ASTR232, Fall 2019
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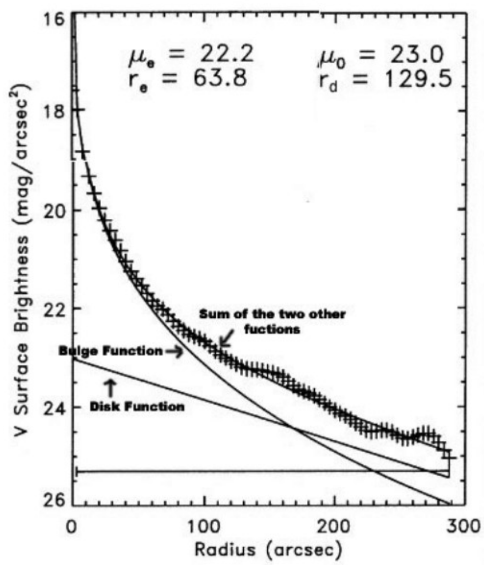
Homework # 3

Problem 1a

Compute the bulge-to-disk ratio of this galaxy.



Solution

From the information that is given in the graph, we see that we can use the following equation:

$$\frac{F_{\text{bulge}}}{F_{\text{disk}}} = (3.16)(\frac{r_e^2}{r_d})(\frac{I_e}{I_0}). \tag{1}$$

We are given $r_e = 63.8$ and $r_d = 129.5$. We need to solve for $\frac{I_e}{I_0}$, which we do with

$$\frac{I_e}{I_0} = 10^{\frac{-(\mu_e - \mu_0)}{2.5}} \tag{2}$$

Using $\mu_e = 22.2$ and $\mu_0 = 23.0$, we plug in to equation 2 and then equation 1 to get:

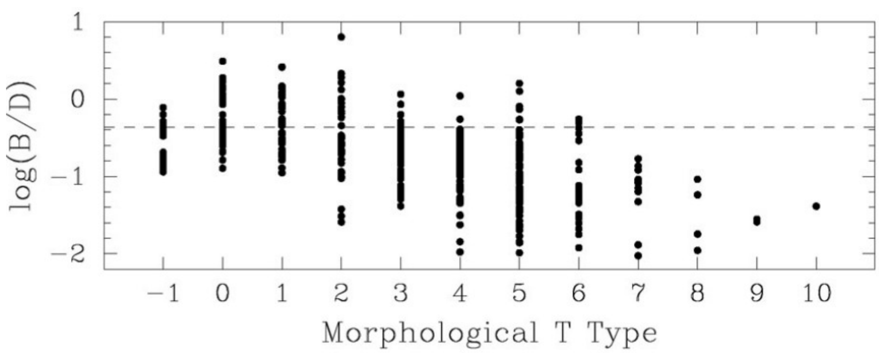
$$\frac{F_{\text{bulge}}}{F_{\text{disk}}} = (3.16)(\frac{63.8^2}{129.5})(10^{\frac{-(22.2-23.0)}{2.5}}) = \boxed{1.83}.$$

Problem 1b

Estimate the galaxy's Hubble type - and justify your answer.

Solution

We compute that $\log(\frac{F_{\text{bulge}}}{F_{\text{disk}}}) = 0.2$. Now, looking at the following plot from lecture 5:



We roughly estimate that the morphological T type of our galaxy is ≈ 2, which corresponds to about a Sa type galaxy.

Problem 2a

What is the central surface brightness $\mu_B(0)$ of this galaxy?

Solution

We are given $r_e = 26.0$ and $\mu_B(r_e) = 21.75$ mag arcsec^{-2}. We plug into the de Vaucouleurs $r^{1/4}$ law:

$$\mu(r) = \mu_e + 8.326\left((\frac{r}{r_e})^{1/4} - 1\right) \tag{3}$$

Plugging in the values:

$$\mu(0) = 21.75 + 8.326(0 - 1)$$

$$\boxed{\mu(0) = 13.4}$$

Problem 2b

Considering your solution to part (a) and the appearence of the surface brightness suggest a reason why it is more practical to use μ_e rather than μ_0 to characterize the luminosity distributions of ellipticals and spiral galaxy bulges.

