Solution to the Macro A exam paper

Section A

1. (a) The equilibrium vector is $[p_t, E_{t-1}p_t, y_t, E_{t-1}y_t]$ such that (AS) and (AD) are satisfied and expectations are mathematical expectations conditional on the agents' information set.

   (b) It is

$$p_t - E_{t-1}p_t = \frac{1}{2}\left[m_t - E_{t-1}m_t + v_t - u_t\right] \tag{1}$$

$$y_t = \frac{1}{2}\left[m_t - E_{t-1}m_t + v_t + u_t\right] \tag{2}$$

   (c) From (2) it is $(\sigma_u^2 + \sigma_v^2)/4$.

   (d) No if the shocks are uncorrelated, as the government has no informational advantage over private agents. If v_t is autocorrelated the government has an informational advantage has it observes v_{t-1} while private agents do not. Since private agents can only forecast the shock-independent component of the monetary rule the government can reduce the output variance by following the feedback rule $m_t = \overline{m} - \rho v_{t-1}$. This implies

$$y_t = \frac{1}{2}\left[\bar{m} - \rho v_{t-1} - \bar{m} + v_t + u_t\right] = \frac{1}{2}\left[e_t + u_t\right]. \tag{3}$$

   The associated output variance is $(\sigma_u^2 + \sigma_e^2)/4$.

   The result does not contradict the PIP which assumes symmetric information and flexible prices.

2. Lower case letters denote variables in efficiency units of labour.

   (a) The associated Lagrangean is

$$\mathcal{L} = \int_0^\infty \frac{c_t^{1-\theta}}{1-\theta} e^{-[\rho-(1-\theta)g-n]t} dt + \lambda\left[b_0 + \int_0^\infty (w_t - c_t) e^{-R_t+(n+g)t} dt\right]. \tag{4}$$

   Students were not required to be able to derive the intertemporal budget constraint.

   The first order condition for optimal consumption is

$$c_t^{-\theta} = e^{-[R_t-(\rho+\theta g)t]}. \tag{5}$$

   Taking logs and time derivatives the FOC can be rewritten as the Euler equation

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho - \theta g}{\theta}. \tag{6}$$

(b) General equilibrium requires $b_t = k_t$ and factor prices to equal the factor's marginal products. The two differential equations that characterize the evolution of c and k are

$$\dot{k_t} = f(k_t) - c_t - (n+g)k_t. \tag{7}$$

and

$$\frac{\dot{c}}{c} = \frac{f'(k_t) - \rho - \theta g}{\theta}. \tag{8}$$

To these one must add the solvency constraint evaluated at equilibrium $\lim_{t\to\infty} k_t e^{-R(t)+(n+g)t} \geq 0$.

(c) From the Euler equation we obtain that in steady state it is $0.5k^{-.5} = \rho + \theta g = 0.05$ or $k = 100$. The goods market clearing condition implies $c = \sqrt{100} - .06 \times 100 = 4$.

(d) The vertical $\dot{c} = 0$ locus shifts left while the $\dot{k} = 0$ locus is unaffected. c_{t_0} jumps up onto the new saddle path for consumption to fall on the transition to the new steady state ($f(k) < \rho + \theta g$ on the transition path). Capital and consumption fall along the transition path as they converge to lower values in the new steady state.

3. The FOC for the general case can be derived by maximizing the central banker's welfare function subject to the SRAS constraint. One obtains

$$\pi_t = \frac{1}{2}\left[E_{t-1}\pi_t + \gamma - d - v_t\right]. \tag{9}$$

This implies

$$E_{t-1}\pi_t = \gamma - d \tag{10}$$

in the absence of commitment. $d = 0$ in part (b) of the question.

(a) $[p_t, E_{t-1}p_t, y_t, E_{t-1}y_t]$ such that the aggregate supply curve is satisfied, the policymaker FOC is satisfied and expectations are "rational".

(b) Since the commitment is credible it is $\pi_t = E_{t-1}\pi_t = \alpha$ and $y_t = v_t$. Replacing in the government objective function and taking expectations yields $W = -\alpha^2 - \sigma^2 - \gamma^2$ which is minimized for $\alpha = 0$. Too rigid and suboptimal since government dislikes output fluctuations.

(c) From (9) the equilibrium value of inflation is $\pi_t = \gamma - d - v_t/2$. Equilibrium output is $y_t = v_t/2$.

(d) It is optimal for the government to choose $d = \gamma$ which sets the inflation bias to zero. The associated value of government welfare is $W^g = -\sigma^2/2 - \gamma^2$. Relative to case (b) the optimal contract induces the central bank to stabilize output fluctuations which is desirable given that output enters government welfare quadratically.

Section B

4. With an exogenous saving rate the steady state stock of capital in efficiency units satisfies $f(k^*)/k^* = (\delta + n + g)/s$. It is possible that k^* exceeds the golden rule value k_{GR} which maximizes steady state consumption and satisfies $f'(k_{GR}) = \delta + n + g$. In such a case the economy is dynamically inefficient. Increasing the saving rate reduces steady state aggregate consumption.

5. Solvency requires the stock of government debt not to exceed the present value of future surpluses. It is normally assumed that the solvency constraint holds as an equality. If this is the case, for a given path of government expenditure, the present value of taxes is fully pinned down. Therefore current tax cuts imply future tax increases of equal present value. It is clearly a necessary condition for Ricardian equivalence to hold.

6. Weak scale effects are associated with increasing returns to capital and labour but decreasing returns to reproducible factors (capital and the stock of knowledge). They imply that policy does not affect the steady state rate of growth of the economy (no level effects) though the latter depends on the rate of growth of the population.
   Strong scale effects are associated with constant returns to reproducible factor (e.g AK model) and imply that the growth rate of the economy depends on the size of the population and policy variables such as saving rates.
   From an empirical point of view both cannot explain productivity differences in the absence of cross-country barriers to technological adoption (all countries would have access to the same technology).
   As theories of what determines the world (frontier) level of TFP weak scale effects are more plausible since otherwise growth rates should respond to changes in saving rates or population sizes which we do not observe.

7. TFP shocks are the main source of fluctuations. Since output fluctuations are associated with large employment fluctuations and little fluctuations in labour productivity, the model requires a large intertemporal elasticity of labour substitution (one way or another).
   With the measured capital share equal to 0.3 in the data the model generates little persistence in output over and above the persistence of TFP shocks. Also little amplification. TFP shocks must be highly variable and persistent.