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```
syms q1 q2 q1d q2d
question4_func(q1, q2, q1d, q2d)
function []= question4_func(q1, q2, q1d, q2d)

clc
%clear all;
close all;

I1=1; I2 = 1; m1=0.5; r1=0.15; m2=0.5; r2=.15; l1=0.3; l2=0.3;

a = I1+I2+m1*r1^2+ m2*(l1^2+ r2^2);
b = m2*l1*r2;
d = I2+ m2*r2^2;

global Mmat_symb Cmat_symb G_matrix_symb
```

Model equations

```
Mmat_symb = [a+2*b*cos(q1), d+b*cos(q1); d+b*cos(q1), d];
Cmat_symb = [-b*sin(q1)*q2d, -b*sin(q1)*(q1d+q2d); b*sin(q2)*q1d,0];
g = 9.81;
G_matrix_symb = [(m1*r1 + m2*l1)*g * sin(q1) + m2 * r2 * g * sin(q1 +
q2);...
m2 * r2 * g * sin(q1 + q2)];

%xd = input('X coordinate ');
%yd = input('Y coordinate ');
```

position calculations

```
posi = [0.300,0.450]
posf = [-0.300,0.450]

[q1l,qi2] = twoDOFIK(posi(1),posi(2),1);
[qf1,qf2] = twoDOFIK(posf(1),posf(2),-1);
```

```
x0= [qi1,qi2,0,0]; % Initial Condition - Format:
[theta1,theta2,dtheta1,dtheta2]
```

```
tf=10;
```

```
posi =
```

```
    0.3000    0.4500
```

```
posf =
```

```
   -0.3000    0.4500
```

Solve the closed-loop system nonlinear differential equation (PlanarArmODE) via ode45

```
%ode45 solves the differential equation and returns X with respect to
T.
```

```
global torque
```

```
torque=[];
```

```
[T,X] = ode45(@(t,x)planarArmODE(t,x),[0 tf],x0);
```

```
temp = ones(length(X(:,1)));
```

```
[x,y] = twoDOFFK([X(:,1),X(:,2)],l1,l2);
```

```
poses = [x,y];
```

```
linkx = l1*cos(X(:,1));
```

```
linky = l1*sin(X(:,1)) ;
```

```
links = [linkx,linky];
```

```
error_joints = [temp * qf1 - X(:,1),temp * qf1 - X(:,2)];
```

```
error_task = sqrt((posf(1)-x).^2 + (posf(2)-y).^2) ;
```

```
task_velocity = diff(poses);
```

```
t =
```

```
    0
```

```
t =
```

```
  4.6897e-06
```

```
t =
```

7.0346e-06

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2.0843e-05

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$$t = 9.7750$$

$$t = 9.7750$$

$$t = 9.5853$$

$$t = 9.5960$$

$$t = 9.6498$$

$$t = 9.6594$$

$$t = 9.6713$$

$$t = 9.6713$$

$$t = 9.6928$$

$$t = 9.7036$$

$$t = 9.7573$$

$$t = 9.7669$$

$$t = 9.7788$$

$$t = 9.7788$$

$$t = 9.8009$$

$$t = 9.8119$$

$$t = 9.8669$$

$$t = 9.8767$$

```
t =  
  
9.8890
```

```
t =  
  
9.8890
```

```
t =  
  
9.9112
```

```
t =  
  
9.9223
```

```
t =  
  
9.9778
```

```
t =  
  
9.9877
```

```
t =  
  
10
```

```
t =  
  
10
```

Plot Data

```
figure('Name','Model');  
plot([0,links(end,1),poses(end,1)],  
[0,links(end,2),poses(end,2)], 'r-');  
hold on  
plot([0,links(1,1),poses(1,1)], [0,links(1,2),poses(1,2)], 'b-');  
hold on  
  
figure('Name','Task Error');  
plot(T, error_task, 'r-');
```

```
hold on

figure('Name','Positions red is x');
plot(T, x, 'r-');
hold on
plot(T, y, 'b-');
hold on

