M2 Ingénierie et Finance
Pricing lookback options with Monte-Carlo method

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An lookback option depends on the maximum or the minimum of the underlying price during the period from issue of the contract until maturity. A lookback call issued at time T_0 maturing at time T pays the diffrence between the price of the underlying S at maturity and the minimum of $S(t)$ for all times from issue until maturity:

$$S(T) - \min_{T_0 \leq t \leq T} S(t)$$

A lookback put pays the difference between the maximum of the underlying and its price:

$$\max_{T_0 \leq t \leq T} S(t) - S(T)$$

We will compute the price of lookback options with European exercise using a Monte-Carlo method. The numerical computations must be written using the `C++` programming language. The user interface should use `Excel`.

Inputs

1. The computation date, if not present the date of the computation day should be used. The option is supposed to be issued at this date.
2. The option maturity.
3. The option type (Call or Put).
4. The price of the underlying share S_0.
5. The constant risk-free yield r.
6. The volatility σ.
7. The Monte-Carlo method parameters.

Computation method

The Black-Scholes method should be used to describe the underlying dynamics. The Monte-Carlo method will be used to perform the numerical computations.

Outputs

Let $P(S, t, r, \sigma)$ be the price of the option. The code should comput the following outputs:

1. The theoretical price of the option: P.
2. The Δ of the option: $\frac{\partial P}{\partial S}$.
3. The Γ of the option : $\frac{\partial^2 P}{\partial S^2}$.
4. The Θ of the option : $\frac{\partial P}{\partial t}$.
5. The ρ of the option : $\frac{\partial P}{\partial r}$.
6. The vega of the option : $\frac{\partial P}{\partial \sigma}$.
7. A plot of the option price $P(S, T_0)$ as a function of the price of the underlying share at computation date T_0.
8. A plot of the option $\Delta(S, T_0)$ s a function of the price of the underlying share at computation date T_0.

References

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[5] Emmanuel Gobet, *Monte-Carlo Methods and Stochastic Processes: From Linear to Non-Linear*, CRC Press, 2016