

§1 Discrete Random Variables

Random Variable	Support	$P[X = i]$	$E[X]$	$\text{Var}(X)$
Bernoulli(p) (Indicator)	$\{0, 1\}$	$P[X = 1] = p$ $P[X = 0] = 1 - p$	p	$p(1 - p)$
$\text{Bin}(n, p)$	$\{0, 1, \dots, n\}$	$\binom{n}{i} p^i (1 - p)^{n-i}$	np	$np(1 - p)$
Hypergeometric(N, B, n)	$\{0, 1, \dots, n\}$	$\frac{\binom{B}{i} \binom{N-B}{n-i}}{\binom{N}{n}}$	$n \frac{B}{N}$	$n \frac{B}{N} (1 - \frac{B}{N}) (\frac{N-n}{N-1})$
Uniform $\{1, \dots, n\}$	$\{1, 2, \dots, n\}$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n^2 - 1}{12}$
Geometric(p)	$\{1, 2, 3, \dots\}$	$(1 - p)^{i-1} p$	$\frac{1}{p}$	$\frac{1 - p}{p^2}$
Poisson(λ)	$\{0, 1, 2, \dots\}$	$\frac{\lambda^i}{i!} e^{-\lambda}$	λ	λ

§2 Continuous Random Variables

Random Variable	Support	p.d.f., $f_X(x)$	c.d.f., $P[X \leq x]$	$E[X]$	$\text{Var}(X)$
Uniform $[0, \ell]$	$[0, \ell]$	$\frac{1}{\ell}$	$\frac{x}{\ell}$	$\frac{\ell}{2}$	$\frac{\ell^2}{12}$
Exp(λ)	$[0, \infty)$	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\mathcal{N}(\mu, \sigma^2)$ (Gaussian)	$(-\infty, \infty)$	$\frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	μ	σ^2

§3 Concentration Inequalities

Theorem (Markov's Inequality)

For nonnegative finite mean random variable X ,

$$P[X \geq c] \leq \frac{E[X]}{c} \quad \forall c > 0$$

Theorem (Chebyshev's Inequality)

For finite mean random variable X ,

$$P[|X - E[X]| \geq c] \leq \frac{\text{Var}(X)}{c^2} \quad \forall c > 0$$

Theorem (Weak Law of Large Numbers)

For i.i.d. random variables X_1, X_2, \dots with common expectation $E[X_i] = \mu \forall i$,

$$P\left[\left|\frac{1}{n}(X_1 + X_2 + \dots + X_n) - \mu\right| < \varepsilon\right] \rightarrow 1 \quad \text{as } n \rightarrow \infty \quad \forall \varepsilon > 0$$