**ASTR221: Problem Set 3**
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**Due: Thursday, Feb. 28 by midnight.**

**Problem 1.** There is an error in the book on page 108? What correction is required?

**Answer 1.** In page 108 of the textbook, we see that the author states the Rayleigh-Jeans equation as

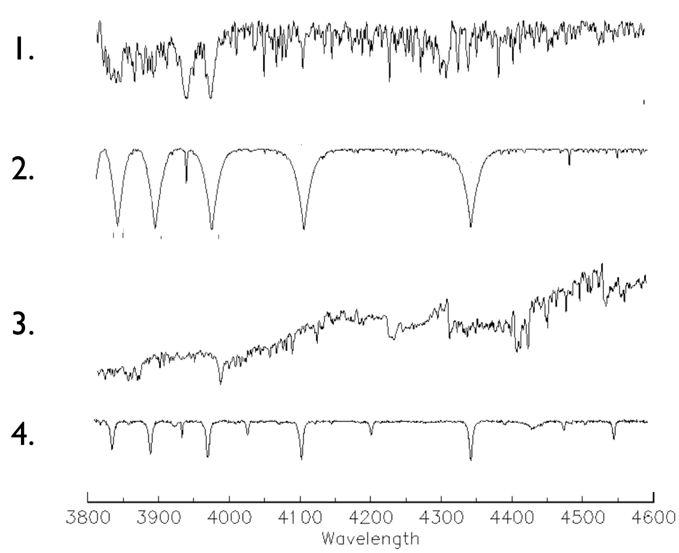
$$B_\nu(T) = \frac{2kT\nu^2}{hc^2}.$$

While close, this is, in fact, the wrong equation. The correct equation is

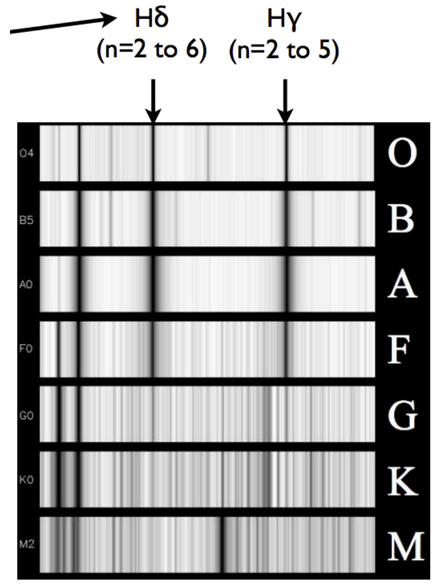
$$B_\nu(T) = \frac{2kT\nu^2}{c^2}.$$

We can do a dimnesional analysis test to see that the equation stated in the textbook had the extra $h$ term. The correct form omits the $h$ so that the units match up.

**Problem 2.** Become familiar enough with stellar spectra that you can easily classify, at a glance, almost any spectrum into one of the main classes (OBAFGKM) to within plus or minus 1 class. Note that I am not talking about subclasses here, i.e telling a G2 star from a G3 star, which is difficult. I am talking about telling a G star from an A star or an M star. As an illustration of your prowess, how would you classify each of the stars shown below.




**Answer 2.** For all of the spectra, we use the following figure which shows the intensity of lines of different stars:




We also note that the H$\gamma$ line is at 4340Åand the H$\delta$ is at 4100Å. Now, we will use this information to classify the spectra given.

1. Looking at the first spectra, we see a lot of lines at all wavelengths, but the two strongest features come at the lower wavelengths, around 4000Åand 4050Å. We can immidietaly narrow our options to be between a K star or an M star. The deciding factor is the missing line prominent in M stars present in the middle of the spectra. Therefore, we conlcude that spectra 1 belongs to a $\boxed{\text{K star.}}$

2. The second spectra has few prominent lines that are very prominent. We see that is has very strong H$\gamma$ and H$\delta$, so we can immediately rule out K,M, and G stars. We see that to the left of the H$\delta$ line, there are two more prominent lines, a broad one at 4000Åand a much narrower one at 3950Å. Notice that the spectra for the A star in the image above also has a similar pattern. An F star does too but there are narrower lines present in between the more intense lines as well. Therfore, we conclude that spectra 2 belongs to an $\boxed{\text{A star}}$.

3. Spectra 3 is filled with a lot of noise at all wavelengths and only has a slightly more prominent line at the middle of its spectra. We can therefore elimiate O,B,A, and F stars. We then note that there are no prominent lines in the left side of the spectra, which are charecteristic of G and K stars. Therefore, by process of elimination and because it fits our description, we conclude the spectra belongs to an $\boxed{\text{M star}}$.

