Angular spectrum with attenuation and dispersion and flux-conservative shock wave propagation

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Objective: to introduce the equations, numerical methods, and code usage for the angular spectrum code with frequency-dependent attenuation and dispersion and flux-conservative shock wave propagation capabilities.

1 Description of code

The sample code is based around the propagation of an imaging pulse that is propagating in homogeneous tissue. Here we scanned a clinical imaging transducer with a hydrophone with a distance of 2mm between the hydrophone and the trasducer face. The scan plane was parallel to the transducer face and the recording was performed as a function of time (Fig.1)

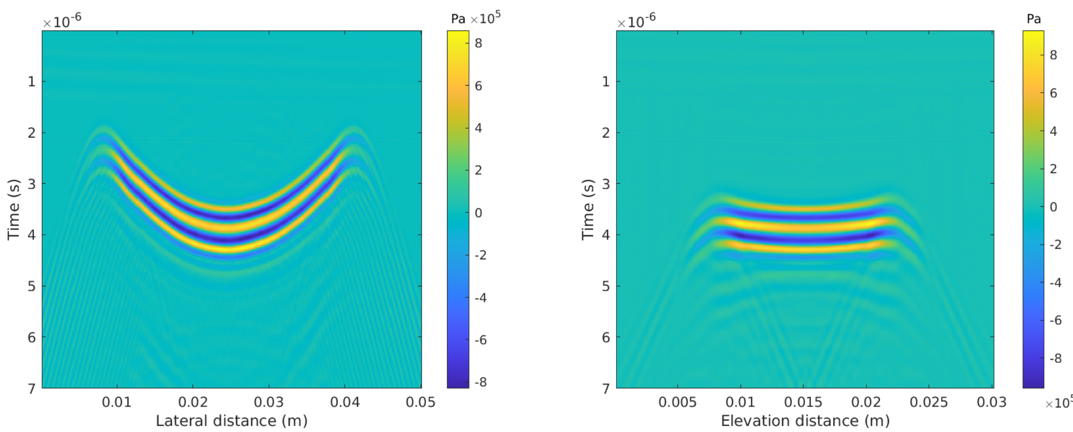

Figure 1: Sections of a 2D space, 1D time hydrophone scan of an imaging pulse emitted by a clinical transducer. Lateral plane (left) and elevational plane (right).

The sampling grid of the hydrophone measurements generally differs from the native simulation grid sampling in both space and time. The hydrophone measurement is thus interpolated to the numerical grid first in time, using a 1D interpolation function, `interp1easy.m`, and then using a 2D interpolation function in space, `interp2easy.m`. A hydrophone scan is not required to run the code. You can, for example, use analytical representations of different acoustical fields.

The grid spacing is defined in terms of the characteristic wavelength, λ, or period, T. By default the simulation `launch_asr3.m` has a grid size of $\lambda/5$ in the $x-y$ plane and 1.6λ in z, the propagation dimension. The time step is by default $T/25$. Smaller grid sizes in space and time will have broader spectral supports that allows the resolution of higher nonlinearities by the simulation. It will also lengthen the computational time.

The propagation, attenuation, and absorbing boundary layer operators are pre-calculated for a given grid size. The propagation operator is pre-calculated in the `precalculate_mas` and it is based on the angular spectrum method. Its output is a propagation matrix `HH` which is then used in the Fourier domain multiplication.

The attenuation can be calculated using two functions. In the first, simpler case, `precalculate_ad` the attenuation is proportional to the f^2, as you would expect in thermoviscous fluids such as water. In this case there is no dispersion. The attenuation coefficient has units of dB/MHz^2/cm. In the second case, `precalculate_ad_pow2`, the attenuation law is assumed to be a power law which has an attenuation/dispersion relationship determined by the Kramers-Kronig relation. The definition of this power law is meant to be conformal to the ultrasound literature. The power in the power law is defined by the variable `pow` and the attenuation is determined by the coefficient α_0, which has units dB/MHz$^{\text{pow}}$/cm. For example,$\alpha_0 = 0.3$ and `pow`=1 is equivalant to a 0.3 dB/MHz/cm attenuation law. The function will automatically calculate the Kramers Kronig dispersion based on the definition of attenuation and there is no need to calculate it explicitly.

Absorbing boundary layers are calculated in `precalculate_abl` using a simple approach. The attenuation with the outer 1/5 of the simulation domain has an attenuation that increases as a quadratic function (Fig. 2). This attenuation boundary layered is multiplied with all three dimensions of the pressure field for each propagation step.

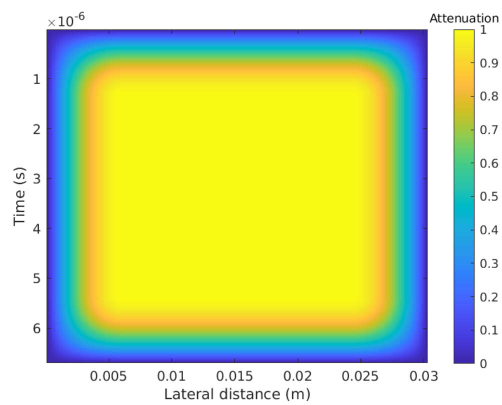

Figure 2: Absorbing boundary layer.