Solution

Looking at the graph, at μ_0, the bulge function accounts for most of the surface brightness function, and there is little input from the disk function. This means the entire galaxy is not fully accounted for. On the other hand, at μ_e, has a more even split between the bulge function and the disk function, meaning it incorporates more than just the nucleus for the brightness calculation, giving a more complete picture of the total brightness.

Problem 2c

Using the relationship given in class for the total flux,

$$F_{\text{tot}} = \int_0^{\infty} 2\pi r I(r) dr = 7.22\pi r_e^2 I_e \tag{4}$$

and the definitions for magnitudes and surface brightness, show that the total apparent magnitude of an early-type galaxy can be expressed as follows:

$$m_{\text{tot}} = \mu_e - 2.5 \log(7.22\pi r_e^2). \tag{5}$$

Then compute the total integrated B magnitude of the observed galaxy.

Solution

We start with the following equation

$$A = \frac{F}{I}. \tag{6}$$

Plugging in equation 4 into equation 6, we get that

$$A = \frac{7.22\pi r_e^2 I}{I} = 7.22\pi r_e^2.$$

We now plug A into

$$m = \mu - 2.5\log(A) \tag{7}$$

to get that

$$\boxed{m = \mu - 2.5\log(7.22\pi r_e^2)}.$$

which matches equation 6, as desired.

We now plug in $r_e = 26.0$ and $\mu = 21.75$ to solve for the total integrated B magnitude of the galaxy:

$$m = 21.75 - 2.5\log(7.22\pi(26^2)) = \boxed{11.29}$$

Problem 2d

If the galaxy is 12.0 Mpc away, what is its absolute B magnitude?

Solution

We use the distance modulus and solve for M.

$$M = m - 5\log(\frac{d}{10}) \tag{8}$$

Using $m = 11.29$ and $d = 12 \times 10^6$ pc, we get an absolute B magnitude of

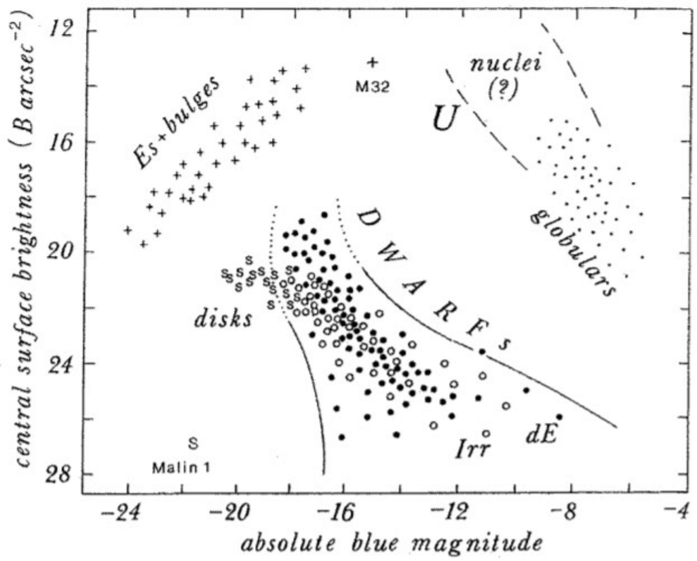
$$M = 11.29 - 5\log(\frac{12 \times 10^6}{10}) = \boxed{-19.12}$$

Problem 2e

Determine the location of this galaxy on the $\mu_B(0)$ vs. MB plot shown during lecture. How does it compare to the locations of other early-type galaxies?

