ICPP'17, Bristol, 15 August

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Flexible-Size Batched LU for Small Matrices and its Integration into Block-Jacobi Preconditioning

Hartwig Anzt, Jack Dongarra, Goran Flegar, Enrique S. Quintana-Ortí

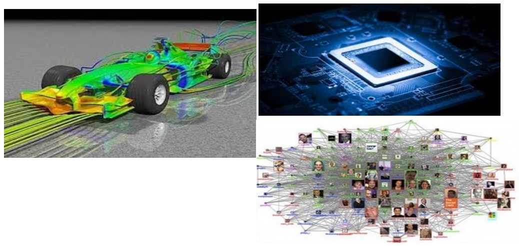
Scan me for slides!

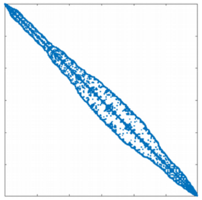
github.com/gflegar/talks/icpp_2017

flegar@uji.es

MAGMA-sparse software library

- GPU-accelerated sparse linear algebra library
  - Focus: linear systems





$$Ax = b$$

MAGMA SPARSE

Joint effort: Innovative Computing Lab at University of Tennessee, Knoxvile; Karlsruhe Institute of Technology; University Jaume I

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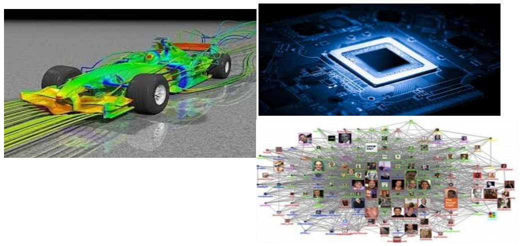

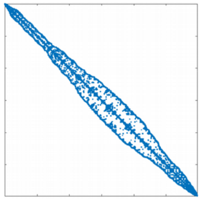

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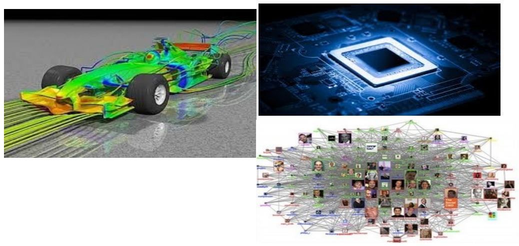

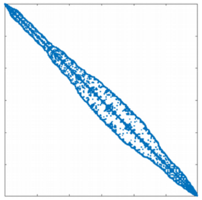

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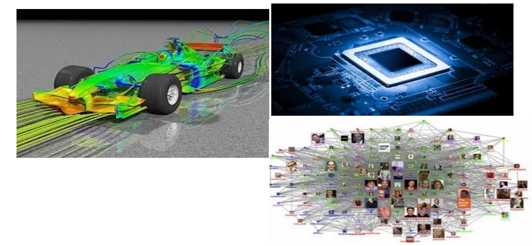

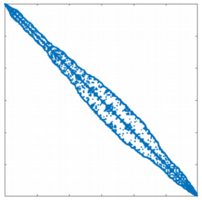

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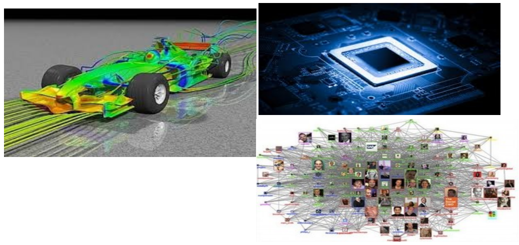

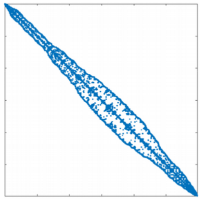

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Preconditioning

$$Ax = b, \; A \in \mathbb{R}^{n \times n}$$

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Preconditioner application

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Preconditioner setup

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Preconditioner setup **Preconditioner application**

Trade-off:
faster convergence,
but more work per iteration

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- Current focus: improve performance for problems with inherent block structure
  - Usually up to 30 unknowns per block (blocks can be of different sizes!)

