

12.1 Food manufacture 1

A food is manufactured by refining raw oils and blending them together. The raw oils are of two categories:

Vegetable oils	VEG 1
	VEG 2
Non-vegetable oils	OIL 1
	OIL 2
	OIL 3

Each oil may be purchased for immediate delivery (January) or bought on the futures market for delivery in a subsequent month. Prices at present and in the futures market are given below in (£/ton):

	VEG 1	VEG 2	OIL 1	OIL 2	OIL 3
January	110	120	130	110	115
February	130	130	110	90	115
March	110	140	130	100	95
April	120	110	120	120	125
May	100	120	150	110	105
June	90	100	140	80	135

The final product sells at £150 per ton.

Vegetable oils and non-vegetable oils require different production lines for refining. In any month, it is not possible to refine more than 200 tons of vegetable oils and more than 250 tons of non-vegetable oils. There is no loss of weight in the refining process, and the cost of refining may be ignored.

It is possible to store up to 1000 tons of each raw oil for use later. The cost of storage for vegetable and non-vegetable oil is £5 per ton per month. The final product cannot be stored, nor can refined oils be stored.

There is a technological restriction of hardness on the final product. In the units in which hardness is measured, this must lie between 3 and 6. It is assumed that hardness blends linearly and that the hardnesses of the raw oils are

VEG 1	8.8
VEG 2	6.1
OIL 1	2.0
OIL 2	4.2
OIL 3	5.0

What buying and manufacturing policy should the company pursue in order to maximise profit?

At present, there are 500 tons of each type of raw oil in storage. It is required that these stocks will also exist at the end of June.

This problem, and the subsequent problem, is based on a larger model built for the margarine producer Van den Berghs and Jurgens and discussed in Williams and Redwood (1974).

12.2 Food manufacture 2

It is wished to impose the following extra conditions on the food manufacture problem:

1. The food may never be made up of more than three oils in any month.
2. If an oil is used in a month, at least 20 tons must be used.
3. If either of VEG 1 or VEG 2 are used in a month then OIL 3 must also be used.

Extend the food manufacture model to encompass these restrictions and find the new optimal solution.

12.6 Refinery optimisation

An oil refinery purchases two crude oils (crude 1 and crude 2). These crude oils are put through four processes: distillation, reforming, cracking and blending, to produce petrols and fuels that are sold.

12.6.1 Distillation

Distillation separates each crude oil into fractions known as *light naphtha*, *medium naphtha*, *heavy naphtha*, *light oil*, *heavy oil* and *residuum* according to

their boiling points. Light, medium and heavy naphthas have octane numbers of 90, 80 and 70, respectively. The fractions into which one barrel of each type of crude splits are given in the following table:

	Light naphtha	Medium naphtha	Heavy naphtha	Light oil	Heavy oil	Residuum
Crude 1	0.1	0.2	0.2	0.12	0.2	0.13
Crude 2	0.15	0.25	0.18	0.08	0.19	0.12

N.B. There is a small amount of wastage in distillation.

12.6.2 Reforming

The naphthas can be used immediately for blending into different grades of petrol or can go through a process known as *reforming*. Reforming produces a product known as *reformed gasoline* with an octane number of 115. The yields of reformed gasoline from each barrel of the different naphthas are given as follows:

- 1 barrel of light naphtha yields 0.6 barrels of reformed gasoline;
- 1 barrel of medium naphtha yields 0.52 barrels of reformed gasoline;
- 1 barrel of heavy naphtha yields 0.45 barrels of reformed gasoline.

12.6.3 Cracking

The oils (light and heavy) can either be used directly for blending into *jet fuel* or *fuel oil* or be put through a process known as *catalytic cracking*. The catalytic cracker produces *cracked oil* and *cracked gasoline*. Cracked gasoline has an octane number of 105.

- 1 barrel of light oil yields 0.68 barrels of cracked oil and 0.28 barrels of cracked gasoline;
- 1 barrel of heavy oil yields 0.75 barrels of cracked oil and 0.2 barrels of cracked gasoline.

