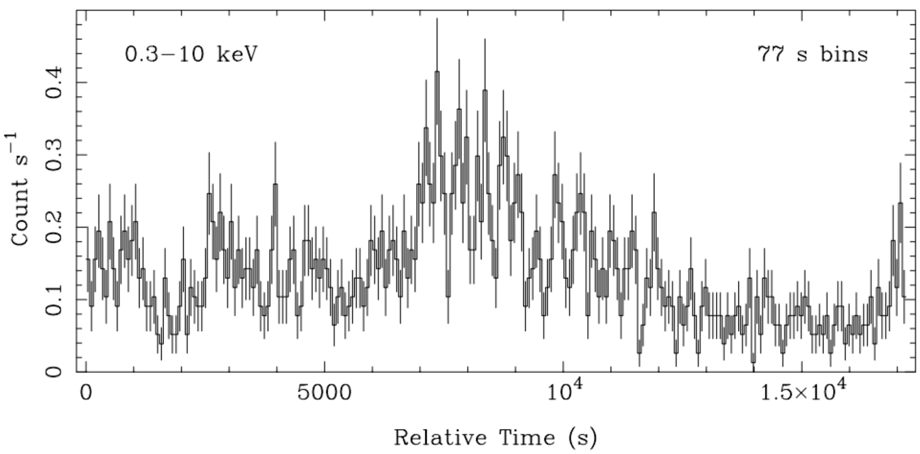
ASTR232, Fall 2019

Gil Garcia[1]

Homework # 5

Problem 1a

Examine the light curve and estimate Δt_{min}. Then compute x_{max}, the upper limit to the size of the emmiting region in cm and AU.



Solution

From the light curve, we estimate a Δt_{min} of 1000 seconds from 2500 seconds to 3500 seconds. Using this, we can calculate the size of the emitting region with the following equation:

$$x = c\Delta t \tag{1}$$

where c is the speed of light. Plugging in our values, we get:

$$x_{max} = c\Delta t_{min} = (2.998\times10^{10}\text{cm/s})(1000\text{s}) = \boxed{2.998\times10^{13}\text{cm} = 2.004\text{ AU}}$$

Problem 1b

Using the value of x_{max} you obtained in (1a), you could compute a limit for the mass of the black hole in this object. (i) How would you approach this and what assumptions would have to be made? (ii) Would you obtain an upper limit or a lower limit? (iii) Explain why this approach would not be particularly useful.

Solution

(i) From 1a, we can assume that the x_{max} is the innermost stable orbit. We can also assume that the orbit of the emmiting region is circular and that the region uniformly emits at the same rate. Thus, we can define our x_{max} to be roughly equal to $3R_s$. Under this assumption, we can use the following equation:

$$R_s = \frac{2GM_{BH}}{c^2} \tag{2}$$

Rearranging this equation and plugging in $x_{max} = 3R_s$, we get the mass of the black hole:

$$M_{BH} = \frac{x_{max}c^2}{6G}$$

(ii) The equation above will produce an upper limit. We are assuming that the what we have resolved is indeed the innermost circular orbit, but it is possible that the we can not resolve an emmiting region that has a smaller x value than what we have resolved. A smaller x value would produce a smaller M_{BH} Additionally, we are assuming that the emmiting region uniformly emits at the same rate. In actuality, it is possible for some regions to not emit. This will cause the Δt value to be less than what we have calculated and hence the values of x and M_{BH} will be smaller.

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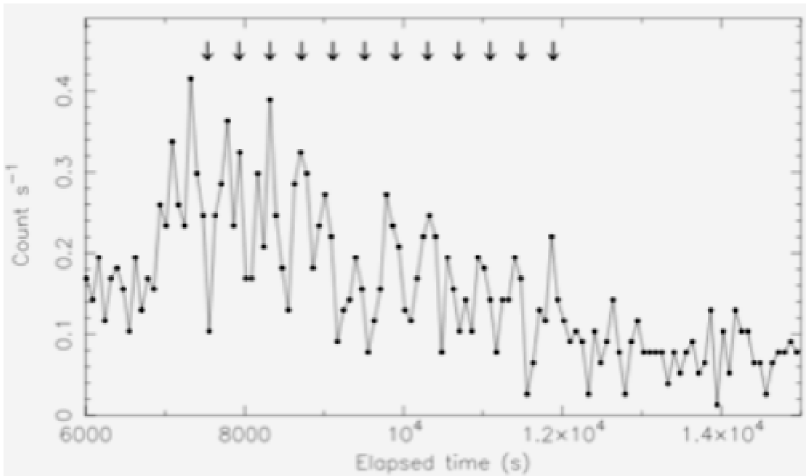
[1] This homework was done without the assistance of any witches and/or witchcraft.

