(KTXFI2EBNF) Physics II. Lecture

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Where are we? I

- ~~Motion of charged particles in electromagnetic fields.~~
- ~~Elements of quantum mechanics. Heisenberg's uncertainty principle. The stationary Schrödinger equation and its applications.~~
- ~~Limits of the classical conceptual framework. Thermal radiation. Photoelectric effect. Compton effect. The dual nature of electromagnetic radiation. The dual nature of particles.~~
- ~~Moving reference frames. Inertial forces in accelerating reference frames. Elements of special relativity. Dirac-equation, antimatter.~~
- **The classical theory of atomic structure (Rutherford, Franck-Hertz experiment, Bohr model, quantum numbers, Pauli exclusion principle).**
- Physics of condensed matter. Metallic bonding. Electrical conduction in metals based on the free electron model and the wave model. Hall effect. Band theory of solids.

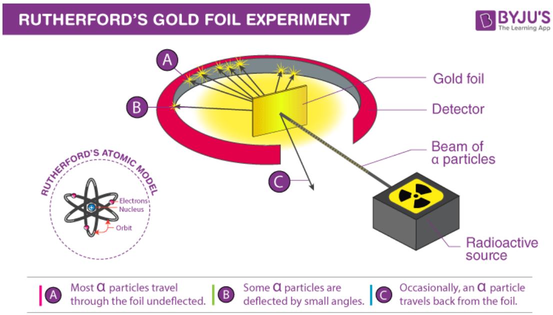
Where are we? II

- Semiconductors. Elements of Fermi-Dirac statistics. Thermoelectric phenomena. Magnetic properties.
- Ferroelectricity. Piezoelectricity and electrostriction. Liquid crystals. Superconductivity.
- Luminescence. Lasers. Basic knowledge of nuclear physics. Basic knowledge of particle physics.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle I

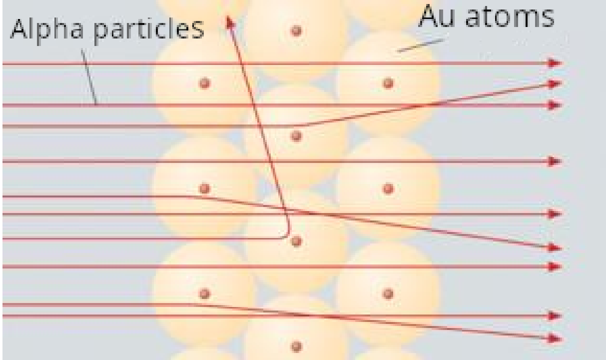
- Plum Pudding Model
  - **J.J. Thomson (1897):** The atom's "plum pudding" model.
  - The atom is overall neutral: positive charge distributed uniformly in a sphere.
  - Analogy:
    - Dough $\rightarrow$ positive charge
    - Raisins $\rightarrow$ negative electrons

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle II



- Rutherford Atomic Model (1909–1911)
  - University of Manchester: Hans Geiger, Ernest Marsden under the direction of Ernest Rutherford.
  - Gold foil scattering experiment using alpha particles.
  - **Expected:** Alpha particles pass through with small deflections.
  - **Observed:** Some alpha particles scattered at large angles.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle III



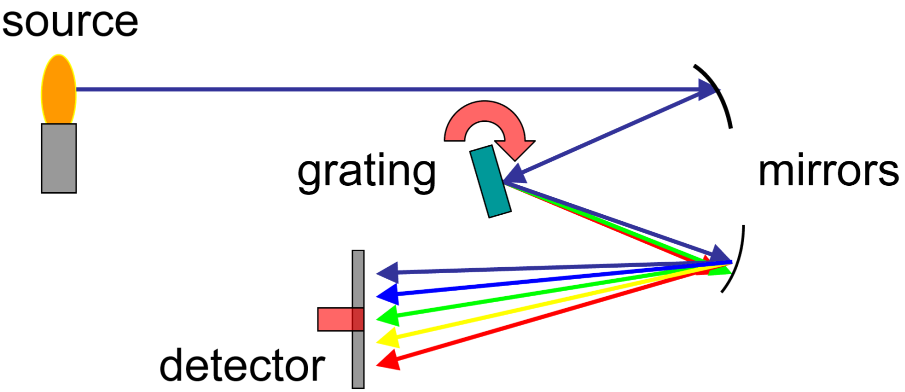
- Explanation of the experiment
  - If the atom followed the Plum Pudding Model, alpha particles would not scatter strongly.
  - Observation implies the existence of a massive, positively charged, localized scattering center - the nucleus.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle IV