Cracked oil is used for blending *fuel oil* and *jet fuel*; cracked gasoline is used for blending *petrol*.

Residuum can be used for either producing *lube-oil* or blending into *jet fuel* and *fuel oil*:

- 1 barrel of residuum yields 0.5 barrels of lube-oil.

12.6.4 Blending

12.6.4.1 Petrols (motor fuel)

There are two sorts of petrol, *regular* and *premium*, obtained by blending the naphtha, reformed gasoline and cracked gasoline. The only stipulations concerning them are that regular must have an octane number of at least 84 and that premium must have an octane number of at least 94. It is assumed that octane numbers blend linearly by volume.

12.6.4.2 Jet fuel

The stipulation concerning jet fuel is that its vapour pressure must not exceed 1 kg cm^2 . The vapour pressures for light, heavy, cracked oils and residuum are 1.0, 0.6, 1.5 and 0.05 kg cm^2 , respectively. It may again be assumed that vapour pressures blend linearly by volume.

12.6.4.3 Fuel oil

To produce fuel oil, light oil, cracked oil, heavy oil and residuum must be blended in the ratio 10:4:3:1.

There are availability and capacity limitations on the quantities and processes used as follows:

1. The daily availability of crude 1 is 20 000 barrels.
2. The daily availability of crude 2 is 30 000 barrels.
3. At most 45 000 barrels of crude can be distilled per day.
4. At most 10 000 barrels of naphtha can be reformed per day.
5. At most 8000 barrels of oil can be cracked per day.
6. The daily production of lube oil must be between 500 and 1000 barrels.
7. Premium motor fuel production must be at least 40% of regular motor fuel production.

The profit contributions from the sale of the final products are (in pence per barrel) as follows:

Premium petrol	700
Regular petrol	600
Jet fuel	400
Fuel oil	350
Lube-oil	150

How should the operations of the refinery be planned in order to maximise total profit?

12.13 Market sharing

A large company has two divisions, D1 and D2. The company supplies retailers with oil and spirit. This is a much smaller version of the problem British Petroleum and Shell faced when they were forced to demerge – one of the largest demergers in history. The original model proved impossible to solve in 1972.

It is desired to allocate each retailer to either division D1 or division D2. This division will be the retailer's supplier. As far as possible, this division must be made so that D1 controls 40% of the market and D2 the remaining 60%. The retailers are listed below as M1 to M23. Each retailer has an estimated market for oil and spirit. Retailers M1 to M8 are in region 1; retailers M9 to M18 are in region 2 and retailers M19 to M23 are in region 3. Certain retailers are considered to have good growth prospects and categorised as group A and the others are in group B. Each retailer has a certain number of delivery points as given below. It is desired to make the 40/60 split between D1 and D2 in each of the following respects:

1. Total number of delivery points
2. Control of spirit market
3. Control of oil market in region 1
4. Control of oil market in region 2
5. Control of oil market in region 3
6. Number of retailers in group A
7. Number of retailers in group B.

There is a certain flexibility in that any share may vary by $\pm 5\%$. That is, the share can vary between the limits 35/65 and 45/55.

The primary aim is to find a feasible solution. If, however, there is some choice then possible objectives are (i) to minimise the sum of the percentage deviations from the 40/60 split and (ii) to minimise the maximum such deviation.

Build a model to see if the problem has a feasible solution and if so find the optimal solutions.

The numerical data are given in Table 12.5.

