'The Set of finite length Strings is countable"

CS 70 Discrete Mathematics and Probability Theory Spring 2021 DIS 7A

1 Countability and the Halting Problem

Prove the Halting Problem using the set of all programs and inputs.

a) What is a reasonable representation for a computer program? Using this definition, show that the set of all programs are countable. (*Hint: Python Code*)

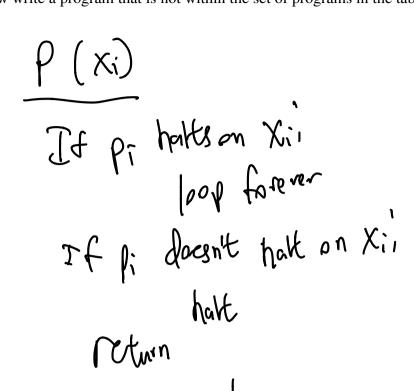
b) We consider only finite-length inputs. Show that the set of all inputs are countable.

c) Assume that you have a program that tells you whether or not a given program halts on a specific input. Since the set of all programs and the set of all inputs are countable, we can enumerate them and construct the following table.

	χ_1	x_2	x_3	<i>x</i> ₄	
p_1	H	L	Н	L	
p_2	L	L	L	Н	
<i>p</i> ₃	Н	N	H	L	
p_4	L	Н			
:	:	:	÷	1	<i>;</i>

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An H (resp. L) in the ith row and jth column means that program p_i halts (resp. loops) on input x_i . Now write a program that is not within the set of programs in the table above.



d) Find a contradiction in part a and part c to show that the halting problem can't be solved.

2 Fixed Points

Consider the problem of determining if a function F has any fixed points. That is, given a function F that takes inputs from some (possibly infinite) set \mathscr{X} , we want to know if there is any input $x \in \mathscr{X}$ such that F(x) outputs x. Prove that this problem is undecidable.

3 Hello World!

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program *P* on input *x* prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

(b)	You want to determine whether a program <i>P</i> prints "Hello World!" before running the <i>k</i> th line in the program. Is there a computer program that can perform this task? Justify your answer.
(c)	You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task? Justify your answer.