figure('Name','Theta_1 Error');
plot(T, error_joints(:,1), 'r-');
hold on

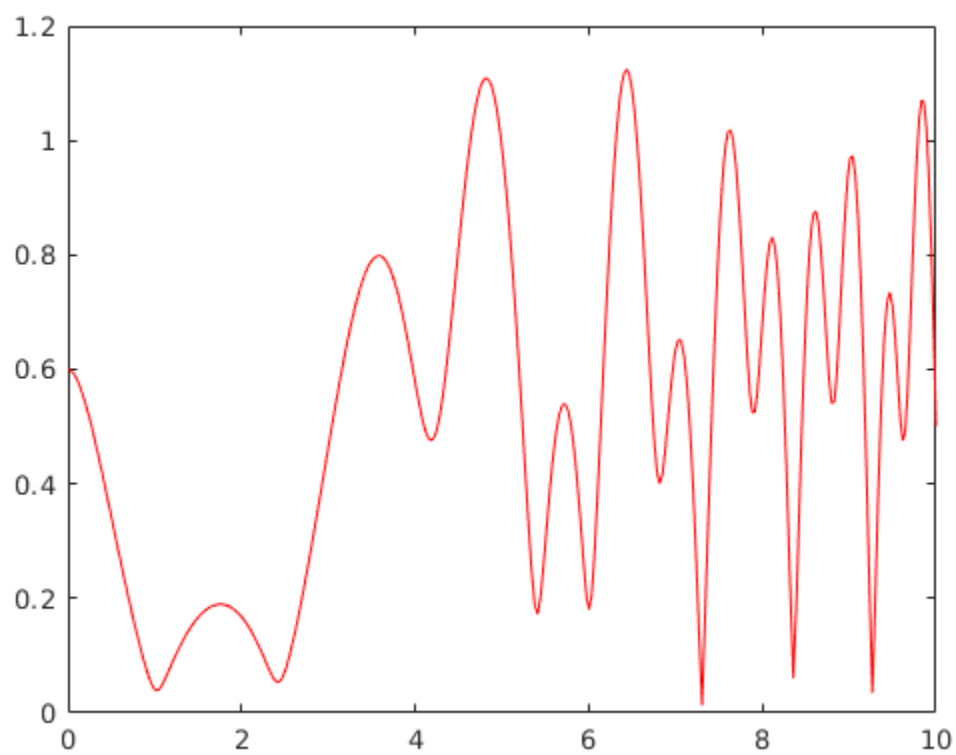
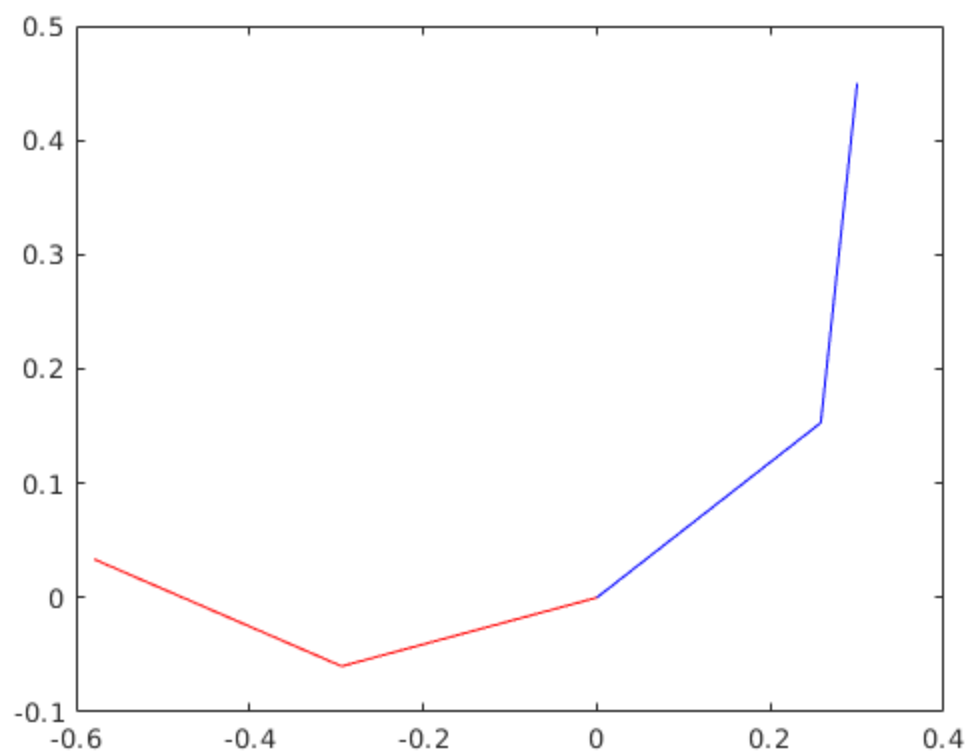
figure('Name','Theta_2 Error');
plot(T, error_joints(:,2), 'r-');
hold on

