Dahl hysteresis modeling and position control of piezoelectric digital manipulator. **Supplementary material**

Gerardo Flores, *Member, IEEE*, and Micky Rakotondrabe, *Member, IEEE*

SIMULATION STUDY

The simulation environment is Matlab Simulink with 30 seconds of simulation time and an automatic solver with variable-step. The system parameters used for simulation correspond to the actuator in [?] with hysteresis model of [?]. They are shown in Table I.

TABLE I: Set of system parameters used in the simulation. The super-index $(\cdot)^n$ means nominal value.

Parameter	Value	Parameter	Value
m_p (g)	$m_p^n = 0.1828$	c_p (N/s/m)	$c_p^n = 2.5973 \times 10^3$
d_p (N/V)	$d_p^n = 0.0468$	k_p (N/m)	$k_p^n = 2.6065 \times 10^4$
γ	$\gamma^n = 0.01$	α	$\alpha^n = 1.1773 \times 10^6$
β	$\beta^n = 121.9874$		

The control parameters are as described in Table II.

TABLE II: Set of control parameters used in the simulation.

Parameter	Value	Parameter	Value
l_1	2×10^3	ϵ	0.0001
l_2	1×10^6	k_1	25
l_3	3×10^5	k_2	14
l_4	0.1		

The desired trajectory is $x_1^d = 80 \sin \frac{2\pi}{10} t$ with first-time and second derivatives, $\dot{x}_1^d = x_2^d$ and $\ddot{x}_1^d = \dot{x}_2^d$, respectively. The unknown exogenous force $F(t)$ is chosen as,

$$\begin{aligned} F(t) &= 50e^{-0.08(t-0.03)}\mathcal{H}(t-6) + 100 \\ 80e^{(-0.05(t-1))}&\mathcal{H}(t-12) + 56e^{(-0.03(t-1.3))}\mathcal{H}(t-22), \end{aligned} \tag{1}$$

where $\mathcal{H}(\cdot)$ is the Heaviside step function. A picture of $F(t)/m_p$ is depicted in Fig. 3.

The closed-loop system response under the effect of the proposed control, i.e., $u(\hat{x}_1, \hat{x}_2, \hat{\Delta})$ with a structure similar to the u of Proposition **??**, is depicted in Fig. 1. There are slight differences near zero in finite points due to the minor differences in the unknown signal Δ and its estimate $\hat{\Delta}$. Besides, the observer errors (e_1, e_2) are depicted in Fig. 2. Notice how the high-gains l_1, l_2 produce a rapid convergence to zero. The unknown term $\Delta(x,t) = -\frac{\gamma}{m_p}\,\mathrm{sgn}\,(x_2)z - \frac{1}{m_p}F(t)$ and its estimate with the proposed observer is depicted in Fig. 3, where it is shown that despite the abrupt changes the convergence is achieved.

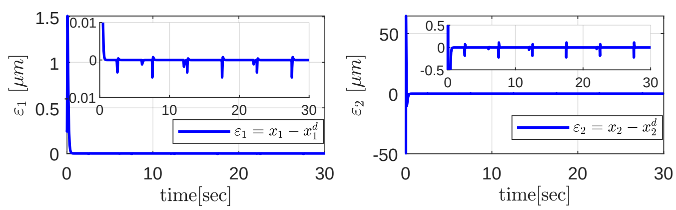

Fig. 1: The tracking errors (**??**) under the proposed control and a zoom near zero. Recall that only $y = x_1$ is the only available output.

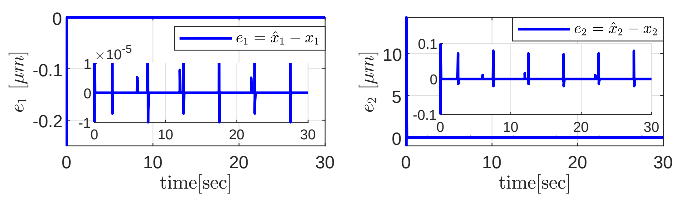

Fig. 2: The observer errors (e_1, e_2) from (**??**) and a zoom near zero. The error magnitude can be minimized by choosing higher gains.

Finally, the control response $u(\hat{x}_1, \hat{x}_2, \hat{\Delta})$ is depicted in Fig. 4. In the same figure, the closed-loop response map, y vs. y_d, shows a perfectly linear response, as expected.

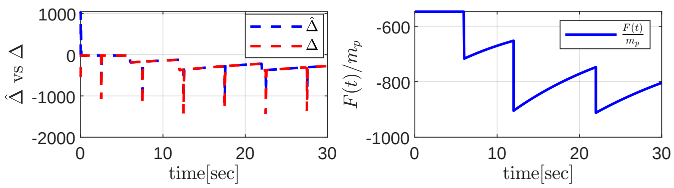

Fig. 3: On the LHS is the unknown term $\Delta(x,t)$ of (**??**) and its estimate $\hat{\Delta}$ obtained with the observer (**??**). On the RHS is the unknown exogenous force $F(t)/m_p$ in (**??**).

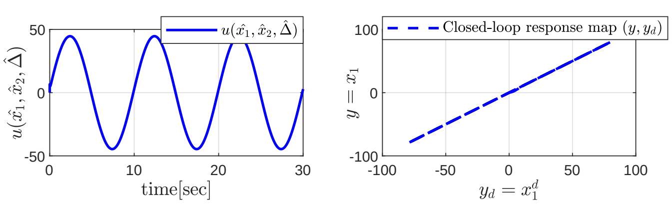

Fig. 4: The control algorithm described by $u(\hat{x}_1, \hat{x}_2, \hat{\Delta})$, and the closed-loop response map showing a clear linear behavior.