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UNIVERSITAT JAUME I

Batched Generation of Block-Jacobi Preconditioners for Iterative Sparse Linear System Solvers on GPUs

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Problem setting

- **Solve sparse linear system using an iterative Krylov method**

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$$M^{-1}Ax = M^{-1}b$$

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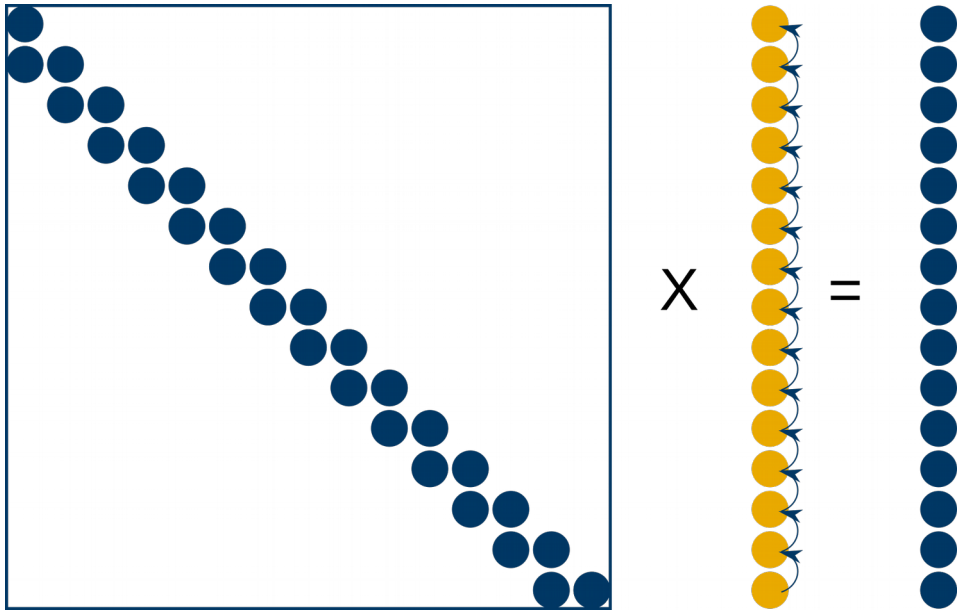
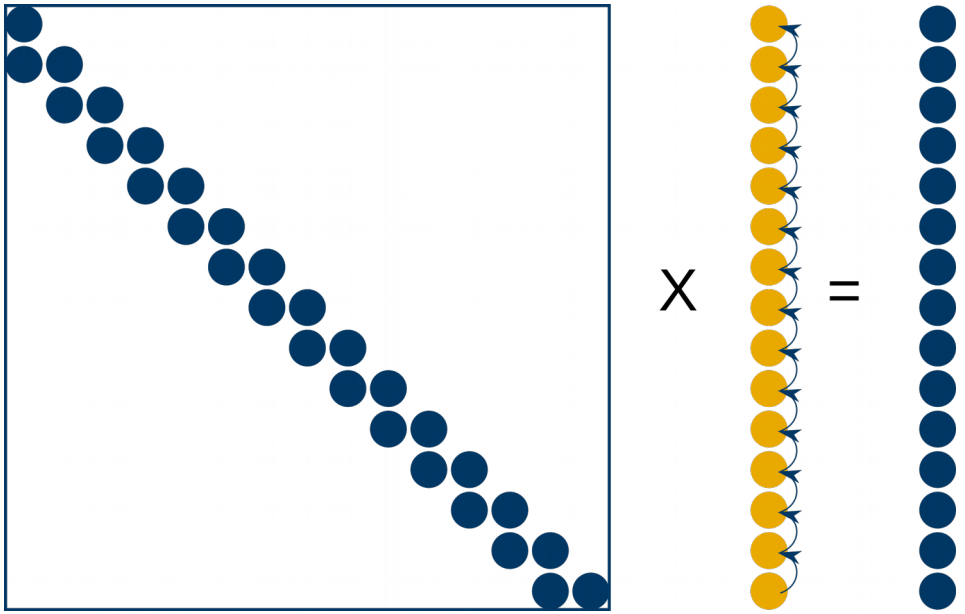


source: devblogs.nvidia.com/parallelforall/

Problem setting

- **Solve sparse linear system using an iterative Krylov method**
- **Convergence typically benefits from using a preconditioner**
- **Conventional ILU-type preconditioners offer limited concurrency**
- **Need high degree of parallelism to use a GPU effectively**
  - ILU on GPUs is an area of active research



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GPU architecture (NVIDIA GP100)

- **56 SMs x 64 cores = 3584 cores!**
- **Need to oversubscribe to hide memory latency!**