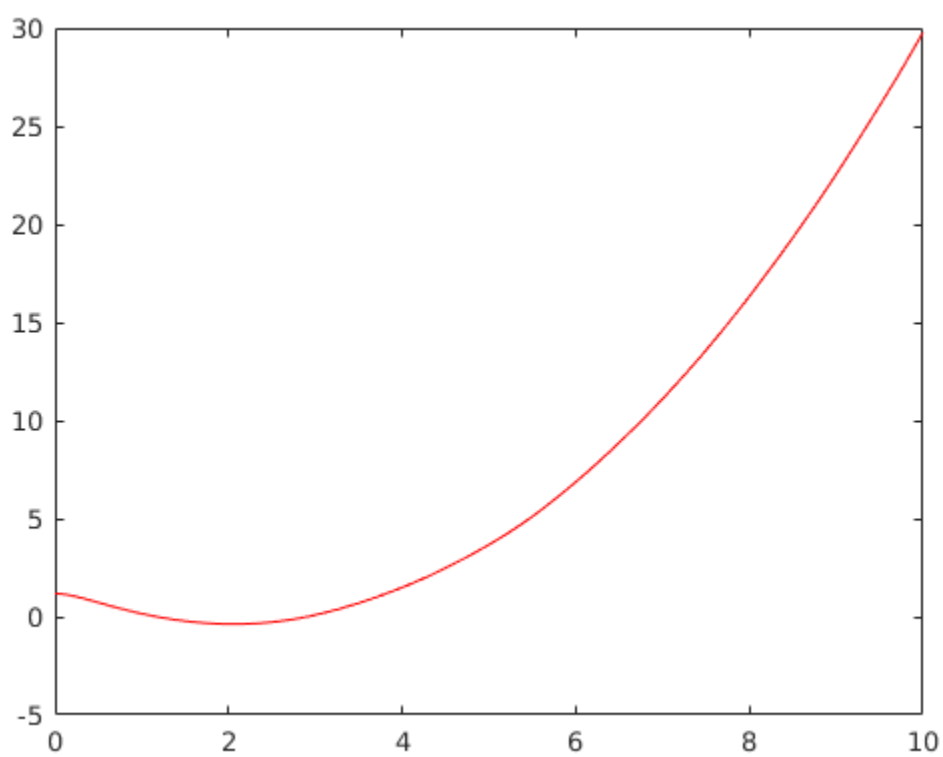
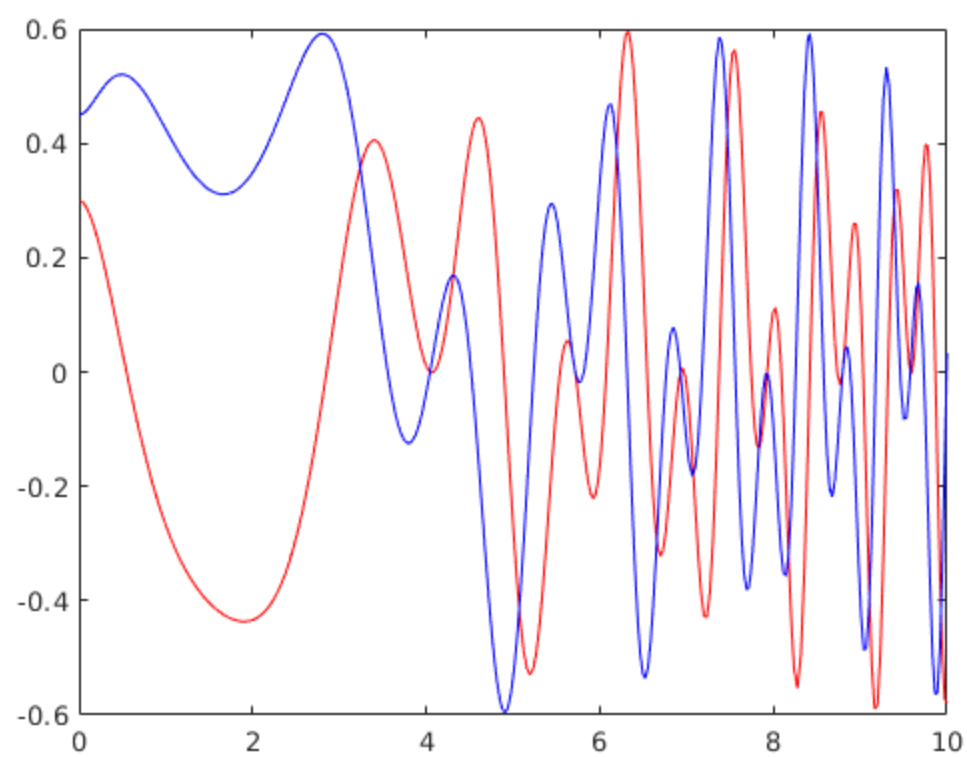
figure('Name','Theta_1 Position');
plot(T, X(:,1), 'r-');
hold on

