AST 221: Problem Set 1

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Due: Thursday, Feb. 7 by midnight. Late papers are not accepted. If you cannot complete the assignment, hand in what you have completed before the deadline. Consider the deadline to be like the boarding time for an airplane, or the deadline for a grant submission to NASA or NSF. If you miss the deadline, you do not get on the airplane, no matter how good your excuse is. If you miss an NSF or NASA deadline, you do not get the grant, no matter how good your project is. The best advice is ... finish early. You can submit multiple times, right up to the deadline. Whatever your latest submission is, when the deadline occurs, is what will be graded.

Problem 1. Convert the commonly used speed unit of km s^{-1} into a useful unit system for galactic astronomy, pc My^{-1}. That is, 1 kilometer per second equals how many parsecs per million years?

Answer 1. Using the appropriate conversion rates, we get the following:

$$\frac{1\text{km}}{\text{sec}} \times \frac{60\text{sec}}{1\text{min}} \times \frac{60\text{min}}{1\text{hr}} \times \frac{24\text{hr}}{1\text{day}} \times \frac{365\text{day}}{1\text{yr}} \times \frac{1,000,000\text{yr}}{1\text{My}} \times \frac{3.2408 \times 10^{-14}\text{pc}}{1\text{km}} = \boxed{1.02\frac{\text{pc}}{\text{My}}}$$

Problem 2. Calculate the value of the constant in the equation:

$$v_t = \text{const}\mu d$$

where v_t has the units km s^{-1}, μ is in arc-seconds y^{-1} and d is in pc.

Answer 2. We write out the formula using the units for each variable. So we get:

$$\frac{\text{km}}{\text{sec}} = (\text{const})\frac{\text{arcsec}}{\text{yr}}\text{pc}.$$

We know that $\dfrac{\text{au}}{\text{arcsec}} = \text{pc}$. So au = (arcsec)(pc) is also true. So we plug substitute that into the equation to get That

$$\frac{\text{km}}{\text{sec}} = (\text{const})\frac{\text{au}}{\text{yr}} \longrightarrow (\frac{\text{km}}{\text{au}})(\frac{\text{yr}}{sec}) = \text{const}.$$

We find that there are 1.496×10^{8}km in an au and 3.171×10^{-8} years in one second. Therefore, we plug in those values into the equation:

$$(1.496 \times 10^{8})(3.171 \times 10^{-8}) = \text{const} \longrightarrow \boxed{\text{const} = 4.74\frac{\text{km}}{\text{sec}}}.$$

Problem 3. Write a computer program that transforms a given RA, Dec (J2000) to galactic coordinates (l,b). Test your program against a Web site that does the same transformation. Be sure to test it for both positive and negative declinations. [Hint: Make sure it works in the tricky region between 0 degrees and -1 degrees, such as RA = 04:37:48, Dec = -0:21:25. Also be sure it gives the right answer near the galactic center, l=0.]

Answer 3. :

```
'''
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ASTR221 - Galactic Astronomy
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In this script, we convert equatorial coordinates into galactic coordinates.
'''
#import the necessary libraries
import numpy as np
from astropy import units as u
from astropy.coordinates import SkyCoord
import matplotlib.pyplot as plt
#ask user to enter the equatorial coordinates
#asking for RA
print('Enter RA in decimal form:')
ra = float(input())
#asking for DEC
print('Enter DEC in decimal form:')
dec = float(input())
#we define some constants, all in degrees
ag,dg = 192.85948, 27.12825 #equatorial coordinates of north galactic pole
l_ncp = 122.93192 #galactic longtitude of the north celestial pole
#we now convert the given equatorial coordinates (ra,dec) into galactic
#   coordinates (l,b) using the following equations:
def eq_to_gal(ra,dec): # must give ra,dec in decimal degrees for fcn to work,
   returns coords in degrees
    rad_ag, rad_dg = np.deg2rad(192.85948),np.deg2rad(27.12825) #equatorial
       coordinates of north galactic pole
    rad_l_ncp = np.deg2rad(122.93192) #galactic longtitude of the north celestial
       pole
    rad_ra,rad_dec = np.deg2rad(ra),np.deg2rad(dec) # converting the given coords
       to radians
    #we use the following formula to calculate b - galactic latitude
    b = np.arcsin(np.sin(rad_dec)*np.sin(rad_dg) +
       np.cos(rad_dec)*np.cos(rad_dg)*np.cos(rad_ra-rad_ag))
    #the function to find l - galactic longtitude - is the arctan of (l_ncp minus
       the fraction(x1/x2))
    x1 = np.cos(rad_dec)*np.sin(rad_ra - rad_ag)
    x2 = np.sin(rad_dec)*np.cos(rad_dg) -
       np.cos(rad_dec)*np.sin(rad_dg)*np.cos(rad_ra - rad_ag)
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    l = rad_l_ncp - np.arctan2(x1,x2) #use arctan2 so that we get the correct
        sign by calculating the angle between x1 and x2
    l,b = np.rad2deg(l),np.rad2deg(b)
    if l < 0: #we want l to be between 0 and 360 degrees
        l = l + 360
    return l,b
print()
print('Equatorial Coordinates (ra,dec):',ra,',',dec)
print()
print('Our Calculation:')
l,b = eq_to_gal(ra,dec)
print("Galactic Coordinates (l,b):",l,',',b)
print()

'''
#checking our answer with the answer given by the astropy library
# setting up the coordinates for conversion
#coord = SkyCoord(ra,dec,unit=(u.hourangle, u.deg))
coord = SkyCoord(ra,dec,frame='icrs',unit='deg')
#converting to galactic coordinates
g_coords = coord.galactic
print('Astropy Calculation:')
print("Galactic Coordinates (l,b):",g_coords.l.degree,',',g_coords.b.degree)

#we now want to make sure that our function works for all possible equatorial
    coordinates
ras = np.arange(0,360,1) # create an array of all possible ra's
decs = np.arange(-90,90,0.5) # create an array of all possible dec's
counter = 0
#create a for loop that checks all combinations of coordinates
for ra in ras:
    for dec in decs:
        lo,bo = eq_to_gal(ra,dec) #convert using our function
        coord = SkyCoord(ra,dec,frame='icrs',unit='deg') #convert unsing astropy
        g_coords = coord.galactic
        if abs(lo - g_coords.l.degree) > 1: #if the answer is wrong, tell me what
            coordinate doesn't work
            print(ra,dec)
    print('Done',counter)
    counter += 1

#we verify that it works for all coordinates so our function works!
'''
```

