UJ1 UNIVERSITAT JAUME I

Flexible-Size Batched LU for Small Matrices and its Integration into Block-Jacobi Preconditioning
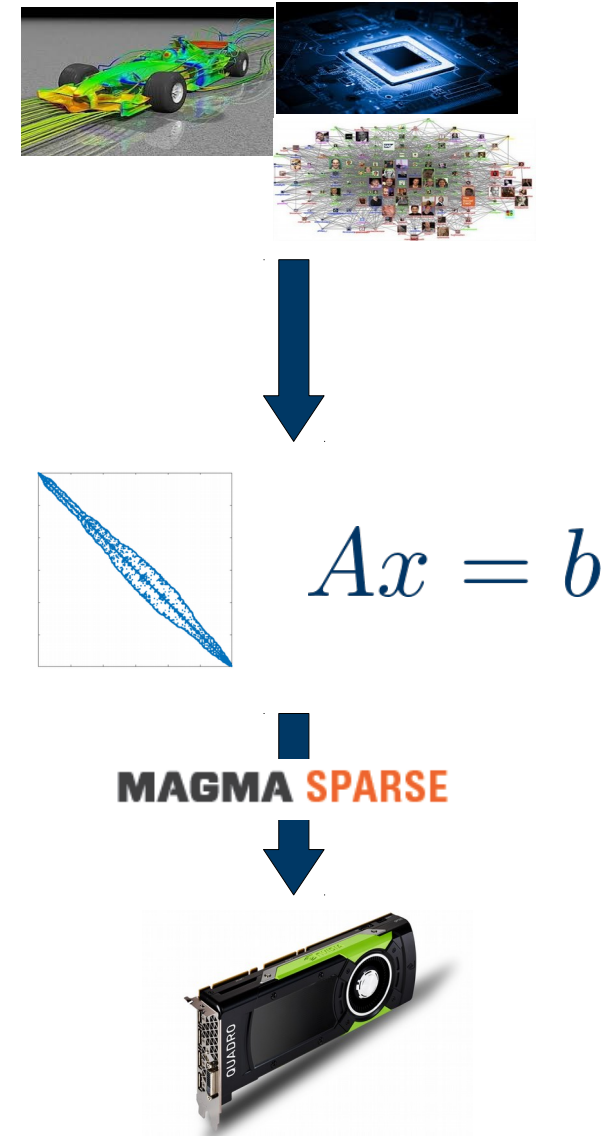
Hartwig Anzt, Jack Dongarra, Goran Flegar, Enrique S. Quintana-Ortí

Scan me for slides!

github.com/gflegar/talks/icpp_2017

flegar@uji.es

MAGMA-sparse software library

- GPU-accelerated sparse linear algebra library
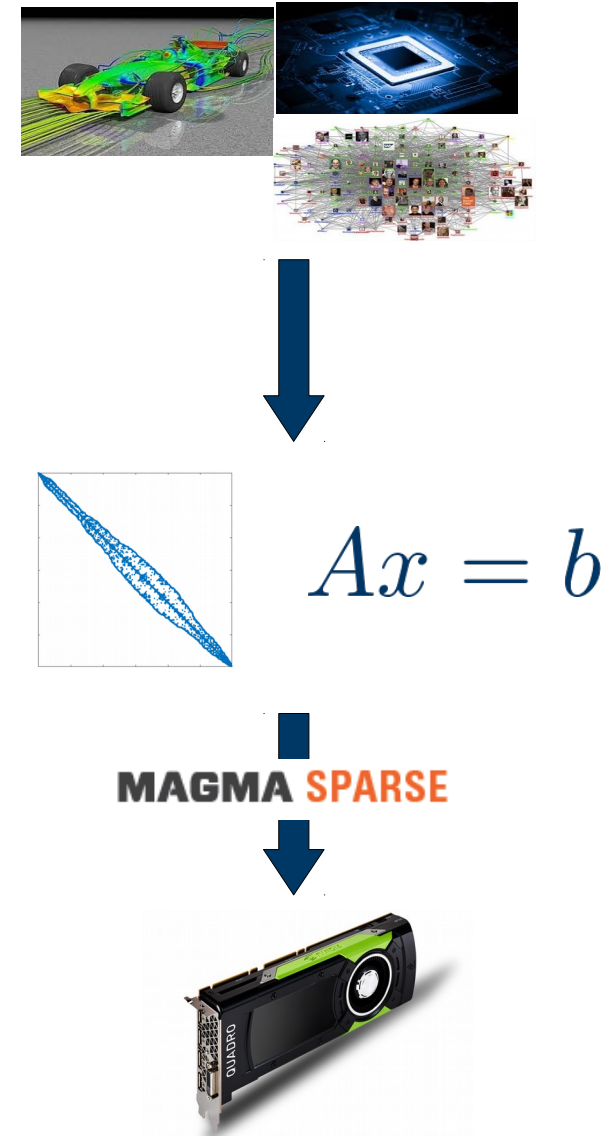
  – Focus: linear systems

$$Ax = b$$

MAGMA SPARSE

Joint effort: Innovative Computing Lab at University of Tennessee, Knoxvile; Karlsruhe Institute of Technology; University Jaume I