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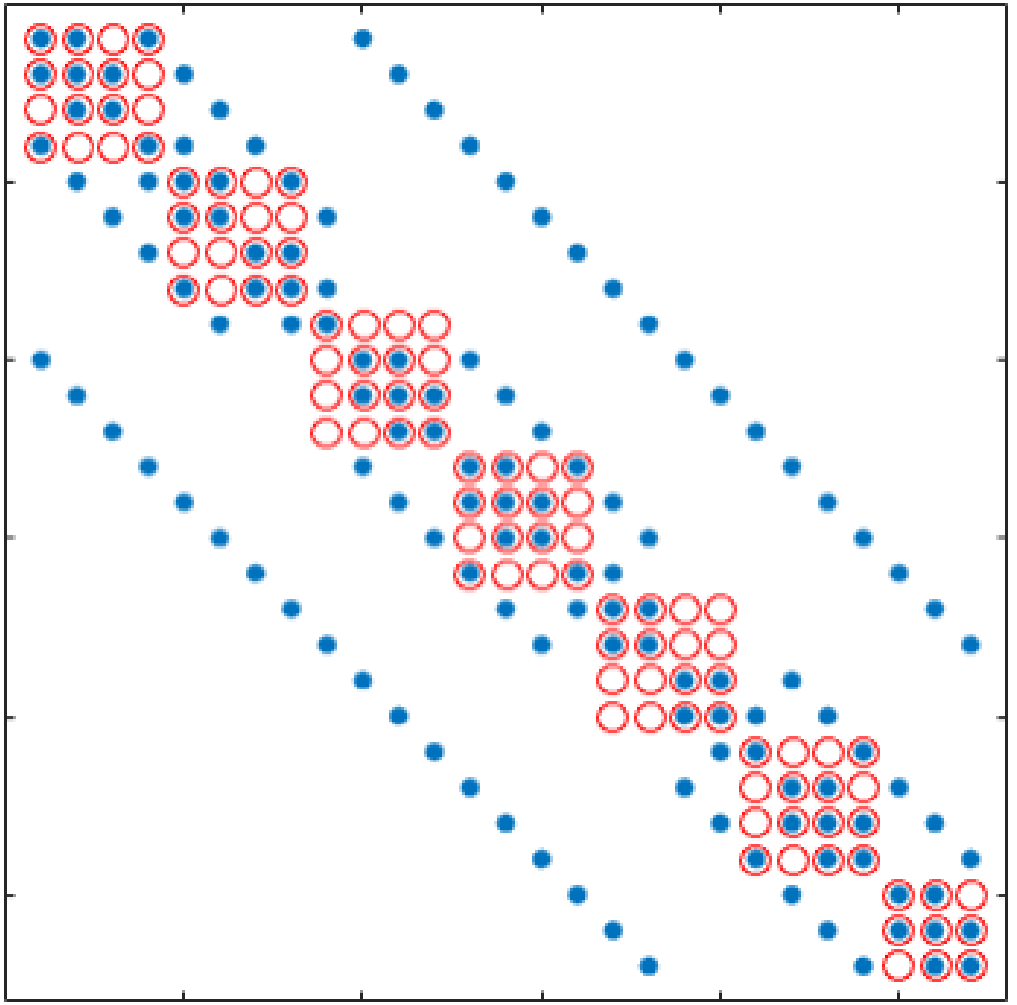
Different approach

- **Scalar Jacobi**
  - Scale with inverses of diagonal elements
- **Block-Jacobi**
  - Scale with inverses of diagonal blocks (possibly of different sizes!)
  - Can reflect the block structure of the problem
  - Often superior to scalar Jacobi
  - Lower convergence rate improvement compared to ILU
- **Can proces each block independently!**



General Ideas

- **Restrict block size to 32x32**
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  - Large block sizes require more memory to store the preconditioner matrix
- **Use a single warp to process the whole block (one thread per column)**
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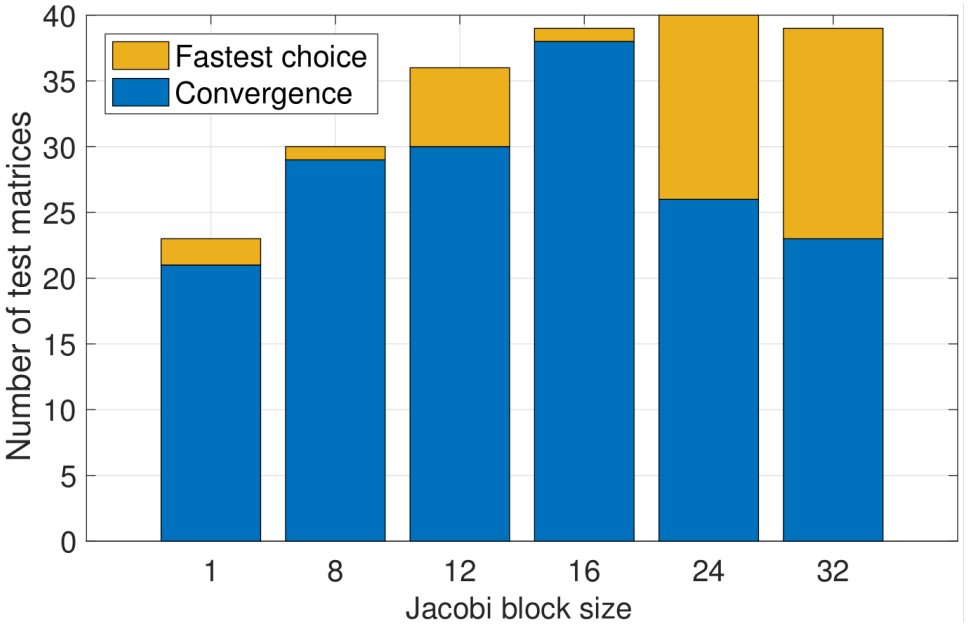
- **Use the large register file to store the entire block**
  - Read/write from mem. once
  - Avoids load/store instructions
  - Com. via warp shuffles