Problem 4. A fictional G2V star has the following known properties:

Position = 04:24:46.0, +12:37:22.0 (J2000)

Parallax (p) = 0.025″

Proper motion = -5.0 mas yr^{-1} (RA), +24.0 mas yr^{-1} (Dec)
Heliocentric radial velocity (v_r) = +28.0 km/s

Determine the following information for this star. Use appropriate units for galactic astronomy

Galactic Coordinates (l,b)
Distance (d)
Magnitude of the proper motion vector (μ)
Position Angle of the proper motion vector (PA)
Transverse velocity (v_t)
Speed relative to the Sun (v_{space})
What constellation is this star in?
Is it closer to or further from the center of the Galaxy than the Sun?

Answer 4. :

- We use the program created in problem #3 to find that the galactic coordinates of the star is $\boxed{(182.478, -24.77)}$. We found this using the given RA and declination, where the declination is taken to be in degrees.

- We take the given parallax to compute the distance:

$$\frac{1}{p} = \frac{1}{0.025"} = \boxed{40\text{pc}}.$$

- To find magnitude of the proper motion vector, we use the following equation:

$$|\mu| = \sqrt{\mu_\delta^2 + \mu_{\alpha*}^2}$$

  where $\mu_\delta = 24.0\frac{\text{mas}}{\text{yr}}$ and $\mu_{\alpha*} = -5.0\frac{\text{mas}}{\text{yr}}$.

  Therefore,

$$|\mu| = \sqrt{(24^2) + (-5.0)^2} = \boxed{24.52\frac{\text{mas}}{\text{yr}}}$$

  .

- We find the angle between proper motion vector. Therfore,

$$\tan(\theta) = \frac{24}{-5} \implies \theta = \tan^{-1}(\frac{24}{-5}) = \boxed{-78.23°}$$

- To find the transverse velocity (v_t) we use the following equation:

$$v_t = 4.74\mu d$$

  where $\mu = 24.52\frac{\text{mas}}{\text{yr}} = 0.02452\frac{as}{yr}$ and $d = 40$pc. Therefore,

$$.v_t = 4.74(0.02452)(40) = \boxed{4.64\frac{\text{km}}{\text{sec}}}$$

  .

- The speed relative to the sun (v_{space}) is defined as follows:

$$v_{space} = v_r^2 + v_t^2$$

where $v_r = 28.0\frac{\text{km}}{\text{sec}}$ and $v_t = 4.38\frac{\text{km}}{\text{sec}}$. Therefore,

$$v_t = (28.0^2)(4.38^2) = \boxed{803.18\frac{\text{km}^2}{\text{sec}^2}}$$

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- Using stellerium, we plug in the equatorial coordinates given to find that the star is in the $\boxed{\text{Taurus}}$ constellation.

- We know that the center of the galaxy is towards the Sagittarius constellation. So if we look at the position of Taurus and Sagittarius in the sky, we see that they are on opposite sides of it. From that we can conclude that the sun is in the middle of the fictional G2V star and the center of the galaxy. This means the fictional G2V star is further from the center of the galaxy than the sun.

  Also, by looking at the coordinates, we see that any object with a galactic coordinate greater that 90° longtitude will be to the side or behind us. Since the fictional star is at 182°, then it is behind us. So by the same argument as above, G2V is further from the center of the galaxy than the sun.