4. Spectra 4 has very few lines where its only lines are narrow, including lines at H$\gamma$ and H$\delta$, which seem to be its most intense. Immidieately, we can eliminate all the noisier spectra for M,K,G, and F stars. Because none of the lines are too intense, we conclude that the star must be an $\boxed{\text{O star}}$ since other stars have much more intense H$\gamma$ and H$\delta$ lines.

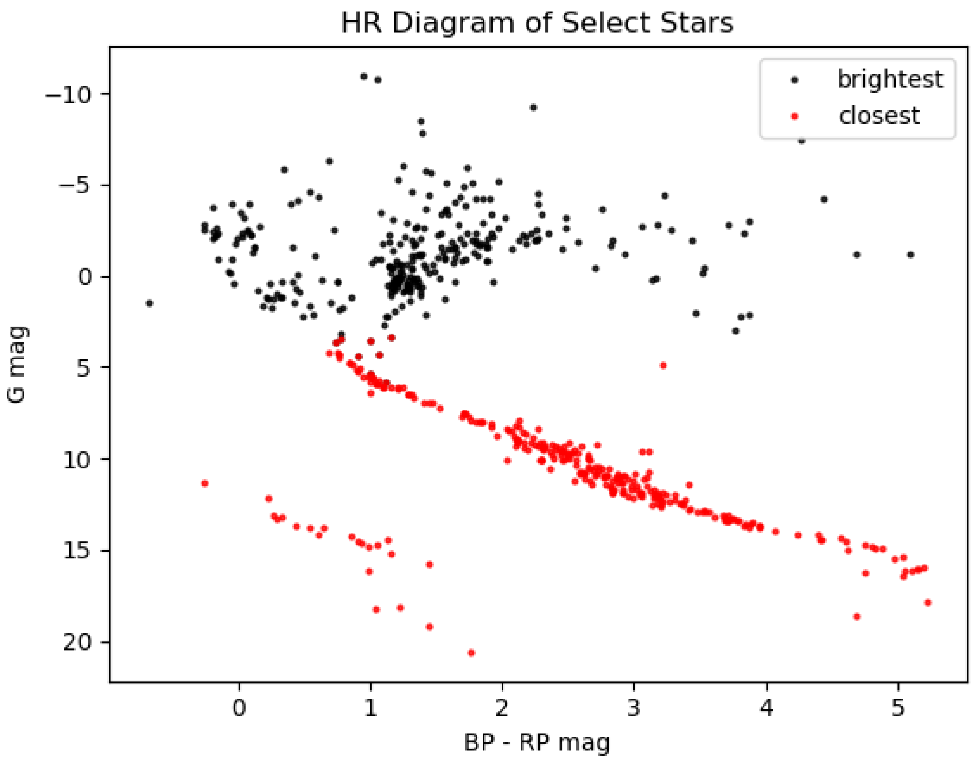
**Problem 3.** Make an HR diagram that compares a sample of the nearest stars (100 or so would do) with a sample of the brightest stars (chosen by apparent magnitude). The Sun, of course, will be in both data sets, since it is the nearest and the brightest. Plot everything on the same diagram and use different symbols and/or colors to differentiate between the nearest and the brightest. There are a variety of catalogs you could use for this, including Gaia, Hipparcos, Gliese, Yale, etc. You may choose where to get your data and exactly what quantities you wish to plot. For example, luminosities could be in terms of $M_V$, $M_{bol}$, $L/L_{\odot}$, etc. and effective temperatures could be in terms of B-V, some other color index, $T_e$, etc. These are up to you! Make a great looking plot and then describe it and what it tells you about the stellar population in the Galaxy (and other galaxies, as it turns out).