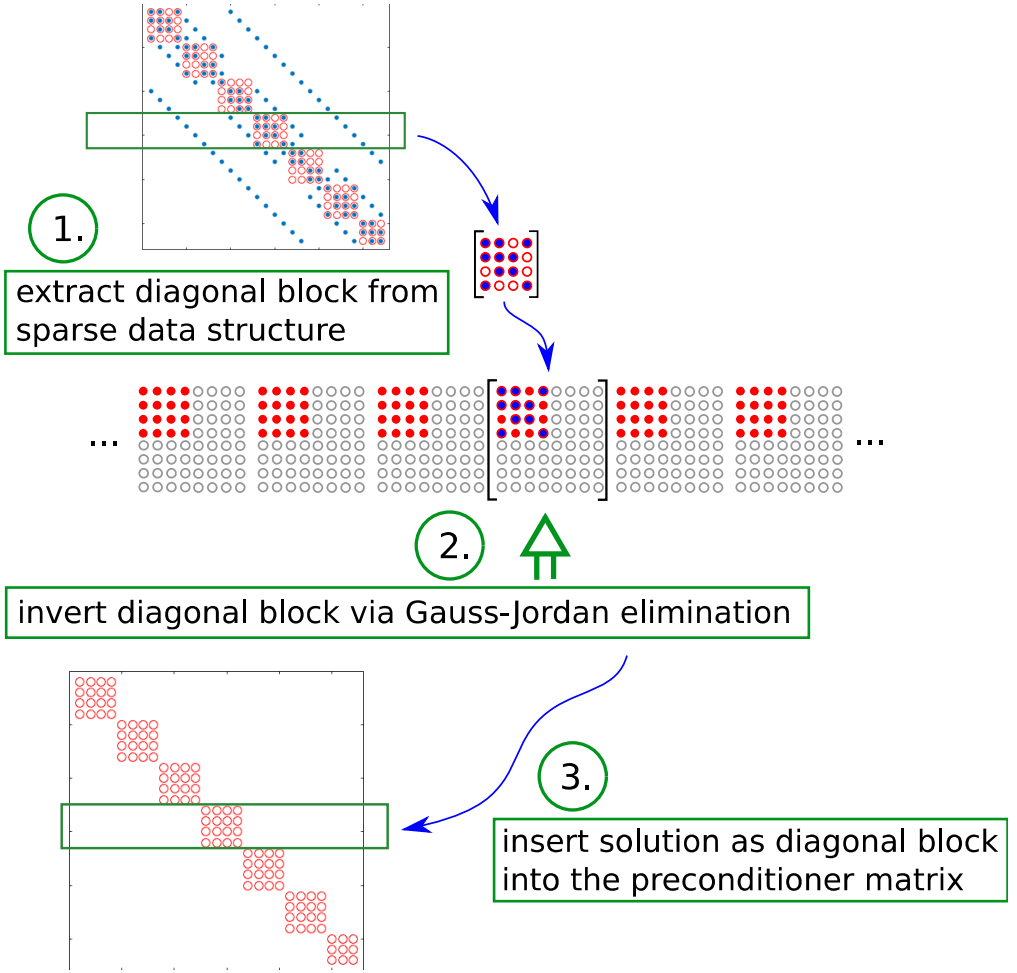
Benefits of block-Jacobi

- **40 matrices from SuiteSparse**
- **MAGMA-sparse open source library**
  - IDR solver
  - Jacobi preconditioner
  - Supervariable blocking
- **Block-Jacobi improves the robustness of the solver**
  - More problems converge
- **Decreases time-to-solution**



Implementation options

- **Inversion in preconditioner setup + matrix-vector product in application**
  - (FLOPS: $2n^3$ setup, $2n^2$ app.)
  - H. Anzt et all., “Batched Gauss-Jordan Elimination for Block-Jacobi Preconditioner Generation on GPUs”, PMAM’17