Table 12.5

	Retailer	Oil market (10^6 gallons)	Delivery points	Spirit market (10^6 gallons)	Growth category
Region 1	M1	9	11	34	A
	M2	13	47	411	A
	M3	14	44	82	A
	M4	17	25	157	B
	M5	18	10	5	A
	M6	19	26	183	A
	M7	23	26	14	B
	M8	21	54	215	B
Region 2	M9	9	18	102	B
	M10	11	51	21	A
	M11	17	20	54	B
	M12	18	105	0	B
	M13	18	7	6	B
	M14	17	16	96	B
	M15	22	34	118	A
	M16	24	100	112	B
	M17	36	50	535	B
	M18	43	21	8	B
Region 3	M19	6	11	53	B
	M20	15	19	28	A
	M21	15	14	69	B
	M22	25	10	65	B
	M23	39	11	27	B

12.15 Tariff rates (power generation)

A number of power stations are committed to meeting the following electricity load demands over a day:

12 p.m. to 6 a.m.	15 000 MW
6 a.m. to 9 a.m.	30 000 MW
9 a.m. to 3 p.m.	25 000 MW
3 p.m. to 6 p.m.	40 000 MW
6 p.m. to 12 p.m.	27 000 MW

There are three types of generating unit available: 12 of type 1, 10 of type 2 and five of type 3. Each generator has to work between a minimum and a maximum level. There is an hourly cost of running each generator at minimum level. In addition, there is an extra hourly cost for each megawatt at which a unit is operated above the minimum level. Starting up a generator also involves a cost. All this information is given in Table 12.6 (with costs in £).

In addition to meeting the estimated load demands there must be sufficient generators working at any time to make it possible to meet an increase in load of up to 15%. This increase would have to be accomplished by adjusting the output of generators already operating within their permitted limits.

Table 12.6

	Minimum level	Maximum level	Cost per hour at minimum	Cost per hour per megawatt above minimum	Cost
Type 1	850 MW	2000 MW	1000	2	2000
Type 2	1250 MW	1750 MW	2600	1.30	1000
Type 3	1500 MW	4000 MW	3000	3	500

Which generators should be working in which periods of the day to minimise total cost?

What is the marginal cost of production of electricity in each period of the day; that is, what tariffs should be charged?

What would be the saving of lowering the 15% reserve output guarantee; that is, what does this security of supply guarantee cost?

12.16 Hydro power

This is an extension of the Tariff Rates (Power Generation) problem of Section 12.15. In addition to the thermal generators, a reservoir powers two hydro generators: one of type A and one of type B. When a hydro generator is running, it operates at a fixed level and the depth of the reservoir decreases. The costs associated with each hydro generator are a fixed start-up cost and a running cost per hour. The characteristics of each type of generator are shown in Table 12.7.

For environmental reasons, the reservoir must be maintained at a depth of between 15 and 20 m. Also, at midnight each night, the reservoir must be 16 m deep. Thermal generators can be used to pump water into the reservoir. To increase the level of the reservoir by 1 m, it requires 3000 MWh of electricity. You may assume that rainfall does not affect the reservoir level.

At any time, it must be possible to meet an increase in demand for electricity of up to 15%. This can be achieved by any combination of the following: switching on a hydro generator (even if this would cause the reservoir depth to fall below 15 m); using the output of a thermal generator, which is used for pumping water into the reservoir; and increasing the operating level of a thermal generator to its maximum. Thermal generators cannot be switched on instantaneously to meet increased demand (although hydro generators can be).

Table 12.7

	Operating level	Cost per hour	Reservoir depth reduction per hour (m)	Start-up cost
Hydro A	900 MW	£90	0.31	£1500
Hydro B	1400 MW	£150	0.47	£1200

Which generators should be working in which periods of the day, and how should the reservoir be maintained to minimise the total cost?

12.23 Milk collection

A small milk processing company is committed to collecting milk from 20 farms and taking it back to the depot for processing. The company has one tanker lorry with a capacity for carrying 80000 litres of milk. Eleven of the farms are small and need a collection only every other day. The other nine farms need a collection every day. The positions of the farms in relation to the depot (numbered 1) are given in Table 12.16 together with their collection requirements.

Find the optimal route for the tanker lorry on each day, bearing in mind that it has to (i) visit all the 'every day' farms, (ii) visit some of the 'every other day' farms and (iii) work within its capacity. On alternate days, it must again visit the 'every day' farms and also visit the 'every other day' farms not visited on the previous day.