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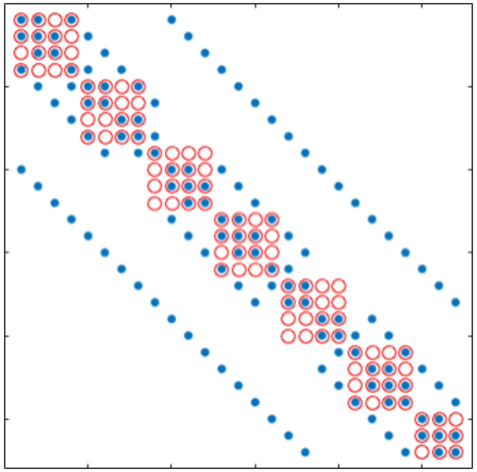

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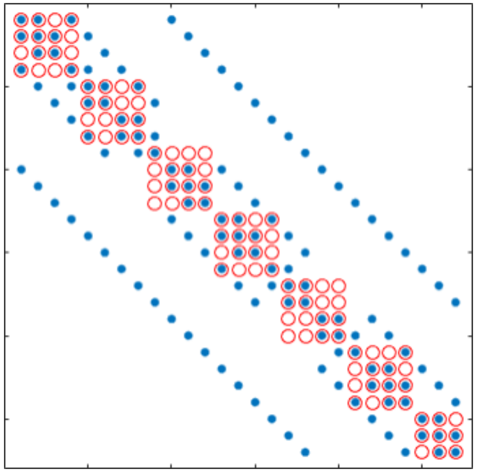

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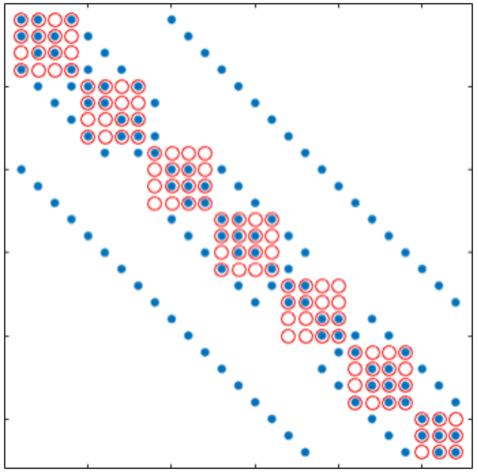

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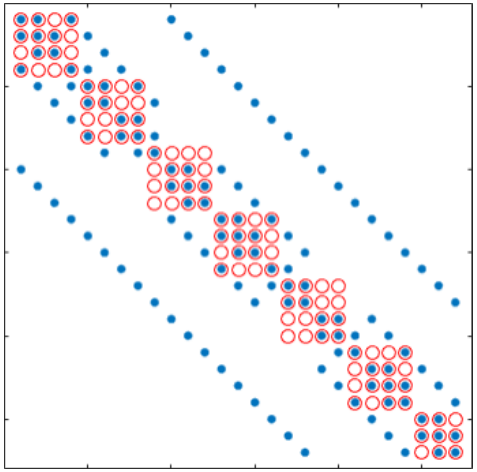

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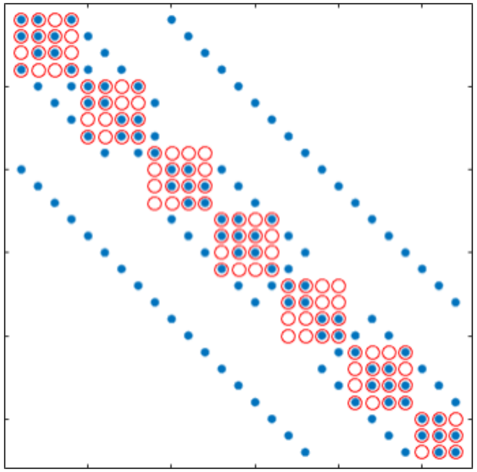

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Setup

$$U_i y_i = w_i$$

$$L_i w_i = z_i$$

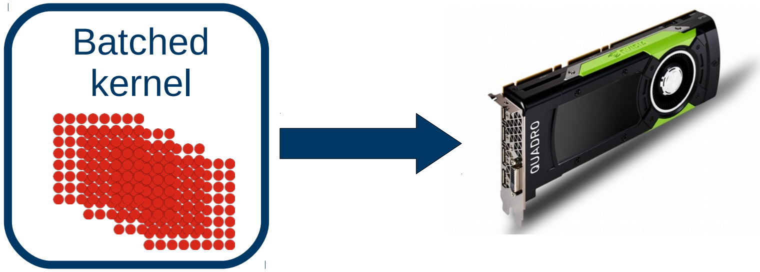
Application

Block-Jacobi Preconditioning

Solving 30-by-30 systems in sequence on a GPU with several thousand cores wastes computational resources!

Batched routines

Launch a single kernel which applies an operation to multiple independent data entities in parallel.



