Macroeconomics B

Solution to problem set 4

Part of the purpose of this problem set is to help you read Bradford DeLong's note on Barro and Reitz's solution to the equity premium puzzle.

1. Denote by $u(c_i) = 1 - \frac{1}{c_i}$ the consumer's felicity function[1]. We use the general notation for ease of comparison with last week's lecture notes. We substitute the specific functional form later. The consumer problem is

$$\max_{C_1,C_2,L_1,N_1,L_2,N_2} u(c_1) + \frac{1}{1+\rho}\mathbb{E}_1 u(c_2) \tag{53}$$

$$\text{s.t. } Y = c_1 + L_1 R_1^{-1} + N_1 P_1 \tag{54}$$

$$c_2 + L_2 R_2^{-1} + N_2 P_2 = L_1 + N_1(P_2 + Y_2), \tag{55}$$

$$L_2,\ N_2 \geq 0. \tag{56}$$

2. Non-satiation implies that the constraints $L_2,\ N_2 \geq 0$ are satisfied as equalities. There are two ways of obtaining the relevant Euler equations.

   One is to use the budget identity at time 2 to obtain

$$L_1 = c_2 - N_1(P_2 + Y_2)$$

   and replace in the budget identity at time 1 to obtain the intertemporal budget constraint

$$Y = c_1 + R_1[c_2 - N_1(P_2 + Y_2)] + N_1 P_1.$$

   One can use the IBC to replacing for c_2 (or c_1) and maximize with respect to N_1 and c_1 (or c_2).

   Alternatively, one can replace for c_1 and c_2 using the two constraints (54)-(55) and maximize with respect to L_1 and N_1. The associated FOCs are

$$-u'(c_1)R_1^{-1} + \frac{1}{1+\rho}\mathbb{E}_1 u'(c_2) = 0 \tag{57}$$

$$-u'(c_1)P_1 + \frac{1}{1+\rho}\mathbb{E}_1[u'(c_2)(P_2 + Y_2)] = 0. \tag{58}$$

   Note that these are the same equations as last week.

3. Replacing for $u'(c_i) = \frac{1}{c_i^2}$ and rearranging we can write

$$R_1^{-1} = \frac{1}{1+\rho}\mathbb{E}_1\left[\frac{c_1}{c_2}\right]^2 \tag{59}$$

$$P_1 = \frac{1}{1+\rho}\mathbb{E}_1\left[\left(\frac{c_1}{c_2}\right)^2 (P_2 + Y_2)\right]. \tag{60}$$

[1] Note that the utility function is CRRA with coefficient of relative risk aversion equal to 2.

4. Because the demand for N_2 is zero, its ex-dividend price P_2 has to equal zero (no bubble). Furthermore, given identical agents in equilibrium it is $c_i = Y_i$ and $L_1 = 0$. Imposing the equilibrium conditions in (116) and (117) yields

$$R_1^{-1} = \frac{1}{1+\rho}\mathbb{E}_1\left[\frac{Y}{Y_2}\right]^2 = \frac{1}{1+\rho}\frac{1}{2}\left[\frac{(1-\sigma)^2}{(1+g)^2} + \frac{(1+\sigma)^2}{(1+g)^2}\right] = \frac{1}{1+\rho}\frac{1+\sigma^2}{(1+g)^2} \tag{61}$$

$$\frac{P^1}{Y} = \frac{1}{1+\rho}\mathbb{E}_1\left[\frac{Y}{Y_2}\right] = \frac{1}{1+\rho}\frac{1}{2}\left[\frac{1-\sigma}{1+g} + \frac{1+\sigma}{1+g}\right] = \frac{1}{1+\rho}\frac{1}{1+g}. \tag{62}$$

Note that the risk-free rate of return satisfies

$$R_1 = (1+\rho)\frac{(1+g)^2}{1+\sigma^2.} \tag{63}$$

It has to ensure that consumers optimally choose to consume their endowments in every period (neither save nor dissave). If $g = \sigma = 0$, endowments are flat across time and certain. It has to be $r = \rho$ for agents not to be willing to borrow or land. If $g > 0$ and $\sigma = 0$, it has to be $r > \rho$ in equilibrium, for consumption tilting to offset the desire to borrow against higher future income (consumption smoothing). If $g = 0$ and $\sigma > 0$ it is $r < \rho$ for consumption in equilibrium, for consumption tilting to offset the precautionary saving motive.

5. The ratio of the two prices can be written as

$$\frac{P_1}{R_1^1} = Y\frac{1+g}{1+\sigma^2}. \tag{64}$$

A higher σ reduces the risk-free rate R_1 because of the precautionary saving motive. A higher g increases the risk-free rate R_1 because it makes people more impatient (consumption smoothing). In both cases, the risk-free rate has to adjust to offset that to ensure that the Euler equation is satisfied at the original endowment point.