(iii) This approach would not be particularly useful because the assumptions we have made can cause some problems. In particular, we are only allowed to use equation 2 for non-rotating black holes, but it is likely that this black hole could be rotating. Thus, our method is useless.

Problem 1c

The local maxima in the more coarsely binned version of the light curve (shown below) are seperated by 396 seconds. Assuming this period is associated with a bright transient feature orbiting within the accretion disk, use Kepler's third law to obtain a more useful upper limit to the mass of the black hole.



Solution

Kepler's third law is

$$P^2 = \frac{4\pi^2}{GM} a^3 \rightarrow M = \frac{4\pi^2}{GP^2} a^3 \tag{3}$$

We are given that $P = 396$ seconds. We solve for a by using equation 1, but this time with $\Delta t_{max} = 396$. We get:

$$a = c\Delta t = (2.998 \times 10^8 \text{m})(396\text{s}) = 1.19 \times 10^{11} \text{ m}$$

Plugging in our values, we calculate:

$$M = \frac{4\pi^2}{GP^2} a^3 = \frac{4\pi^2}{(6.674 \times 10^{-11})(396)^2} (1.19 \times 10^{11})^3 = \boxed{6.32 \times 10^{39} \text{ kg} = 3.18 \times 10^9 M_\odot}$$

Problem 2a

A $7M_\odot$ star comes close enough to a SMBH that the star becomes tidally locked. The gas from the star forms an accretion disk around the black hole.

Estimate the amount of radiant energy that will be released as a result of the accretion of this gas onto the black hole.

Solution

The amount of energy released is given by the following equation:

$$\Delta U = \frac{mc^2}{6} \tag{4}$$

Plugging in the mass of the sun and the speed of light, we get:

$$\Delta U = \frac{(7)(1.99 \times 10^{30})(2.998 \times 10^8)}{6} = \boxed{2.09 \times 10^{47} \text{J} = 2.09 \times 10^{54} \text{ erg}}$$

Problem 2b

If $M_{BH} = 10^{7.7} M_\odot$, what is the Eddington-limit luminosity L_{Edd}?

Solution

To solve for the Eddington-limit luminosity, we use the following equation:

$$L_{Edd} = 1.3 \times 10^{38} (\frac{M_{BH}}{M_\odot}) \text{ erg/s} \tag{5}$$

Plugging in our mass, we get:

$$L_{Edd} = 1.3 \times 10^{38} (10^{7.7}) = \boxed{6.5 \times 10^{45} \text{ erg/s}}$$

Problem 2c

If $L_{bol} = L_{Edd}$, how long will the AGN phase associated with this particular tidal disruption event last?

Solution

We can find how long it will be in this AGN phase by dividing the total amount of energy released from 2a by the energy release rate from 2b. Thus, we will compute that the black hole will be in this AGN phase for:

$$\text{AGN phase time} = \frac{2.09 \times 10^{54}}{6.5 \times 10^{45}} = 3.22 \times 10^{8} \text{ seconds.}$$

Problem 3a

Consider an AGN with the following properties: $d = 32.3$ Mpc, $\sigma = 264$ km s^{-1}, $M_{BH} = 4.1 \times 10^{8} M_\odot$.

What is the physical radius of the black hole's sphere of influence in this AGN?

Solution

We find the radius of the SOI with the following equation:

$$R_{\text{SOI}} = \frac{GM_{BH}}{\sigma^2} \approx 11.2 (\frac{M_{BH}}{10^8 M_\odot})(\frac{\sigma}{200\text{km/s}})^{-2} \text{pc} \tag{6}$$

Plugging in our values, we compute the following radius for the SOI:

$$R_{\text{SOI}} = 11.2(4.1)(\frac{264}{200})^{-2} = \boxed{26.35 \text{ pc}}$$

Problem 3b

Calculate the minimum diameter a diffraction-limited telescope would have to have in order to resolve the SOI in the V band ($\lambda \approx 5550$Å)

Solution

To find the diameter needed, we use the diffraction-limited aperture equation for an airy disk:

$$\theta = \frac{1.22\lambda}{d} \rightarrow d = \frac{1.22\lambda}{\theta} \tag{7}$$

We plug in $\theta = \frac{26.35}{32.3 \times 10^6} = 8.16 \times 10^{-7}$ and $\lambda = 5550\text{Å} = 5.55 \times 10^{-7} m$. Solving for d,

$$d = \frac{(2)1.22(5.55 \times 10^{-7})}{(8.16 \times 10^{-7})} = 1.66\text{m}$$