Solution

Looking at plot below from lecture 5:



We see that our galaxy lies at the point (-19.12,13.4), which corresponds to the tip of the Es + bulges cluster. Thus, it has one of the greatest central surface brightness of any galaxy. Comparing to the other Es+ bulges, it a lower absolute magnitude, but it has one of the brightest absolute magnitude comparing it to the disks.

Problem 3a

The stellar mass-to-light ratio of a galaxy can be expressed as

$$\log \frac{M}{L} = a\lambda + b\lambda \times \text{color} \tag{9}$$

Derive an expression for $\log(M_*)$.

Solution

We start off with equation 9, rearrange using log rules and express M and L in terms of stellar mass and stellar luminosity:

$$\log(M) - \log(L) = a\lambda + b\lambda \times \text{color}$$

$$\log(\frac{M_*}{M_\odot}) - \log(\frac{L_*}{L_\odot}) = a\lambda + b\lambda \times \text{color}$$

We now manipulate the following equation:

$$M_2 - M_1 = 2.5\log(\frac{L_1}{L_2}) \tag{10}$$

in order to solve for $\frac{L_1}{L_2}$, where 1 denotes our galaxy and 2 denotes the sun. Thus,

$$M_\odot - M_* = 2.5\log(\frac{L_*}{L_\odot})$$

$$0.4(M_\odot - M_*) = \log(\frac{L_*}{L_\odot})$$

$$\frac{L_*}{L_\odot} = 10^{0.4(M_\odot - M_*)}$$

We now plug the final expression back into our derivation:

$$\log(\frac{M_*}{M_\odot}) - \log(10^{0.4(M_\odot - M_*)}) = a\lambda + b\lambda \times \text{color}$$

$$\log(\frac{M_*}{M_\odot}) - 0.4(M_\odot - M_*) = a\lambda + b\lambda \times \text{color}$$

We now refer back to the distance modulus (equation 8):

$$m_* - M_* = 5\log(d) - 5$$

Adding $M_\odot$ on both sides, we get:

$$M_\odot + m_* - M_* = 5\log(d) - 5 + M_\odot$$

$$M_\odot - M_* = 5\log(d) - 5 + M_\odot - m_*$$

We plug in our final expression back into our derivation:

$$\log(\frac{M_*}{M_\odot}) - 0.4(5\log(d) - 5 + M_\odot - m_*) = a\lambda + b\lambda \times \text{color}$$

$$\log(\frac{M_*}{M_\odot}) = a\lambda + b\lambda \times \text{color} + 0.4(5\log(d) - 5 + M_\odot - m_*)$$

We arrive at our final, desired equation:

$$\boxed{\log(\frac{M_*}{M_\odot}) = 0.4\Big(2.5(a\lambda + b\lambda \times \text{color}) + 5\log(d) - 5 + M_\odot - m_*\Big)}$$

Problem 3b

Go to NED and get data for NGC7331 and then use 3a to compute the galaxy's stellar mass.

Solution

Using NED, we get the following data for NGC7331:

	Hubble flow dist. (Mpc)	J_{tot}	B_T^0	V_T^0	B-V	a_J	b_J
NGC7311	14.4	7.058	9.33	8.75	0.63	-0.261	0.433

We now plug our values into our answer for 3a:

$$\log(\frac{M_*}{M_\odot}) = 0.4\Big(2.5(-0.261+0.433\times0.63)+5\log(14.4\times10^6)-5+4.75-7.058\Big)$$

$$\log(\frac{M_*}{M_\odot}) = 11.4$$

$$\boxed{M_* = 2.54 \times 10^{11} M_\odot}$$

Problem 3c

Obtain the total HI 21 cm flux density of NGC7311 reported in NED. Then compute the HI gas mass of the galaxy.

