1 Representations of quadratics

Consider the quadratic equation $ax^2+bx+c=0$ (and/or its graph, $y = ax^2+bx+c$). Given the quadratic in this form, and assuming that $a = 1$ and also that the quadratic will factorise (these things are assumed throughout this section), we can find other useful information about (or if you want to think about it this way, forms of) this quadratic.

1.1 Y-intercept

The point at which the quadratic's graph crosses the y-axis, or rather, its coordinate when $x = 0$. It is c from $y = ax^2 + bx + c$, because when $x = 0$, $ax^2 + bx$ must also equal 0.

$$\begin{aligned} \text{Equation:} \quad & y = x^2 + 2x - 24 \\ \text{Y-intercept:} \quad & (0, -24) \end{aligned} \tag{1}$$

1.2 Factorisation

A factorised version of the quadratic in the form $(x+m)(x+n)$. Useful for finding the roots. For $y = ax^2 + bx + c$, m and n are two numbers that sum to a and multiply to b.

$$\begin{aligned} &\text{Equation:} && y = x^2 + 2x - 24 \\ &\text{Factorisation:} && (x+6)(x-4) \end{aligned} \tag{2}$$

1.3 Roots

The roots (or solutions) are the values of x that satisfy $ax^2+bx+c = 0$. They are also the x-coordinates at which $y = ax^2 + bx + c$ crosses the x-axis. To find them, take m and n from the quadratic's factorisation, and multiply by -1.

$$\begin{aligned} &\text{Factorisation:} && (x+6)(x-4) \\ &\text{Roots:} && x = -6 \text{ or } x = 4 \end{aligned} \tag{3}$$

1.4 Completed the square

This version takes the from $(x+p)^2+q$. It's useful for finding the turning points of the graph. For $y=ax^2+bx+c$, $p=\frac{b}{2}$, and $q=c-q^2$.

$$\begin{aligned} \text{Equation:} \quad & y=x^2+2x-24 \\ \text{Completed the square:} \quad & (x+1)^2-25 \end{aligned} \tag{4}$$

1.5 Turning point

The turning point of the graph, found using the completed the square form of the equation. The x-coordinate is $-p$, because this would make the entire expression as small as possible (0, because no square term can be smaller), and the y-coordinate is simply q, because that will be all that's left when $(x+p)^2=0$.

$$\begin{aligned} \text{Completed the square:} \quad & (x+1)^2-25 \\ \text{Turning point:} \quad & (-1,-25) \end{aligned} \tag{5}$$

1.6 Sketch

Sketches of the graph should have their intercepts labelled, as well as the turning point.

So for $y = x^2 + 4x + 3$ (the other one was too big to draw nicely):