ASTR221: Problem Set 2
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Due: Thursday, Feb. 21 by midnight.

Problem 1. There is a small error in the book in the first paragraph of Section 2.3. What is the correction required?

Answer 1. In Section 2.3 of the textbook states that the flux per unit frequency interval f_ν is in units of $Wm^{-2}s^{-1}Hz^{-1}$. The mistake here is that there is too many seconds $[s]$ units. Watts has units of seconds $[\frac{J}{s}]$ and we also have the s^{-1}. So in fact the time unit is counted for once more than actually needed. To fix this, we can either (1) change watts $[W]$ to just joules $[J]$, or (2) get rid of the s^{-1} unit.

Problem 2. Estimate the effective temperature of a star with the following properties, by fitting a black body curve to its flux density distribution:

B = 9.31, V = 8.94, J= 8.11, H = 7.93, K = 7.84

Plot the data and the black body that you chose as your best fit to the data and attach the plot to your answer. You can make this fit simply by eye, or you can use a more sophisticated fitting process – it is up to you. Based on the color of the star, estimate its spectral type, neglecting interstellar reddening. Compare the effective temperature based on the black body fit to the one based on the spectral type of the star and comment on the difference. Assuming that the star has luminosity class V, what is its distance, again neglecting interstellar reddening?

Answer 2. We use an HR - diagram to say that because it is in the V luminosity class, the luminosity will be $\approx 0.1L_\odot$. We know $M_\odot = 4.83$. Therefore, we have:

$$M - M_\odot = -2.5\log(\frac{L}{L_\odot})$$
$$M = -2.5\log(0.1) + 4.83 = 7.33$$
$$V - M_V = 5\log(d)$$
$$d = 10^{\frac{8.94-7.33}{5}} = \boxed{2.09pc}$$

For the code, we fit our data point to Planck's law:

$$B_\lambda(\lambda, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

```python
'''
In this program, we fit a blackbody curve to given data
'''

import numpy as np
import matplotlib.pyplot as plt
from scipy import optimize


#plotting our data
mag = np.array([9.31, 8.94, 8.11, 7.93, 7.84]) #b, v, j, h, k mags
f_v0 = np.array([4000,3600,1570,1020,636]) #flux of star in jansky
wavlen = np.array([445,551,1220,1630,2190]) #wavelength of each band respectively
wavlen = wavlen * 10**(-9) #convert nm to m
f_v0 = f_v0 * (10**(mag/-2.5)) #converting our maggies into flux values


plt.plot(wavlen,f_v0,'k.',label='Data Points') #plotting our data
plt.xlabel('Wavelength [m]') #labeling x axis
plt.ylabel(r'Flux $[Wm^{-2}Hz^{-1}]$') #labeling y axis

#defining the black body curve function
def black_body(lam,t):
    #defining constants
    c=2.9979*10**8 #speed of light [m/s]
    k=1.3806*10**(-23) #Boltzmann constant [m^2kgs^-2K^-1]
    h=6.6261*10**(-34) #Planck constant [m^2kgs^-1]
    #the equation
    term1 = (2.*h*c**2.)/(lam**5.)
    exp = ((h*c)/(lam*k*t))
    bttm = np.exp(exp) - 1.
    term2 = (1./bttm)
    return term1 * term2

lams = np.linspace(10**(-20),0.0000025,1000) #creating an array for wavlengths
curve = [] #creating an empty list to append fluxes into
scalar = 1/(1.9e12) #scalar to line up our curve with our data
temp = 3700 # guess and check until we line up the peaks of our data
for i in lams:
    curve.append(scalar*black_body(i,temp)) #finding the intensity for every
        wavelength in the array

plt.plot(lams,curve,color='k',label='Blackbody Curve') #plotting our curve
plt.legend()
plt.title('Given Data vs Blackbody Curve')
plt.show()
```