* Measured on NVIDIA P100

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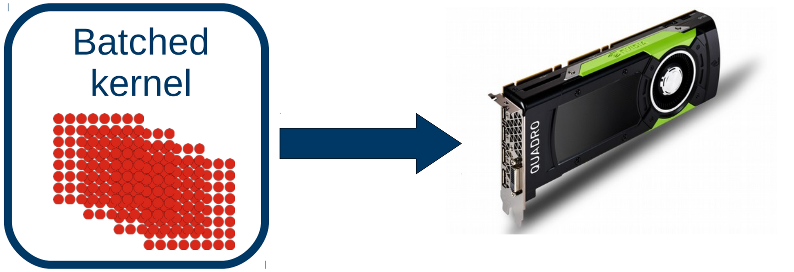

- There is no standard BLAS & LAPACK interface
- Most implementations only support problems of equal sizes
- High performance libraries are not optimized for small blocks
  - cuBLAS batched trsv: ~25 Gflop/s *
  - MAGMA-sparse SpMV: 60 – 90 Gflop/s *

** Measured on NVIDIA P100*

MAGMA-sparse LU & trsv outline

- Assign one warp to each problem
  - hardware SIMD unit, represented as a group of 32 threads in CUDA

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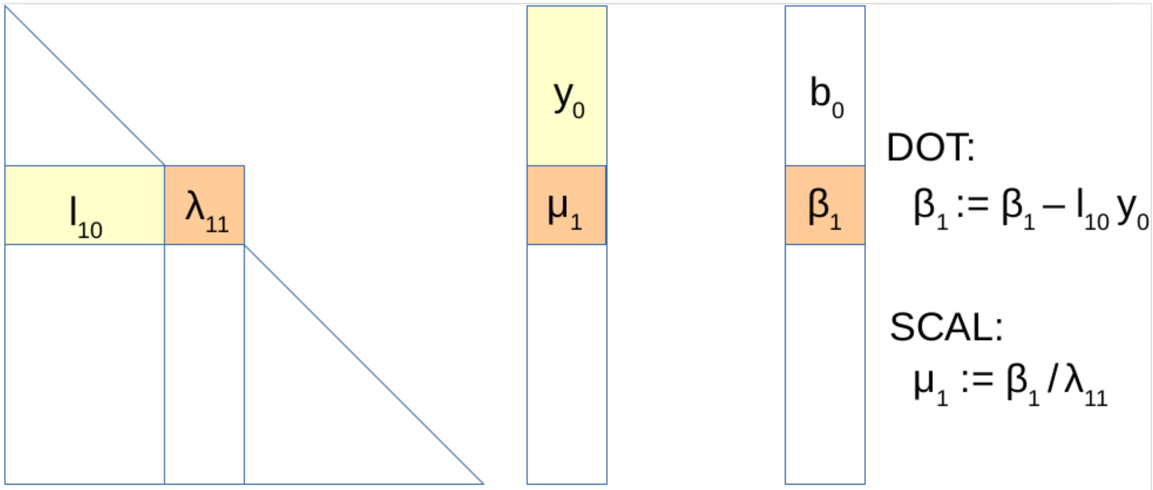
- Use implicit pivoting
  - Do not explicitly swap rows, "re-assign" the threads instead

MAGMA-sparse LU & trsv outline

- Use "eager" triangular solves
  - Cast solution vector updates in terms of axpy, not in terms of dot product!