Problem 5. The Gaia Mission is able to measure positional accuracy in the best cases to about 10 μas (micro arc-seconds)!

a) Give an example of what 10 μas is in terms of everyday experience, i.e. something you could a general audience with, to explain to them how good this instrument is at resolving angles.

b) Given its ability to detect proper motions of about 10 μas y^{-1} would Gaia be capable of observing grass growing on the Moon, (assuming grass could grow on the Moon)? If not, from what distance could it observe grass growing? (Be sure to state your assumptions.)

c) Could Gaia detect the rotation of the Andromeda galaxy? (Again, be sure to state any assumptions you make.)

d) Is Gaia capable of detecting the proper motion of a typical quasar? (.... assumptions...)

Answer 5. :

(a) We want to find out how far away we can read a paper with 12pt font using Gaia's resolution power. So we set up the following trigonometric function

$$\tan(\theta) = \frac{\text{font}}{\text{distance}}$$

We know $\theta = 10\mu\text{as}$ which is equal to $\frac{1.389 \times 10^{-6\,\circ}}{2}$ We also know that 12pt font $= \frac{0.00423\text{m}}{2}$. We divided both by two since we want to create a right triangle and not an isosceles triangle. Dividing by 2 will accomplish this while conserving the distance. Therefore, we have the following expression where we also use the small angle approximation for $\tan(\theta)$:

$$5 \times 10^{-7} = \frac{0.002115\text{m}}{\text{distance}} \Longrightarrow \text{distance} = \frac{0.002115}{5 \times 10^{-7}} = \boxed{1.522 \times 10^7\text{m}}.$$

We turn that answer into yards to get 1.664×10^7y. Dviding by 100 yards will give us number of football fields. Thefore, we conclude that using Gaia's data, we can read a paper with 12pt font letters $166,400$ football fields away.

(b) We find that grass grows at $1.325\frac{\text{m}}{\text{yr}} = 4.2 \times 10^{-11}\frac{\text{km}}{\text{sec}}$. So we take that value as our v_t and using $\mu = 10^{-5}\frac{\text{arcsec}}{\text{yr}}$ we use the following formula:

$$v_t = 4.74\mu * d \Longrightarrow 4.2 \times 10^{-11} = (4.74)(10^{-5})(d)$$

$$d = 8.86 \times 10^{-7}\text{pc} = 2.73 \times 10^{10}\text{m}$$

The true distance from Earth to the moon is 3.044×10^8m. Since Gaia can resolve grass growing up to a much larger distance, we conclude that Gaia is capable of observing grass groq on the moon.

(c) We find that the rotation on Andromeda galaxy is $270\frac{\text{km}}{\text{sec}}$. We take that as out v_t and once again, we use $\mu = 4.74$ and plug into the equation:

$$v_t = 4.74\mu * d \Longrightarrow 270 = (4.74)(10^{-5})(d)$$

$$d = 5.69 \times 10^6\text{pc}.$$

The true distance from Earth to Andromeda is 2.78×1^5pc. Therefore, since Gaia can observe this movement up a much greater distance than the distance to Andromeda, we conclude that Gaia can detect the rotation of Andromeda galaxy.

(d) Given that Gaia, itself, uses quasars as the refrence frame[1] to detect proper motion in closer objects like stars, we conclude that Gaia can not detect the proper motion of distant quasars.

Problem 6. With the precision that the Gaia Mission reaches for many stars, a level of about 60 μas per measurement, could it detect the wobble of the Sun due to Jupiter from a distance of 10 pc, assuming it made enough observations over a sufficiently long period of time?

Answer 6. :

We use the center of mass formula which is defined as follows:

$$r_c = \frac{m_1}{m_1 + m_2}d$$

[1]https://www.aanda.org/articles/aa/pdf/2016/05/aa27935-15.pdf

where $m_1 = 1.898 \times 10^{27}$ kg is the mass of Jupiter, $m_2 = 1.989 \times 10^{30}$ kg is the mass of the sun, and $d = 778547200$ km is the distance between the sun and Jupiter. When we plug in, we get:

$$r_c = \frac{1.898 \times 10^{27}}{1.898 \times 10^{27}} + 1.989 \times 10^{30}(778547200) = 742219.1\text{km}$$

We can create a triangle where 7442219.1×2 (the wobble of the sun) is the opposite side and $10\text{pc} = 3.086 \times 10^{14}\text{km}$ is the adjacent side. To solve for θ, we have:

$$\theta = \frac{7442219.1 \times 2}{3.086 \times 10^{14}} = 7.11 \times 10^{-9}\text{deg} = 2.55 \times 10^{-5}\text{as} = 25.5\mu\text{as}.$$

Since the wobble is less than the 60μas, Gaia can not detect the wobble of the sun due to Jupiter.