Different options

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  - (FLOPS: $2n^3$ setup, $2n^2$ app.)
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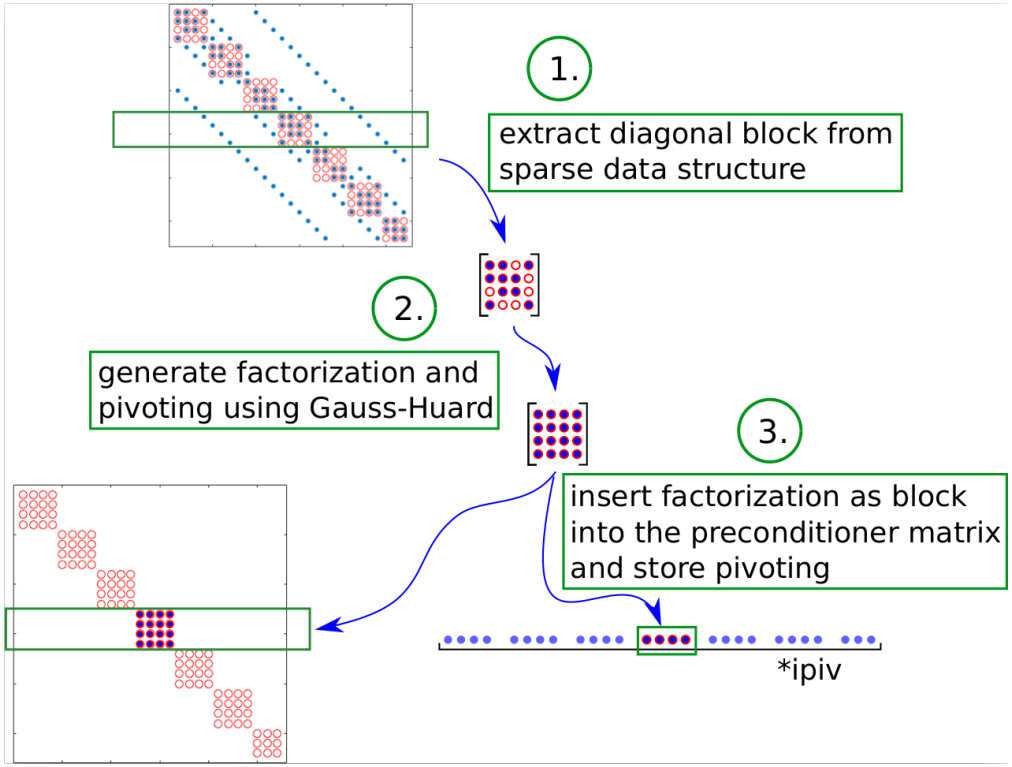
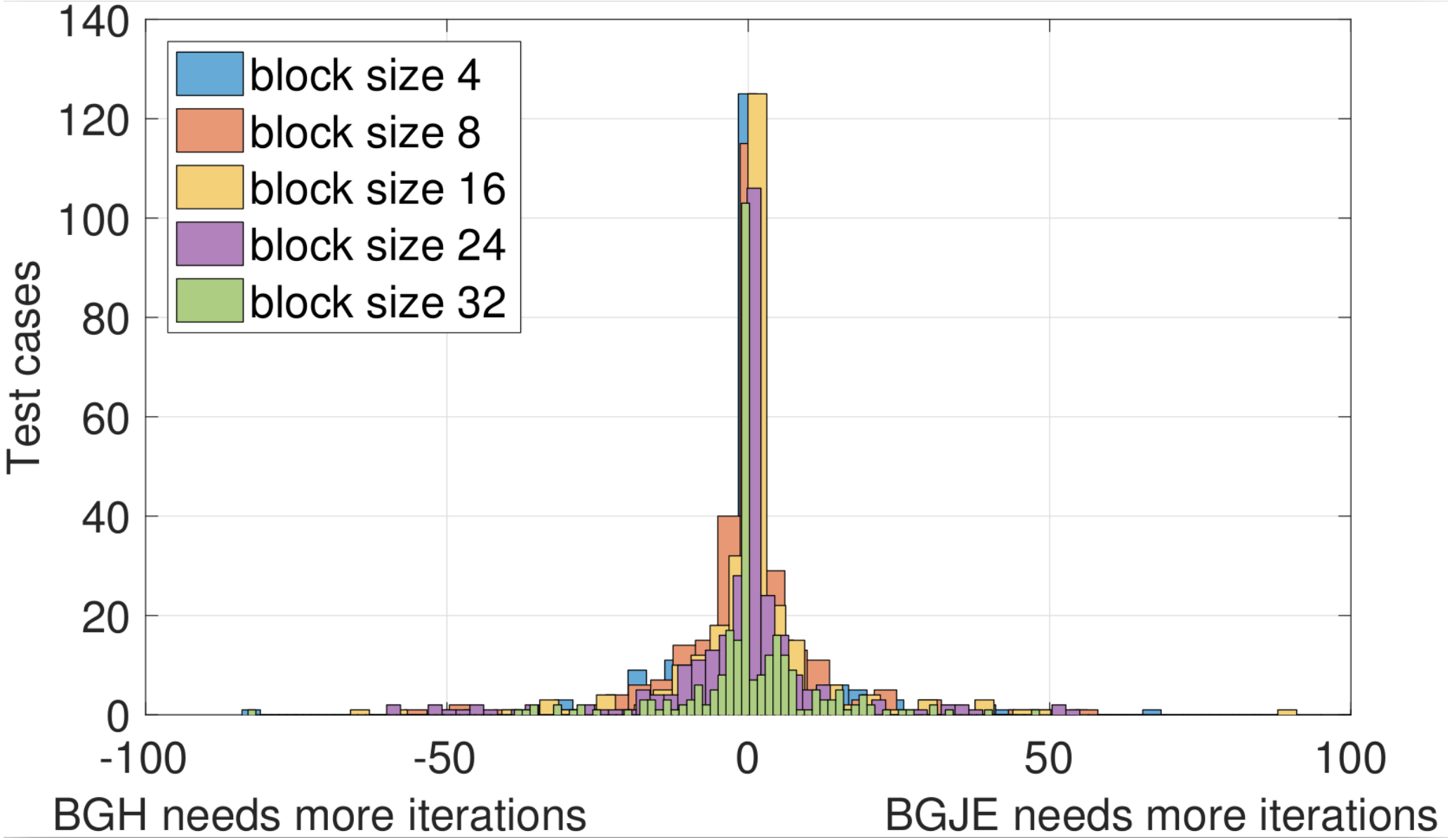
- **Matrix decomposition in setup + solve in application**
  - (FLOPS: $2/3n^3$ setup, $2n^2$ app.)
  - Gauss-Huard decomposition