Solution

From NED, we get that the total HI 21cm flux density is 269.46 Jy km s^{-1}. Now, to find the mass of HI gas, we use the following formula:

$$\frac{M_{\text{HI}}}{M_\odot} = (2.36 \times 10^5)(d^2)(\delta_{21}) \tag{11}$$

Plugging in d = 14.4 Mpc and $\delta_{21} = 269.46$ Jy km s^{-1}, we get

$$\frac{M_{\text{HI}}}{M_\odot} = (2.36 \times 10^5)(14.4^2)(269.46)$$

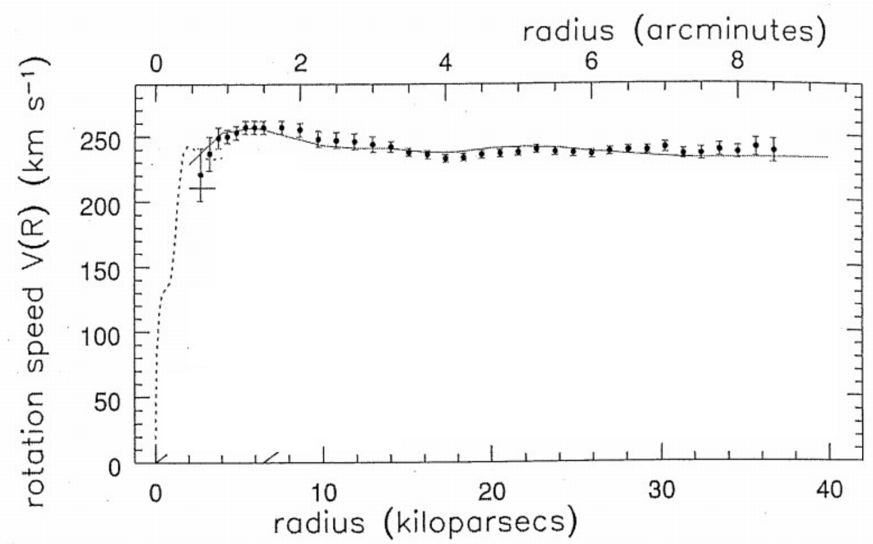
$$\boxed{M_{\text{HI}} = 1.32 \times 10^{10} M_\odot}$$

Problem 3d

Compute the dynamical mass of the galaxy.

Solution

We use the following plot to determine the physical radius of NGC7311:



Points give the rotation curve of NGC 7331

To get the physical radius, we use $\theta = 8.5' = 0.00247$ rad and d $= 14.4 \times 10^3$ Kpc in the following equation:

$$r = \theta d \tag{12}$$

$$r = (0.00247)(35.47) = 35.47 \text{ Kpc}.$$

We then use the dynamical mass equation

$$M = \frac{V_{\text{rot}}^2 r}{G} \tag{13}$$

Plugging in $r = 35.57$ Kpc $= 1.097{\times}10^21$ m, $v_{rot} = 240 \times 10^3$ m/s (estimated from the plot above), and $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, we get:

$$M = \frac{(240 \times 10^3)^2 (1.097 \times 10^{21})}{6.67 \times 10^{-11}} = 9.47 \times 10^{41} \text{ kg}$$

$$\boxed{M = 4.77 \times 10^{11} M_\odot}$$

Problem 3e

Add your answers to parts (b) and (c), and compare the sum to the dynamical mass you calculated in part (d). How closely do they agree? Offer an explanation for the results of your comparison.

Solution

Adding answers from b and c, we get a mass of

$$M = 2.54 \times 10^{11} + 1.32 \times 10^{10} = 2.67 \times 10^{11} M_\odot.$$

Our answer for part d, $4.77 \times 10^{11} M_\odot$, is about 1.8 times larger than just adding b + c. Thus, we can conclude that they do not agree very well. Some possible explanations are that we are only accounting for two objects in the galaxy when adding mass of stars plus mass of HI and ignoring mass contributions from everything else such as planets and black holes. On the other hand, the dynamical mass accounts for all possible factors, including *dark matter*. However, a source of error here comes when picking the exact physical radius of the galaxy. If we overestimate, it is possible that we calculate more mass than is actually in the galaxy.