"Lazy" triangular solve



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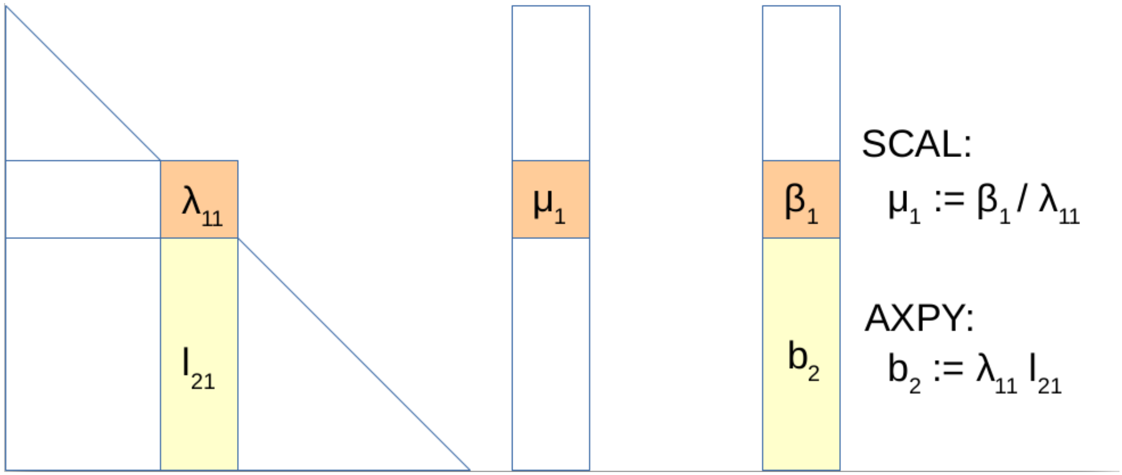
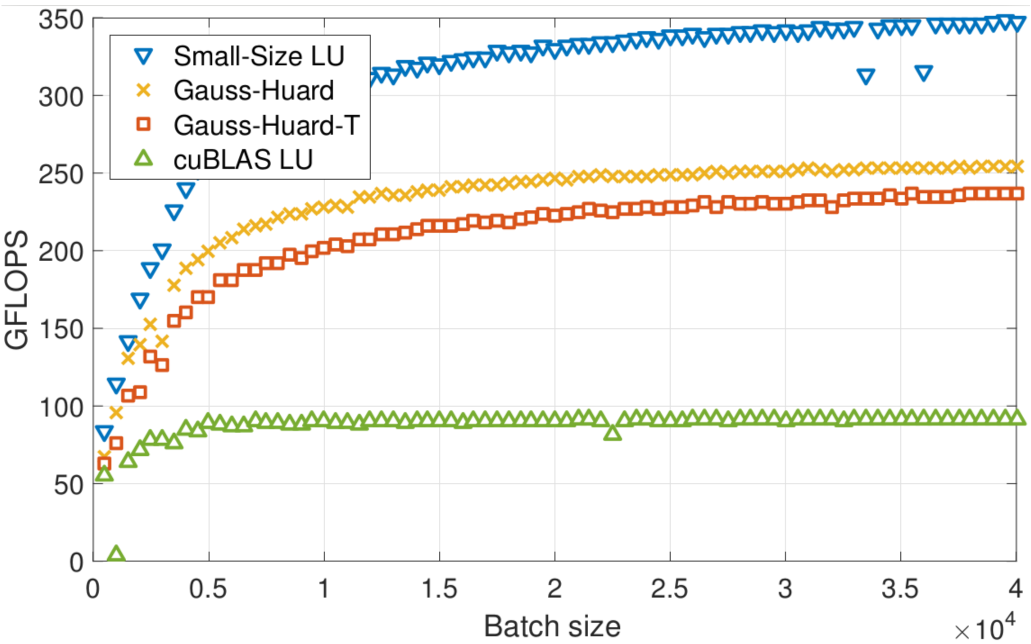


LU decomposition performance

- Comparisson of MAGMA-sparse vs cuBLAS batched LU decomposition
- Gauss-Huard(-T) is a similar approach, using a different algorithm for decomposition/solves *

Fixed problem size (32)



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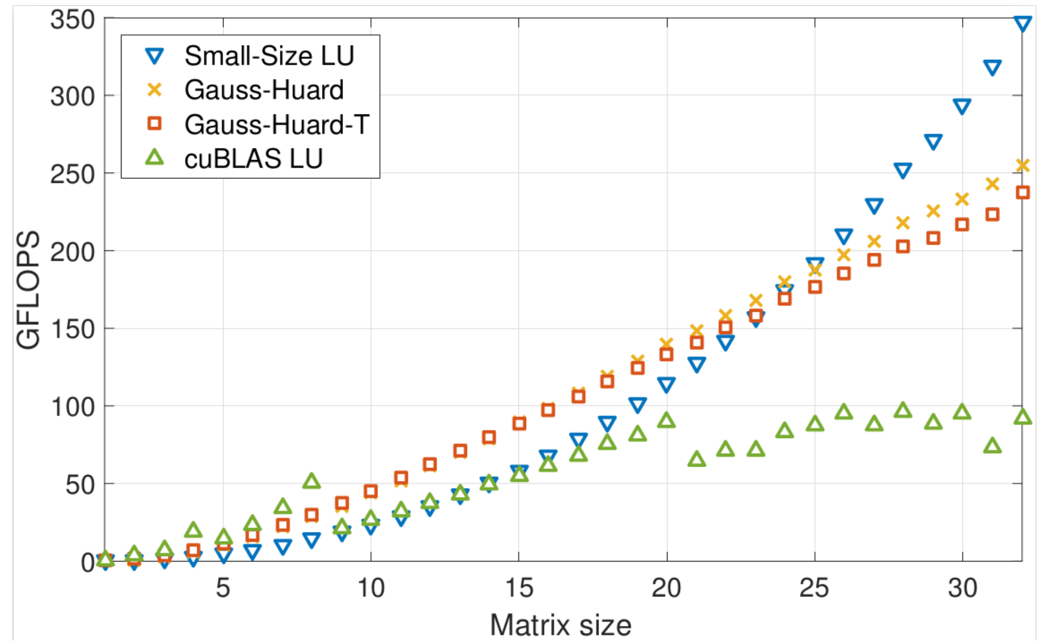

