AST 221: Problem Set 5

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Due: Thursday, April 11 by midnight.

Problem 1. Is it likely that two stars in some galactic cluster within our Galaxy have ever collided? Use the same approach as illustrated in lecture but substitute the radius of a typical star for the strong encounter radius. Consider both the cluster as a whole and the core of the cluster. Note that I want a quantitative analysis here, including such information as what is the average time between stellar collisions in globular clusters.

Answer 1. In the universe, we can assume that the average star's mass is $0.5M_{\odot}$. Thus, we use the following equation to calculate the average star's radius:

$$M_{\odot}^{\frac{1}{8}} = R_{\odot} \rightarrow 0.5M_{\odot}^{\frac{1}{8}} = 0.57R_{\odot} = 1.289 \times 10^{-8}\text{pc}.$$

We will assume this radius value for all scenarios. For open clusters, we can consider the cluster as a whole and the core. Thus, for the whole cluster and the core of the the cluster, we will assume a stellar density of 10pc^{-3} and 100pc^{-3} and a velocity of 1 km/s and 10 km/s, respectively. We convert velocity and thus get a velocity of 3.24×10^{-14}pc/s and 3.24×10^{-13}pc/s. We now have all the components for the following equation to calculate the time needed for two stars to get within $0.57R\odot$ from each other (and hence collide):

$$t_s = \frac{1}{\pi r^2 v n}$$

Plugging in for the values we presented above we get that

Time for star's to collide at the <u>core of an open cluster</u>: 1.87×10^{9} Gyrs

Time for star's to collide in the <u>whole of an open cluster</u>: 1.87×10^{11} Gyrs

Thus, we see that (assuming the age of the universe is 13×10^{9} billion years) there will be on average 0 collision in the core of the open cluster and 0 in the shell as well. The chances are very slim.

Problem 2. Repeat the calculation in problem 1 for a typical globular cluster. Is it likely that there has ever been a stellar collision within a globular cluster during the lifetime of the Milky Way galaxy?

Answer 2. For globular clusters, we will apply the same method and equation as above. Again, we will consider the core and the shell of the globular cluster sepereratetly. We assume a stellar density of 10^5 and 10^4 stars per pc^{-3} respectively. We then assume a velocity of 50km/s for the core and 10km/s for the shell. This equals to 1.62×10^{-12}pc/s and 3.24×10^{-14}pc/sec, respectively. We plug into the t_s function above with the stated numbers to get that

Time for star's to collide at the <u>core of an globular cluster</u>: 1.87×10^6 Gyrs

Time for star's to collide in the <u>whole of an globular cluster</u>: 1.87×10^8 Gyrs

Again, we see that there will be on average 0 collisions in the core and in the shell since a collision once in a time period longer than the age of the universe.

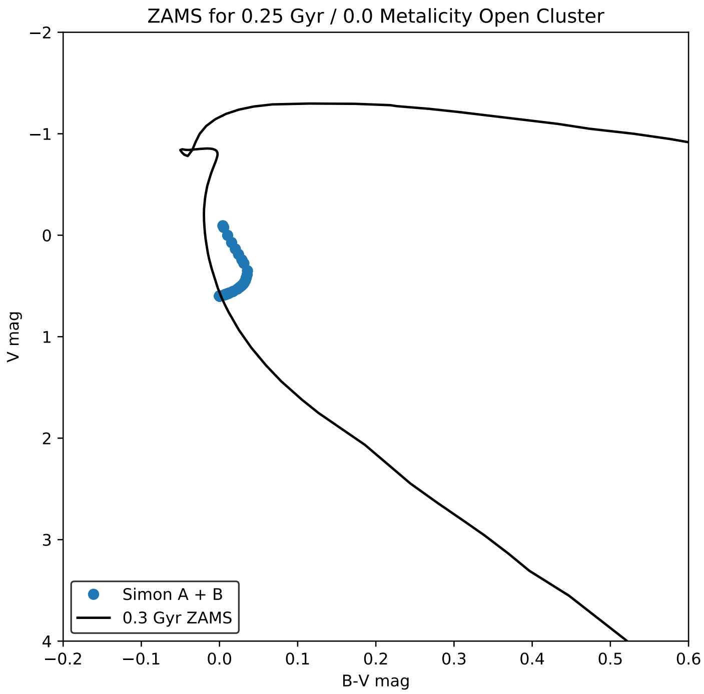
Problem 3. Answer the question from the mid-term exam by writing a computer program and plotting the results on an HR diagram as similar as possible to the one shown on the test. You can plot the ACTUAL ZAMS from the Dartmouth models instead of the approximate ZAMS that I put on the test figure. So that you don't have to look it up again, here are the question and figure from the test:

Consider two stars that are close enough together to form an unresolved binary, which we call Simon (after my cat). The components are known as Simon A and Simon B. Suppose Simon A has the spectral type A0V, corresponding to a color of B-V= 0.0. Plot, on the CM diagram below the path that Simon follows, as we change the spectral class of Simon B from A0 to M8, with it always being a dwarf. Neglect interstellar reddening.

