AST 231: Problem Set 1

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February 7, 2020

Problem 1. The Solar Constant is about 1365 W/m^2. Calculate the distance from a 100 W light bulb at which its flux has the same value as the Solar Constant. Verify your result by holding your hand at that distance from the light bulb, but PLEASE do not BURN yourself due to an incorrect calculation!

Answer 1. We know that flux, luminosity, and distance are related by the following equation

$$f = \frac{L}{4\pi d^2} \tag{1}$$

To find the distance at which a 100 W light bulb will have the same flux as the solar constant, we rearrange equation one to solve for d and plug in $L = 100$ W and $f = 1365$ W m^{-2}. Doing this, we get:

$$d = \sqrt{\frac{L}{4\pi f}}$$

$$d = \sqrt{\frac{100}{4\pi 1365}}$$

$$\boxed{d = 0.076 \text{ m}}$$

Thus, for a light bulb to have the same flux as the solar constant, it needs to be at a distance of $d = 0.076$m.

Problem 2. A star that appeared to be single was measured to have V = 12.13 and B-V = 1.28. On closer inspection with HST it was found that this star is actually a visual binary, with component B being 0.90 mag fainter in V and 1.80 mag fainter in B than component A. Calculate the V magnitude and B-V color of each component.

Answer 2. We are given that the V magnitude of the system is 12.13 and that $V_A - V_B = 0.9$. From these two quantities, we can determine the total flux and flux ratio of the stars using the following equations:

$$m_{\text{total}} = -2.5 \log_{10}\left(\frac{f_{\lambda,\text{total}}}{\text{zero mag flux}}\right) \tag{2}$$

and

$$m_A - m_B = -2.5\log_{10}\left(\frac{f_{\lambda,\mathrm{A}}}{f_{\lambda,\mathrm{B}}}\right) \tag{3}$$

We can then set up a system of equations our results from which we can find the flux of each component:

$$f_{\lambda,\mathrm{A}} + f_{\lambda,\mathrm{B}} = f_{\lambda,\mathrm{total}} \tag{4}$$

$$\frac{f_{\lambda,\mathrm{A}}}{f_{\lambda,\mathrm{B}}} = \mathrm{const.} \tag{5}$$

We use the NASA/IPAC Infrared Science Archive [1] to find the zero mag fluxes in the V and B band: 3781 Jy and 4130 Jy, respectively. We follow the steps above to find the B and V mags of both stars.

V mag:

Using V_{total}=12.13 and zero mag flux in V=3781 Jy, we plug into equation 2:

$$12.13 = -2.5\log\left(\frac{f_{\mathrm{V,total}}}{3781}\right)$$

$$\rightarrow f_{\mathrm{V,total}} = 0.0053 \text{ Jy}$$

Using V_B-V_A=0.9, we use equation 3 to find the flux ratio:

$$0.9 = -2.5\log_{10}\left(\frac{f_{\mathrm{V,B}}}{f_{\mathrm{V,A}}}\right)$$

$$\rightarrow \frac{f_{\mathrm{V,B}}}{f_{\mathrm{V,A}}} = 0.437$$

We now can create a system of equation:

$$f_{\mathrm{V,A}} + f_{\mathrm{V,B}} = f_{\mathrm{V,total}} = 0.0053 \text{ Jy}$$

$$f_{\mathrm{V,B}} = 0.437 \cdot f_{\mathrm{V,A}}$$

From this, we get that $f_{\mathrm{V,B}} = 0.0161$ Jy and $f_{\mathrm{V,A}} = 0.0369$ Jy. We plug these values into equation 2 to get the V mags:

$$\boxed{V_B = -2.5\log\left(\frac{0.0161}{3781}\right) = 13.43}$$

$$\boxed{V_A = -2.5\log\left(\frac{0.0369}{3781}\right) = 12.53}$$

B mag:

Using B_{total}=13.41 and zero mag flux in B=4130 Jy, we plug into equation 2:

$$13.41 = -2.5\log\left(\frac{f_{\mathrm{B,total}}}{4130}\right)$$

$$\rightarrow f_{\mathrm{B,total}} = 0.0179 \text{ Jy}$$

Using B_B-B_A=1.80, we use equation 3 to find the flux ratio:

$$1.80 = -2.5\log_{10}\left(\frac{f_{\mathrm{B,B}}}{f_{\mathrm{B,A}}}\right)$$

$$\rightarrow \frac{f_{\mathrm{B,B}}}{f_{\mathrm{B,A}}} = 0.191$$

We now can create a system of equation:

$$f_{\mathrm{B,A}} + f_{\mathrm{B,B}} = f_{\mathrm{B,total}} = 0.0179 \text{ Jy}$$

$$f_{\mathrm{B,B}} = 0.191 \cdot f_{\mathrm{B,A}}$$

From this, we get that $f_{\mathrm{B,B}} = 0.0029$ Jy and $f_{\mathrm{B,A}} = 0.015$ Jy. We plug these values into equation 2 to get the B mags:

$$B_B = -2.5\log\left(\frac{0.0029}{4130}\right) = 15.38$$

$$B_A = -2.5\log\left(\frac{0.015}{4130}\right) = 13.6$$

[1] https://irsa.ipac.caltech.edu/data/SPITZER/docs/spitzermission/missionoverview/spitzertelescopehandbook/19/