** Anzt et al., Variable-Size Batched Gauss-Huard for Block-Jacobi Preconditioning, ICCS'17*

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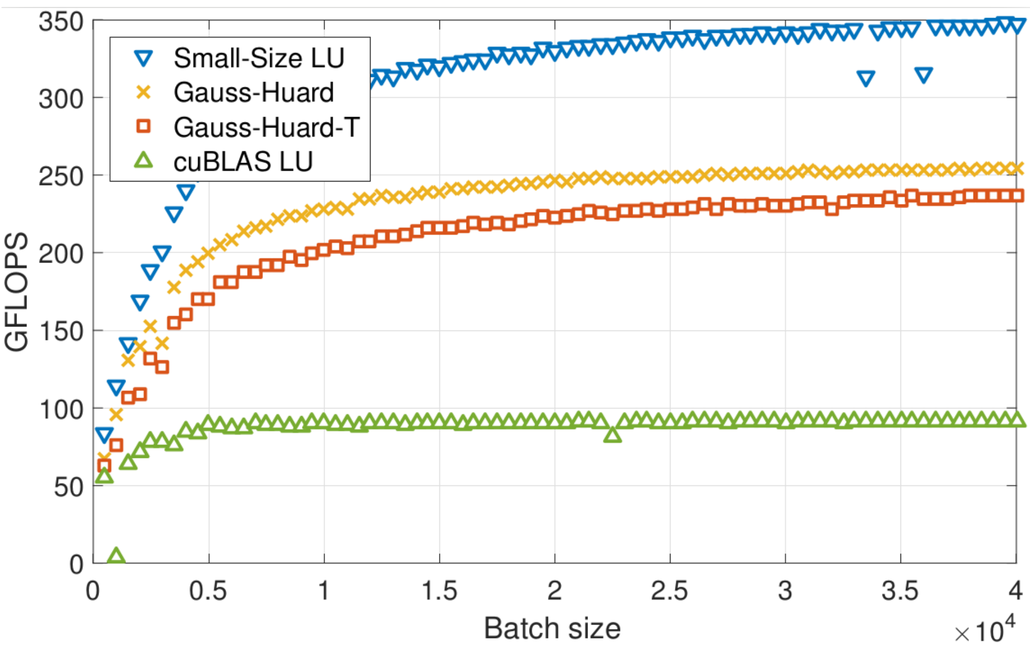

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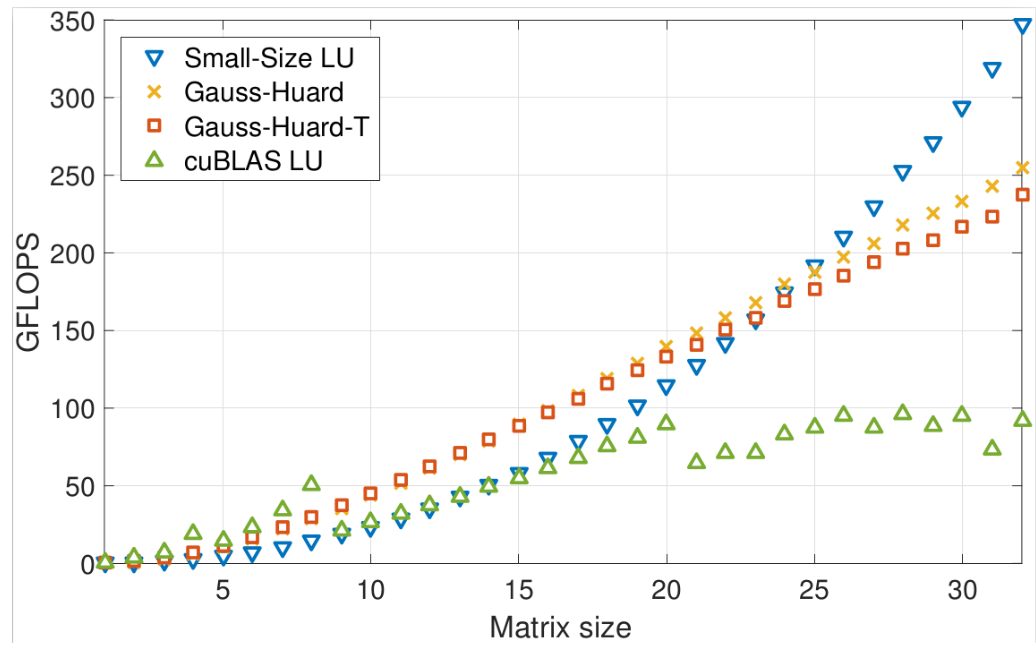

MAGMA-sparse LU can also:

- Handle problems of different sizes
- Integrate diagonal block extraction and diagonal block decomposition into a single kernel