- Rutherford Model Characteristics
  - The atom's mass is concentrated in the nucleus.
  - Electrons revolve in circular orbits held by electrostatic (Coulomb) attraction.
  - **Flaw:** Accelerating charges radiate energy $\rightarrow$ the electron would spiral into the nucleus.
  - Since this does not occur, the model is incomplete.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle V

- Spectral Analysis



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VI

- Precursors to the Bohr Model
  - **Johann Balmer (1825–1898):** Hydrogen shows a line spectrum.

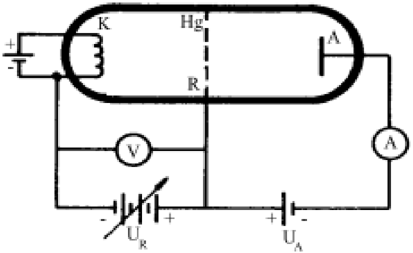
$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \quad n = 3, 4, 5, \ldots$$

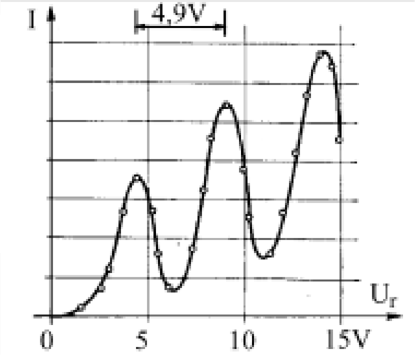
  - **Johannes Rydberg (1854–1919):** Extended formula to other atoms.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VII

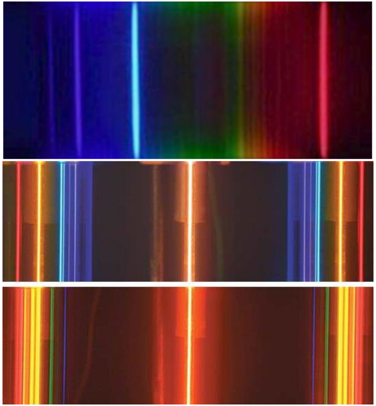
- **Franck–Hertz Experiment:** Mercury atoms absorb discrete energies, e.g. $hf = 4.9\,\text{eV}$.





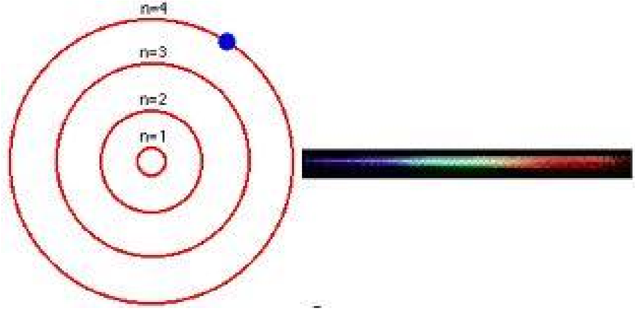
Atomic Models, Quantum Numbers, and Pauli Exclusion Principle VIII

- Spectra: H, He, and Ne atoms, respectively



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle IX

- Bohr Atomic Model
  - Atoms exist in stationary states with definite energies $E_1, E_2, \ldots$ — no radiation occurs.
  - Electrons orbit the nucleus only on specific paths with discrete energies.