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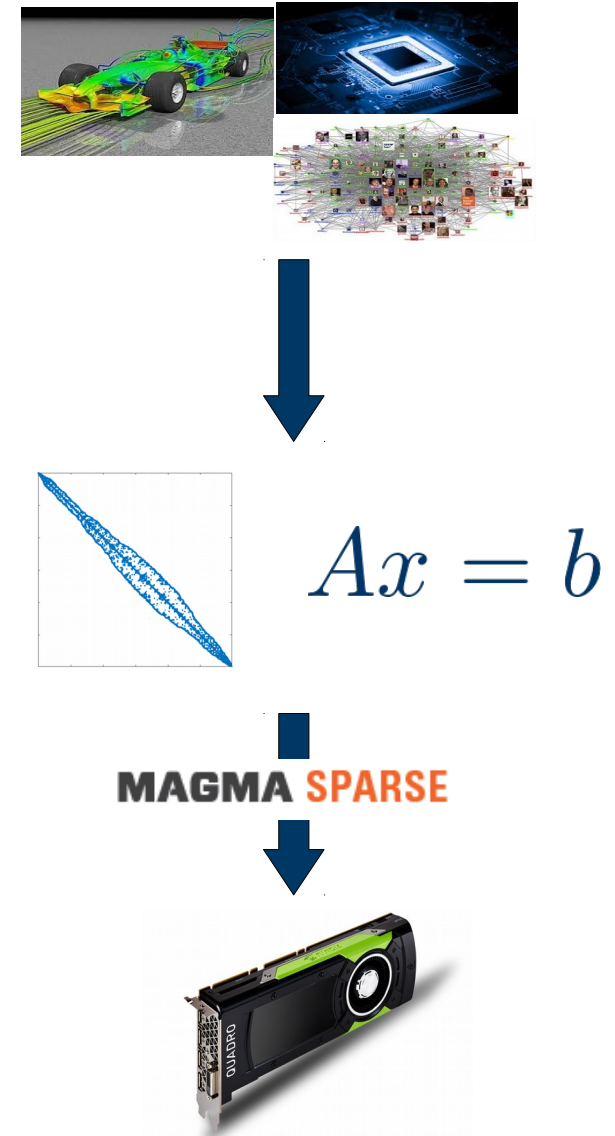
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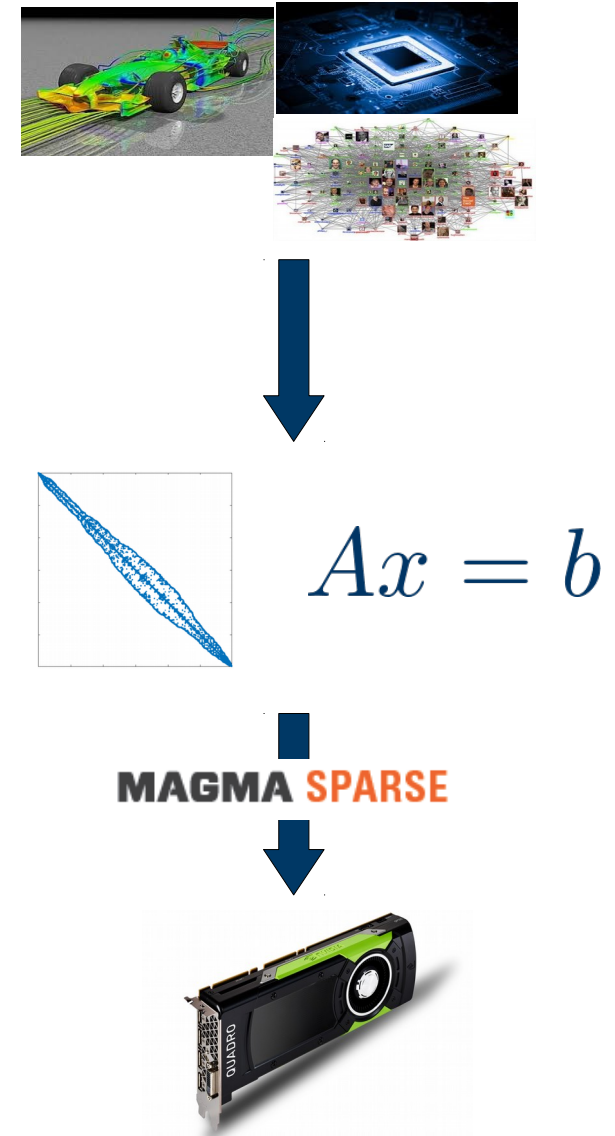# MAGMA-sparse software library

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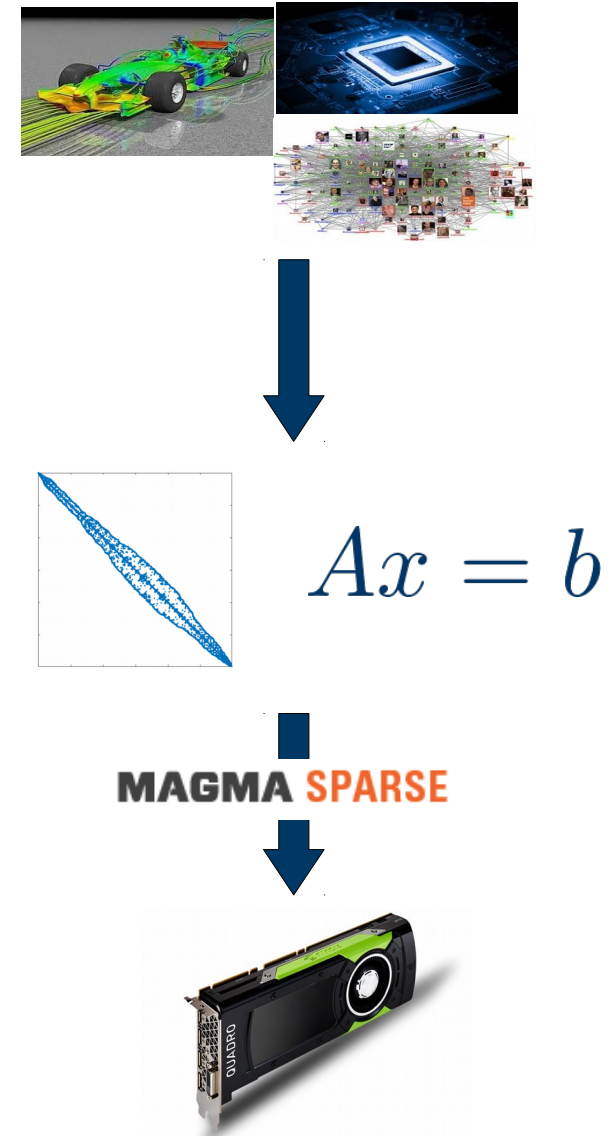
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Preconditioning