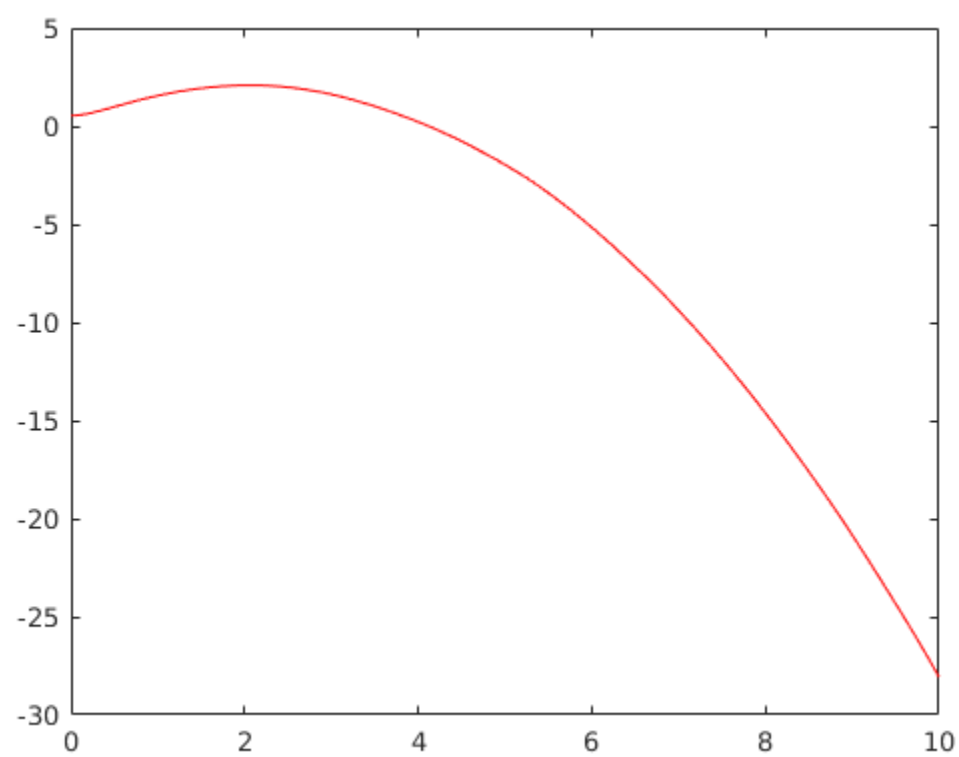
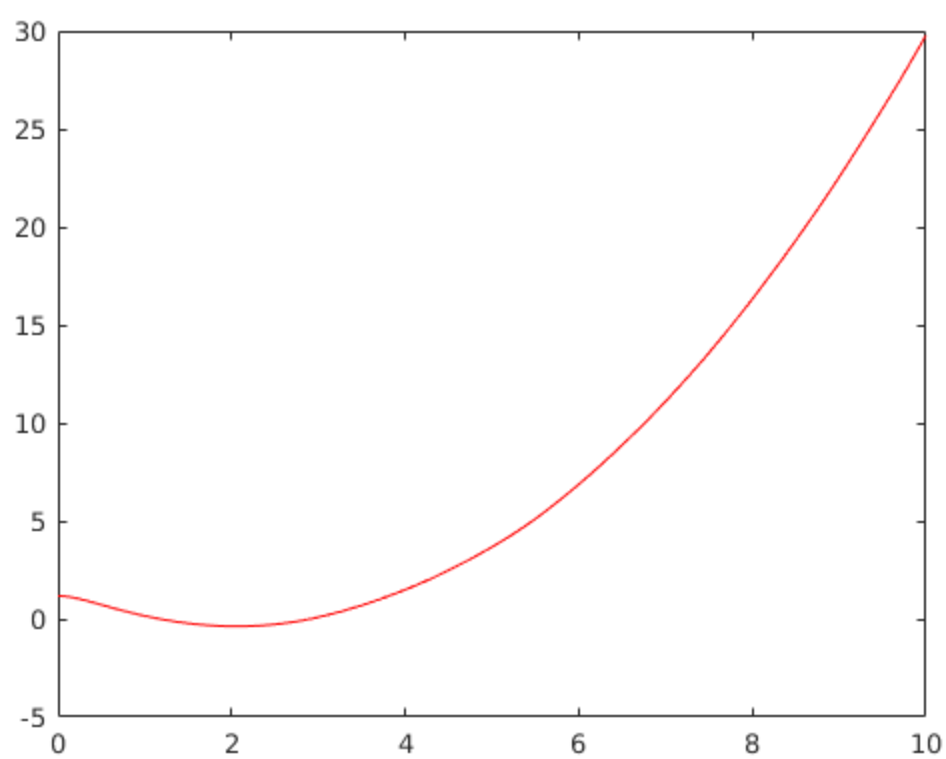
figure('Name','Theta_1 Velocity ');
plot(T, [0;diff(X(:,1))], 'r-');
hold on
%}
figure('Name','Theta_2 under PD SetPoint Control');
plot(T, X(:,2), 'r--');
hold on
figure('Name','Theta_2 Velocity ');
plot(T, [0;diff(X(:,2))], 'r-');
hold on

