Finite-time stabilization of the generalized Bouc-Wen model for piezoelectric systems. **Supplementary material**

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Simulation Study

In this section, we illustrate the effectiveness of our proposed controller. The simulations were conducted in MATLAB/Simulink using a fixed-step solver (ode5) constrained to a step size of 0.0001s. The idea is to check the hysteresis before the control and after implementing the latter by plotting the state x versus the input.

Let us consider plant (Σ, Ω) and output feedback controller (6), (17), and (22) for the simulation study. Notice that from remark 1 that in our simulation we take the hysteresis estimate $\hat{h}$ for feedback in our control. Besides, we consider exogenous unknown disturbances $\delta_x = -3 - 10\cos(0.2t + 10)$, and $\delta_h = 8\sin(t)$. The system parameters are $\tau = 0.001$s, $A = -\frac{1}{\tau}$, $B = \frac{1}{\tau}$, $C = 1$, $d_p = 1.0773$, while the hysteresis model parameters are given by $\alpha = 0.40648$, $\beta_1 = 0.0032$, $\beta_2 = 0.0035$, $\beta_3 = -0.0010$, $\beta_4 = -0.0003$, $\beta_5 = -0.0002$, $\beta_6 = 0.0004$. The control parameters are set as $k_h = 20$, $k_{x_1} = 800$, $k_{x_2} = 200$, $L_1 = 450$, $M_1 = 500$ with estimated bound in (3) chosen as $q = 0.0095$. The observer parameters are taken as $l_1 = 500$, $l_2 = 200$, $Q = \begin{pmatrix} 50 & 0 \\ 0 & 30 \end{pmatrix}$, and then $P = \begin{pmatrix} 0.0560 & -0.0150 \\ -0.0150 & 0.3175 \end{pmatrix}$, $\mu_1 = 0.2$, $\mu_2 = 1.2$, and $p = 1$. The initial conditions are chosen as $x(0) = 3$, $h(0) = -2$, $\hat{x}(0) = \hat{h}(0) = 0$. The reference signal is $x_d = 80\sin\left(\frac{2\pi}{10}t\right)$.

Fig. 1 shows the output performance of $(\hat{x}, \hat{h})$. We see how such observer states converge to the real states (x, h). The same figure on the right-hand side illustrates the convergence of the states (x, h) to its desired values (x_d, h_d) despite the presence of disturbances and unknown of the hysteresis parameters, h_d being computed by the virtual control (17).

The left-hand side of Fig. 2 shows the control law u given by the integral of (6). The right-hand side of Fig. 2 depicts the open-loop and closed-loop map, i.e., the graph showing the relation between x and x_d in the case where $u = x_d$ and in the closed-loop case, where our control is implemented. Both maps are under the effect of external disturbances. It clearly shows that the hysteresis when without the proposed control law is completely linearized when using the latter along with a unity gain is observed, i.e. $x = x_d$.

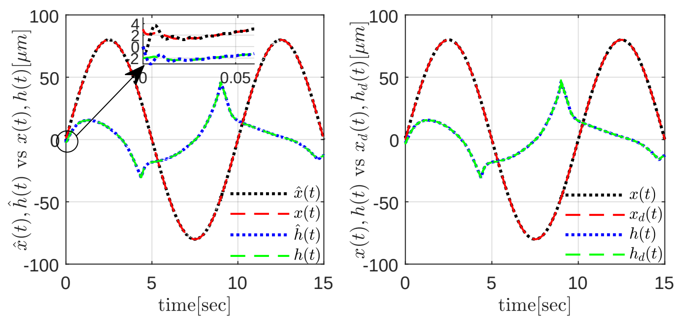

Fig. 1: The observer's performance is on the left-hand side, and we depicted the system's performance states under the proposed controller on the right-hand side. All is in μm.

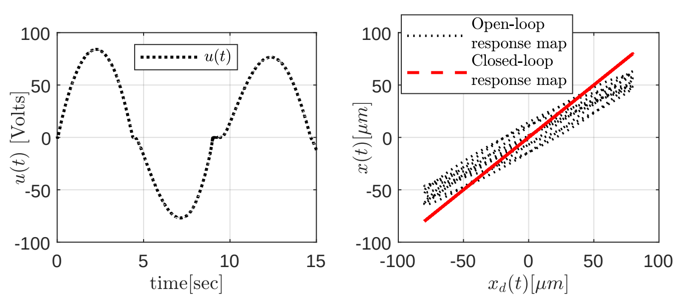

Fig. 2: The controller u and the open-loop and closed-loop response maps under disturbed system.