

1 A-level prep (holiday work)

Work from the weebly page.

1.1 Laws of indices

When multiplying indices, add the exponents. When dividing indices, subtract them.

$$x^m \times x^n = x^{m+n} \quad (1)$$

$$\frac{x^m}{x^n} = x^{m-n} \quad (2)$$

For nested indices, multiply the exponents.

$$(x^m)^n = x^{mn} \quad (3)$$

Fractional indices take the form $\frac{\text{power}}{\text{root}}$.

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad (4)$$

Negative indices indicate “one over”.

$$x^{-m} = \frac{1}{x^m} \quad (5)$$

Anything raised to the power 0 is 1.

$$x^0 = 1 \quad (6)$$

1.2 Surds

1.2.1 Key surd rules

Multiplying

$$\sqrt{x}\sqrt{y} = \sqrt{xy} \quad (7)$$

Dividing

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}} \quad (8)$$

1.2.2 Simplifying surds

To simplify a surd, find the largest square number that divides it.

$$\begin{aligned} \sqrt{50} &= \sqrt{25 \times 2} \\ &= 25\sqrt{2} \end{aligned} \quad (9)$$

1.2.3 Rationalising the denominator

To rationalise the denominator of a fraction containing a surd, multiply by a fraction with that surd top and bottom (which is equal to 1).

$$\begin{aligned}\frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}}{3}\end{aligned}\tag{10}$$

For more complicated instances, the difference of two squares can be used.

$$\begin{aligned}\frac{3}{2 + \sqrt{5}} &= \frac{3}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}} \\ &= \frac{3(2 - \sqrt{5})}{(2 + \sqrt{5})(2 - \sqrt{5})} \\ &= \frac{6 - 3\sqrt{5}}{-1} \\ &= 3\sqrt{5} - 6\end{aligned}\tag{11}$$

1.3 Completing the Square

Completing the square rearranges a quadratic from the form $ax^2 + bx + c$ into the form $p(x + q)^2 + r$. If $a \neq 1$, then this is done by factorising a out of the first 2 terms.

$$\begin{aligned} 2x^2 - 5x + 1 &= 2 \left(x^2 + \frac{5}{2}x \right) + 1 && // \text{ factorise out } a \\ &= 2 \left[\left(x - \frac{5}{4} \right)^2 - \left(\frac{5}{4} \right)^2 \right] + 1 && // \text{ CTS inner part} \\ &= 2 \left(x - \frac{5}{4} \right)^2 - \frac{25}{8} + 1 && // \text{ multiply out} \\ &= 2 \left(x - \frac{5}{4} \right)^2 - \frac{7}{8} + 1 && // \text{ simplify} \quad (12) \end{aligned}$$