or move uniformly in a straight line relative to each other are equivalent for all physical laws.
- Principle of the Constancy of the Speed of Light: The speed of light is a universal constant, independent of the motion of the source, the observer, or the reference frame. Its value is $c \approx 3 \cdot 10^8\,\mathrm{m/s}$. The speed of light in vacuum is the ultimate speed limit — it cannot be reached or exceeded by any object.
- Comment: Cherenkov radiation :)
- Applying these principles to the Michelson–Morley experiment, we obtain $t_{\parallel} \neq t_{\perp}$. *Conclusion:* time flows at different rates.

Relativistic Dynamics I

- Relativistic mass increase:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where m_0 is the rest mass, m is the relativistic mass, c is the speed of light, and v is the velocity.

- Momentum:

$$p = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

- Total energy:

$$E = W = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Relativistic Dynamics II

- Rest energy:

$$E_0 = W_0 = m_0 c^2.$$

- Kinetic energy:

$$W_{\mathrm{kin}} = W - W_0 = mc^2 - m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2.$$

- Fundamental equation of relativistic dynamics:

$$\mathbf{F} = \frac{d\left(m\mathbf{v}\right)}{dt} = \frac{dm}{dt}\mathbf{v} + m\frac{d\mathbf{v}}{dt} = \frac{dm}{dt}\mathbf{v} + m\mathbf{a}.$$

Relativistic Dynamics III

- Equivalence of mass and energy: Mass and energy are inseparable manifestations of matter.

The Dirac Equation and Its Physical Meaning I

- Dirac equation (in covariant form):

$$(i\hbar\gamma^\mu\partial_\mu - mc)\,\psi = 0$$

- Explanation of symbols:
  - ψ : Dirac spinor (a four-component wave function describing spin-$\frac{1}{2}$ particles, e.g. electrons)
  - m : rest mass of the particle
  - c : speed of light in vacuum
  - $\hbar$: reduced Planck constant ($\hbar = \frac{h}{2\pi}$)
  - ∂_μ : four-gradient operator

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right)$$

The Dirac Equation and Its Physical Meaning II

- γ^μ : Dirac gamma matrices ($\mu = 0, 1, 2, 3$), satisfying

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I,$$

where $g^{\mu\nu}$ is the Minkowski metric tensor and I is the identity matrix.

- Alternative (Hamiltonian) form:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-i\hbar c\,\boldsymbol{\alpha} \cdot \nabla + \beta mc^2\right)\psi,$$

where $\boldsymbol{\alpha}$ and β are 4×4 Dirac matrices defined by $\alpha^i = \gamma^0\gamma^i$ and $\beta = \gamma^0$.

The Dirac Equation and Its Physical Meaning III

- Physical meaning:
  - The Dirac equation unifies **quantum mechanics** and **special relativity**.
  - It correctly describes particles with **spin** $\frac{1}{2}$ and predicts their intrinsic **magnetic moment**.
  - It naturally introduces the concept of **antimatter** — particles with the same mass but opposite charge (e.g. the positron).
  - The equation ensures that the probability density remains positive and conserved under Lorentz transformations.
  - At low velocities ($v \ll c$), it reduces to the Schrödinger equation, ensuring consistency with non-relativistic quantum mechanics.

Consequences of the Dirac Equation I

- Existence of spin:
  - The Dirac equation naturally describes particles with **intrinsic spin** $\frac{1}{2}$.
  - The spin is a purely quantum mechanical property with no classical analogue.
  - The spin arises from the four-component structure of the Dirac spinor.
- Magnetic moment:
  - The equation predicts the correct magnetic moment of the electron:

$$\boldsymbol{\mu} = g \frac{e\hbar}{2m} \mathbf{S}, \quad \text{with } g = 2.$$

  - This prediction agrees very closely with experimental data (after quantum electrodynamic corrections).

Consequences of the Dirac Equation II

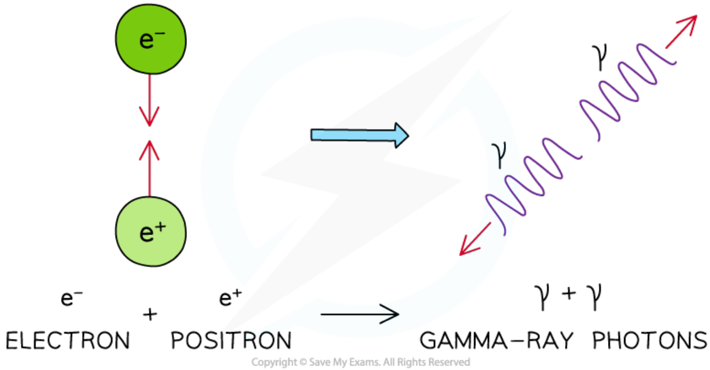
- <u>Negative energy solutions:</u>
  - The Dirac equation allows both positive and negative energy states:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}.$$

  - Initially a theoretical puzzle, later interpreted as the existence of **antiparticles**.
- <u>Antimatter:</u>
  - The negative-energy solutions correspond to particles with the same mass but opposite charge.
  - This led to the prediction — and later experimental discovery — of the **positron** in 1932 (by Carl D. Anderson).

Consequences of the Dirac Equation III

- Annihilation



Consequences of the Dirac Equation IV

- Relativistic quantum field theory:
  - The Dirac equation laid the foundation for **Quantum Electrodynamics (QED)**.
  - It describes interactions between charged spin-$\frac{1}{2}$ particles and electromagnetic fields.
  - This framework explains atomic structure, scattering, and vacuum fluctuations with exceptional precision.
- Summary:
  - The Dirac equation unifies:
    - Quantum mechanics (wave–particle duality),
    - Special relativity (Lorentz invariance),
    - and Intrinsic spin (a fundamental property of matter).
  - It remains one of the cornerstones of modern physics.

The End

Thank you for your attention!