Vector Space Definitions

Practice Quiz • 15 min • 3 total points

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1. Which set of three-by-one matrices (with real number scalars) is not a vector space? **1 / 1 point**

- ○ The set of three-by-one matrices with zero in the second row.
- ◉ The set of three-by-one matrices with the sum of all the rows equal to one.
- ○ The set of three-by-one matrices with the first row equal to the third row.
- ○ The set of three-by-one matrices with the first row equal to the sum of the second and third rows.

Correct

2. Which one of the following sets of vectors is linearly independent? **1 / 1 point**

- ○ $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$
- ○ $\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} \right\}$
- ○ $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$
- ◉ $\left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$

Correct

3. Which one of the following is an orthonormal basis for the vector space of all three-by-one matrices with the sum of all rows equal to zero? **1 / 1 point**

- ○ $\left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$
- ◉ $\left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \right\}$
- ○ $\left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$
- ○ $\left\{ \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \right\}$

Correct

Gram-Schmidt Process

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1. In the fourth step of the Gram-Schmidt process, the vector 1 / 1 point

$$u_4 = v_4 - \frac{(u_1^T v_4)u_1}{u_1^T u_1} - \frac{(u_2^T v_4)u_2}{u_2^T u_2} - \frac{(u_3^T v_4)u_3}{u_3^T u_3}$$

is always orthogonal to

- (●) v_1
- ( ) v_2
- ( ) v_3
- ( ) v_4

Correct

This is because $u_1 = v_1$.

2. The Gram-Schmidt process applied to 1 / 1 point

$$\{v_1, v_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

results in

- (●) $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$
- ( ) $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$
- ( ) $\{\hat{u}_1, \hat{u}_2\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$
- ( ) $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\}$

Correct

3. The Gram-Schmidt process applied to

1 / 1 point

$$\{v_1, v_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

results in

- $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$
- (selected) $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \right\}$
- $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}$
- $\{\hat{u}_1, \hat{u}_2\} = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

Correct

Fundamental Subspaces

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Go to next item

1. Which of the following sets of vectors form a basis for the null space of

1 / 1 point

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 1 \\ 3 & 6 & 1 & 1 \end{pmatrix}?$$

- $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 0 \\ 0 \\ -3 \\ 2 \end{pmatrix} \right\}$
- (selected) $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

Correct

2. The general solution to the system of equations given by **1 / 1 point**

$$x_1 + 2x_2 + x_4 = 1,$$

$$2x_1 + 4x_2 + x_3 + x_4 = 1,$$

$$3x_1 + 6x_2 + x_3 + x_4 = 1,$$

is

- $a \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- **(selected)** $a \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- $a \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -3 \\ 2 \end{pmatrix}$
- $a \begin{pmatrix} 0 \\ 0 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Correct

3. What is the rank of the matrix **1 / 1 point**

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 1 \\ 3 & 6 & 1 & 1 \end{pmatrix}?$$

- 1
- 2
- **(selected)** 3
- 4

Correct

Orthogonal Projections

Practice Quiz • 15 min • 3 total points

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Grade received 100% **To pass** 100% or higher

Go to next item

1. Which vector is the orthogonal projection of $\mathrm{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ onto $W = \operatorname{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \right\}$? **1 / 1 point**

- $\frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$
- (selected) $\frac{1}{3}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$
- $\frac{1}{3}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$
- $\frac{1}{3}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

Correct

2. Suppose we have data points given by $(x_n, y_n) = (1,1), (2,1)$, and $(3,3)$. If the data is to be fit by the line $y = \beta_0 + \beta_1 x$, which is the overdetermined equation for β_0 and β_1? **1 / 1 point**

- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
- (selected) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

Correct

3. Suppose we have data points given by $(x_n, y_n) = (1,1), (2,1)$, and $(3,3)$. Which is the best fit line to the data? **1 / 1 point**

- $y = \frac{1}{3} + x$
- (selected) $y = -\frac{1}{3} + x$
- $y = 1 + \frac{1}{3}x$
- $y = 1 - \frac{1}{3}x$

Correct

Week Three Assessment
Graded Quiz • 30 min

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Go to next item

1. Which set of three-by-one matrices (with real number scalars) is not a vector space? **1 / 1 point**

- The set of three-by-one matrices with zero in the third row.
- The set of three-by-one matrices with the first row one larger than the third row.
- The set of three-by-one matrices with the sum of all the rows equal to zero.
- The set of three-by-one matrices with the first row equal to the negative of the third row.

Correct

2. Which of the following sets of vectors are linearly independent? **1 / 1 point**

- $\left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}$

Correct

3. What is the dimension of the vector space consisting of five-by-one column matrices where the rows sum to zero and the first row is equal to the second row? **1 / 1 point**

- 5
- 4
- 3
- 2

Correct

4. Which of the following is NOT an orthonormal basis for the vector space of all three-by-one matrices with the first row equal to twice the third row? **1 / 1 point**

- $\left\{ \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$
- $\left\{ \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \right\}$
- $\left\{ \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\}$

Correct

5. The Gram-Schmidt process applied to **1 / 1 point**

$$\{v_1, v_2\} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

results in

- (●) $\{u_1, u_2\} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$
- ( ) $\{u_1, u_2\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$
- ( ) $\{u_1, u_2\} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$
- ( ) $\{u_1, u_2\} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

Correct

6. Which of the following sets of vectors form a basis for the null space of **1 / 1 point**

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 4 & -4 & 3 & 6 \\ 2 & -2 & 1 & 3 \end{pmatrix}?$$

- ( ) $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \right\}$
- ( ) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\}$
- ( ) $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\}$
- (●) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

Correct

7. The general solution to the system of equations given by **1 / 1 point**

$$x_1 - x_2 + x_3 + x_4 = 1,$$

$$4x_1 - 4x_2 + 3x_3 + 6x_4 = 0,$$

$$2x_1 - 2x_2 + x_3 + 3x_4 = 0,$$

is

- $a \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ 0 \\ -2 \end{pmatrix}$
- $a \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ 0 \\ -2 \end{pmatrix}$
- $a \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 3 \\ 0 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- $a \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 3 \\ 0 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

Correct

8. What is the rank of the following matrix: **1 / 1 point**

$$\begin{pmatrix} 1 & -2 & 0 & 1 \\ 2 & -4 & 1 & 2 \\ 3 & -6 & 1 & 3 \end{pmatrix}?$$

- 1
- 2
- 3
- 4

Correct

9. Which vector is the orthogonal projection of $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ onto $W = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \right\}$? **1 / 1 point**

- ○ $\begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix}$
- ◉ $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
- ○ $\begin{pmatrix} \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{6}} \\ \frac{2}{\sqrt{3}} - \frac{4}{\sqrt{6}} \\ -\frac{2}{\sqrt{3}} - \frac{2}{\sqrt{6}} \end{pmatrix}$
- ○ $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

✓ Correct

10. Suppose we have data points given by $(x_n, y_n) = (0,0), (1,2)$, and $(2,1)$. Which is the best fit line to the data? **1 / 1 point**

- ○ $y = 1$
- ○ $y = x$
- ◉ $y = \frac{1}{2} + \frac{1}{2}x$
- ○ $y = \frac{3}{2} - \frac{1}{4}x$

✓ Correct