Answer 3. We take the ZAMS for a 0.25 Gyr open cluster with a metallicity of 0.0 from the Dartmouth Data Base. We then plot the path of Simon A (an A0V star) + Simon B as Simon B goes from an A0 Star to an M8 star. To do this, we used the following equations where m_A is the absolute magnitude of Simon A and m_B is the absolute magnitude of Simon B.

$$B = -2.5\log(10^{-0.4m_A} + 10^{-0.4m_B})$$
$$V = -2.5\log(10^{-0.4m_A} + 10^{-0.4m_B})$$

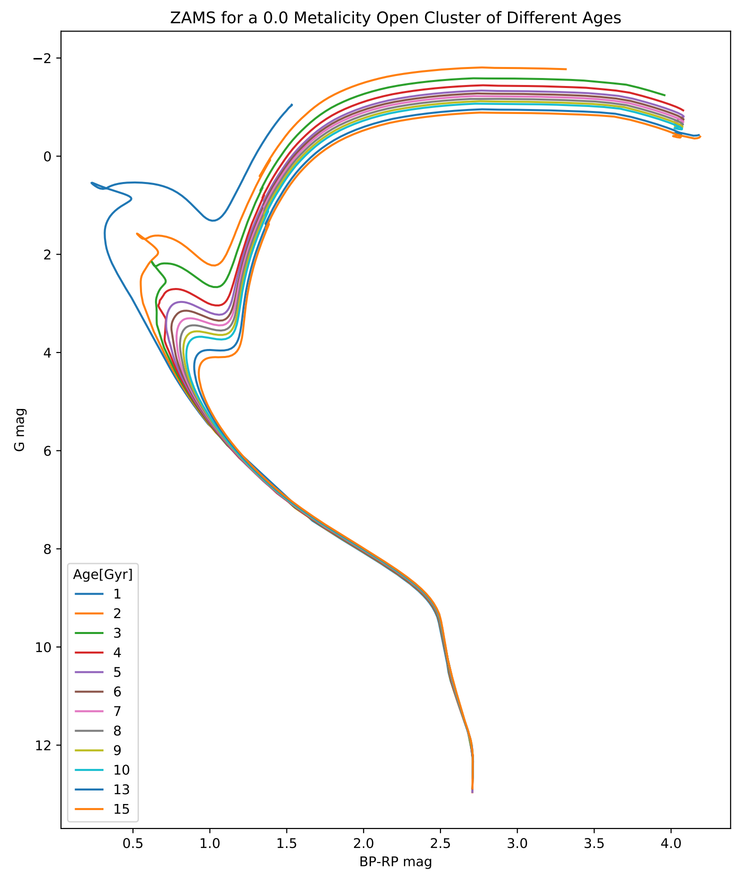
Plugging in magnitude values from M8 to A0 for Simon B, we produce the following:



From this plot, we see the path the combined magnitudes of Simon A and B take in the CM diagram. When Simon B is an A0 star, the Simon system is at the tip of the curve. This makes sense since we will have the greatest V brightness and it will still be at B-V=0 since both stars are A0 stars. As Simon B changes spectral type, Simon B starts to become redder and dimmer. This explains the diagnal drop in the combined path. After a while, however, the Simon B becomes too dim and too insignificant to contribute much in the CM diagram. Thus, the path starts to return to the left. That is, the path moves toward where one one find an A0V star in the CM diagram, which is at B-v=0 and V=0.6, as seen in the plot. At the end, we can only measure the brightness of Simon A, the A0V star and thus, ending the path at such point.

Problem 4. How accurately can the age of an open cluster be determined using the main sequence turn-off method and typical Gaia data? To address this, make a plot of a sequence of stellar isochrones of varying age with metallicity fixed at 0 from the Dartmouth models. Use that, plus the accuracy of Gaia magnitudes and colors, plus any other considerations that occur to you about potential sources of error, to justify your final answer.