$$Ax = b, \quad A \in \mathbb{R}^{n \times n}$$

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Trade-off:
faster convergence,
but more work per iteration

Block-Jacobi Preconditioning

- Current focus: improve performance for problems with inherent block structure
  - Usually blocks are smaller than 30x30 (blocks can be of different sizes!)

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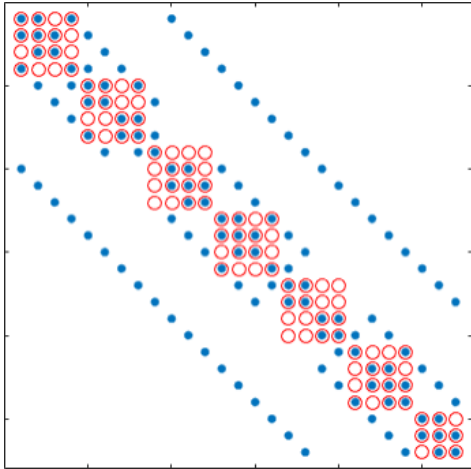 - Use only diagonal blocks for approximation

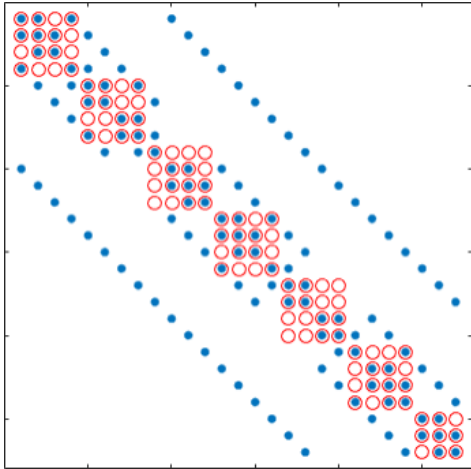$$\mathrm{diag}(A) = [D_1, \ldots, D_k]$$
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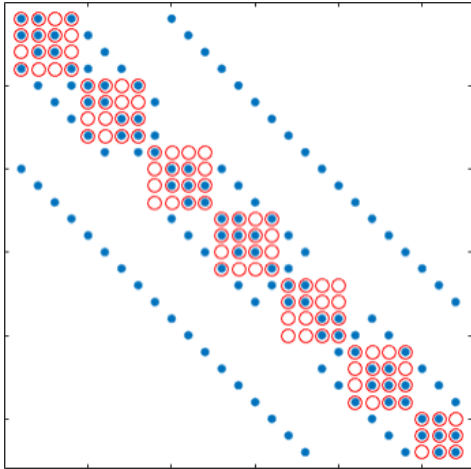
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$$D_i = L_i U_i \quad \textbf{Setup}$$

$$\begin{array}{l} U_i y_i = w_i \\ L_i w_i = z_i \end{array} \quad \textbf{Application}$$

Block-Jacobi Preconditioning

Solving 30-by-30 systems in sequence on
a GPU with several thousand cores
wastes computational resources!

Batched routines

Launch a single kernel which applies an operation to multiple independent data entities in parallel.
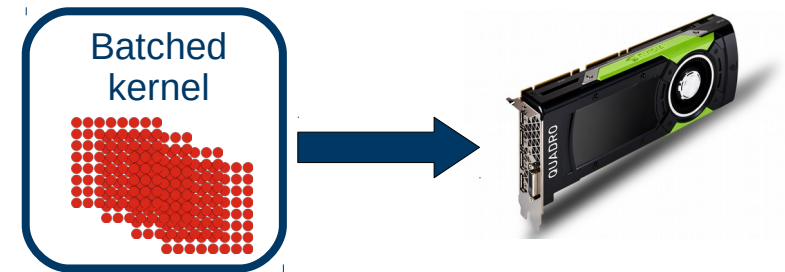
Batched kernel

* Measured on NVIDIA P100

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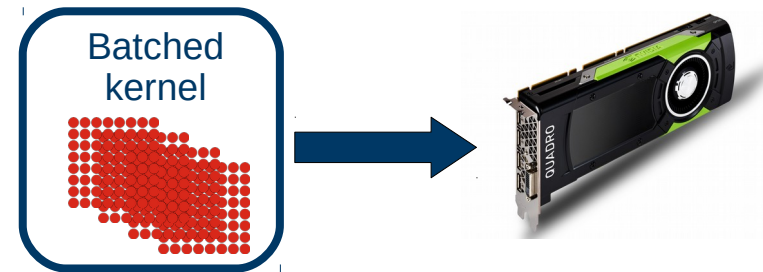
- There is no standard BLAS & LAPACK interface

