



(KTXFI2EBNF) Physics II. Lecture

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Where are we? I

- ▶ ~~Motion of charged particles in electromagnetic fields.~~
- ▶ ~~Elements of quantum mechanics. Heisenberg's uncertainty principle. The stationary Schrödinger equation and its applications.~~
- ▶ ~~Limits of the classical conceptual framework. Thermal radiation. Photoelectric effect. Compton effect. The dual nature of electromagnetic radiation. The dual nature of particles.~~
- ▶ ~~Moving reference frames. Inertial forces in accelerating reference frames. Elements of special relativity. Dirac equation, antimatter.~~
- ▶ **The classical theory of atomic structure (Rutherford, Franck-Hertz experiment, Bohr model, quantum numbers, Pauli exclusion principle).**
- ▶ Physics of condensed matter. Metallic bonding. Electrical conduction in metals based on the free electron model and the wave model. Hall effect. Band theory of solids.

Where are we? II

- ▶ Semiconductors. Elements of Fermi-Dirac statistics. Thermoelectric phenomena. Magnetic properties.
- ▶ Ferroelectricity. Piezoelectricity and electrostriction. Liquid crystals. Superconductivity.
- ▶ Luminescence. Lasers. Basic knowledge of nuclear physics. Basic knowledge of particle physics.

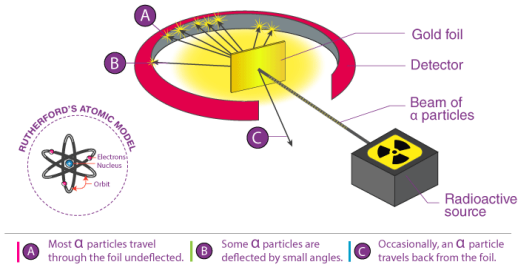
Atomic Models, Quantum Numbers, and Pauli Exclusion Principle I



- ▶ Plum Pudding Model
 - ▶ **J.J. Thomson (1897):** The atom's "plum pudding" model.
 - ▶ The atom is overall neutral: positive charge distributed uniformly in a sphere.
 - ▶ Analogy:
 - ▶ Dough → positive charge
 - ▶ Raisins → negative electrons

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle II

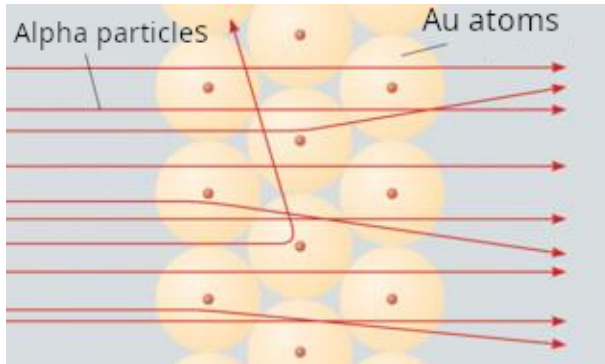
RUTHERFORD'S GOLD FOIL EXPERIMENT



► Rutherford Atomic Model (1909–1911)

- University of Manchester: Hans Geiger, Ernest Marsden under the direction of Ernest Rutherford.
- Gold foil scattering experiment using alpha particles.
- **Expected:** Alpha particles pass through with small deflections.
- **Observed:** Some alpha particles scattered at large angles.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle III



- ▶ Explanation of the experiment
 - ▶ If the atom followed the Plum Pudding Model, alpha particles would not scatter strongly.
 - ▶ Observation implies the existence of a massive, positively charged, localized scattering center - the nucleus.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle IV

▶ Rutherford Model Characteristics

- ▶ The atom's mass is concentrated in the nucleus.
- ▶ Electrons revolve in circular orbits held by electrostatic (Coulomb) attraction.
- ▶ **Flaw:** Accelerating charges radiate energy → the electron would spiral into the nucleus.
- ▶ Since this does not occur, the model is incomplete.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle V

► Spectral Analysis

source



grating

mirrors

detector

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VI

- ▶ Precursors to the Bohr Model

- ▶ **Johann Balmer (1825–1898):** Hydrogen shows a line spectrum.