Answer 4. From the Dartmouth Stellar Evolution Data Base, we import the ZAMS data file for 0.0 metallicity at different ages for open clusters. We plot the data below:

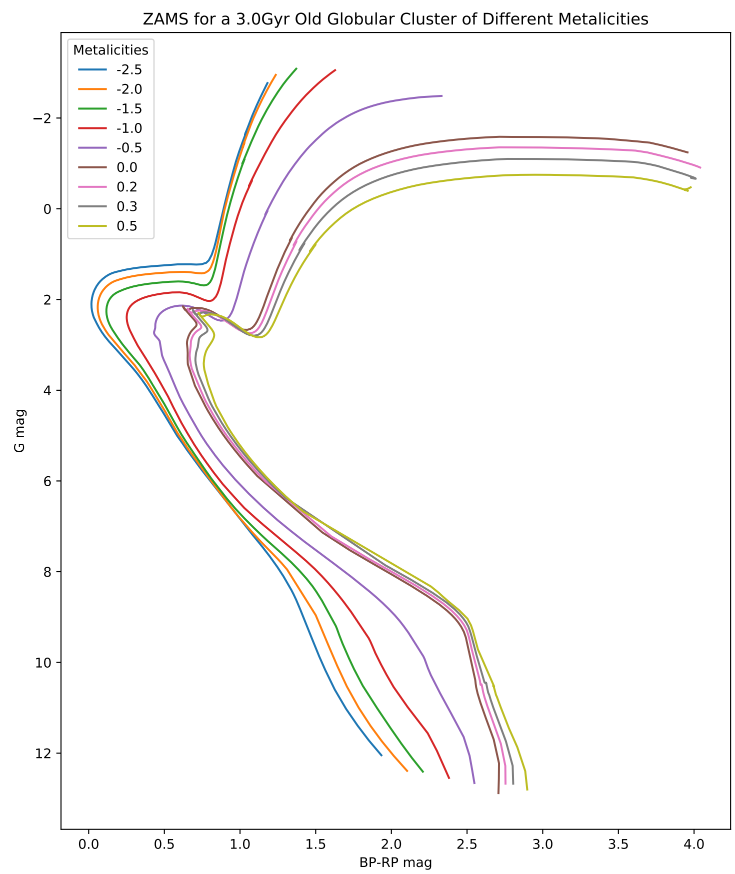


We see here that while there is a visible difference in the ZAMS for different ages, there can still be sources of error since the lines are all very close together. Specifically, if we consider very young open clusters such that they do not yet have a turn-off point and most stars are main sequence stars, then trying to determine an age for the cluster will be difficult. We see in the plot that the ZAMS for all ages of open clusters have converge very close to one another, making it hard to decipher the ages. Once we start to consider older open clusters, such that they have a turn off point, we see that determining the age becomes a bit easier. This is so because the plot has different locations for the turn off points for different aged open clusters. One caveat to this, however, is the ages available in the Dartmouth Data Base. That is, while in this plot, we only consider whole numbers, the data base offers ZAMS for ages incrementing by 0.25 giga years. This gives us some certainty, but not complete certainty as there can be clusters in between ages for which we have ZAMS lines available. At the tip of HR diagram, there is some seperation between the lines but not much. So if it is the case that we have an open cluster that is entirely in at the tip (almost always wont be the case since open clusters are young and dissociate over time), then we can get a rough estimate for the age but not as precise as when they have a turn off point.

We the consider the Gaia uncertainty. Like any observation/detection made, there is a range of certainty in the measurement, usually due to absorption/extinction effects or other effects that do not allow the capture of all the light. If it is the case that Gaia has measurements less precise than the seperation between the ZAMS lines, then it will be hard to categorize the open cluster into one or the other.

Problem 5. How accurately can the metallicity of a globular cluster of known age be deternined using main sequence fitting? Again, use the Dartmouth models to make a plot of isochrones of clusters with the same age but different metallicities to help you answer this question. Include discussion of any other things you believe may impact on the answer.

Answer 5. We take the ZAMS for globular clusters of the same age but different metallicities from the Dartmouth Stellar Evolution Data Base and plot the data below:



The differences in the lines is fairly apparent throughout most metallicities. For the more metalic stars ([FE/H]>0) the main sequence regions are very close to each other, so it is harder to determine the metallicity of the globular cluster there. Below that, there is also a clump in main sequence at very small metallicities, so it is also hard to determine the metallicity there. At the turn off point, again there is a clump for [FE/H]>0, so it is harder to determine metallicity there, but it is easier for the other metallicities since the ZAMS lines are more seperated.

Next, we also have to consider Gaia uncertainties which can occur when measuring the magnitude of very crowded stellar regions such globular clusters. This will affect the magnitude values we get and therefore throw off the location of the cluster. Hence, it will make it harder to determine the metallicity of the cluster.