- Most implementations only support problems of equal sizes

- High performance libraries are not optimized for small blocks

  - cuBLAS batched trsv: ~25 Gflop/s *

  - MAGMA-sparse SpMV: 60 – 90 Gflop/s *

Measured on NVIDIA P100

MAGMA-sparse LU & trsv outline

- Assign one warp to each problem

  - hardware SIMD unit, represented as a group of 32 threads in CUDA

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  – Able to support problems of size up to 32-by-32
  – keep the entire row in thread's registers
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  – Current implementation: use padding for problems of smaller sizes
  – Future work: multiple smaller problems per warp

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- Use implicit pivoting
  - Do not explicitly swap rows, "re-assign" the threads instead
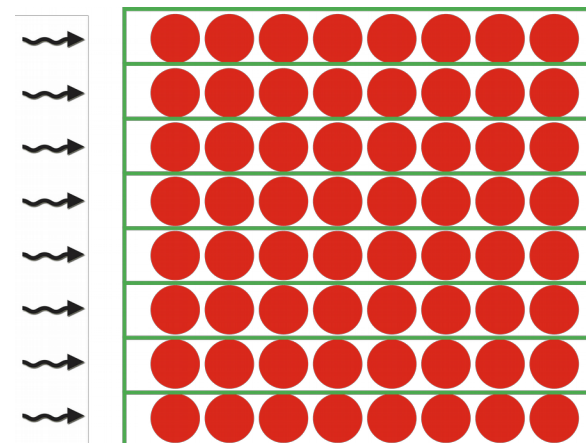
MAGMA-sparse LU & trsv outline

- Use "eager" triangular solves
  - Cast solution vector updates in terms of axpy, not in terms of dot product!

"Lazy" triangular solve
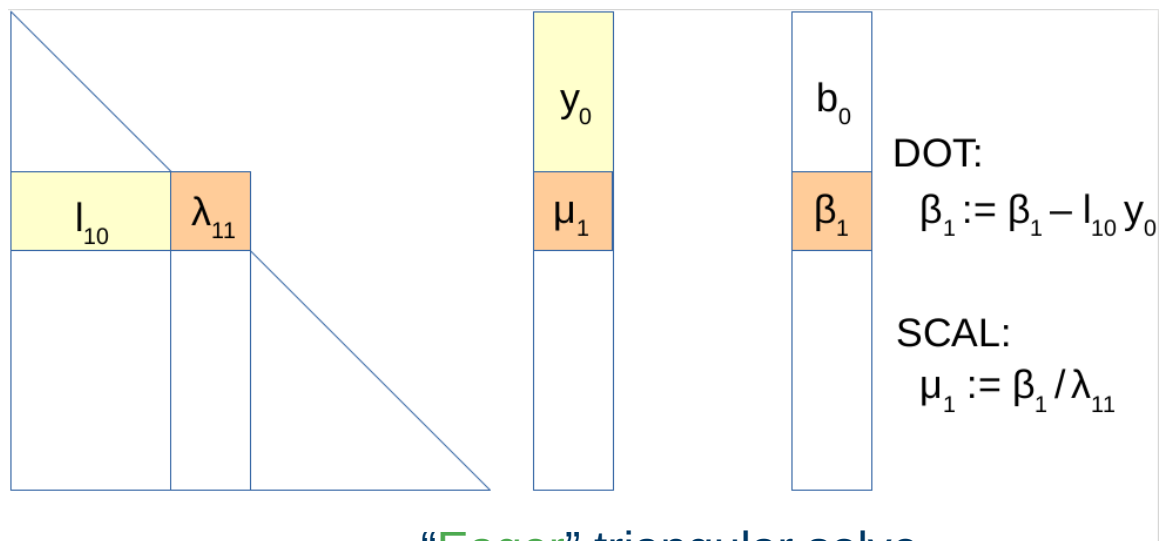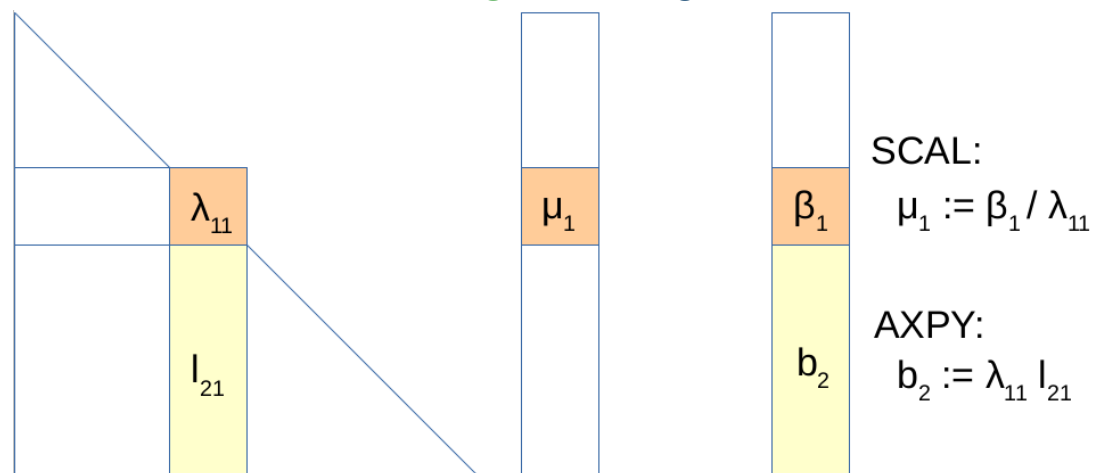
$$\begin{array}{c} l_{10} \quad \lambda_{11} \end{array}$$

y_0

μ_1

b_0

β_1

DOT:
$$\beta_1 := \beta_1 - l_{10}\, y_0$$

SCAL:
$$\mu_1 := \beta_1 / \lambda_{11}$$

"Eager" triangular solve

λ_{11}

l_{21}

μ_1

β_1

b_2

SCAL:
$$\mu_1 := \beta_1 / \lambda_{11}$$

AXPY:
$$b_2 := \lambda_{11}\, l_{21}$$

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LU decomposition performance