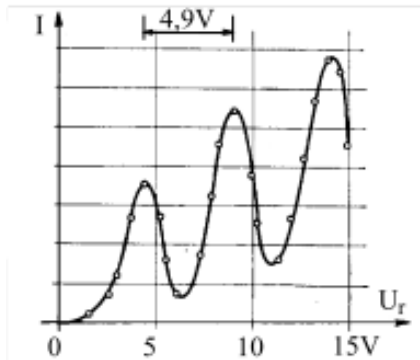
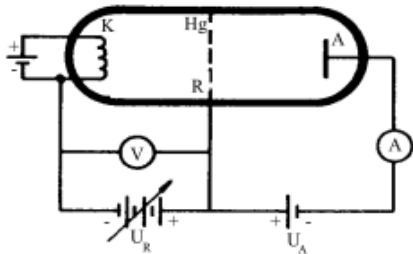
$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \quad n = 3, 4, 5, \dots$$

- ▶ **Johannes Rydberg (1854–1919):** Extended formula to other atoms.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

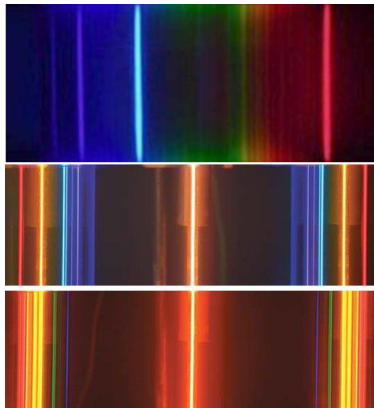
Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VII

- **Franck–Hertz Experiment:** Mercury atoms absorb discrete energies, e.g. $hf = 4.9 \text{ eV}$.



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VIII

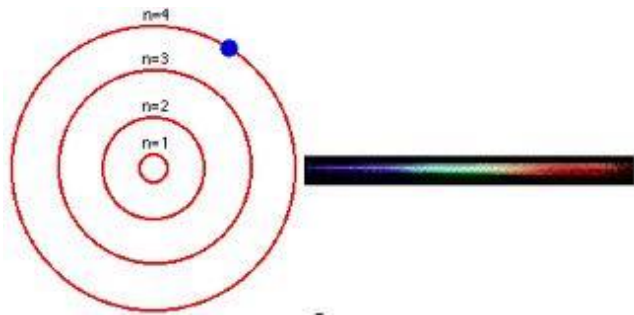
- Spectra: H, He, and Ne atoms, respectively



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle IX

► Bohr Atomic Model

- Atoms exist in stationary states with definite energies E_1, E_2, \dots — no radiation occurs.
- Electrons orbit the nucleus only on specific paths with discrete energies.

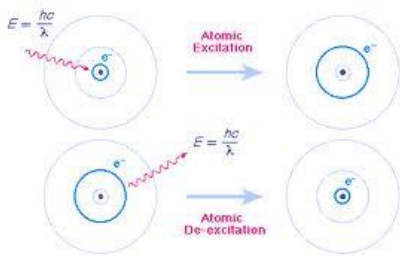


Atomic Models, Quantum Numbers, and Pauli Exclusion Principle X

- ▶ Bohr Model: Energy Transitions
 - ▶ Transition between levels involves photon emission or absorption:

$$W_n - W_k = hf$$

- ▶ The photon energy equals the energy difference between levels.



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XI

- ▶ Bohr Model: Quantization of Angular Momentum

$$mrv = n \frac{h}{2\pi} = n\hbar$$

- ▶ Only orbits where the electron's angular momentum is an integer multiple of \hbar are allowed.
- ▶ **Principal quantum number:** n

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XII

- ▶ Bohr–Sommerfeld Model
 - ▶ Fine structure of spectral lines observed.
 - ▶ Sommerfeld introduced elliptical orbits:

$$L = l \frac{h}{2\pi}$$

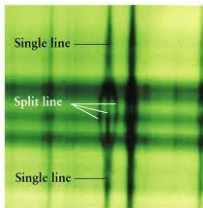
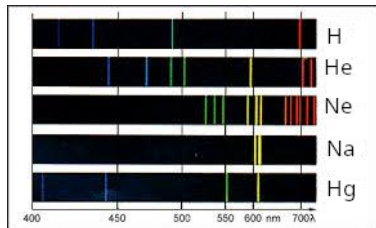
- ▶ **Azimuthal (orbital) quantum number:** $l = 0, 1, 2, \dots, n - 1$

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIII

► Zeeman Effect

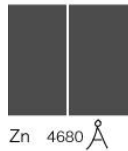
► **Pieter Zeeman (1865–1943):**

- In a strong magnetic field, spectral lines split into components — the normal Zeeman effect.