We can now find the color of the two stars:

$$\boxed{B_B - V_B = 1.95}$$

$$\boxed{B_A - V_A = 1.07}$$

Problem 3. Photometric data on a star are given in the table below. For your convenience, I also provide the flux density of a zero magnitude star and effective wavelengths of the filters for the photometric system in which the star was observed. The flux densities are in Jansky's (Jy), a favorite unit of infrared and radio astronomers. Be careful, because the Jy is a unit of F_ν, not F_λ. There is a link on the course Moodle page to a Web site that will help you do the conversions correctly. Plot the tabulated data on a F_λ versus log (λ) diagram. Make the scale be W/m^2/micron, as on the example, with wavelength expressed in microns. On the same diagram plot the blackbody curve (Planck function) with the temperature that you think best fits the data. In other words, you are using the spectral energy distribution (fondly called the SED by most astronomers) to estimate the effective temperature of the star. Be sure to state the effective temperature that you derive for the star by this method.

Filter	V	I	J	H	K
Magnitude	16.11	14.47	13.42	12.74	12.47
Effective Wavelength (in microns)	0.545	0.798	1.25	1.65	2.20
Zero mag Flux Density (Jy)	3636.0	2416.0	1670.0	980.0	620.0

Answer 3. Using the the data given, we can use the zero mag flux density and the magnitude for each filter to find frequency flux (f_ν) values in Jy units using the following equation:

$$f_\nu = (\text{zero mag flux density}) \cdot 10^{\frac{\text{-mag}}{2.5}} \tag{6}$$

or

```
df['f_nu(Jy)'] = df['zero_mag_flux_density(Jy)']*10**(df['magnitude'] / (-2.5))
```

We then can turn the f_ν (Jy) into a f_λ (W m^{-2} μm^{-1}) from the next equation:

$$f_\lambda = f_\nu \cdot 10^{-26} \cdot \left(\frac{c}{\lambda^2}\right) \tag{7}$$

or

```
df['f_lambda(W/m2/micron)'] = (df['f_nu(Jy)'] * (10**-26) * c) / (df['eff_wav(
    micron)']**2)
```

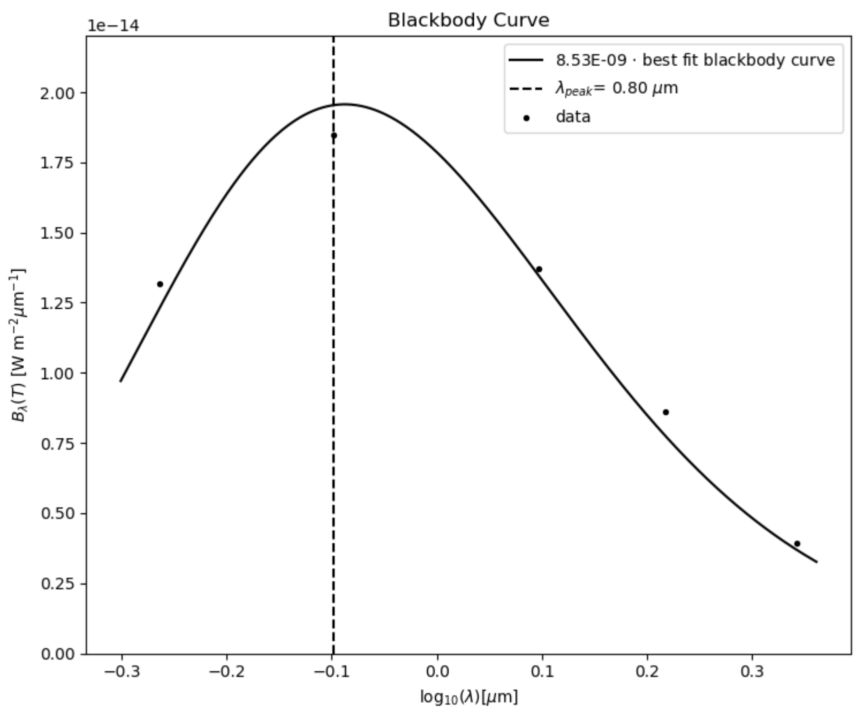
where c=2.988×10^{14} μm s^{-1} and effective λ in units of microns. All this is done in python, where we plot these f_λ values with their corresponding wavelength values and fit the data points with the `planck function`:

$$B_\lambda(T, \lambda, \text{scale factor}) = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda KT}} - 1}\right) \cdot \text{scale factor} \tag{8}$$

or

```python
def planck(wavelength,temperature,scale):
    #default parameters / constants
    h=6.626e-34
    c=2.998e14
    k=1.3806e-23
    #function
    term1 = (2 * h * c**2) / wavelength**5
    exponential_term = (h*c) / (wavelength * k * temperature)
    term2_bttm = np.exp(exponential_term) - 1
    term2 = 1 / term2_bttm
    return term1 * term2 * scale
```

To do the fitting, we imported the `scipy.optimize` package into python and used the `curve_fit` function, so that we get an optimal value for the temperature and scale factor, which allows us to fit the blackbody curve to our data and create the following plot:



Our fit reports an effective temperature of $\boxed{T_{\rm eff} = 3545\ {\rm K}}$ and a scale factor of 8.53×10^{-9}. The effective temperature classifies this star as an M2V star (Table 4 of Pecaut & Mamajek (2013, ApJS, 208, 9)[1].

To make sure that our Python program gives a reasonable result, assume that the highest flux value observed is our peak of the blackbody curve. This makes the second data point

[1] https://arxiv.org/pdf/1307.2657.pdf

in the plot the peak of the blackbody curve. Therefore, $\lambda_{\text{peak}} = 80\mu\text{m}$. We can then use the wein's displacement law to also get T_{color}:

$$\lambda_{\text{peak}} = \frac{b}{T} \tag{9}$$

where b = $2898\mu\text{m}\cdot\text{K}$.

Plugging in $\lambda_{\text{peak}} = 80\mu\text{m}$, we get $\boxed{T_{\text{color}} = 3631\text{K}}$, very close to our program's result.

Code

```python
import pandas as pd
import numpy as np
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt

df = pd.read_csv('pset1_q3_data.csv')

#functions

'''
Planck function for creating blackbody curves.
Input temperature in kelvins and wavelengths in meters unless you change
the default parameters to fit your units.
h = planck constant, default units in J*s
c = speed of light, default units in micron/s
k = boltzmann constant, default units in J/K.
Returns intensity in W/m^2 unless default parameters changed.
'''
def planck(wavelength,temperature,scale):
    #default parameters / constants
    h=6.626e-34
    c=2.998e14
    k=1.3806e-23
    #function
    term1 = (2 * h * c**2) / wavelength**5
    exponential_term = (h*c) / (wavelength * k * temperature)
    term2_bttm = np.exp(exponential_term) - 1
    term2 = 1 / term2_bttm
    return term1 * term2 * scale


#constants
c = 2.99e14 #speed of light in meters per seconds

#data manipulation
    #using zero mag fluxes & mags to solve for flux in frequency
df['f_nu(Jy)'] = df['zero_mag_flux_density(Jy)']*10**(df['magnitude'] / (-2.5))
    # solving for flux in W/m^2
df['f_lambda(W/m2/micron)'] = (df['f_nu(Jy)'] * (10**-26) * c) / (df['eff_wav(micron)']**2)

'''
fitting our data w the plank function
'''
pp = 5000,8.5e-9 #an initial guess for the temperature and scale factor
```

```
popt,pcov = curve_fit(planck,df['eff_wav(micron)'],df['f_lambda(W/m2/micron)'],pp) #scipy curve fitting

print('the temperature[k] of star is: \t',popt[0])
print('the scale factor is: \t',popt[1])

#plotting our fit line and data points
wavelengths = np.linspace(0.5,2.3,1000)
intensities = planck(wavelengths,popt[0],popt[1])
plt.plot(np.log10(wavelengths),intensities,color='black',label='best fit blackbody curve',ls='-')
plt.scatter(np.log10(df['eff_wav(micron)']),df['f_lambda(W/m2/micron)'],s=8,color='black',label='data')
plt.axvline(x=np.log10(df['eff_wav(micron)'].iloc[1]),color='k',ls='--',label=r'$\lambda_{peak}$= %0.2f $\mu$m'%df['eff_wav(micron)'].iloc[1])
plt.xlabel(r'$\log_{10}(\lambda)$[micron]')
plt.ylabel(r' %0.2E $\cdot B_{\lambda}(T)$ [W m$^{-2}\mu$m$^{-1}$ ]' %popt[1])
plt.legend()
plt.title('Blackbody Fit Over Star Data')
plt.show()
print('temperature using peak wavelength and wein displacement law:', 2898 / df['eff_wav(micron)'].iloc[1] )
```