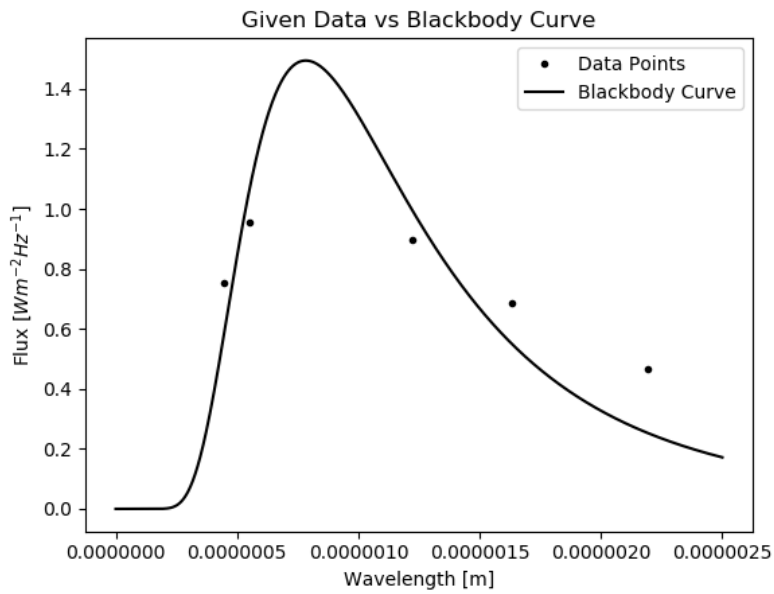


Figure 1: Blackbody curve fitting our data.

We see in our plot that while the curve doesn't fit exactly, it is a fair fit and one which we can use to get a good approximation for the temperature. We find that the temperature is $\boxed{3700K}$.

Problem 3. Given the information on a fictitious star, as listed in the table below, determine its other properties. Present your results as a table and explain each calculation in comments below the table.

Table of data on a fictious star

α, δ	13:42:25.6, -7:13:42.1 (J2000.00)
μ	13.7 mas y^{-1}
PA	122°
v_r	-13 km s^{-1}
V	10.86
B-V	1.63
SpT	K2V

Properties for you to determine and tabulate: (l,b), M_V, E(B-V), A_V, d, the magnitude of heliocentric space velocity (v_{space}), M_{bol}, $L/L_\odot$, T_e, $R/R_\odot$, and $M/M_\odot$. Explain how you obtained each of the requested quantities in comments below the table. In addition, comment on the likely age and chemical composition of this star and whether it is likely to have a planetary system. As always, justify your answers.

Answer 3. We calculate to get the following:

Calculated

(l,b)	(324.5°, 53.49°)
M_V	6.19
E(B-V)	0.746
A_V	2.3
d	29.78 pc
v_{space}	13.14 $\frac{\text{km}}{\text{sec}}$
M_{bol}	5.47
$\frac{L}{L_\odot}$	0.286
T_e	5040 K
$\frac{R}{R_\odot}$	0.699
$\frac{M}{M_\odot}$	0.731

(l,b): We use our program created in problem set #1 to convert the equatorial coordinates into galactic coordinates.

M_V: We locate the theoretical M_V value for an K2V star online at pas.rochester.edu under the row for K2V star.

E(B-V): To find E(B-V) value we have the following:

$$\text{E(B-V)} = \text{(B-V)} - \text{(B-V)}_0$$

We find the$(\text{B-V})_0$ in the pas.rochester.edu page, where we get a value of 0.746 mag. Therefore,

$$\text{E(B-V)} = 1.63 - 0.88 = \boxed{0.746\text{mag}}.$$

A_v: To find A_V, we use the formula A_V = 3.1E(B-V). Therefore, A_V =(3.1)(0.746)=$\boxed{2.3}$.