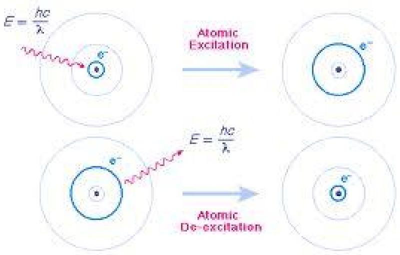


Atomic Models, Quantum Numbers, and Pauli Exclusion Principle X

- Bohr Model: Energy Transitions
  - Transition between levels involves photon emission or absorption:

$$W_n - W_k = hf$$

  - The photon energy equals the energy difference between levels.



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XI

- Bohr Model: Quantization of Angular Momentum

$$mrv = n\frac{h}{2\pi} = n\hbar$$

  - Only orbits where the electron's angular momentum is an integer multiple of $\hbar$ are allowed.
  - **Principal quantum number:** n

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XII

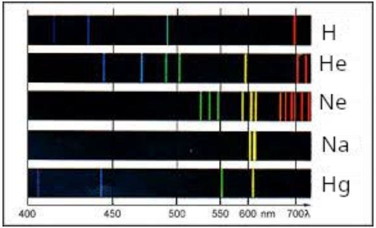
- Bohr–Sommerfeld Model
  - Fine structure of spectral lines observed.
  - Sommerfeld introduced elliptical orbits:

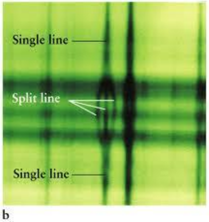
$$L = l\frac{h}{2\pi}$$

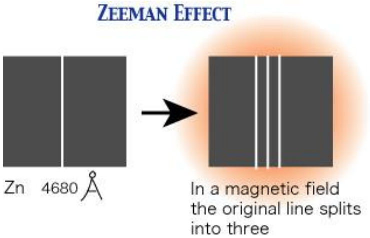
  - **Azimuthal (orbital) quantum number:** $l = 0, 1, 2, \ldots, n-1$

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIII

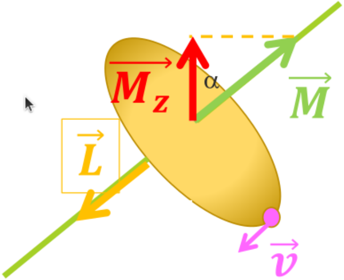
- Zeeman Effect
  - **Pieter Zeeman (1865–1943):**
  - In a strong magnetic field, spectral lines split into components — the normal Zeeman effect.







Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIV



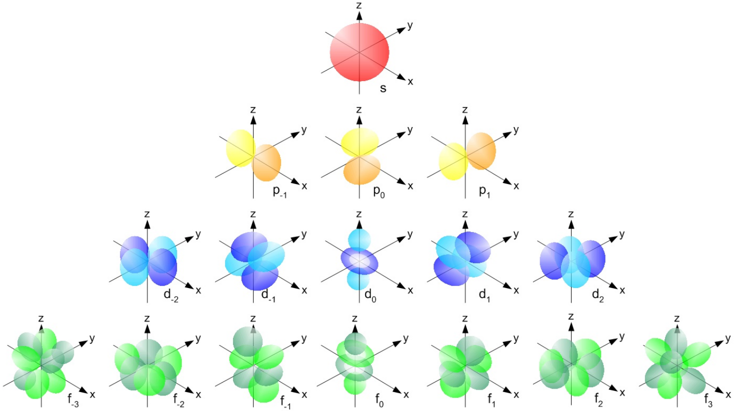
- Magnetic Quantum Number
  - Bohr magneton: $M_B = \dfrac{eh}{4\pi m_e}$
  - Magnetic dipole moment: $M = M_B l$
  - z-component: $M_z = M_B l \cos\alpha$
  - **Magnetic quantum number:** $m = -l, \ldots, 0, \ldots, +l$
- Spin
  - Spin angular momentum: $L_S = \pm \dfrac{1}{2} \dfrac{h}{2\pi}$
  - **Spin quantum number:** $s = \pm \dfrac{1}{2}$

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XV

- Pauli Exclusion Principle
  - No two electrons in an atom can share the same set of four quantum numbers.
  - Only one electron can occupy a given quantum state.

Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVI

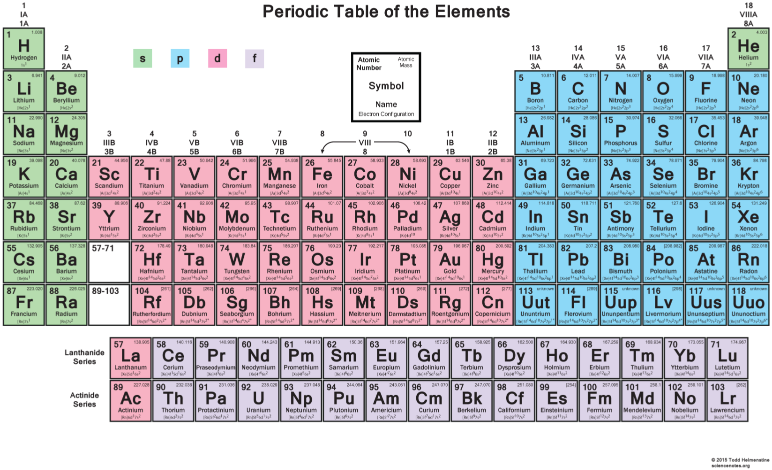
- Electron structure



Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVII

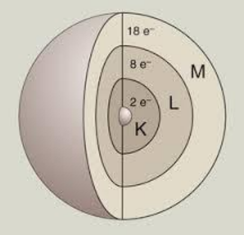
Principal QN	Orbital QN	Magnetic QN	Subshell	Spin QN	Max No of electrons
1	0	0	1s	$\frac{1}{2}$ $+\frac{1}{2}$	2
2	0	0	2s	$\frac{1}{2}$ $+\frac{1}{2}$	2 } 8
	1	1 , 0, +1	2p	$\frac{1}{2}$ $+\frac{1}{2}$	6
3	0	0	3s	$\frac{1}{2}$ $+\frac{1}{2}$	2
	1	1 , 0, +1	3p	$\frac{1}{2}$ $+\frac{1}{2}$	6 } 18
	2	2 , 1, 0, +1, +2	3d	$\frac{1}{2}$ $+\frac{1}{2}$	10
4	0	0	4s	$\frac{1}{2}$ $+\frac{1}{2}$	2
	1	1 , 0, +1	4p	$\frac{1}{2}$ $+\frac{1}{2}$	6 } 32
	2	2 , 1, 0, +1, +2	4d	$\frac{1}{2}$ $+\frac{1}{2}$	10
	3	3 , 2, 1 , 0, +1, +2, +3	4f	$\frac{1}{2}$ $+\frac{1}{2}$	14

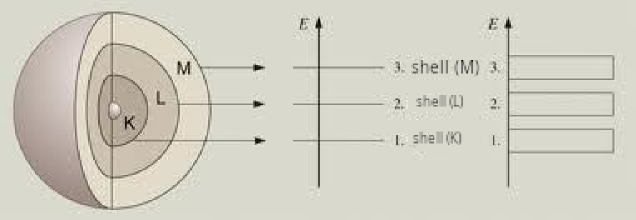
Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XVIII

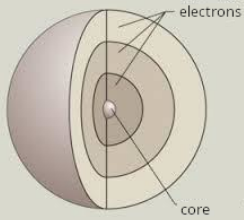


Atomic Models, Quantum Numbers, and Pauli Exclusion Principle XIX

Atom	Atomic Orbits	Electron Formula
$_1H$	$1s^1$	H·
$_2He$	$1s^2$	He|
$_3Li$	$(1s^2)\ 2s^1$	Li·
$_4Be$	$(1s^2)\ 2s^2$	Be|
$_5B$	$(1s^2)\ 2s^2, 2p^1$	|B·
$_6C$	$(1s^2)\ 2s^2, 2p^2$	|Ċ·
$_7N$	$(1s^2)\ 2s^2, 2p^3$	|Ṅ·
$_8O$	$(1s^2)\ 2s^2, 2p^4$	|Ō·
$_9F$	$(1s^2)\ 2s^2, 2p^5$	|F̄·
$_{10}Ne$	$(1s^2)\ 2s^2, 2p^6$	|N̄e|