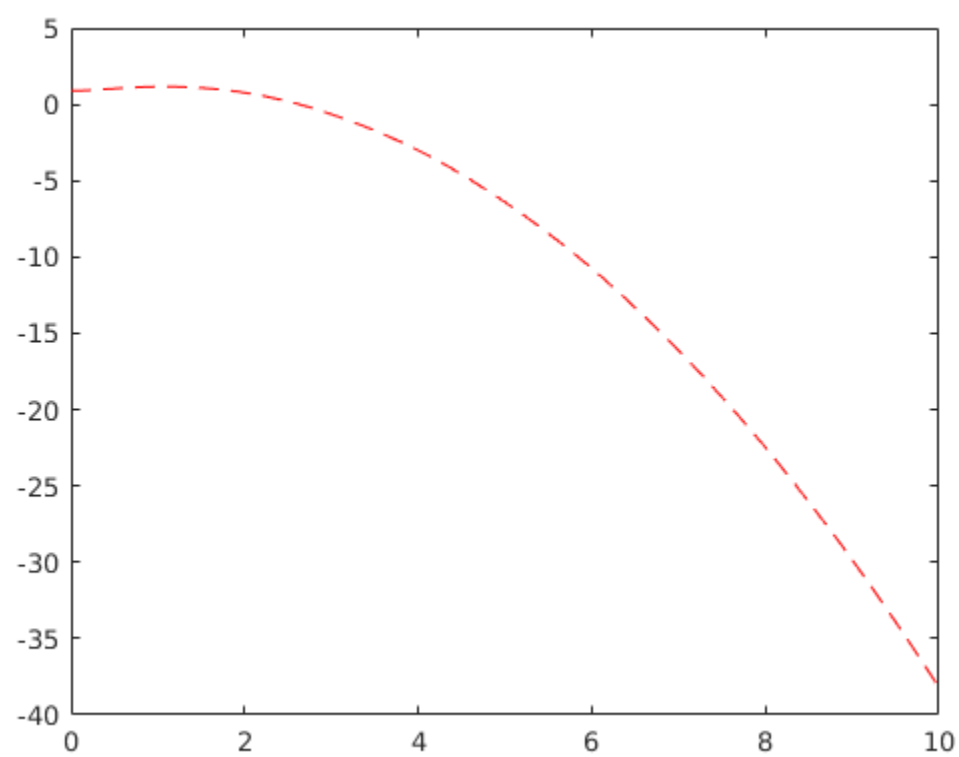
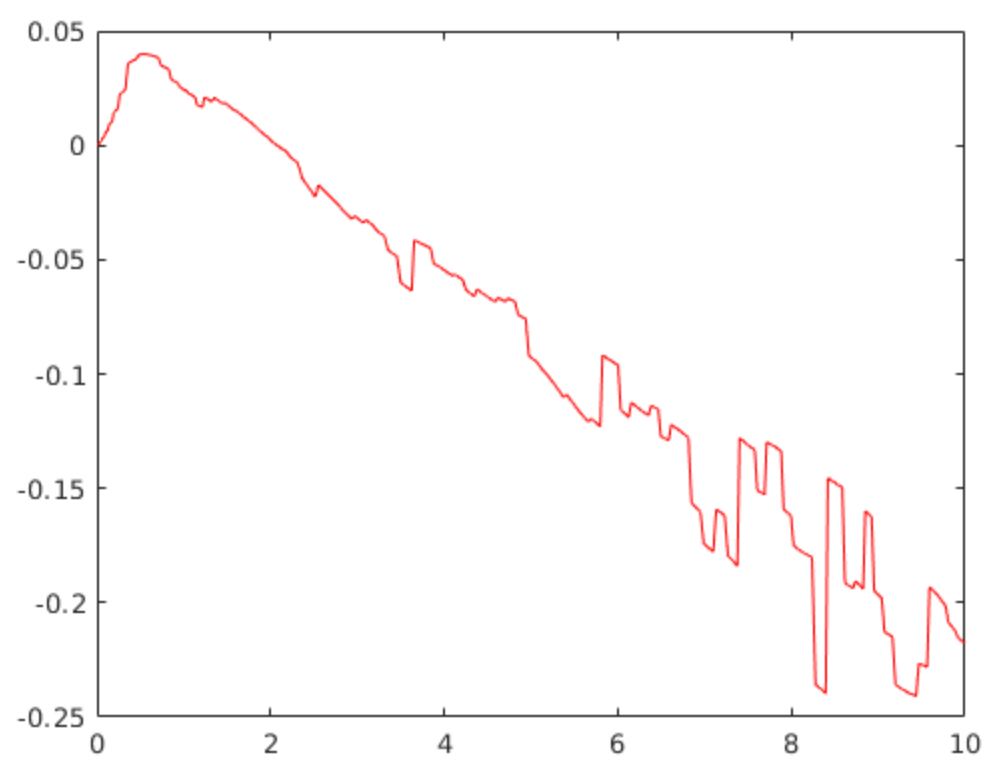
figure('Name','Input_PD control');
plot(T, torque(1,1:size(T,1)), '- ');
hold on
plot(T, torque(2,1:size(T,1)), 'r--');