d: Knowing V=10.86,M_V=6.19, and A_V=2.3, we can use the following:

$$\text{V-}M_V = \text{5log(d) - 5 + } A_V \rightarrow d = 10^{\frac{V - M_V + 5 - A_V}{5}}$$

$$\text{d} = 10^{\frac{10.86 - 6.19 + 5 - 2.3}{5}} = \boxed{29.78\text{pc}}$$

v_{space}: We will use the formula

$$v_{space}^2 = v_t^2 + v_r^2.$$

So we first find v_t using $v_t = 4.74\mu d = 4.74(13.7 \times 10^{-3})(29.78) = 1.93$. We have that $v_r = -13$km/s. Therefore,

$$v_{space} = \sqrt{(1.93)^2 + (-13)^2} = \boxed{13.14\text{km/s}}$$

M_{bol}: To find M_{bol}, we have the formula

$$M_{bol} = M_V + BC_V$$

where BC_V is the bolometric correction. We find the BC_V value on the same paged linked above to find a value of $BC_V = -0.29$. Therefore,

$$M_{bol} = 6.19 - 0.29 = \boxed{5.9}$$

$\frac{L}{L_\odot}$ Using $M_{odot} = 4.83$, and $M_V = 6.19$, we can use

$$M_V - M_\odot = -2.5\log(\frac{L}{L_\odot}) \rightarrow \frac{L}{L_\odot} = 10^{\frac{6.19 - 4.83}{-2.5}} = \boxed{\frac{L}{L_\odot} = 0.286}$$

T_e: We locate the T_e in the linked page above. We get a value of $\boxed{5040K}$.

$\frac{R}{R_\odot}$: We first manipulate the T_e equation:

$$T_e^4 = \frac{L}{4\pi r^2 \sigma} \rightarrow r = \sqrt{\frac{L}{t_e^4 * 4\pi\sigma}}$$

This gives us:

$$r = \sqrt{\frac{0.286 * 2.8 \times 10^{26}}{5040^4 * 4\pi 5.67 \times 10^{-8}}} = 4.86 \times 10^8\text{m}$$

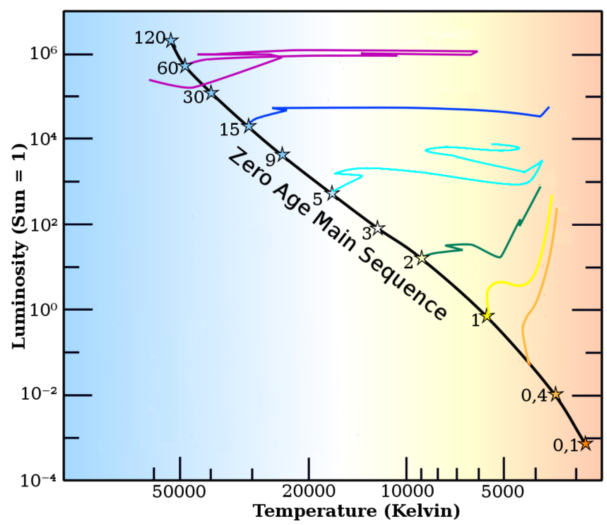
. We now divide by the radius of the sun $R_\odot = 6.95 \times 10^8$m:

$$\boxed{\frac{R}{R_\odot} = 0.69}$$

$\frac{M}{M_\odot}$: We have the relationship

$$(\frac{L}{L_\odot}) = (\frac{M}{M_\odot})^4 \rightarrow (0.286)^{1/4} = \frac{M}{M_\odot} = \boxed{\frac{M}{M_\odot} = 0.731}$$

Using the temperature, mass ratio and luminosity ratio, we can use the following plot:



We see that our star lines up very nicely in the zero age main sequece, to the right the sun, so we can say that the star is very young, $\approx 500,000$ years. Therfore, we know by definition that our star will only have hydrogen, not have started fusion, and it will be metal poor. We can assume that because of its very young age, the star does not have a planetary system.

Problem 4. Fun with magnitudes and colors! If a star with a parallax of 25 mas that initially appears to be single and is measured to have V=6.50, later turns out to be a spectroscopic binary with equal mass components, what is the apparent magnitude of each component? Neglecting reddening, what is the absolute magnitude of each component? If that same star has a measured B-V = 1.25, what is the B-V of each component?