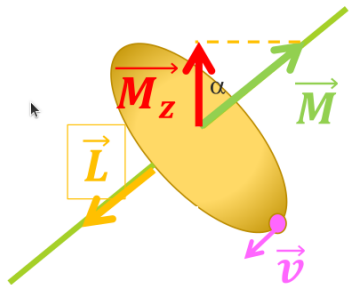
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ZEEMAN EFFECT



In a magnetic field
the original line splits
into three

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIV



► Magnetic Quantum Number

- Bohr magneton: $M_B = \frac{eh}{4\pi m_e}$
- Magnetic dipole moment: $M = M_B l$
- z-component: $M_z = M_B l \cos \alpha$
- **Magnetic quantum number:**
 $m = -l, \dots, 0, \dots, +l$

► Spin

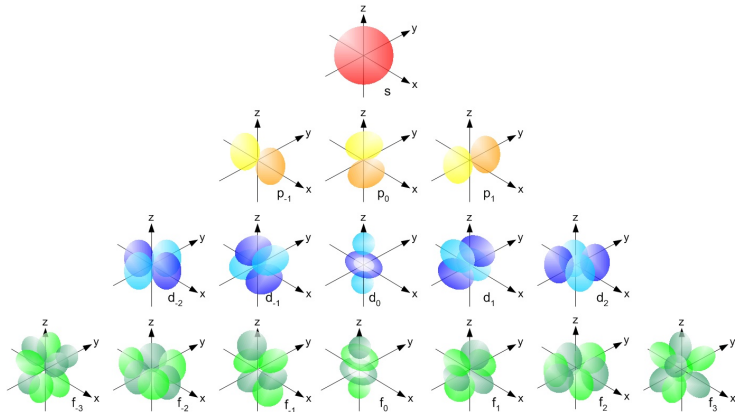
- Spin angular momentum: $L_S = \pm \frac{1}{2} \frac{h}{2\pi}$
- **Spin quantum number:** $s = \pm \frac{1}{2}$

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XV

- ▶ Pauli Exclusion Principle
 - ▶ No two electrons in an atom can share the same set of four quantum numbers.
 - ▶ Only one electron can occupy a given quantum state.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVI

► Electron structure



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVII

Principal QN	Orbital QN	Magnetic QN	Subshell	Spin QN	Max No of electrons
1	0	0	1s	$\frac{1}{2}, +\frac{1}{2}$	2
2	0	0	2s	$\frac{1}{2}, +\frac{1}{2}$	2
	1	1, 0, +1	2p	$\frac{1}{2}, +\frac{1}{2}$	6
3	0	0	3s	$\frac{1}{2}, +\frac{1}{2}$	2
	1	1, 0, +1	3p	$\frac{1}{2}, +\frac{1}{2}$	6
	2	2, 1, 0, +1, +2	3d	$\frac{1}{2}, +\frac{1}{2}$	10
4	0	0	4s	$\frac{1}{2}, +\frac{1}{2}$	2
	1	1, 0, +1	4p	$\frac{1}{2}, +\frac{1}{2}$	6
	2	2, 1, 0, +1, +2	4d	$\frac{1}{2}, +\frac{1}{2}$	10
	3	3, 2, 1, 0, +1, +2, +3	4f	$\frac{1}{2}, +\frac{1}{2}$	14