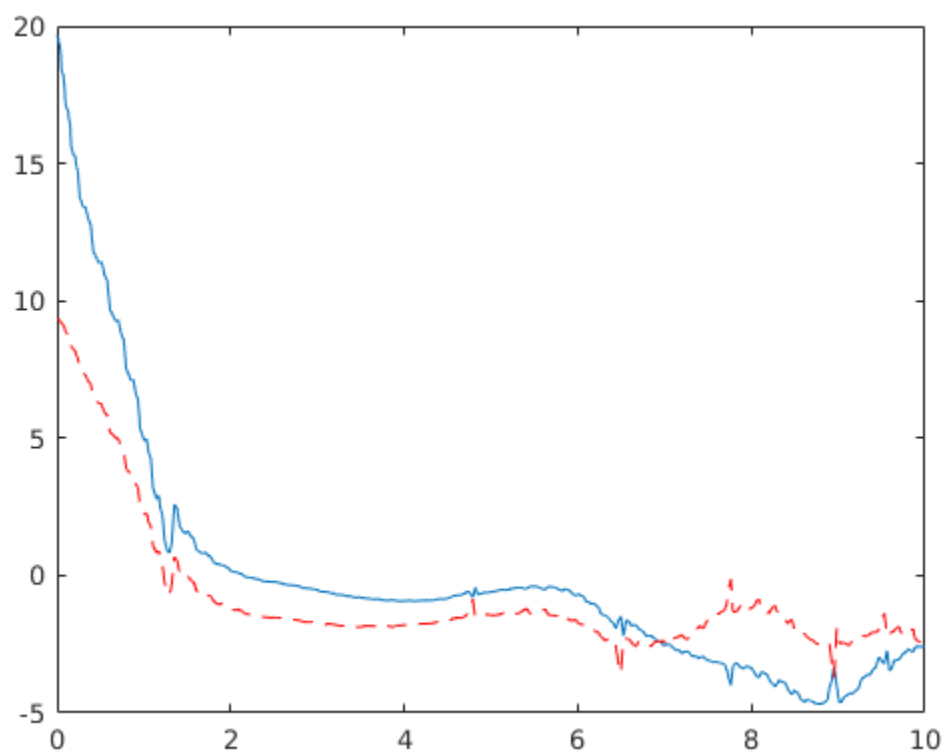
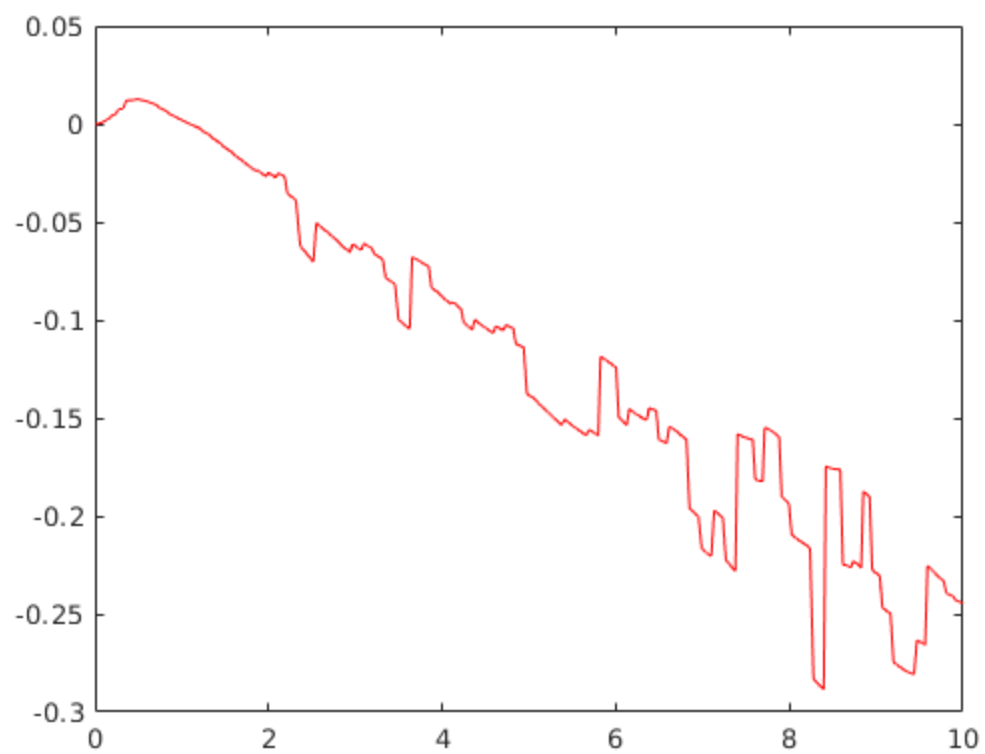
torque=[];
```











Defining Functions

