AP CALCULUS BC
Stuff you MUST Know Cold

l'Hopital's Rule

If $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $= \frac{\infty}{\infty}$,

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Average Rate of Change (slope of the secant line)

If the points $(a, f(a))$ and $(b, f(b))$ are on the graph of $f(x)$ the average rate of change of $f(x)$ on the interval $[a,b]$ is

$$\frac{f(b)-f(a)}{b-a}$$

Definition of Derivative (slope of the tangent line)

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x$$

$$\frac{d}{dx}(\csc x) = -\cot x \csc x$$

$$\frac{d}{dx}(ln\, u) = \frac{1}{u}du$$

$$\frac{d}{dx}\left(e^u\right) = e^u du$$

$$\frac{d}{dx}\left(log_a x\right) = \frac{1}{x\, ln\, a}$$

$$\frac{d}{dx}\left(a^u\right) = a^x\left(ln\, a\right)du$$

Properties of Log and Ln

1. $ln 1 = 0$
2. $ln\, e^a = a$
3. $e^{ln\, x} = x$
4. $ln\, x^n = n\, ln\, x$
5. $ln(ab) = ln\, a + ln\, b$
6. $ln\left(a/b\right) = ln\, a - ln\, b$

Differentiation Rules

Chain Rule

$$\frac{d}{dx}\left[f(u)\right] = f'(u)\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Mean Value & Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number x = c on (a, b) such

that $f'(c) = \frac{f(b)-f(a)}{b-a}$

if f(a) = f(b), then $f'(c) = 0$.

Curve sketching and analysis

y = $f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined.

local minimum: $\frac{dy}{dx}$ goes (-,0,+) or (-,und,+) or $\frac{d^2y}{dx^2} > 0$

local maximum: $\frac{dy}{dx}$ goes (+,0,-) or (+,und,-) or $\frac{d^2y}{dx^2} < 0$

Absolute Max/Min.: Compare local extreme values to values at endpoints.

pt of inflection : concavity changes.

$\frac{d^2y}{dx^2}$ goes (+,0,-),(-,0,+), (+,und,-), or (-,und,+)

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$

2nd Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_{\#}^{g(x)} f(x)dx = f(g(x)) \cdot g'(x)$$

Average Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exist on the interval (a, b), then there exists a number x = c on (a, b) such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

$f(c)$ is the average value

Euler's Method

If given that $\frac{dy}{dx} = f(x, y)$ and

that the solution passes through

(x_0, y_0), then

$$x_{new} = x_{old} + \Delta x$$

$$y_{new} = y_{old} + \left.\frac{dy}{dx}\right|_{(x_{old}, y_{old})} \cdot \Delta x$$

Logistics Curves

$P(t) = \frac{L}{1+Ce^{-(Lk)t}}$,

where L is carrying capacity

Maximum growth rate occurs when P = ½ L

$\frac{dP}{dt} = kP(L-P)$ or

$\frac{dP}{dt} = (Lk)P(1-\frac{P}{L})$

Integrals

$\int kf(u)du = k\int f(u)du$

$\int du = u + C$

$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

$\int \frac{1}{u}du = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$

$\int \cos u\,du = \sin u + C$

$\int \sin u\,du = -\cos u + C$

$\int \tan u\,du = -\ln|\cos u| + C$

$\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \csc u\,du = -\ln|\csc u + \cot u| + C$

$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec\left(\frac{|u|}{a}\right) + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$

$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Arc Length

For a function, $f(x)$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

For a polar graph, r(θ)

$$L = \int_{\theta 1}^{\theta 2} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2}\,d\theta$$

Lagrange Error Bound

If $P_n(x)$ is the nth degree Taylor polynomial of $f(x)$ about c, then

$$\left|f(x) - P_n(x)\right| \leq \frac{\max\left|f^{(n+1)}(z)\right|}{(n+1)!}\left|x - c\right|^{n+1}$$

for all z between x and c.

Distance, velocity and Acceleration

Velocity = $\frac{d}{dt}(position)$

Acceleration = $\frac{d}{dt}(velocity)$

Velocity Vector = $\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$

Speed = $|v(t)| = \sqrt{(x')^2 + (y')^2}$.

Distance Traveled =

$$\int_{\text{initial time}}^{\text{final time}} |v(t)|\,dt = \int_{\text{initial time}}^{\text{final time}} \sqrt{(x')^2 + (y')^2}\,dt$$

$x(b) = x(a) + \int_a^b x'(t)dt$

$y(b) = y(a) + \int_a^b y'(t)dt$

Polar Curves

For a polar curve r(θ), the

Area inside a "leaf" is $\frac{1}{2}\int_{\theta 1}^{\theta 2} [r(\theta)]^2\,d\theta$

where θ1 and θ2 are the "first" two times that $r = 0$.

The slope of $r(\theta)$ at a given θ is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}[r(\theta)\sin\theta]}{\frac{d}{d\theta}[r(\theta)\cos\theta]}$$

Ratio Test
(use for interval of convergence)

The series $\sum_{n=0}^{\infty} a_n$ converges if

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

CHECK ENDPOINTS

Alternating Series Error Bound

If $S_N = \sum_{n=1}^{N} (-1)^n a_n$ is the N^th^ partial sum of a convergent alternating series, then

$$\left|S_\infty - S_N\right| \leq \left|a_{N+1}\right|$$

Volume

Solids of Revolution

Disk Method: $V = \pi\int_a^b [R(x)]^2\,dx$

Washer Method:

$$V = \pi\int_a^b \left([R(x)]^2 - [r(x)]^2\right)dx$$

Shell Method: $V = 2\pi\int_a^b r(x)h(x)dx$

Volume of Known Cross Sections

Perpendicular to

x-axis: $V = \int_a^b A(x)dx$

y-axis: $V = \int_c^d A(y)dy$

Taylor Series

If the function f is "smooth" at x = c, then it can be approximated by the nth degree polynomial

$$f(x) \approx f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f'''(c)}{3!}(x-c)^3 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Elementary Functions

Centered at x = 0

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$

$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

Most Common Series

$\sum_{n=1}^{\infty}\frac{1}{n}$ diverges $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges $\sum_{n=0}^{\infty}A(r)^n$ converges to $\frac{A}{1-r}$ if |r|<1