ASTR232, Fall 2019
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Homework # 1

Problem 2a

It is convenient to derive a PLR from a plot of M_v vs. log (P)-1, assuming the following form for the PLR: $M_v = \alpha + \beta$[log(P) - 1]. Determine the value of β that is required if this form is to represent a true linear period-luminosity relation, i.e., one where L $\propto$ P.

Solution

The relation for magnitude and luminosity is

$$M_2 - M_1 = -2.5\log(\frac{L_1}{L_2}) \tag{1}$$

Solving for M_2,

$$M_2 = M_1 - 2.5\log(\frac{L_1}{L_2}) \tag{2}$$

We are given the equation

$$M_v = \alpha + \beta[\log(P) - 1] \tag{3}$$

Additionally, we are told that $L \propto P$. So we replace P in equation 3 with L. Thus, setting equations 2 and 3 equal to each other, we get that

$$M_1 - 2.5\log(\frac{L_1}{L_2}) = \alpha + \beta[\log(L) - 1].$$

Since the left hand side (equation 2) requires that 2 objects be compared, we use the sun as our reference. Thus,

$$M_\odot - 2.5(\log(L_\odot) - \log(L_2)) = \alpha + \beta[\log(L) - 1].$$

We see that the left and right hand sides of the equations are in the same form. Additionally, we see that the constants of both sides are in the same place. Thus, in order for the equation to be true it must be that $\boxed{\beta = -2.5}$.

Problem 2b

Correct the V magnitudes in Table 1 for extinction (please refer to the extinction supplement on Moodle). Then compute the distances to the stars and their absolute magnitudes.

Solution

To correct for the V magnitudes, we use the equation

$$V_{intrinsic} = V_{obs} - A_v \tag{4}$$

The V_{obs} and A_v are given so we can do this calculation for each star. (All calculations done using Python. Code is attached.) We can then find the distance in parsecs using the parallax measurements given. Since the parallax is in milliarcsec, we use the following formula:

$$\text{Distance(pc)} = \frac{1000}{\pi(\text{mas})} \tag{5}$$

Finally, to get absolute magnitudes, we use our results above and plug into the distance modulus:

$$M_V = -5\log(\frac{\text{d(pc)}}{10}) + V_{intrinsic} \tag{6}$$

The final table with the new calculations is

	period(d)	parallax(mas)	v_mag	a_v	v_intrinsic	distance(pc)	abs_mag
0	35.551341	2.01	3.652	0.52	3.132	497.512438	-5.352020
1	10.150730	2.78	3.881	0.06	3.821	359.712230	-3.958776
2	9.842425	3.14	3.731	0.25	3.481	318.471338	-4.034352
3	7.594904	2.28	4.607	0.37	4.237	438.596491	-3.973326
4	7.012877	3.00	4.526	0.58	3.946	333.333333	-3.668394
5	5.773380	2.13	5.593	0.67	4.923	469.483568	-3.435102
6	5.366270	3.66	3.950	0.23	3.720	273.224044	-3.462595
7	4.470916	2.81	5.342	0.64	4.702	355.871886	-3.054468
8	4.435462	1.90	5.632	0.34	5.292	526.315789	-3.314232
9	3.728190	2.40	5.414	0.20	5.214	416.666667	-2.884944

Problem 2c

Plot M_v vs. [log (P) - 1] for all 10 stars. Make sure the plot is correctly labeled and properly scaled. Then use the method of least squares to determine the slope β and zero-point α of the PLR. (See Secs. 8.1–8.2 of An