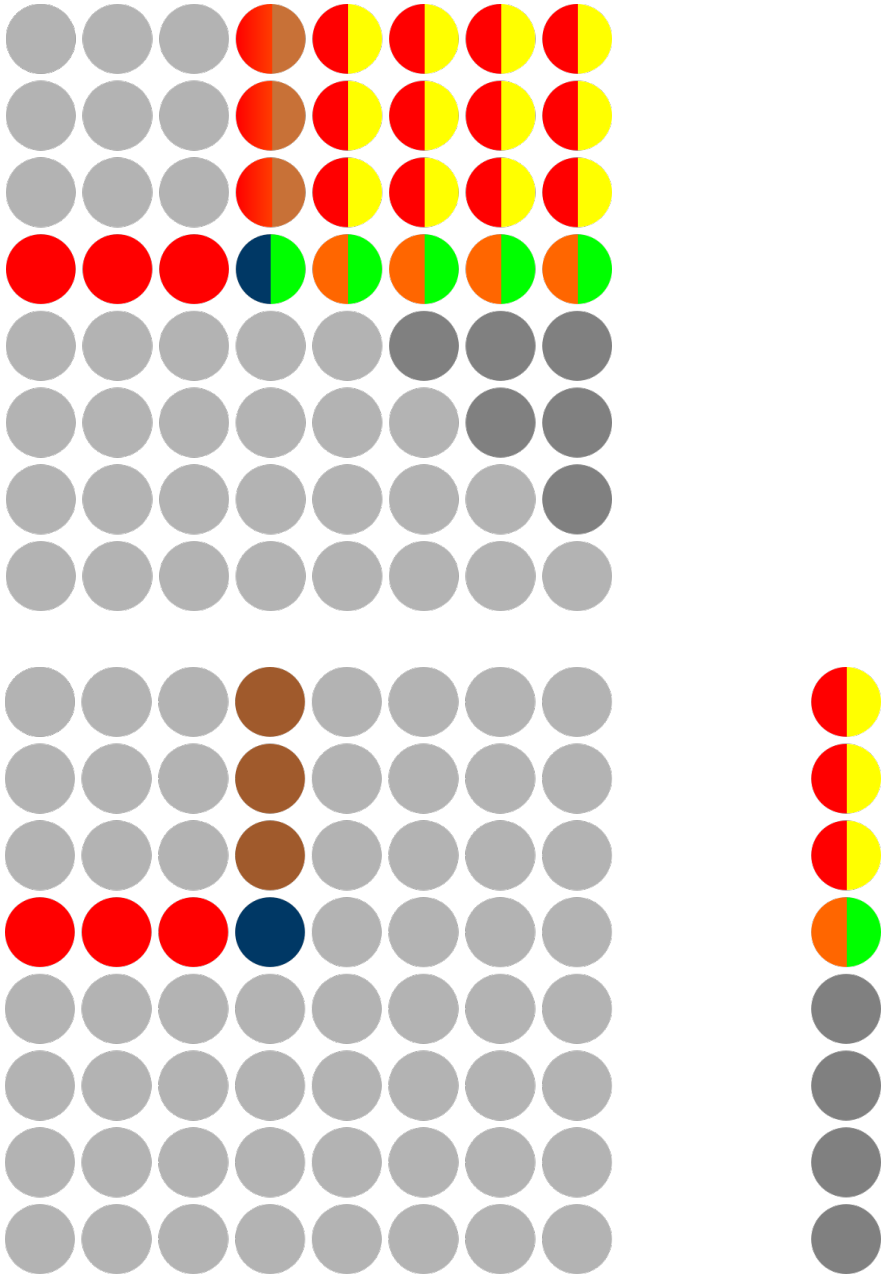


Inversion?!


block size 4
block size 8
block size 16
block size 24
block size 32
Test cases
140
120
100
80
60
40
20
0
-100
-50
0
50
100
BGH needs more iterations
BGJE needs more iterations

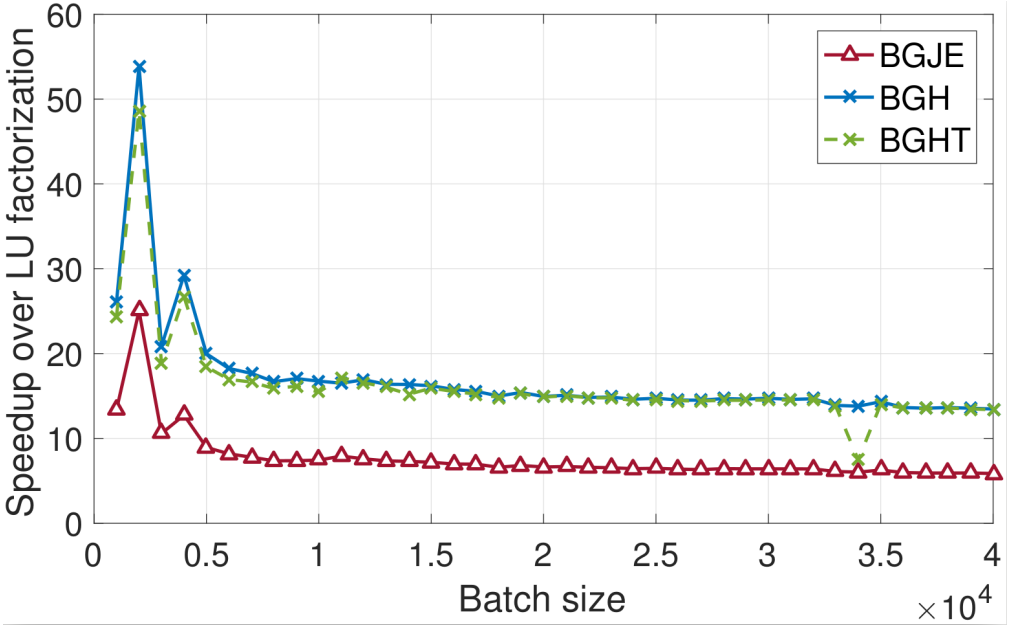
Gauss-Huard decomposition

- **Decomposition**
  - GEMV (G = G - RR)
  - SCAL (O = O / B)
  - GER (Y = Y - BO)
  - Column pivoting
    - Do not swap the columns, just remember which thread holds which column of the result
- **Solve**
  - Store only the solution vector into registers
  - DOT (G = G – RR)
  - SCAL (O = O / B)
  - AXPY (Y = Y - BO)
  - Store lower part transposed wrp. to anti-diagonal for coalesced mem. access