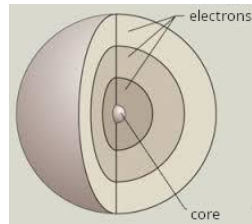
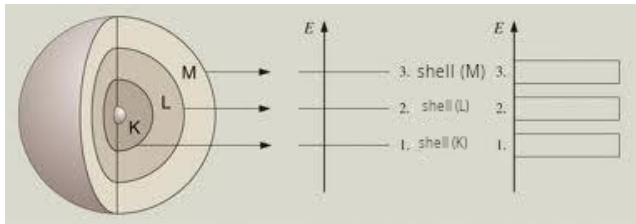
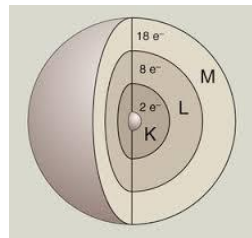
Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVIII

Periodic Table of the Elements																		18 VIIIA 8A					
1 IA 1A																	2 He Helium 4.0026						
1 H Hydrogen 1.00794	2 IIA 2A	s	p	d	f	Atomic Number Symbol Name Electron Configuration												3 IIIA 3A	4 IVA 4A	5 VA 5A	6 VIA 6A	7 VIIA 7A	10 Ne Neon 20.1797
3 Li Lithium 6.941	4 Be Beryllium 9.0122																	5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.1797
11 Na Sodium 22.990	12 Mg Magnesium 24.305	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 9	10 VIII 10	11 IB 1B	12 IIB 2B	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.06	17 Cl Chlorine 35.453	18 Ar Argon 39.948						
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.630	33 As Arsenic 74.922	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 83.796						
37 Rb Rubidium 85.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.94	43 Tc Technetium 98.906	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.905	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.757	52 Te Tellurium 127.6	53 I Iodine 126.905	54 Xe Xenon 131.29						
55 Cs Cesium 132.905	56 Ba Barium 137.327	57-71 Lanthanide Series		72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.222	78 Pt Platinum 195.084	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.384	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium 209	85 At Astatine 210	86 Rn Radon 222.018					
87 Fr Francium 223	88 Ra Radium 226	89-103 Actinide Series		104 Rf Rutherfordium 261	105 Db Dubnium 262	106 Sg Seaborgium 266	107 Bh Bohrium 264	108 Hs Hassium 277	109 Mt Meitnerium 268	110 Ds Darmstadtium 271	111 Rg Roentgenium 272	112 Cn Copernicium 285	113 Nh Nihonium 284	114 Fl Flerovium 289	115 Uup Ununpentium 288	116 Lv Livermorium 293	117 Uus Ununseptium 294	118 Uuo Ununoctium 294					
Lanthanide Series		57 La Lanthanum 138.905	58 Ce Cerium 140.116	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.242	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.50	67 Ho Holmium 164.930	68 Er Erbium 167.256	69 Tm Thulium 168.934	70 Yb Ytterbium 173.054	71 Lu Lutetium 174.967							
Actinide Series		89 Ac Actinium 227.033	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.083	99 Es Einsteinium 252.083	100 Fm Fermium 257.105	101 Md Mendelevium 258.105	102 No Nobelium 259.108	103 Lr Lawrencium 260.105							

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Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIX

Atom	Atomic Orbits	Electron Formula
${}_1\text{H}$	$1s^1$	$\text{H}\cdot$
${}_2\text{He}$	$1s^2$	$\text{He} \uparrow \downarrow$
${}_3\text{Li}$	$(1s^2) 2s^1$	$\text{Li}\cdot$
${}_4\text{Be}$	$(1s^2) 2s^2$	$\text{Be} \uparrow \downarrow$
${}_5\text{B}$	$(1s^2) 2s^2, 2p^1$	$\text{B}\cdot$
${}_6\text{C}$	$(1s^2) 2s^2, 2p^2$	$\text{C}\cdot$
${}_7\text{N}$	$(1s^2) 2s^2, 2p^3$	$\text{N}\cdot$
${}_8\text{O}$	$(1s^2) 2s^2, 2p^4$	$\text{O}\cdot$
${}_9\text{F}$	$(1s^2) 2s^2, 2p^5$	$\text{F}\cdot$
${}_{10}\text{Ne}$	$(1s^2) 2s^2, 2p^6$	$\text{Ne} \uparrow \downarrow$