```
function [q1,q2] = twoDOFIK(x,y,direction)
    q2 = direction * acos((x^2 + y^2 - l1^2 - l2^2)/(2*l1*l2));
    B = atan2((l2*sin(q2)),(l1 + l2*cos(q2)));
    Y = atan2(y,x);
    q1 = Y - direction * B;
end
function [x,y] = twoDOFFK(data,l1,l2)
    q1 = data(:,1);
    q2 = data(:,2);

    x = l1 * cos(q1) + l2*cos(q1 + q2);
    y = l1 * sin(q1) + l2*sin(q1 + q2);
end
function dx = planarArmODE(t,x)
    theta_d=[qf1;-qf2]; % Desired Set-Point Position
    dtheta_d=[0;0]; % Desired velocity (Derivative of theta_d)
    ddtheta_d=[0;0];
    theta= x(1:2,1);
    dtheta= x(3:4,1);

    t
    Mmat = subs(Mmat_symb,[q1;q2;q1d;q2d],x(:,1));
    Cmat = subs(Cmat_symb,[q1;q2;q1d;q2d],x(:,1));
    G_matrix = subs(G_matrix_symb,[q1;q2;q1d;q2d],x(:,1));

    Mmat_D = subs(Mmat_symb,[q1;q2;q1d;q2d],[theta_d;dtheta_d]);
    Cmat_D = subs(Cmat_symb,[q1;q2;q1d;q2d],[theta_d;dtheta_d]);
    G_matrix_D = subs(G_matrix_symb,[q1;q2;q1d;q2d],
[theta_d;dtheta_d]);
%     tau =
    Computed_Torque(theta_d,dtheta_d,ddtheta_d,theta,dtheta,t);
    tau = PDCControl(theta_d,dtheta_d,theta,dtheta,t)...
        + Mmat_D*ddtheta_d + Cmat_D * dtheta_d + G_matrix_D;

    torque =[torque, tau];
    dx=zeros(4,1);
    dx(1) = x(3); %dtheta1
    dx(2) = x(4); %dtheta2
    dx(3:4) = Mmat\(tau-Cmat*x(3:4) - G_matrix);
    %dx(3:4) = tau
end

function tau =
    Computed_Torque(theta_d,dtheta_d,ddtheta_d,theta,dtheta,time)
    Kp=[1500,0;...
        0,14000];
    Kv=[77.46,0;...
        0,236.64];
```

```
time
e=theta_d-theta; % position error
de = dtheta_d - dtheta; % velocity error

tau = Kp*e + Kv*de + ddtheta_d;

end
function tau = PDControl(theta_d,dtheta_d,ddtheta_d,theta,dtheta)
    Kp=10*eye(2);
    Kv=10*eye(2);
    e=theta_d-theta; % position error
    de = dtheta_d - dtheta; % velocity error
    tau = Kp*e + Kv*de;
end

disp('Finish.');
```

Finish.

```
end
```

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