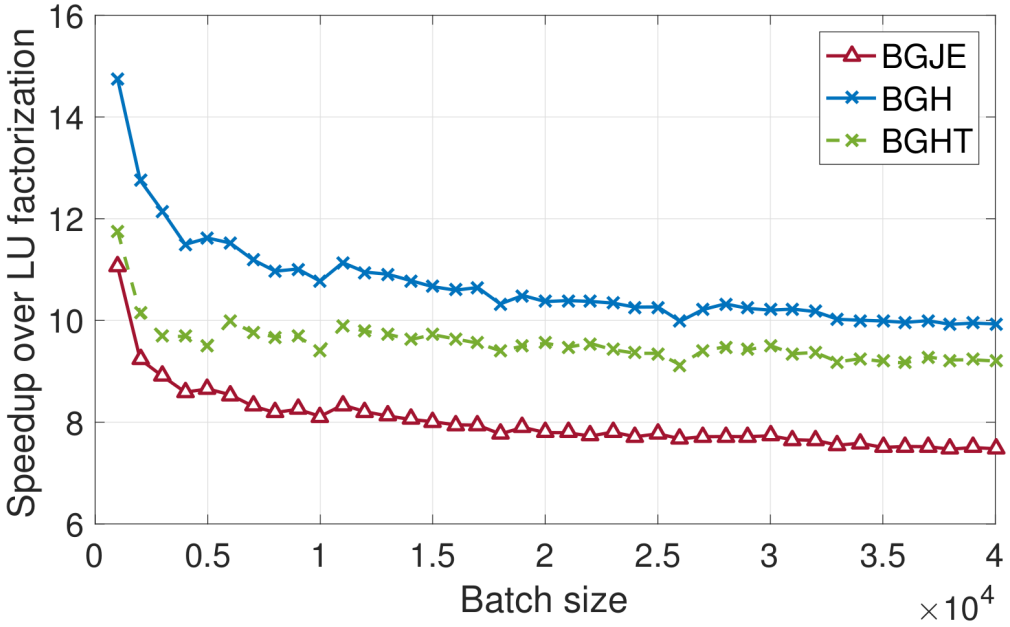


Decomposition comparison to batched LU (MAGMA)