Wave–Particle Duality I

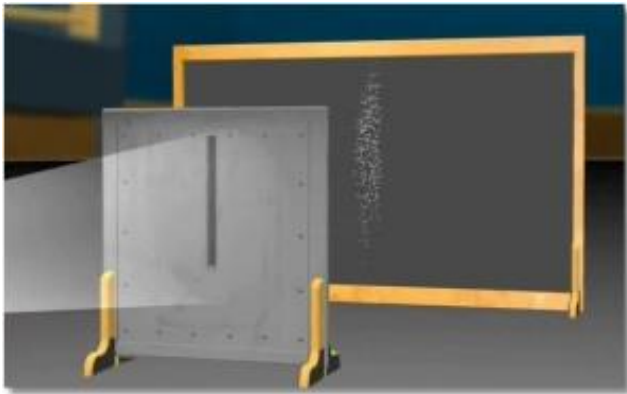
- ▶ Light and matter exhibit both particle and wave properties.
- ▶ De Broglie Matter-Wave Theory
 - ▶ **Louis de Broglie (1892–1987):**
 - ▶ Based on light's dual nature, proposed that particles have wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- ▶ **De Broglie wavelength:** $\lambda = h/p$
- ▶ Verified by the Davisson–Germer experiment (electron diffraction).

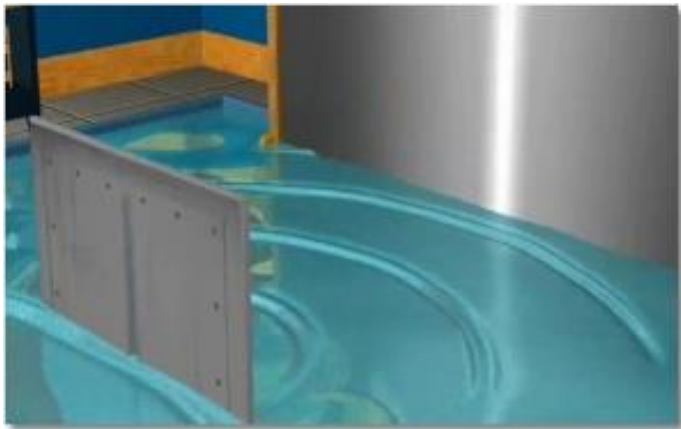
Wave-Particle Duality II

► Wave-Particle Demonstrations



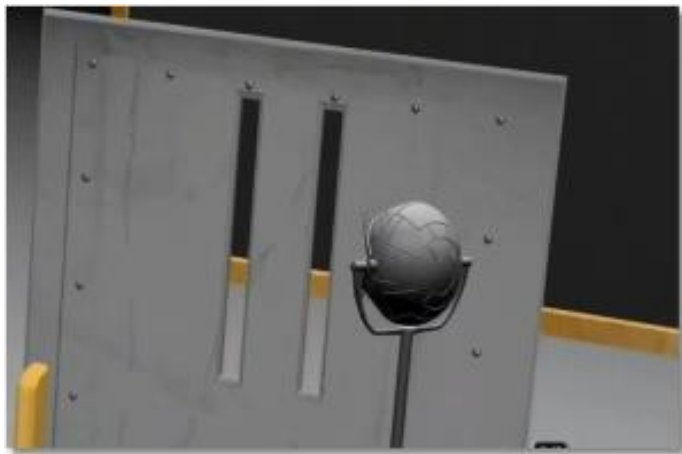
1 slit + light \rightarrow particle pattern