For convenience, a map of the area considered is given in Figure 12.7.

Table 12.15

Garage		Inputs						Outputs		Profit (millions)
		Staff	Show room space (100 m ²)	Population in category 1 (1000 s)	Population in category 2 (1000s)	Enquiries Alpha model (100 s)	Enquiries Beta model (100 s)	Alpha sales (1000 s)	Beta sales (1000s)	
1	Winchester	7	8	10	12	8.5	4	2	0.6	1.5
2	Andover	6	6	20	30	9	4.5	2.3	0.7	1.6
3	Basingstoke	2	3	40	40	2	1.5	0.8	0.25	0.5
4	Poole	14	9	20	25	10	6	2.6	0.86	1.9
5	Woking	10	9	10	10	11	5	2.4	1	2
6	Newbury	24	15	15	13	25	1.9	8	2.6	4.5
7	Portsmouth	6	7	50	40	8.5	3	2.5	0.9	1.6
8	Alresford	8	7.5	5	8	9	4	2.1	0.85	2
9	Salisbury	5	5	10	10	5	2.5	2	0.65	0.9
10	Guildford	8	10	30	35	9.5	4.5	2.05	0.75	1.7
11	Alton	7	8	7	8	3	2	1.9	0.70	0.5
12	Weybridge	5	6.5	9	12	8	4.5	1.8	0.63	1.4
13	Dorchester	6	7.5	10	10	7.5	4	1.5	0.45	1.45
14	Bridport	11	8	8	10	10	6	2.2	0.65	2.2
15	Weymouth	4	5	10	10	7.5	3.5	1.8	0.62	1.6
16	Portland	3	3.5	3	20	2	1.5	0.9	0.35	0.5
17	Chichester	5	5.5	8	10	7	3.5	1.2	0.45	1.3
18	Petersfield	21	12	6	6	15	8	6	0.25	2.9
19	Petworth	6	5.5	2	2	8	5	1.5	0.55	1.55

(continued overleaf)

Table 12.15 (continued)

Garage		Inputs						Outputs		
		Staff	Show room space (100 m ²)	Population in category 1 (1000 s)	Population in category 2 (1000 s)	Enquiries Alpha model (100 s)	Enquiries Beta model (100 s)	Alpha sales (1000 s)	Beta sales (1000 s)	Profit (millions)
20	Midhurst	3	3.6	3	3	2.5	1.5	0.8	0.20	0.45
21	Reading	30	29	120	80	35	20	7	2.5	8
22	Southampton	25	16	110	80	27	12	6.5	3.5	5.4
23	Bournemouth	19	10	90	22	25	13	5.5	3.1	4.5
24	Henley	7	6	5	7	8.5	4.5	1.2	0.48	2
25	Maidenhead	12	8	7	10	12	7	4.5	2	2.3
26	Fareham	4	6	1	1	7.5	3.5	1.1	0.48	1.7
27	Romsey	2	2.5	1	1	2.5	1	0.4	0.1	0.55
28	Ringwood	2	3.5	2	2	1.9	1.2	0.3	0.09	0.4

Table 12.16

Farm	Position 10 miles		Collection frequency	Collection requirement (1000 l)
	East	North		
1 (Depot)	0	0	—	—
2	−3	3	Every day	5
3	1	11	Every day	4
4	4	7	Every day	3
5	−5	9	Every day	6
6	−5	−2	Every day	7
7	−4	−7	Every day	3
8	6	0	Every day	4
9	3	−6	Every day	6
10	−1	−3	Every day	5
11	0	−6	Every other day	4
12	6	4	Every other day	7
13	2	5	Every other day	3
14	−2	8	Every other day	4
15	6	10	Every other day	5
16	1	8	Every other day	6
17	−3	1	Every other day	8
18	−6	5	Every other day	5
19	2	9	Every other day	7
20	−6	−5	Every other day	6
21	5	−4	Every other day	6

