Macroeconomics B
Solution to problem set 1

1. (a) The consumer sequence problem is

$$\max_{c_1,c_2,a_2,a_3} u(c_1) + \beta u(c_2) \tag{1}$$
$$\text{s.t. } a_2 = (1+r)a_1 + y_1 - c_1 \tag{2}$$
$$a_3 = (1+r)a_2 + Gy_1 - c_2 \tag{3}$$
$$a_1 = 0, a_3 \geq 0. \tag{4}$$

Given that utility is strictly increasing in consumption the solvency constraint $a_3 \geq 0$ holds. Replacing for a_1, a_3 and concatenating the two dynamic budget identities (2)-(3) one obtains the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{Gy_1}{1+r}. \tag{5}$$

Replacing for c_2 in (1) the consumer problem can be rewritten as

$$\max_{c_1} u(c_1) + \beta u((1+r)(y_1 - c_1) + Gy_1). \tag{6}$$

This implies the standard first-order condition (Euler equation)

$$u'(c_1) = \beta(1+r)u'(c_2). \tag{7}$$

(b) Replacing for c_2 using the intertemporal budget constraint and using the functional form

$$u'(c) = c^{-\sigma} \tag{8}$$

we can rewrite the Euler equation as

$$c_1^{-\sigma} = \beta(1+r)((1+r)(y_1 - c_1) + Gy_1)^{-\sigma}. \tag{9}$$

Which implies that $c_1 \gtreqless y_1$ is optimal if

$$y_1^{-\sigma} \gtreqless \beta(1+r)(Gy_1)^{-\sigma} \tag{10}$$

or

$$1 \gtreqless \beta(1+r)G^{-\sigma}. \tag{11}$$

The term on the right hand side reflects both the consumption tilting effect ($\beta(1+r) \gtreqless 1$) and the consumption smoothing effect (the income endowment is not smooth if $G \neq 1$).

(c) To obtain the consumption function in period 1, just rearrange equation (9) to solve for c_1

$$c_1 = \frac{\gamma}{\gamma+1}\left(1 + \frac{G}{1+r}\right)y_1 = \frac{1}{1+\gamma^{-1}}\left(1 + \frac{G}{1+r}\right)y_1 \tag{12}$$

with $\gamma = [\beta(1+r)^{1-\sigma}]^{-\frac{1}{\sigma}}$. It follows that

$$\gamma^{-1} = \beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1} \tag{13}$$

is increasing/constant/decreasing in r if σ is smaller than/equal to/larger than 1.

- If $G = 0$ the effect of an increase in r on c_1 is the opposite of the effect of r on γ^{-1}. The value of σ determines whether the substitution or the income effect prevails. If $\sigma < 1$, the substitution effect prevails and a higher r reduces c_1.
- If $G > 0$ a change in r also has a *wealth effect* as it affects the the present value of lifetime income. If r increases, lifetime wealth falls adding to the substitution effect. So if $G > 0$, and increase in r reduces c_1 even for values of σ above, but close enough, to 1.

2. From (12), the change in c_1 associated with a given change in y_1 is

$$\frac{\partial c_1}{\partial y_1} = \frac{1}{1+\gamma^{-1}}\left(1+\frac{G}{1+r}\right) \tag{14}$$

If $G = 0$ the effect of an increase in y_1 on c_1 is effectively half (exactly half if $r = 0$) than in the case in which $G = 1$. The intuition is that in the former case the income increase takes place just in the first period (temporary) and a significant fraction of it is saved to spread it over the second period too. If $G > 1$ the income increase is the same in both periods (permanent) and there is no need to spread it through saving.