Wave–Particle Duality I

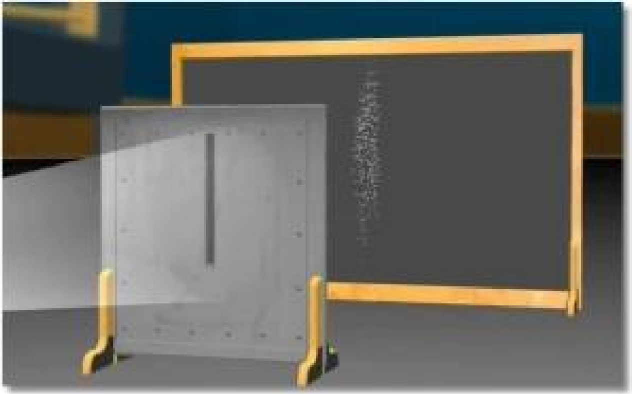
- Light and matter exhibit both particle and wave properties.
- De Broglie Matter-Wave Theory
  - **Louis de Broglie (1892–1987):**
  - Based on light's dual nature, proposed that particles have wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

  - **De Broglie wavelength:** $\lambda = h/p$
  - Verified by the Davisson–Germer experiment (electron diffraction).

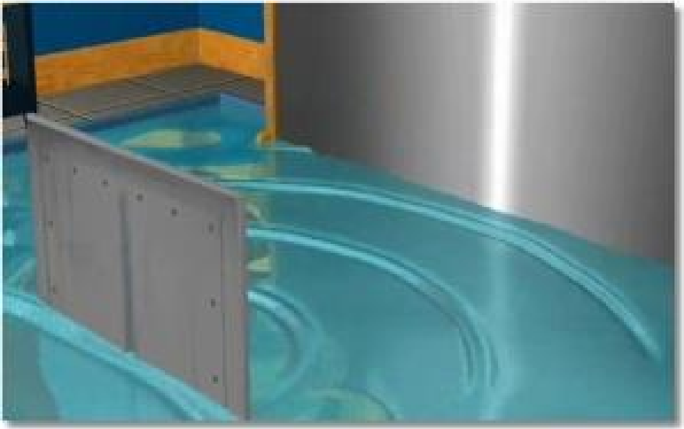
Wave–Particle Duality II

- Wave–Particle Demonstrations



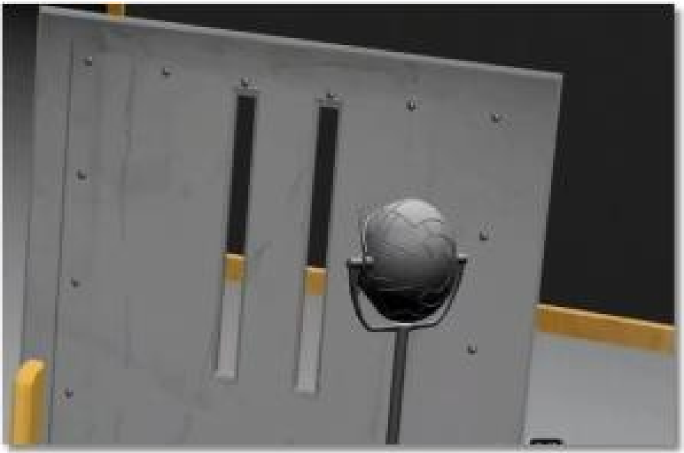
1 slit + light → particle pattern

Wave–Particle Duality III