**Answer 3.** We use Gaia to query the closest and the brightest stars in the DR2 catalog.

```
'''
In this program, we create an HR diagram of the
300 brightest stars and of the 300 closest stars
using Gaia data for both
'''

#importing libraries to run math operations and to plot
import numpy as np
import matplotlib.pyplot as plt
#we import Gaia so we can query Gaia DR2
from astroquery.gaia import Gaia
#we import coordinates to handle our celestial coordinates
from astropy.coordinates import Angle
# prints out all the columns available in gaia dr2 so we know what we have to
    work with
gaia_dr2 = Gaia.load_table('gaiadr2.gaia_source')
#for column in gaia_dr2.get_columns():
#    print(column.get_name())
#function1 - given apparent mag and distance, using the distance modulus to find
    abs mag
#we will use this to find abs mag of the stars that we query
def distane_modulus(apparant_m,distance):
    term2 = 5*(np.log10(distance)-1)
    return (apparant_m - term2)
#we query gaia dr2 using sql
#we first query to get the 100 brightest stars in terms of apparent mag
job = Gaia.launch_job_async("SELECT \
parallax, phot_rp_mean_mag, phot_g_mean_mag, \
bp_rp, bp_g,g_rp \
FROM gaiadr2.gaia_source \
WHERE phot_g_mean_mag < 3.5 \
ORDER BY phot_g_mean_mag\
")
#print(job)
gaia_results = job.get_results()
#only the 100 brightest stars:
gaia_results = gaia_results[0:300]
```

```
#print(gaia_results)
#finding abs G mag using parallax and apparent G mag
g_distance = 1/ (gaia_results['parallax']/1000.)

abs_g_mags = distane_modulus(gaia_results['phot_g_mean_mag'],g_distance)
#creating an HR diagram of the 100 brightest stars as seen from Earth
plt.plot(gaia_results['bp_rp'],abs_g_mags,'.',color = 'black',markersize =
    4,alpha=0.8,label='brightest')
#plt.gca().invert_yaxis()
#plt.xlabel('BP - RP mag')
#plt.ylabel('G mag')
#plt.show()

#now we query the 100 closest stars
job1 = Gaia.launch_job_async("SELECT \
parallax, phot_rp_mean_mag, phot_g_mean_mag, \
bp_rp, bp_g,g_rp \
FROM gaiadr2.gaia_source \
WHERE parallax >100 \
AND parallax < 770 \
AND NOT (phot_g_mean_mag > 18 AND parallax > 100) \
ORDER BY parallax")
gaia_results1 = job1.get_results()
gaia_results1.sort('parallax')
#gaia_results1.reverse()
#only the 100 brightest stars:
gaia_results1 = gaia_results1[0:300]
#print(gaia_results1)
#print(gaia_results1['phot_g_mean_mag'])
#finding abs G mag using parallax and apparent G mag
g_distance1 = 1/ (gaia_results1['parallax']/1000.)
abs_g_mags1 = distane_modulus(gaia_results1['phot_g_mean_mag'],g_distance1)
#creating an HR diagram of the 100 brightest stars as seen from Earth
plt.plot(gaia_results1['bp_rp'],abs_g_mags1,'.',color = 'red',markersize =
    4,alpha=0.8,label='closest')
plt.gca().invert_yaxis()
plt.xlabel('BP - RP mag')
plt.ylabel('G mag')
plt.legend()
plt.title("HR Diagram of Select Stars")
plt.show()
```

Following our query, we produce the following:




From the HR diagram we produce, we can see that the majority in the main sequence stars in our galaxy are less massive than the sun. From the stars that are brighter, we notice that some continue in the main sequence while others break off into the red giant branch. There is some overlap, which is expected.

**Problem 4.** Determine the stellar number density and mass density in the solar neighborhood based on the sample of the 100 or so nearest stars that you used in problem 3. Note that you will need to assign masses to these objects, using a mass-luminosity relationship, which you adopt. Report your results in several different ways, so that you can get a feeling for what they mean. In particular, give the number density in units of stars per $\mathrm{pc}^{-3}$, and the mass density in terms of $\mathrm{M}_\odot\ \mathrm{pc}^{-3}$, $\mathrm{gm\ cm}^{-3}$, and H-atoms $\mathrm{cm}^{-3}$.

**Answer 4.** To find the stellar number density and mass density in the solar neighborhood, we take the 100 closest stars and find their bolometric magnitude so that we can find luminosity and use the mass - luminosity relationship. The equations that were implemented in our code are as follows:

$$M_{bol} = M_v + BC$$
$$M_{bol} - ML_\odot = -2.5\log\frac{L}{L_\odot}$$
$$\frac{L}{L_\odot} = \frac{M}{M_\odot}^{3.5}$$