Wave–Particle Duality III



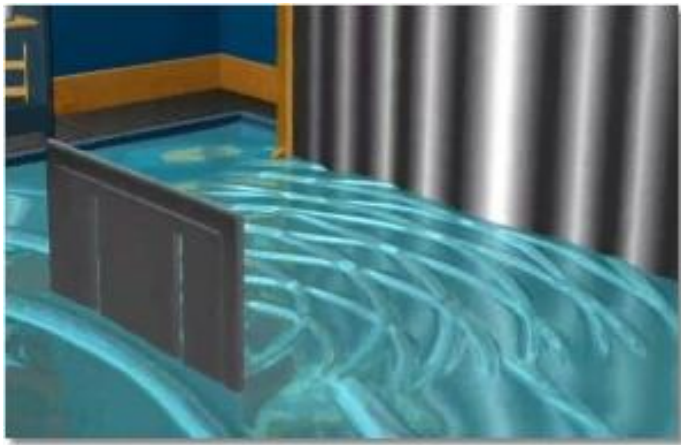
1 slit + light \rightarrow waves

Wave-Particle Duality IV



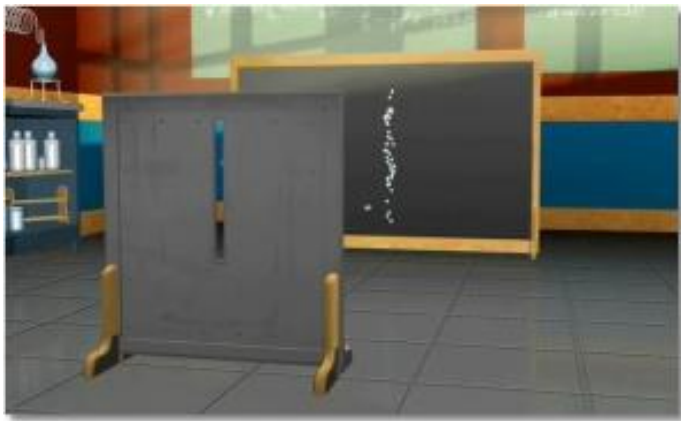
2 slits + light \rightarrow interference fringes

Wave–Particle Duality V



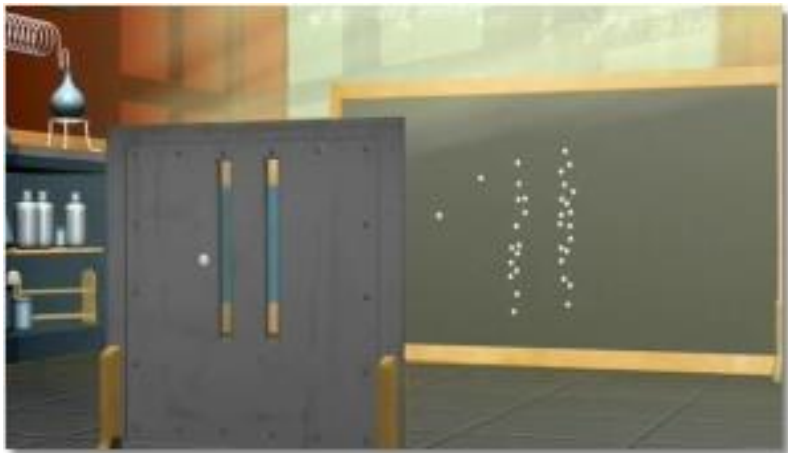
2 slits + light \rightarrow interference fringes

Wave-Particle Duality VI



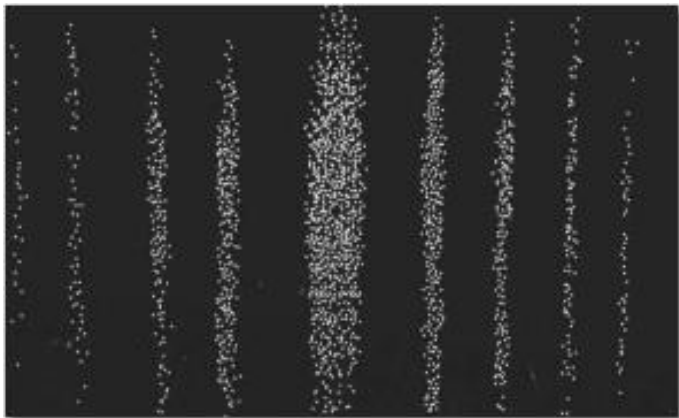
1 slit + electron \rightarrow particle pattern

Wave-Particle Duality VII



2 slits + electrons \rightarrow interference pattern

Wave-Particle Duality VIII



2 slits + electrons \rightarrow interference pattern

Conclusion: Electrons have wave properties!