** Anzt et al., Variable-Size Batched Gauss-Huard for Block-Jacobi Preconditioning, ICCS'17*

Triangular solve performance

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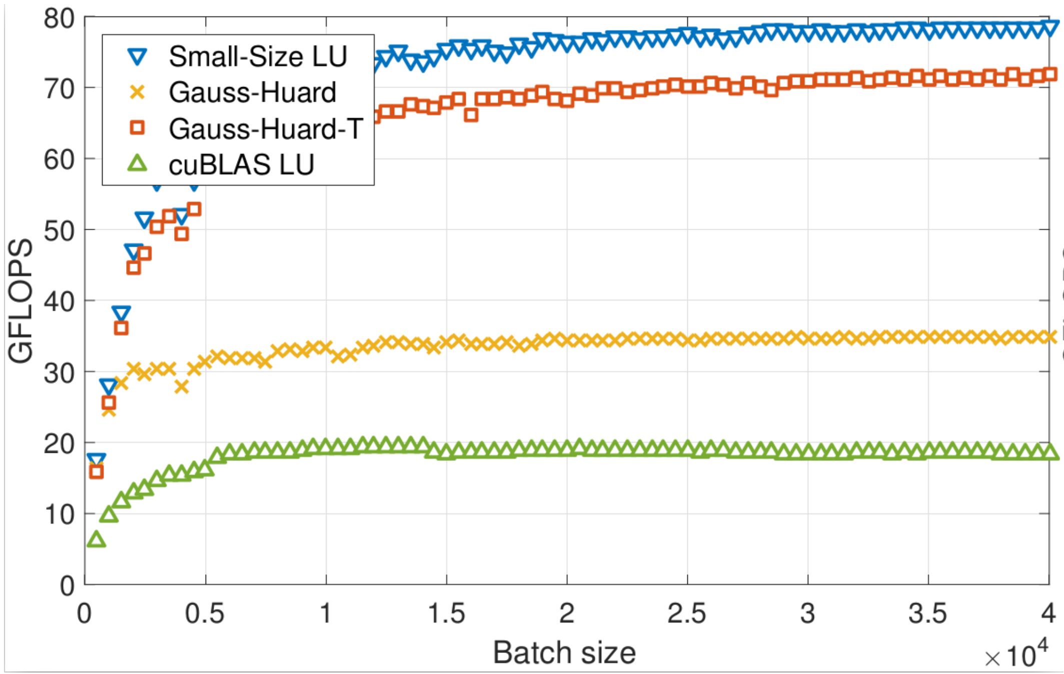

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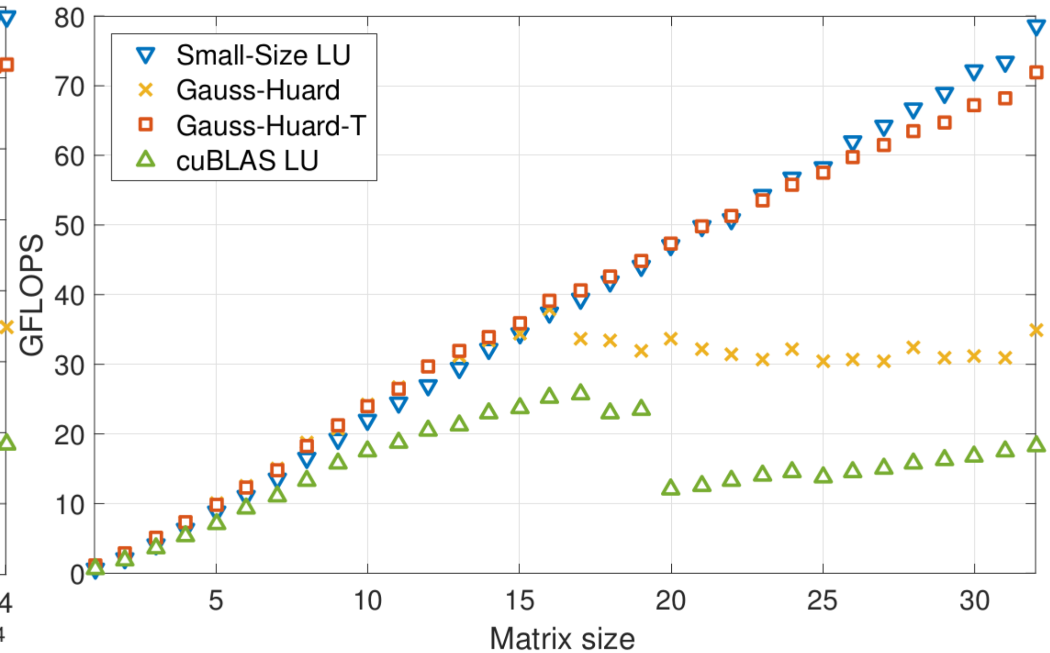
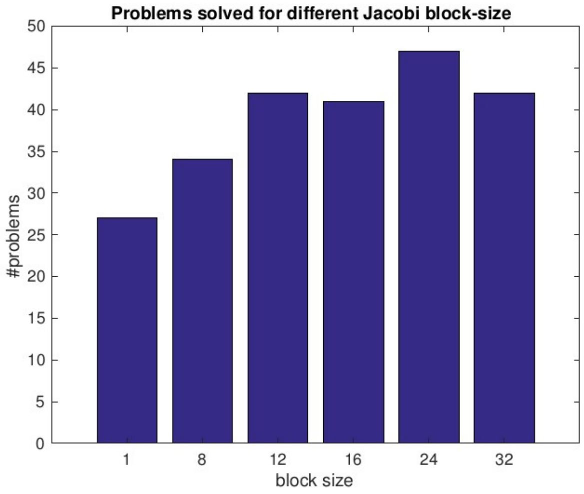