- Comparisson of MAGMA-sparse vs cuBLAS batched LU decomposition

- Gauss-Huard(-T) is a similar approach, using a different algorithm for decomposition/solves *
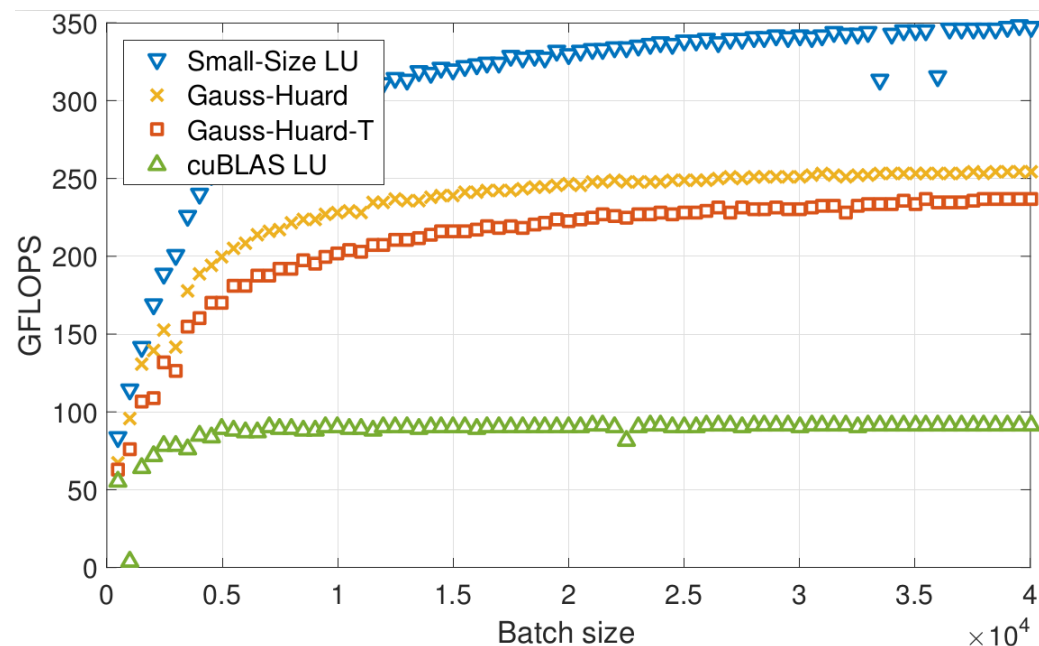
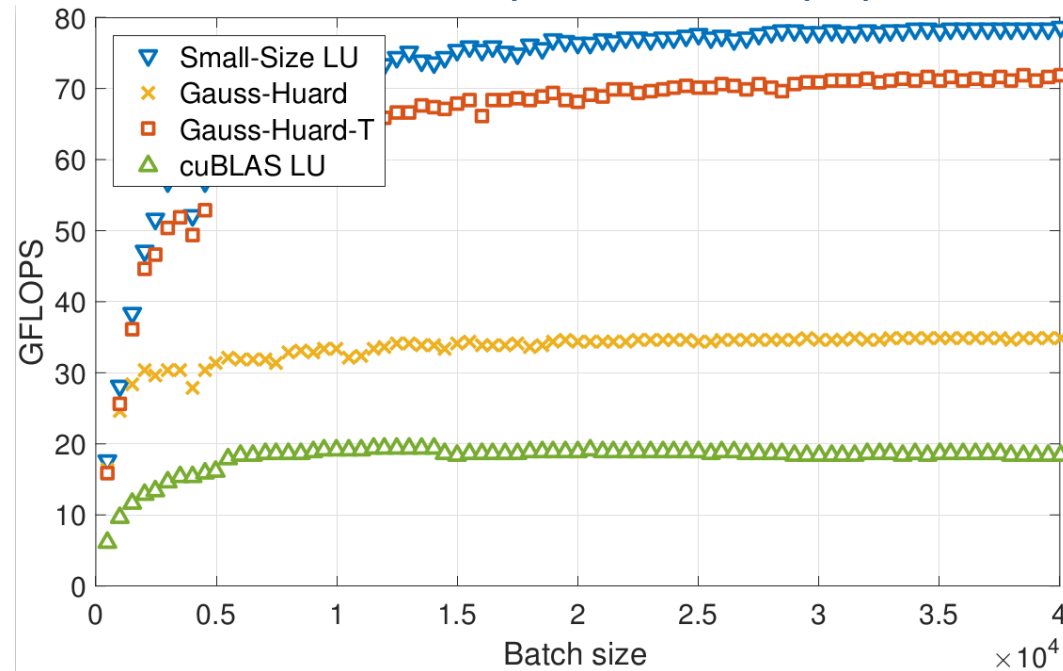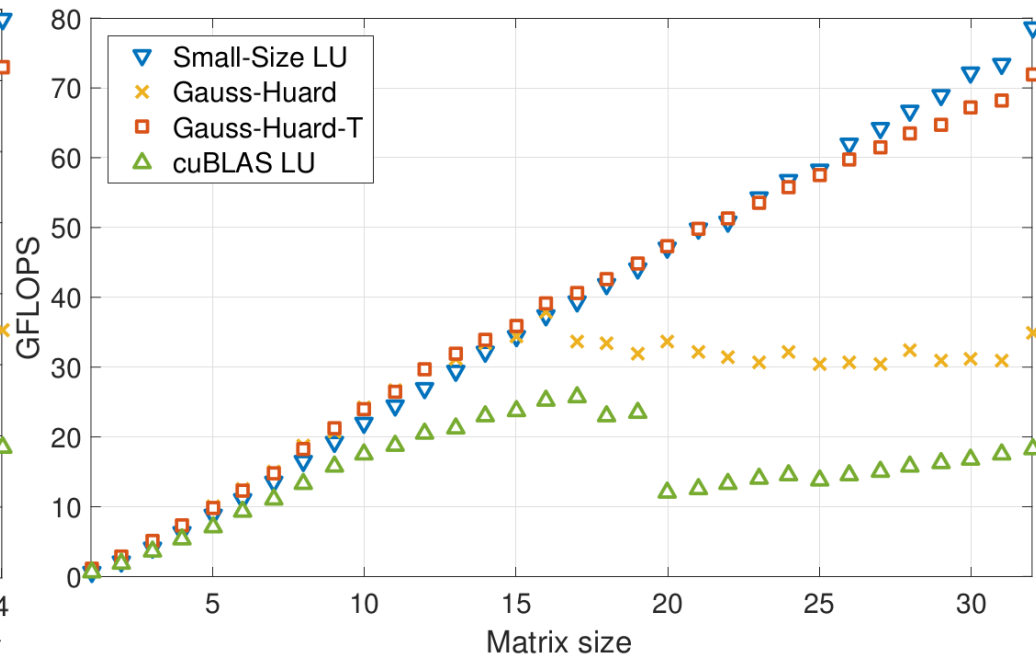Fixed problem size (32)　　　　　　　　　　Fixed batch size (40,000)
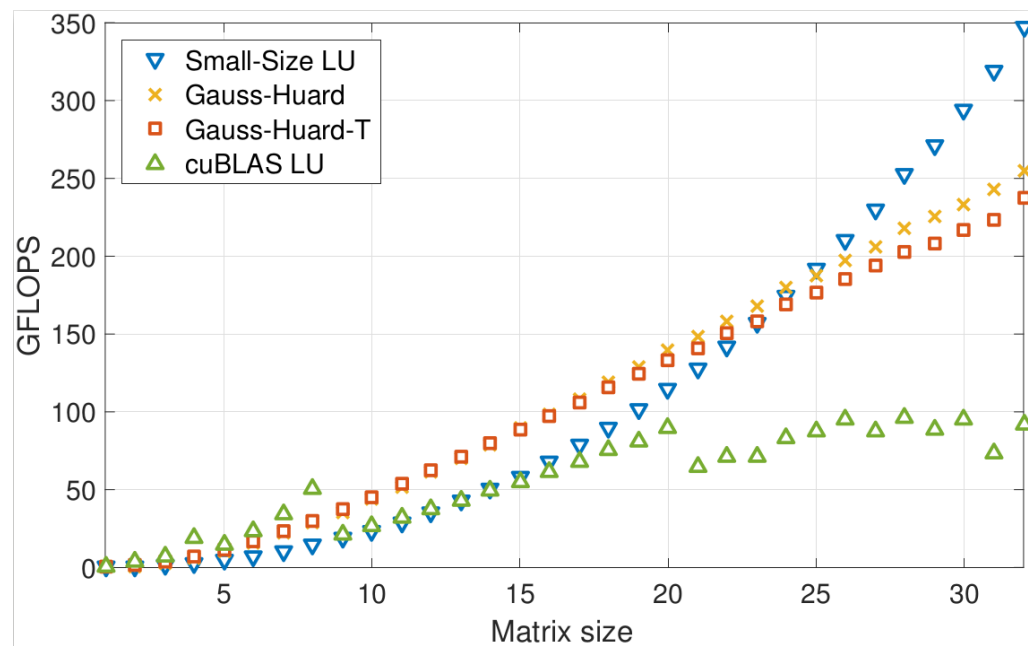
Anzt et al., Variable-Size Batched Gauss-Huard for Block-Jacobi Preconditioning, ICCS'17