Answer 4. We use the parallax to find the distance, which we will plug in to the distance modulus to then find the luminosity.

$$d = \frac{1}{p} = \frac{1}{25\times10^{-3}} = 40\text{pc}$$
$$V - M_V = 5\log(d) - 5 \rightarrow 6.50 - M_V = 5\log(d) - 5$$
$$M_V = 3.489$$

We know the $M_\odot = 4.83$ and $L_\odot = 3.8 \times 10^{26}$, so we use the following equation:

$$M - M_\odot = -2.5\log(\frac{L}{L_\odot})$$
$$3.489 - 4.83 = -2.5\log(\frac{L}{L_\odot})$$
$$\frac{L}{L_\odot} = 3.439 \rightarrow L = 1.3 \times 10^{27}$$
$$\frac{L}{2} = 6.534 \times 10^{26}$$

So we plug that luminosity back in:

$$M - M_{\odot} = -2.5\log(\tfrac{L}{L_{\odot}})$$
$$M - 4.83 = -2.5\log(\tfrac{6.534\times10^{26}}{3.8\times10^{26}})$$
$$M = 4.24$$
$$V - M = 5\log(d) - 5$$
$$V - 4.24 = 5\log(40) - 5$$
$$\boxed{V = 7.25}$$

We know consider the magnitude in the B band so that we can calculate teh $(B - V)$. So we go through the same calculations again:

$$B = V + B_V = 6.50 + 1.25 = 7.75$$
$$B - M_B - 5\log(d) - 5$$
$$7.75 - M_B = 5\log(40) - 5$$
$$M_B = 4.74$$

We know that in the B band, $M_{\odot} = 5.48$. Therefore,

$$M_B - M_{\odot} = -2.5\log(\tfrac{L}{L_{\odot}})$$
$$4.74 - 5.48 = -2.5\log(\tfrac{L}{L_{L_{\odot}}})$$
$$\tfrac{L}{L_{\odot}} = 1.98 \to L = 7.5\times10^{26}$$
$$\tfrac{L}{2} = 3.76\times10^{2}6$$
$$M_B - 5.49 = -2.5\log(\tfrac{3.76\times10^{26}}{3.8\times10^{26}})$$
$$B - M_B = 5\log(d - 5)$$
$$B - 5.49 = 5\log(40) - 5$$
$$B = 8.5$$
$$\boxed{(B - V) = 1.25}$$

Problem 5. More fun with magnitudes and colors! Suppose a binary system is composed of an A star, with V = 7.80 and B-V = 0.00, and a K star, with V = 8.20 and B-V = +1.50. If the stars are so close together on the sky that that they cannot be resolved as individual objects (i.e. an unresolved binary), what will be the measured V magnitude and B-V color of the "star" (that is actually the combined light of both components)?

Answer 5. We start off by finding the flux ratios compared to the flux of the sun for each star and then add them together. We can do this since fluxes can add linearly. We then take the combined flux and find the combined magnitude in the V band. After, we repeat the process in the B band so that we can find the color index.

<u>A star:</u>

$$V - 0 = -2.5\text{log}(\tfrac{f_V}{f_0})$$
$$7.8 = -2.5\text{log}(\tfrac{f_V}{f_0})$$
$$\tfrac{f_V}{f_0} = 0.00076$$

<u>K star:</u>

$$V - 0 = -2.5\text{log}(\tfrac{f_V}{f_0})$$
$$8.2 = -2.5\text{log}(\tfrac{f_V}{f_0})$$
$$\tfrac{f_V}{f_0} = 0.00052$$

$$V = -2.5\log(\frac{f_{VA}+f_{VK}}{f_0})$$
$$V = -2.5\log(0.0013)$$
$$\boxed{7.22mags}$$

We now solve for their combined $(B - V)$.

A star:

$$B = V + (B - V) = 7.8 + 0 = 7.80$$
$$B = 7.8 = -2.5\log(\frac{f_B}{f_0})$$
$$\frac{f_B}{f_0} = 0.00076$$

K star:

$$B = V + (B - V) = 8.2 + 1.5 = 9.7$$
$$B = 9.7 = -2.5\log(\frac{f_B}{f_0})$$
$$\frac{f_B}{f_0} = 0.0.00089$$

$$B = \frac{f_{BA}+f_{BK}}{f_0} = 0.0089$$

Having both B and V, we find $\boxed{(B - V) = 0.406}$