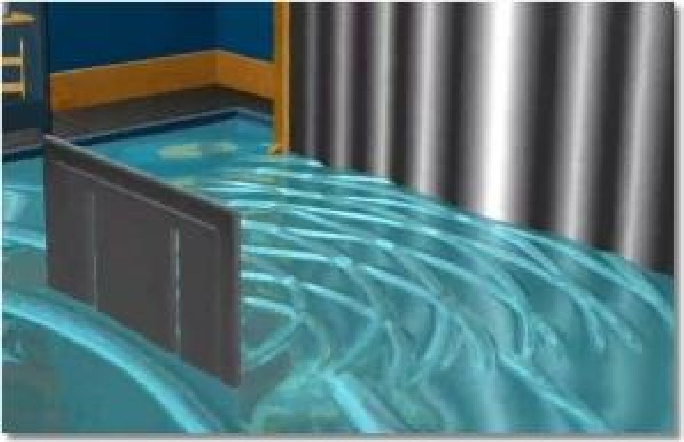
1 slit + light $\rightarrow$ waves

Wave–Particle Duality IV



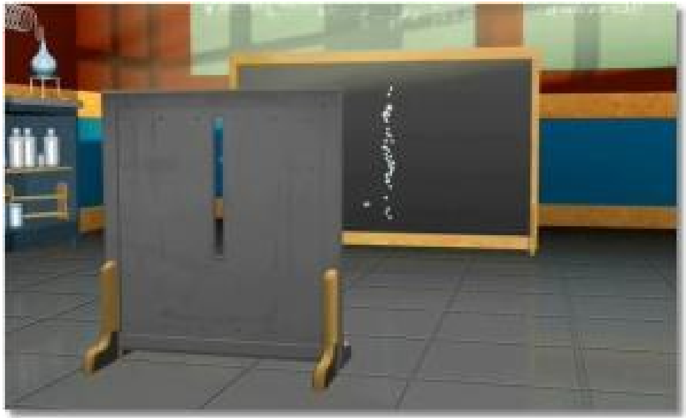
2 slits + light $\rightarrow$ interference fringes

Wave–Particle Duality V



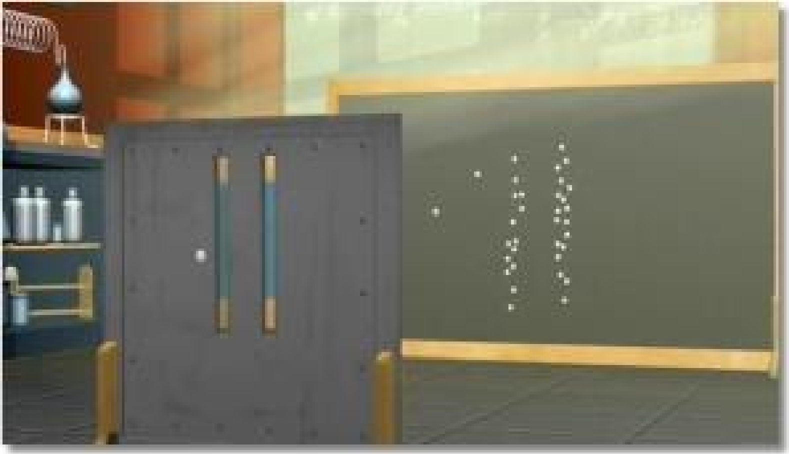
2 slits $+$ light $\rightarrow$ interference fringes

Wave–Particle Duality VI



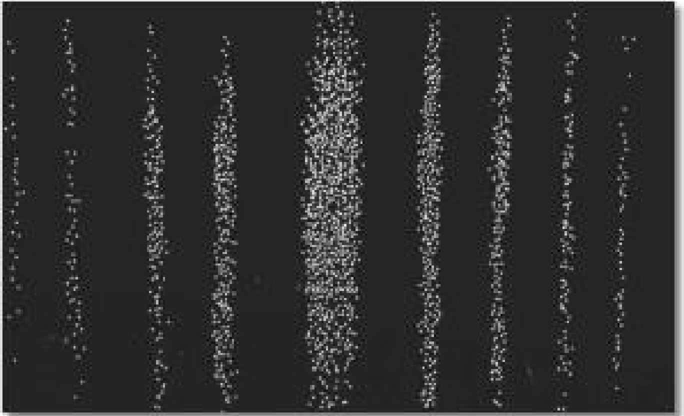
1 slit + electron $\rightarrow$ particle pattern

Wave–Particle Duality VII



2 slits + electrons → interference pattern

Wave–Particle Duality VIII



2 slits + electrons $\rightarrow$ interference pattern

Conclusion: Electrons have wave properties!