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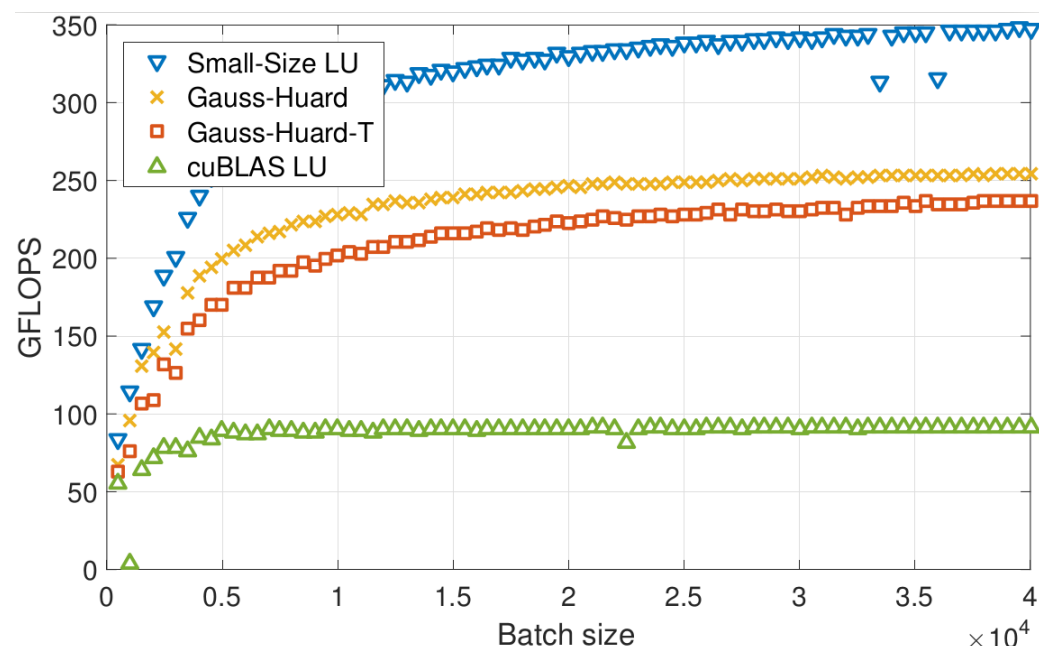
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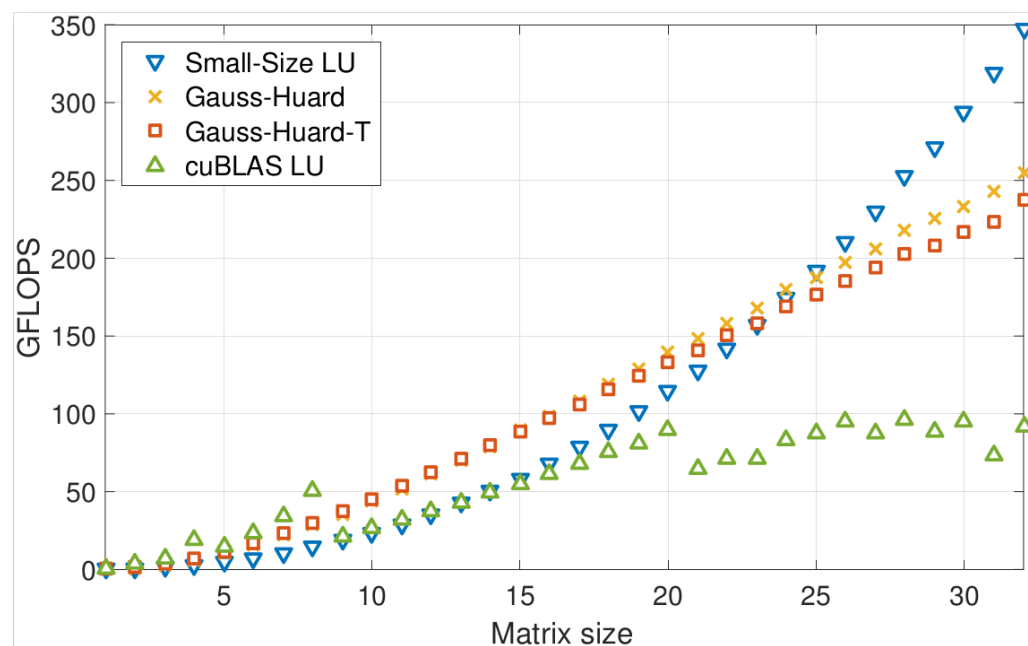
MAGMA-sparse LU can also:
- Handle problems of different sizes
- Integrate diagonal block extraction and diagonal block decomposition into a single kernel