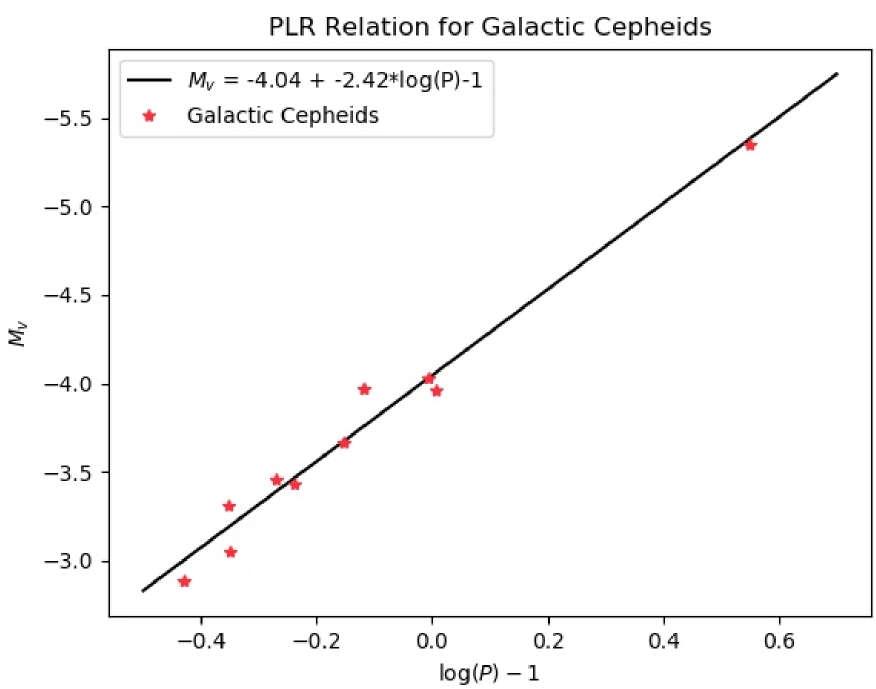
Introduction to Error Analysis by J. R. Taylor if you need a refresher on least-squares fitting.) Report the expression you obtain for the PLR, and add the best-fit line to your plot.

Solution

We plot the results from 2b and using the least squares routine (created for ASTR211), we find the α and β values of the PLR. That is, we find that $\alpha = -4.04$ and $\beta = -2.42$, which is very close to what we expect from 2a. Our equation for the best fit line is thus

$$M_v = -4.04 - 2.42 \log(P) - 1 \tag{7}$$

We plot the best fit line and our data points below:



Problem 3a

The second supplement on Moodle for this assignment contains basic data for Cepheids in an external galaxy. In this file, the extinction is given in terms of the color excess E(B -V). Again, referring to the extinction supplement, apply extinction corrections to the stars' V magnitudes assuming R = 3.1.

Solution

In the dataframe, we are given E(B-V) and told that $R = 3.1$. To find the reddening correction value A_v, we use the following formula:

$$A_v = 3.1\left(E(B - V)\right) \tag{8}$$

We apply equation 8 to each row in the dataframe. The observed magnitudes are given as well, so we then use equation 4 again to find our corrected magnitude values.

Problem 3b

Using the PLR obtained for the Galactic sample in part 2(c), compute the distances (in Mpc) to the 72 stars in the table. Then calculate the average distance $\bar{d}$ and the standard deviation $\sigma_d = \sqrt{\frac{1}{N}\sum(d_i - \bar{d})^2}$.

Solution

To solve for absolute magnitude, we plug in the period for each of our stars into equation 7. We then manipulate the distance modulus to solve for distance. Therfore, we get the formula

$$d = 10^{\frac{m-M+5}{5}} \tag{9}$$

We plug in the absolute and corrected apparent magnitudes for our stars into equation 9 to solve for distance. Using the pandas library functions to solve for the mean and standard deviation, we get that

$$\boxed{\bar{d} = 7.07 \text{ Mpc}} \text{ and } \boxed{\sigma_d = 0.199}.$$

Problem 3c

The measured redshift of this galaxy is z = 0.001494. Assuming a Hubble constant of 73 km s^{-1} Mpc^{-1} and the distance obtained in part 3(a), calculate the radial component of the galaxy's peculiar velocity v_{pec}.