Elements of Quantum Mechanics I

- Fundamental principles governing atomic and subatomic systems.
- Heisenberg Uncertainty Principle
  - Certain pairs of quantities (e.g. position and momentum) cannot be measured simultaneously with arbitrary precision.
  - **Relations:**

$$\Delta t \, \Delta E \geq h, \qquad \Delta x \, \Delta p \geq h$$

Elements of Quantum Mechanics II

- Heisenberg's first uncertainty principle:

$$t = \frac{1}{f} \Rightarrow \Delta t \geq \frac{1}{f}$$

$$W = h \cdot f \Rightarrow \Delta W = h \cdot \Delta f \Rightarrow \frac{1}{\Delta f} = \frac{h}{\Delta W},$$

therefore, $\Delta t \leq \frac{h}{\Delta W}$. So,

$$\Delta t \cdot \Delta W \leq h.$$

In the physical description of micro-objects, the product of the uncertainties in energy and time cannot be smaller than Planck's constant. Example: The exact energy of a photon could only be determined from measurements carried out over an infinitely long period.

Elements of Quantum Mechanics III

- Heisenberg's second uncertainty principle: $f = \frac{\nu}{\lambda} \Rightarrow$ the finite difference with respect to λ: $\frac{\Delta f}{\Delta \lambda} = -\frac{\nu}{\lambda^2}$.
  Rearranging:
  $$-\frac{\Delta f \cdot \lambda^2}{\Delta \lambda} = \nu$$
  $$-\frac{\lambda^2}{\Delta \lambda} = \frac{\nu}{\Delta f}.$$
  The de Broglie equation: $p = \frac{h}{\lambda}$. Take the finite difference with respect to λ: $\frac{\Delta p}{\Delta \lambda} = -\frac{h}{\lambda^2}$.
  Rearranging:
  $$-\frac{\Delta p \cdot \lambda^2}{\Delta \lambda} = h$$

Elements of Quantum Mechanics IV

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{h}{\Delta p}.$$

- Heisenberg's second uncertainty principle:

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{\nu}{\Delta f}$$

and

$$-\frac{\lambda^2}{\Delta\lambda} = \frac{h}{\Delta p}.$$

Therefore

$$\frac{\nu}{\Delta f} = \frac{h}{\Delta p}.$$

So,

Elements of Quantum Mechanics V

$$v \cdot \Delta t \geq \frac{h}{\Delta p}.$$

However,

$$v \cdot \Delta t = \Delta x,$$

hence:

$$\Delta x \geq \frac{h}{\Delta p'},$$

or

$$\Delta x \cdot \Delta p \geq h.$$

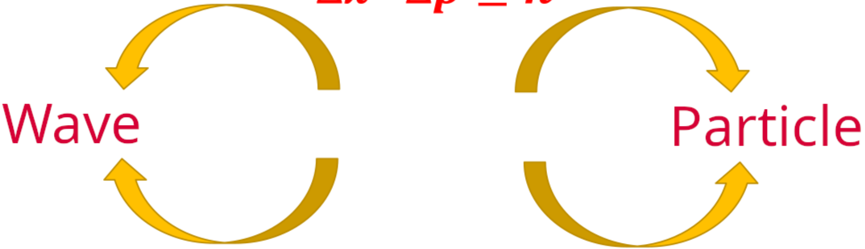
In the physical description of micro-objects, the product of the uncertainties of momentum and position cannot be smaller than Planck's constant.
Example: the orbit of an electron cannot be precisely described. We cannot, for instance, determine its exact velocity at a given position.