Anzt et al., Variable-Size Batched Gauss-Huard for Block-Jacobi Preconditioning, ICCS'17

Triangular solve performance

- Comparisson of MAGMA-sparse vs cuBLAS batched triangular solves

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Fixed problem size (32)

Fixed batch size (40,000)

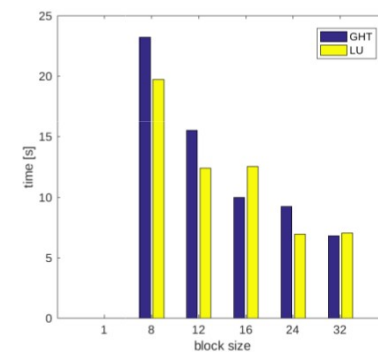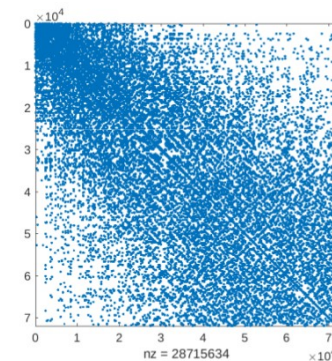# Complete solver runtime
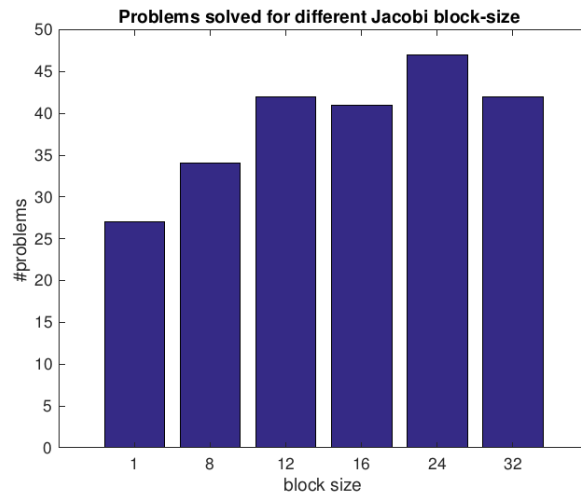
56 problems from SuitSparse