Also note that 1 H atom = $1.6737\times10^{-24}$g and 1 pc = $3.08\times10^{18}$cm. We now calculate our desired values using Python:

```python
'''
In this program, we find the mass - luminosity relationship
to find number density and mass density of stars in the local neighborhood
'''

#we import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
#we import the gaia catalog
from astroquery.gaia import Gaia
from astropy.coordinates import Angle

gaia_dr2 = Gaia.load_table('gaiadr2.gaia_source')

#we query the selected products from Gaia so
job = Gaia.launch_job_async("SELECT \
parallax, phot_rp_mean_mag, phot_g_mean_mag, \
bp_rp, bp_g,g_rp \
FROM gaiadr2.gaia_source \
WHERE parallax BETWEEN 100 AND 780 \
AND NOT (phot_g_mean_mag > 18 AND parallax > 100)\
AND phot_g_mean_mag < 1000\
AND bp_rp < 1000\
ORDER BY parallax DESC\
")
#printing our job results so that we can analyze
gaia_results = job.get_results()
gaia_results = gaia_results[0:100]
#finding the distance to our stars using the parallax given
g_distance = 1 / (gaia_results['parallax']/1000.)
#we find the bolometric correction using the abs mag and temperature of the stars
#finding the temp
temp = 10**((gaia_results['bp_rp'] - 14.551) / -3.684)
#calculating bol_correction
bol_correct = -8.499*(np.log10(temp) - 4)**4 + 13.421*(np.log10(temp)-4)**3 -
    8.131*(np.log10(temp)- 4)**2 - 3.901*(np.log10(temp)- 4) - 0.438
#generating our bol mags
abs_mag = gaia_results['phot_g_mean_mag'] + bol_correct
#calculating luminosity
lum = 10**((abs_mag - 4.75) / 2.5)
#calculating the mass from M-L reln
mass = (lum)**0.2875
#calculating mass in grams
mass_in_grams = mass * 1.989e33
#finding total amount of mass from our query
tot_grams = np.sum(mass_in_grams)
#printing our the total amount of mass
```

```
print("Total amount of mass in grams: ",tot_grams)
#finding the furthest star from our query
far_star = g_distance[99]
print("The furthest star in our catalog in pc: ",far_star)
#finding the volume enclosed in our query
vol = (4/3) * np.pi * (far_star ** 3)
strs_per_c_pc = 100 /vol
print("stars per cubic pc: ",strs_per_c_pc)
#counting the number of white dwarfs and main sequence stars in our query
white_dwarf = 0
main_seq = 0
for item in gaia_results:
    if item[2] > 17:
        white_dwarf += 1
    else:
         main_seq += 1
print("There is %d white dwarfs in our query and %d main sequence stars."
    %(white_dwarf, main_seq))
```

We produce the following results:

$$\text{Stellar number density: } 0.0879\ \frac{\text{stars}}{\text{pc}^3}$$

$$\text{Mass density: } 0.026\ \frac{M_\odot}{\text{pc}^3},$$

$$1.8\ \times 10^{-24} \frac{\text{g}}{\text{cm}^3},$$

$$\text{and } 1.07\ \frac{\text{H-atoms}}{\text{cm}^3}$$

**Problem 5.** Problem 3.1 at the end of Chapter 3 of the textbook.

**Answer 5.** To show that the average value of $\sin^3$(i) for the binaries is 0.59, consider the average integral function, That is, consider:

$$\langle f(x) \rangle = \int_{LB}^{UB} f(x)\omega(x)\ dx$$

where $f(x)$ is the given function and $\omega(x)$ is the nomalization function. We define $\omega(x)$ by finding a function such that when we integrate it from our desired bounds of integration, the result is 1. So we want to find a function such that

$$\int_0^{\frac{\pi}{2}} \omega(x)\ dx = 1$$

We notice that such a function is the sine function since integrating it gives us -cos$(x)|_0^{\frac{\pi}{2}} = 1$, as desired. Therfore, we have our average function defined as

$$< \sin^3 i >= \int_0^{\frac{\pi}{2}} \sin^3(i)\sin(i)\ di = \int_0^{\frac{\pi}{2}} \sin^4(i)\ di$$

Evaluating this integral, we get:

$$< \sin^3 i >= \int_0^{\frac{\pi}{2}} \sin^4(i)\ di = \frac{3i}{8} - \frac{\sin 2i}{4} + \frac{\sin 4i}{32}|_0^{\frac{\pi}{2}} = \frac{3\pi}{16} \approx 0.59$$

Therefore, $< \sin^3 i >= 0.59$, as desired.

**Problem 6.** Problem 3.2 at the end of Chapter 3 of the textbook.

**Answer 6.** We know that the $T_{eff}$ of the sun is $5777K$. From page 87 of the textbook, we have that the average radius of a pulsar star is about 10km = 10,000 m. Therefore, we can plug in to the following function to get luminosity:

$$T_{eff} = \frac{L}{4\pi\sigma R^2}^{1/4}$$

$$T_{eff} = \frac{L}{4\pi(5.67 \times 10^{-8})(10,000^2)^{1/4}}$$

$$L = 7.936 \times 10^{16}$$

Knowing this, we can plug into the the following function, where $M_\odot = 4.83$ and $L_\odot = 3.8 \times 10^{26}$:

$$M - M_\odot = -2.5\log(\frac{L}{L_\odot})$$

$$M - 4.83 = -2.5\log(\frac{7.94 \times 10^{16}}{3.8 \times 10^{26}})$$

$$\boxed{M_V = 29.03}$$