Elements of Quantum Mechanics VI

- Bohr's Complementarity Principle
  - Wave and particle properties are complementary aspects of microscopic objects.
  - Both are needed for a complete description.

$$\Delta t \cdot \Delta W \geq h$$

$$\Delta x \cdot \Delta p \geq h$$



The Dirac Equation and Its Physical Meaning I

- Dirac equation (in covariant form):

$$(i\hbar\gamma^\mu\partial_\mu - mc)\,\psi = 0$$

- Explanation of symbols:
  - ψ : Dirac spinor (a four-component wave function describing spin-$\frac{1}{2}$ particles, e.g. electrons)
  - m : rest mass of the particle
  - c : speed of light in vacuum
  - $\hbar$: reduced Planck constant ($\hbar = \frac{h}{2\pi}$)
  - ∂_μ : four-gradient operator

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right)$$

The Dirac Equation and Its Physical Meaning II

- γ^μ : Dirac gamma matrices ($\mu = 0, 1, 2, 3$), satisfying

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I,$$

where $g^{\mu\nu}$ is the Minkowski metric tensor and I is the identity matrix.

- Alternative (Hamiltonian) form:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-i\hbar c\,\boldsymbol{\alpha}\cdot\nabla + \beta mc^2\right)\psi,$$

where $\boldsymbol{\alpha}$ and β are 4×4 Dirac matrices defined by $\alpha^i = \gamma^0\gamma^i$ and $\beta = \gamma^0$.

The Dirac Equation and Its Physical Meaning III

- Physical meaning:
  - The Dirac equation unifies **quantum mechanics** and **special relativity**.
  - It correctly describes particles with **spin** $\frac{1}{2}$ and predicts their intrinsic **magnetic moment**.
  - It naturally introduces the concept of **antimatter** — particles with the same mass but opposite charge (e.g. the positron).
  - The equation ensures that the probability density remains positive and conserved under Lorentz transformations.
  - At low velocities ($v \ll c$), it reduces to the Schrödinger equation, ensuring consistency with non-relativistic quantum mechanics.

Consequences of the Dirac Equation I

- Existence of spin:
  - The Dirac equation naturally describes particles with **intrinsic spin** $\frac{1}{2}$.
  - The spin is a purely quantum mechanical property with no classical analogue.
  - The spin arises from the four-component structure of the Dirac spinor.
- Magnetic moment:
  - The equation predicts the correct magnetic moment of the electron:

$$\boldsymbol{\mu} = g\frac{e\hbar}{2m}\mathbf{S}, \quad \text{with } g = 2.$$

  - This prediction agrees very closely with experimental data (after quantum electrodynamic corrections).

Consequences of the Dirac Equation II

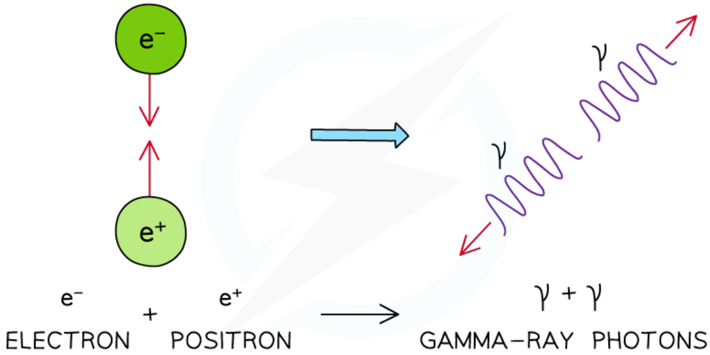
- Negative energy solutions:
  - The Dirac equation allows both positive and negative energy states:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}.$$

  - Initially a theoretical puzzle, later interpreted as the existence of **antiparticles**.
- Antimatter:
  - The negative-energy solutions correspond to particles with the same mass but opposite charge.
  - This led to the prediction — and later experimental discovery — of the **positron** in 1932 (by Carl D. Anderson).

Consequences of the Dirac Equation III

- Annihilation



Consequences of the Dirac Equation IV

- Relativistic quantum field theory:
  - The Dirac equation laid the foundation for **Quantum Electrodynamics (QED)**.
  - It describes interactions between charged spin-$\frac{1}{2}$ particles and electromagnetic fields.
  - This framework explains atomic structure, scattering, and vacuum fluctuations with exceptional precision.
- Summary:
  - The Dirac equation unifies:
    - Quantum mechanics (wave–particle duality),
    - Special relativity (Lorentz invariance),
    - and Intrinsic spin (a fundamental property of matter).
  - It remains one of the cornerstones of modern physics.

The End

Thank you for your attention!