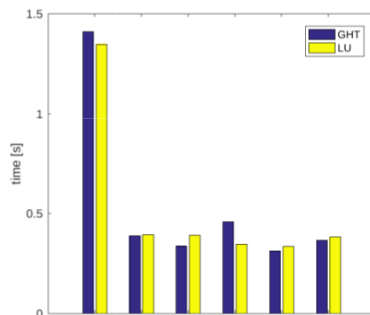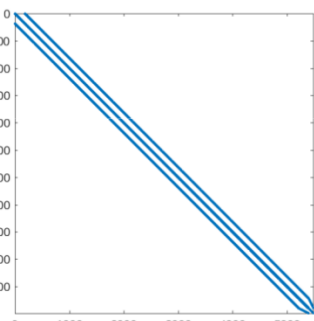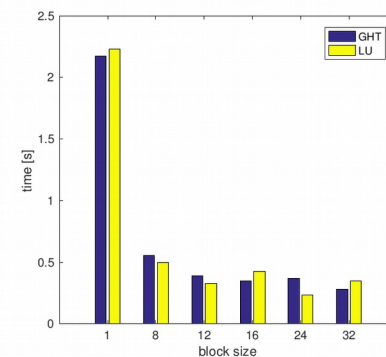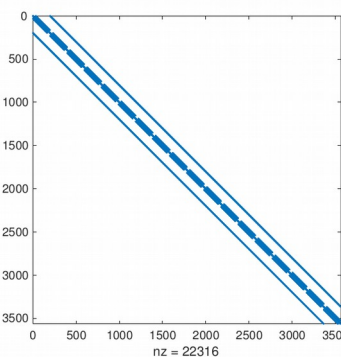Problems solved for different Jacobi block-size

Complete solver runtime

56 problems from SuitSparse
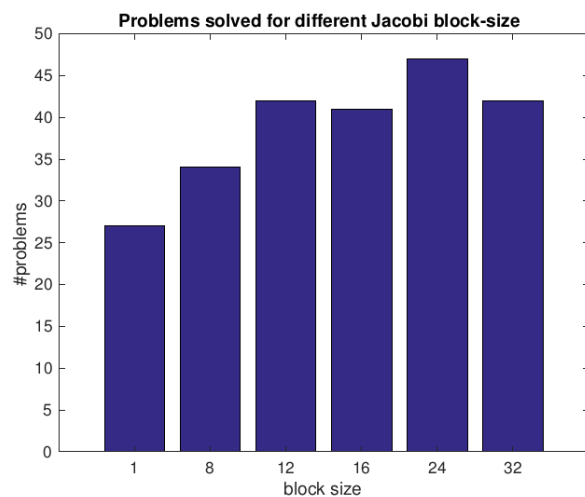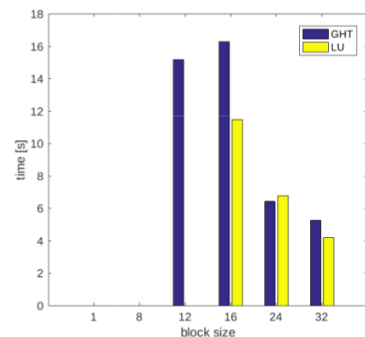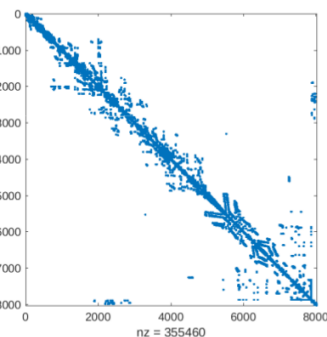
Conclusion

"Solving many small problems in sequence on manycore hardware wastes computational resources! We can design batched routines which apply the same operation on a set of independent data entities."

Workshop on batched BLAS*
- Effort to standardize the batched BLAS interface
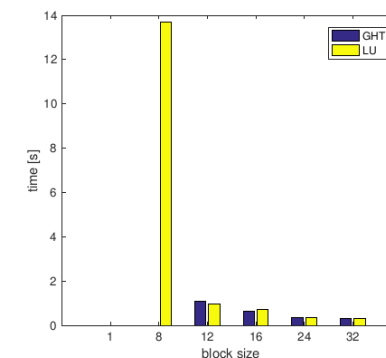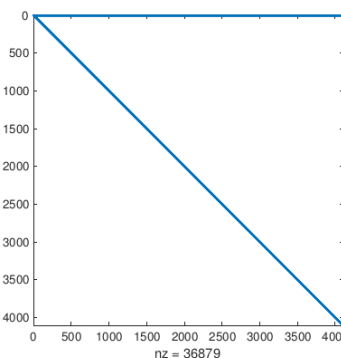
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- Or should we have more routines?

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- Global synchronization between two batched calls.

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Instead provide BLAS which operates on a part of memory/core hierarchy?
- E.g. block, warp, thread level BLAS for CUDA.
- Let users build their own batched routines from these building blocks.

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Thank you! Questions?

All functionalities are part of the MAGMA-sparse project.

MAGMA SPARSE

ROUTINES	BiCG, BiCGSTAB, Block-Asynchronous Jacobi, CG, CGS, GMRES, IDR, Iterative refinement, LOBPCG, LSQR, QMR, TFQMR
PRECONDITIONERS	ILU / IC, Jacobi, ParILU, ParILUT, Block Jacobi, ISAI
KERNELS	SpMV, SpMM
DATA FORMATS	CSR, ELL, SELL-P, CSR5, HYB

http://icl.cs.utk.edu/magma/

Scan me for slides!

github.com/gflegar/talks/icpp_2017

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