Complete solver runtime

56 problems from SuitSparse



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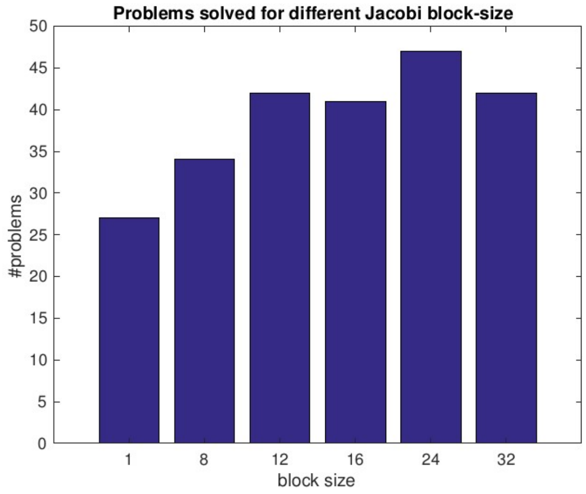

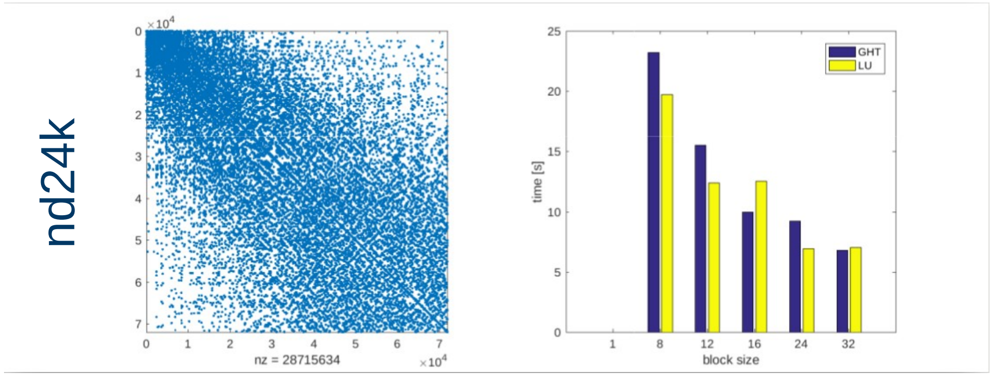

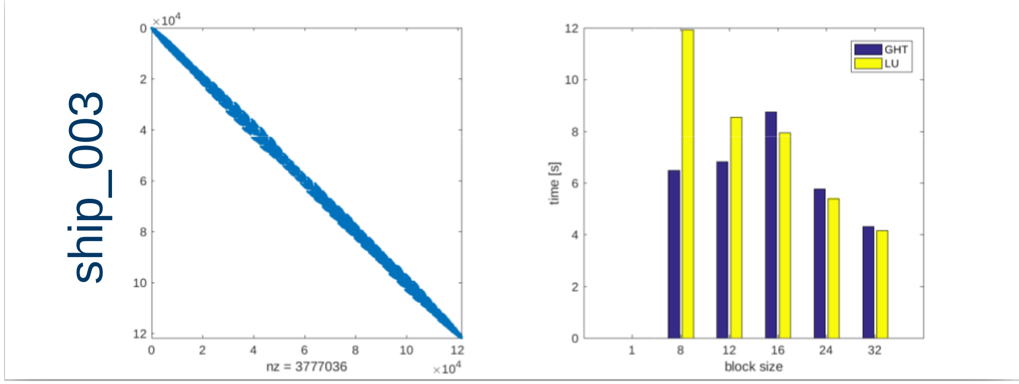

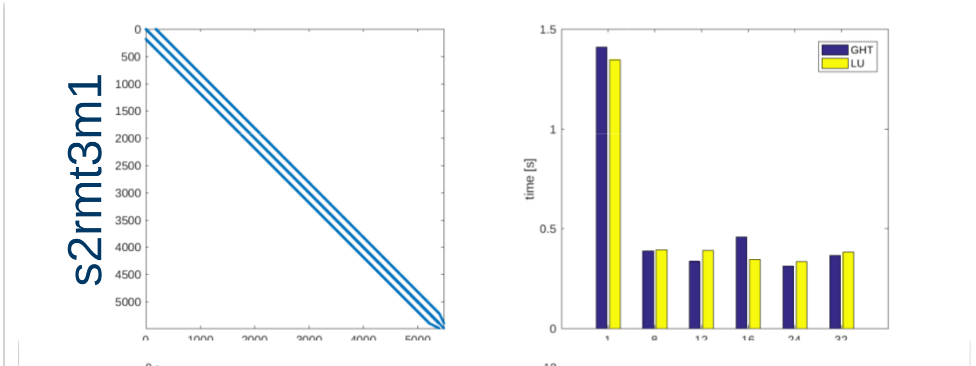

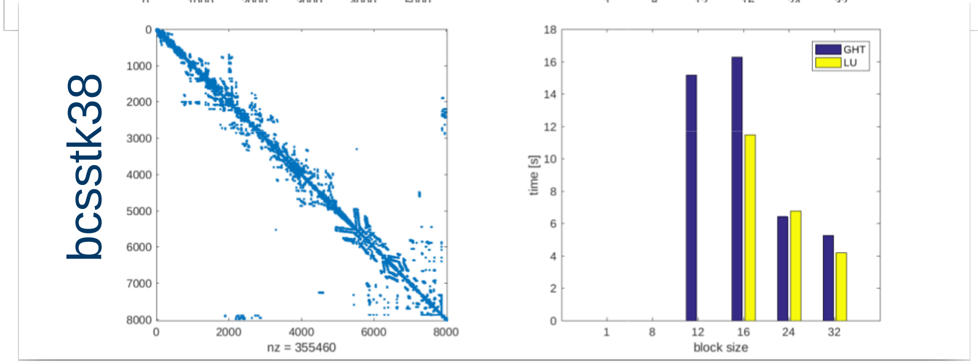

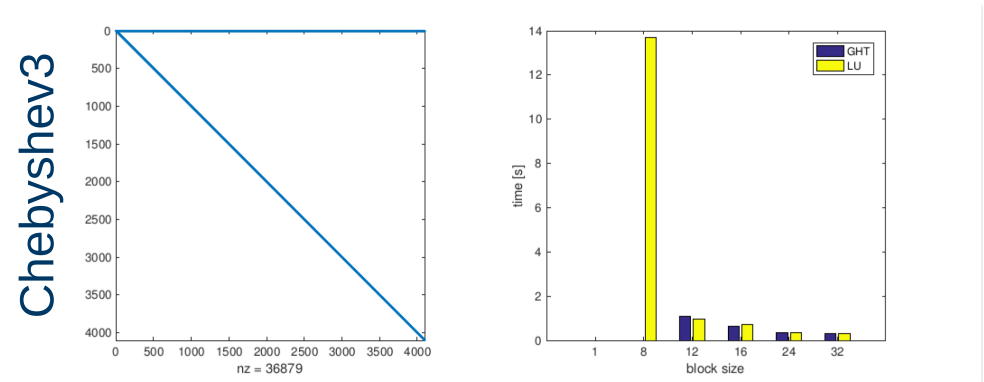

Conclusion

“Solving many small problems in sequence on manycore hardware wastes computational resources! We can design batched routines which apply the same operation on a set of independent data entities.”

Workshop on batched BLAS*

- Effort to standardize the batched BLAS interface

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- Batched gemm proposal has 16 input arguments!
- Global synchronization between two batched calls.

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Instead provide BLAS which operates on a part of memory/core hierarchy?
- E.g. block, warp, thread level BLAS for CUDA.
- Let users build their own batched routines from these building blocks.

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Thank you! Questions?

All functionalities are part of the MAGMA-sparse project.

MAGMA SPARSE

ROUTINES	BiCG, BiCGSTAB, Block-Asynchronous Jacobi, CG, CGS, GMRES, IDR, Iterative refinement, LOBPCG, LSQR, QMR, TFQMR
PRECONDITIONERS	ILU / IC, Jacobi, ParILU, ParILUT, Block Jacobi, ISAI
KERNELS	SpMV, SpMM
DATA FORMATS	CSR, ELL, SELL-P, CSR5, HYB

`http://icl.cs.utk.edu/magma/`

Scan me for slides!

github.com/gflegar/talks/icpp_2017

This research is based on a cooperation between Hartwig Anzt, Jack Dongarra (University of Tennessee), Goran Flegar and Enrique S. Quintana-Ortí (Universidad Jaume I).

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