Solution

We know that the total observed velocity is made up of two components: radial velocity (v_r) and peculiar velocity (v_{pec}). Thus the formula for observed velocity is given by

$$v_{obs} = v_r + v_{pec} \tag{10}$$

Since we want to solve for peculiar velocity in this problem, we rearrange equation 10 to get

$$v_{pec} = v_{obs} - v_r \tag{11}$$

Now, to get v_{obs}, we use the formula

$$v_{obs} = cz \tag{12}$$

where c is speed of light and z is redshift. Plugging in $c = 2.998 \times 10^5$km/s and $z = 0.001494$ into equation 12, we get that

$$v_{obs} = (2.998 \times 10^5)(0.001494) = 447.9\text{km/s}.$$

Next, to solve for radial velocity, we use Hubble's law:

$$v_r = H_0 d \tag{13}$$

Using a Hubble constant of 73 km/s/Mpc and a distance of 7.07 Mpc, we get that

$$v_r = (73)(7.07) = 516.11\text{km/s}.$$

Plugging v_{obs} and v_r back into equation 11, we can solve for the peculiar velocity.

$$v_{pec} = 447.9 - 515.11$$

$$\boxed{v_{pec} = -68.2\text{km/s}}.$$

Problem 4

A galaxy has been discovered to contain H_2O masers within a nearly edge-on molecular disk in its nucleus. A high-velocity (or radial) maser is located 8.0 mas (that's milliarcseconds) from the black hole. Its orbital velcocity is 770 km s^{-1}. The proper motion of a systemic maser has been measured at 31.5 μas yr^{-1}. Deprojection of the maser disk indicates the systemic maser is 4.1 mas from the black hole. Use these data to compute the geometric distance to the galaxy.

Solution

We are given a lot of information in different units, so we first gather all our numbers and write them in units that we can use in our calculations:

$$v_r = 770 \text{ km/s} = 7.7 \times 10^7 \text{ cm}$$
$$\dot{\phi} = 31.5\ \mu\text{as/yr} = 4.843 \times 10^{-18} \text{ rad/s}$$
$$\theta_{sys} = 4.1 \text{ mas} = 2 \times 10^{-8} \text{ rad}$$
$$\theta_{rad} = 8.0 \text{ mas} = 3.88 \times 10^{-8} \text{ rad}$$

Using the variables defined above, we can use the following equations to solve for distance:

$$k = v_r \sqrt{\theta_{rad}} \tag{14}$$

and

$$d = \frac{k/\sqrt{\theta_{sys}}}{\dot{\phi}} \tag{15}$$

We first solve for k using equation 14:

$$k = (7.7 \times 10^7)(\sqrt{3.88 \times 10^{-8}}) = 15167.2 \text{ km/s} \cdot \text{ rad}$$

We then plug in our k value into equation 15 along with the other required variables:

$$d = \frac{15167.2/\sqrt{2 \times 10^{-8}}}{4.843 \times 10^{-18}} = 2.215 \times 10^{25} \text{cm}.$$

