Implications, 
$$P \Rightarrow Q$$

Vacuously

F

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Stanford thinks Stanford > Berkeley

P  $\Rightarrow Q$ 

Converse

P  $\Rightarrow Q$ 

Converse

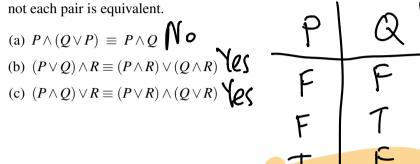
P  $\Rightarrow Q$ 

PNQ

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## 1 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.



## TTTT

## 2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

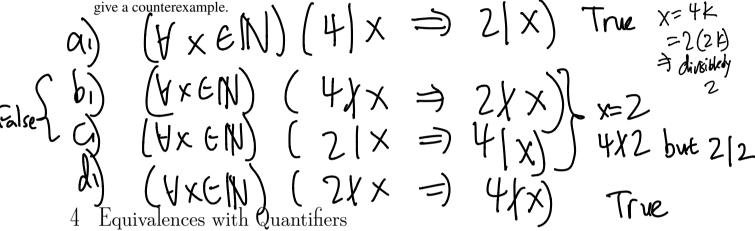
- (a) There is a real number which is not rational.
- (b) All integers are natural numbers or are negative, but not both.
- (c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.
- (d)  $(\forall x \in \mathbb{Z}) (x \in \mathbb{Q})$
- (e)  $(\forall x \in \mathbb{Z}) (((2 \mid x) \lor (3 \mid x)) \implies (6 \mid x))$
- (f)  $(\forall x \in \mathbb{N}) ((x > 7) \implies ((\exists a, b \in \mathbb{N}) (a + b = x)))$

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Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

- (a) Write the statement in propositional logic. Prove that it is true or give a counterexample.
- (b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication  $P \Longrightarrow Q$  is  $\neg P \Longrightarrow \neg Q$ .)
- (c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.



Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers.

(a) 
$$\forall x ((\exists y Q(x,y)) \Rightarrow P(x))$$
  $\forall x \exists y (Q(x,y) \Rightarrow P(x))$   $\forall x ((\exists y P(x,y)) \land (\exists y Q(x,y)))$   $\forall x ((\exists y P(x,y)) \land (\exists y Q(x,y)))$   $\forall x (P(x) \Rightarrow (\exists y Q(x,y))$ 

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 $\neg \left(\neg P(x_i y) \lor \neg Q(x_i y)\right)$ LHS; Yx Jy \n [P(x,y) =>nQ(x,y))  $= \forall x \exists y (P(x_i y) \land Q(x_i y))$ BAR, AX (JY B(XX)) V (JY B(XX)) not necesarily same value of y Not the same

$$= -\left(p(x) \wedge p(x)\right)$$

$$= -\left(p(x) \wedge p(x) \wedge \dots\right)$$

$$= -\left(p(x) \wedge p(x) \wedge \dots\right)$$