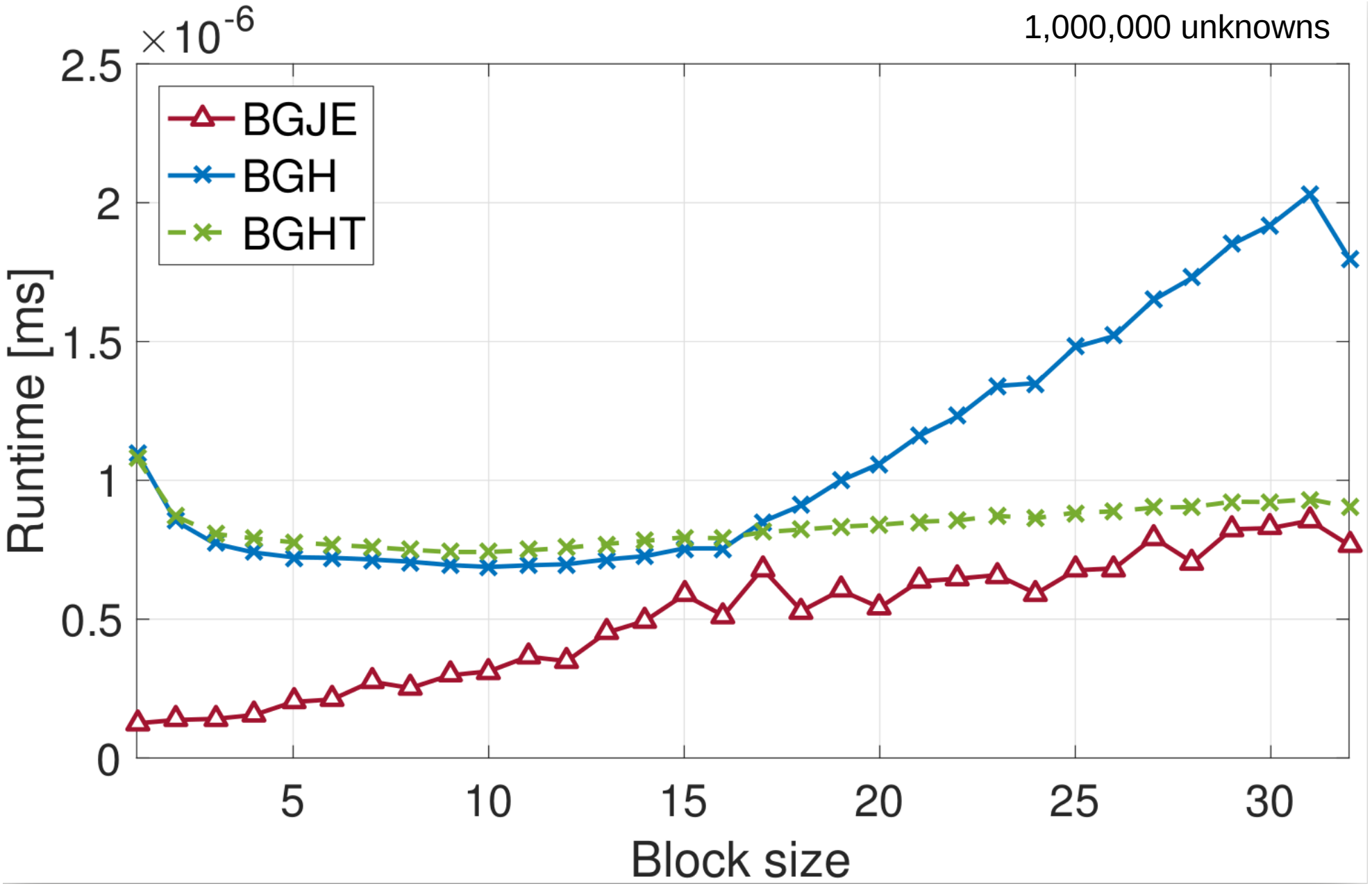
Block size 16



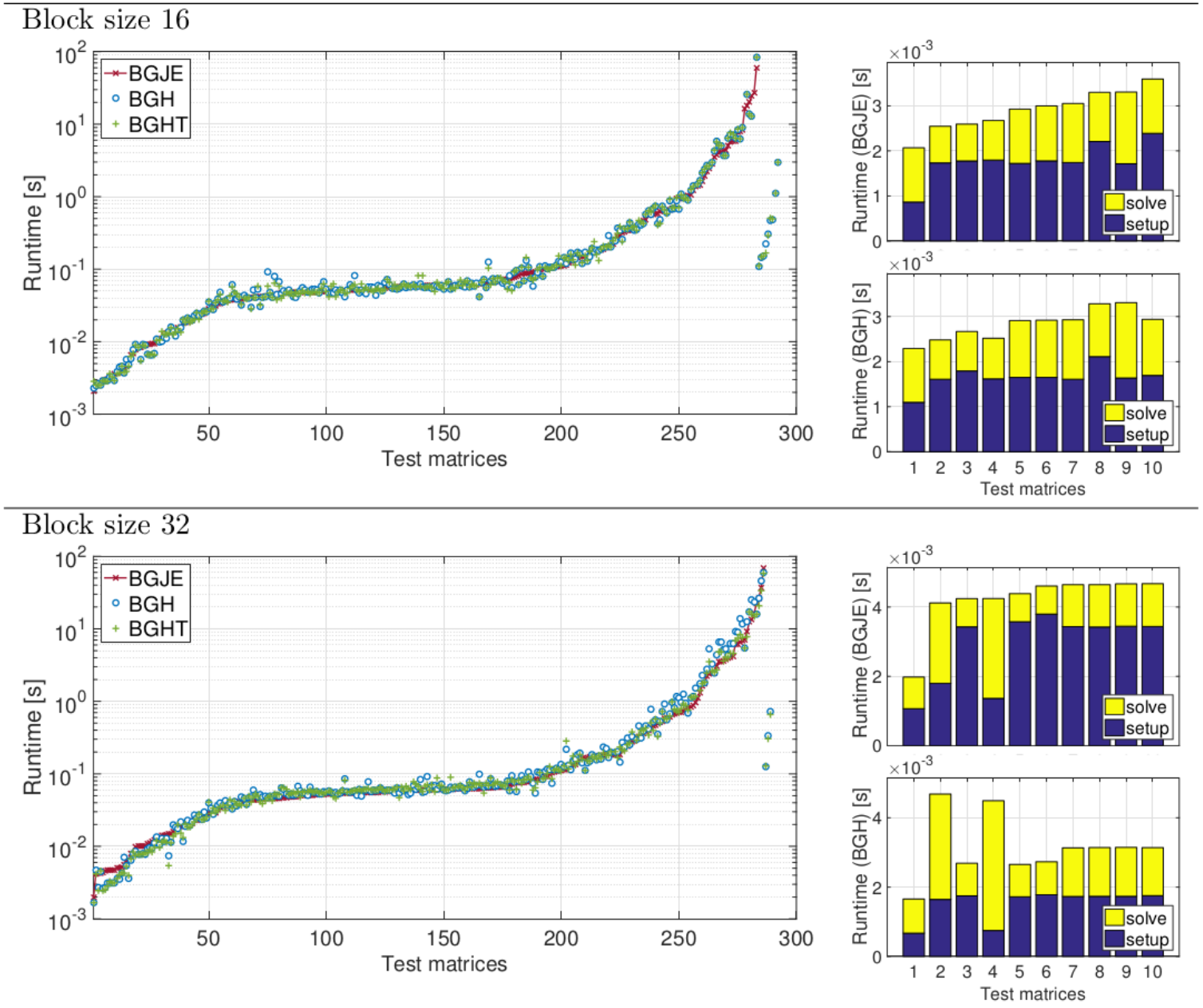
Block size 32



Application time



Total runtime of block-Jacobi preconditioned BiCGSTAB



Thank you! Questions?

All functionalities are part of the MAGMA-sparse project.

MAGMA SPARSE

ROUTINES	BiCG, BiCGSTAB, Block-Asynchronous Jacobi, CG, CGS, GMRES, IDR, Iterative refinement, LOBPCG, LSQR, QMR, TFQMR
PRECONDITIONERS	ILU / IC, Jacobi, ParILU, ParILUT, Block Jacobi, ISAI
KERNELS	SpMV, SpMM
DATA FORMATS	CSR, ELL, SELL-P, CSR5, HYB

http://icl.cs.utk.edu/magma/

This research is based on a cooperation between Hartwig Anzt, Jack Dongarra (University of Tennessee), Goran Flegar, Enrique S. Quintana-Ortí and Adrés E. Tomás (Universidad Jaume I).

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