Elements of Quantum Mechanics I

- ▶ Fundamental principles governing atomic and subatomic systems.
- ▶ Heisenberg Uncertainty Principle
 - ▶ Certain pairs of quantities (e.g. position and momentum) cannot be measured simultaneously with arbitrary precision.
 - ▶ **Relations:**

$$\Delta t \Delta E \geq h, \quad \Delta x \Delta p \geq h$$

Elements of Quantum Mechanics II

- Heisenberg's first uncertainty principle:

$$t = \frac{1}{f} \Rightarrow \Delta t \geq \frac{1}{f}$$

$$W = h \cdot f \Rightarrow \Delta W = h \cdot \Delta f \Rightarrow \frac{1}{\Delta f} = \frac{h}{\Delta W},$$

therefore, $\Delta t \leq \frac{h}{\Delta W}$. So,

$$\Delta t \cdot \Delta W \leq h.$$

In the physical description of micro-objects, the product of the uncertainties in energy and time cannot be smaller than Planck's constant. Example: The exact energy of a photon could only be determined from measurements carried out over an infinitely long period.

Elements of Quantum Mechanics III

- Heisenberg's second uncertainty principle: $f = \frac{\nu}{\lambda} \Rightarrow$ the finite difference with respect to λ : $\frac{\Delta f}{\Delta \lambda} = -\frac{\nu}{\lambda^2}$.

Rearranging:

$$-\frac{\Delta f \cdot \lambda^2}{\Delta \lambda} = \nu$$
$$-\frac{\lambda^2}{\Delta \lambda} = \frac{\nu}{\Delta f}.$$

The de Broglie equation: $p = \frac{h}{\lambda}$. Take the finite difference with respect to λ :

$$\frac{\Delta p}{\Delta \lambda} = -\frac{h}{\lambda^2}.$$

Rearranging:

$$-\frac{\Delta p \cdot \lambda^2}{\Delta \lambda} = h$$

Elements of Quantum Mechanics IV

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{h}{\Delta p}.$$

- Heisenberg's second uncertainty principle:

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{\nu}{\Delta f}$$

and

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{h}{\Delta p}.$$

Therefore

$$\frac{\nu}{\Delta f} = \frac{h}{\Delta p}.$$

So,

Elements of Quantum Mechanics V

$$v \cdot \Delta t \geq \frac{h}{\Delta p}.$$

However,

$$v \cdot \Delta t = \Delta x$$

, hence:

$$\Delta x \geq \frac{h}{\Delta p'}$$

, or

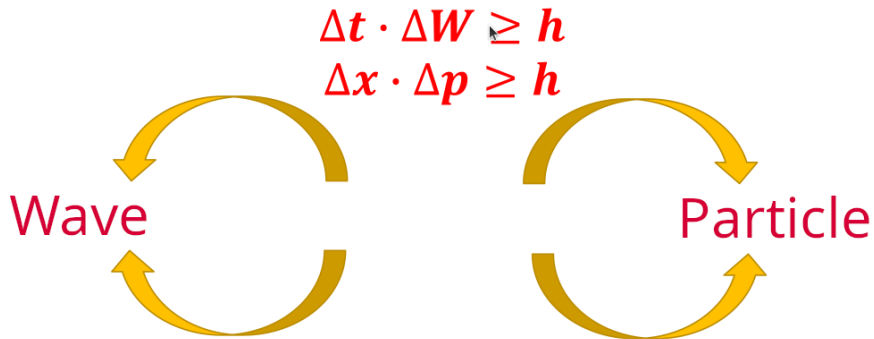
$$\Delta x \cdot \Delta p \geq h.$$

In the physical description of micro-objects, the product of the uncertainties of momentum and position cannot be smaller than Planck's constant.

Example: the orbit of an electron cannot be precisely described. We cannot, for instance, determine its exact velocity at a given position.

Elements of Quantum Mechanics VI

- ▶ Bohr's Complementarity Principle
 - ▶ Wave and particle properties are complementary aspects of microscopic objects.
 - ▶ Both are needed for a complete description.



The Dirac Equation and Its Physical Meaning I

- ▶ Dirac equation (in covariant form):

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

- ▶ Explanation of symbols:

- ▶ ψ : Dirac spinor (a four-component wave function describing spin- $\frac{1}{2}$ particles, e.g. electrons)
- ▶ m : rest mass of the particle
- ▶ c : speed of light in vacuum
- ▶ \hbar : reduced Planck constant ($\hbar = \frac{h}{2\pi}$)
- ▶ ∂_μ : four-gradient operator

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, -\nabla \right)$$

The Dirac Equation and Its Physical Meaning II

- ▶ γ^μ : Dirac gamma matrices ($\mu = 0, 1, 2, 3$), satisfying

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}I,$$

where $g^{\mu\nu}$ is the Minkowski metric tensor and I is the identity matrix.

