

Time allowed: 2 hours + 15 minutes reading time

Answer ONE question from Section A and TWO questions from Section B.

Do not turn over until you are told to do so by the Invigilator.

Section A

1. (60 points) Assume an economy in which there are two activities: production of ideas or knowledge, A , and production of a final and intermediate good Y . Output is produced using capital K , labor L and knowledge A according to the technology

$$Y = K^\alpha (A(1 - a_L)L)^{1-\alpha}, \quad (1)$$

where $0 < \alpha < 1$ and a_L is the constant proportion of workers employed in the good sector. The exogenous rate of growth of the total labour force is n . In each period, a constant fraction s of output Y is invested in new machines and the depreciation rate of the existing stock of machines is zero. New knowledge is produced using researchers and existing ideas according to the technology

$$\dot{A} = \delta(a_L L)^\lambda A^\varphi, \quad (2)$$

where $0 < \lambda < 1$, $\varphi > 0$.

- (a) Write down the equation describing the accumulation of capital per efficiency unit of labour.
 - (b) Derive the steady state rates of growth of the stock of ideas A , capital per worker K/L and income per capita Y/L in this economy when $\varphi < 1$ and $n > 0$. What happens to these rates of growth when φ tends to one?
 - (c) Suppose $s = 0.12$, $n = 0.02$, $\lambda = 0.3$ and $\varphi = 0.7$. Derive the steady state value of Y/K for the economy in point (b).
 - (d) Derive the steady state rates of growth of the stock of ideas A , capital per worker K/L and income per capita Y/L in this economy when $\varphi = 1$ and $n = 0$.
2. (60 points) Suppose the aggregate supply and demand curves are given by

$$\begin{aligned} \text{AS } y_t &= \alpha(p_t - E_{t-1}p_t) \\ \text{AD } y_t &= m_t - p_t + v_t \end{aligned}$$

where $v_t = v_{t-2} + \epsilon_t$ and ϵ_t is a white noise error with zero mean and variance σ_ϵ^2 . The expectation operator E_{t-1} is conditional on all information available to private agents up to the *beginning* of period $t-1$; i.e. two periods out of date. On the other hand, the policy maker observes shocks with just one period delay. So at time t the policy maker cannot observe v_t but does observe v_{t-i} for all $i > 0$.

- (a) Write down the equilibrium vector for this economy.
- (b) Obtain an expression for the equilibrium level of output as a function of the money stock m_t and the shock v_t .
- (c) What is the variance of output if the policymaker follows the publicly known policy rule $m_t = \bar{m}$ with \bar{m} constant?

- (d) Suppose instead the policymaker follows the publicly known linear policy rule $m_t = \bar{m} + \gamma_0 v_{t-1} + \gamma_1 v_{t-2}$. Calculate the values of γ_0 and γ_1 that minimize the variance of output.

3. (60 points) Consider the Ramsey model. Households maximize

$$U_0 = \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta} L_t e^{-\rho t} dt,$$

where C_t denotes consumption per household member, $L_t = e^{nt}$ is the household size and $\rho > 0$ is the subjective discount rate. There is just one household in the economy. Output is produced according to the production function $F(K_t, A_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}$ where K_t is the aggregate capital stock and $A_t = e^{gt}$ the efficiency of labour. Capital does not depreciate.

- Write down the Lagrangean for the household problem and derive the Euler equation.
- Impose general equilibrium and derive the equations that characterize the evolution of capital and consumption per unit of effective labour.
- At time t_0 the economy has a level of capital per head above its steady state value. Use a phase diagram to derive the equilibrium path for capital and consumption per unit of effective labour from time t_0 onwards.
- Suppose the economy is initially in steady state equilibrium and that at time t_1 a demographic shock reduces n permanently. Use a phase diagram to explain how the shock affects the time path of capital per unit of effective labour from time t_1 onwards.

Section B

- (20 points) State the Lucas-Sargent-Wallace Policy Ineffectiveness Proposition? What does it imply about the usefulness of macroeconomic stabilization?
- (20 points) Discuss how the interplay of the consumption smoothing and consumption tilting motives shapes the response of capital accumulation to a permanent total factor productivity shock in a Real Business Cycle model with fixed labour supply.
- (20 points) What technological features are necessary to generate endogenous growth? Discuss formally in the context of Arrow learning-by-doing model.
- (20 points) What is time inconsistency? What does it imply for the conduct of monetary policy?

END OF PAPER