Converting our distance to Mpc, we get a final answer of

$$\boxed{d = 7.17 \text{ Mpc}}.$$

Python Code:

```
'''
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ASTR232 - Cosmology
Sept 11, 2019
HW1
'''

### routines needed for the assignment ###


#To find the line of best fit's slope and y-int, I created a method
    of least squares fxn
def least_squares(x,y):
   N=float(len(x))
   #We know will use the equations given in class to find A and B
   #A= y-int, B= slope
   delta=(N*(np.sum(x**2)))-((np.sum(x))**2)
   A_top=((np.sum(y))*(np.sum(x**2)))-((np.sum(x))*(np.sum(x*y)))
   A=(A_top)/(delta)
   B_top=((N*(np.sum(x*y)))-((np.sum(x))*(np.sum(y))))
   B=(B_top)/(delta)
   return A,B

### importing the required libraries ###
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

#####################################
#####################################
#######        #2         ########
#####################################
#####################################

    #manually entering the data for table 1
period = [35.551341, 10.150730, 9.842425, 7.594904, 7.012877,
    5.773380, 5.366270, 4.470916, 4.435462, 3.728190]
parallax = [2.01, 2.78, 3.14, 2.28, 3.00, 2.13, 3.66, 2.81, 1.90,
    2.40]
v_mag = [3.652, 3.881, 3.731, 4.607, 4.526, 5.593, 3.950, 5.342,
    5.632, 5.414]
a_v = [0.52,0.06,0.25,0.37,0.58,0.67,0.23,0.64,0.34,0.20]
```

```python

   #converting the lists into a python dictionary
dict = {
'period(d)':period,
'parallax(mas)':parallax,
'v_mag':v_mag,
'a_v':a_v
}

   #converting the dictionary into a pandas dataframe for easier handling
df = pd.DataFrame.from_dict(dict)

#####################################
#####################################
#######        #2b        ########
#####################################
#####################################

    # creating a new column for the corrected magnitudes using v_intrinsic = v_mag - a_v
df['v_intrinsic'] = df['v_mag'] - df['a_v']
    # using parallax (mas) to find distance in pc and making a new column for it
df['distance(pc)'] = 1000/ (df['parallax(mas)'])
    # calculating the abs mags using distance modulus and making a new column for it
df['abs_mag'] = -5*np.log10(df['distance(pc)']/10) + df['v_intrinsic']

#####################################
#####################################
#######        #2c        ########
#####################################
#####################################

   #plotting period against abs mag
plt.scatter( np.log10(df['period(d)'])-1, df['abs_mag'],color='k',s=12)
plt.xlabel(r'$ \log(P) -1 $')
plt.ylabel(r'$M_v$')
plt.close()

   #using the least_squares routine on our period and abs mag data to find the line of best fit
```

```
yint, slope = least_squares( np.log10(df['period(d)'])-1, df['abs_mag'] )

   #creating the line of best fit in order to plot it over our data
x = np.linspace( -0.5,0.7,100 )
plt.plot(x,(slope*x)+yint,'black',label=r'$M_v$ = '+str(yint)[:5]+' + '+str(slope)[:5]+'*log(P)-1' )
plt.plot( np.log10(df['period(d)'])-1, df['abs_mag'],'.',label='Galactic Cepheids',c='#F5313F',marker = "*")
plt.xlabel(r'$ \log(P) -1 $')
plt.ylabel(r'$M_v$')
plt.legend()
plt.title('PLR Relation for Galactic Cepheids')
   #inverting the y axis so that brighter stars are higher in the plt
plt.gca().invert_yaxis()
   #saving plt as a jpg file
plt.savefig('2c.jpg')
plt.close()

######################################
######################################
#######        #3         ########
######################################
######################################

   #reading in table 2 into a pandas dataframe
table2 = pd.read_csv('hw1_table2.txt',sep='\s+',header=0,index_col=None)

######################################
######################################
#######        #3a        ########
######################################
######################################

   #creating a new column of a_v values found using a_v = 3.1 * E(B-V)
table2['A_v'] = 3.1 * table2['E(B-V)']
   #creating a new column of corrected v magnitudes
table2['v_intrinsic'] = table2['V_obs'] - table2['A_v']

######################################
######################################
#######        #3b        ########
```

```
#####################################
#####################################

   #creating a column of abs mag values using the PLR best fit line found in question 2
table2['abs_mag'] = yint + slope*(np.log10(table2['P(d)']) -1)
   #creating a column of distances using the distance modulus
table2['distance(pc)'] = 10** (( table2['v_intrinsic'] - table2['abs_mag'] +5 )/5)
   #converting the distances from pc to Mpc
table2['distance(Mpc)'] = table2['distance(pc)'] / 1e6
   #using pandas inherit fxns to find the mean and st.dev of our distances
print('avg distance to stars in Mpc: ',table2['distance(Mpc)'].mean())
print('std distance to stars in Mpc: ',table2['distance(Mpc)'].std())
```