- ▶ Alternative (Hamiltonian) form:

$$i\hbar \frac{\partial \psi}{\partial t} = (-i\hbar c \boldsymbol{\alpha} \cdot \nabla + \beta mc^2) \psi,$$

where $\boldsymbol{\alpha}$ and β are 4×4 Dirac matrices defined by $\alpha^i = \gamma^0 \gamma^i$ and $\beta = \gamma^0$.

The Dirac Equation and Its Physical Meaning III

► Physical meaning:

- The Dirac equation unifies **quantum mechanics** and **special relativity**.
- It correctly describes particles with **spin** $\frac{1}{2}$ and predicts their intrinsic **magnetic moment**.
- It naturally introduces the concept of **antimatter** — particles with the same mass but opposite charge (e.g. the positron).
- The equation ensures that the probability density remains positive and conserved under Lorentz transformations.
- At low velocities ($v \ll c$), it reduces to the Schrödinger equation, ensuring consistency with non-relativistic quantum mechanics.

Consequences of the Dirac Equation I

► Existence of spin:

- The Dirac equation naturally describes particles with **intrinsic spin** $\frac{1}{2}$.
- The spin is a purely quantum mechanical property with no classical analogue.
- The spin arises from the four-component structure of the Dirac spinor.

► Magnetic moment:

- The equation predicts the correct magnetic moment of the electron:

$$\boldsymbol{\mu} = g \frac{e\hbar}{2m} \mathbf{S}, \quad \text{with } g = 2.$$

- This prediction agrees very closely with experimental data (after quantum electrodynamic corrections).

Consequences of the Dirac Equation II

- ▶ Negative energy solutions:

- ▶ The Dirac equation allows both positive and negative energy states:

$$E = \pm \sqrt{p^2 c^2 + m^2 c^4}.$$

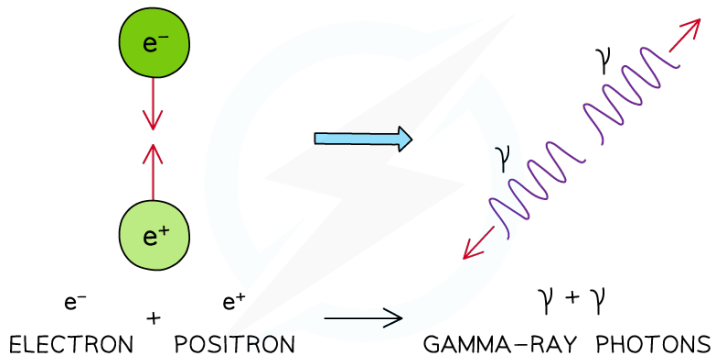
- ▶ Initially a theoretical puzzle, later interpreted as the existence of **antiparticles**.

- ▶ Antimatter:

- ▶ The negative-energy solutions correspond to particles with the same mass but opposite charge.
 - ▶ This led to the prediction — and later experimental discovery — of the **positron** in 1932 (by Carl D. Anderson).

Consequences of the Dirac Equation III

► Annihilation



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Consequences of the Dirac Equation IV

- ▶ Relativistic quantum field theory:
 - ▶ The Dirac equation laid the foundation for **Quantum Electrodynamics (QED)**.
 - ▶ It describes interactions between charged spin- $\frac{1}{2}$ particles and electromagnetic fields.
 - ▶ This framework explains atomic structure, scattering, and vacuum fluctuations with exceptional precision.
- ▶ Summary:
 - ▶ The Dirac equation unifies:
 - ▶ Quantum mechanics (wave–particle duality),
 - ▶ Special relativity (Lorentz invariance),
 - ▶ and Intrinsic spin (a fundamental property of matter).
 - ▶ It remains one of the cornerstones of modern physics.

The End

Thank you for your attention!