PHYS340: Lab 3
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February 18, 2019

Introduction:

In this assignment, we set out to computationally solve Netwon's equation of motion. That is, we approximate $x(t)$ from the following differential equation:

$$ma = -kx$$

Procedure:

To solve the differential equation:

$$m\frac{d^2x}{dt^2} = -kx$$

we break up the second order differential equation into two first order differential equations. That is, we split it up to get:

$$m\frac{dv}{dt} = -kx \qquad \text{and} \qquad \frac{dx}{dt} = v$$

We manipulate the two above equations so that we can use the C programming language to calculate the solutions to them. So we get the following equations over which we iterate:

$$dv = \frac{-kxdt}{m} \qquad v = v + dv$$

$$dx = vdt \qquad x = x + dx$$

$$t = t + dt$$

In order to use these equations, we ask the user for initial conditions: x_i, v_i, k, and m. We also define a step size (dt) to determine by how much we will be jumping forward after each iteration. Here, we defined $dt = 0.00001$. Once aquired, we can update the values as many times as we want by iterating over the functions with a for loop. Here, we iterated 600,000 times. It is important that we print out the values of t and x at each iteration since that is what we are solving for. Therefore, the final step was to print out these values (x, t) into a file. To later analysis the accuracy of our solution, we also define a cosine function which we plot on the same graph as our solution.

Discussion:

After running our program, we use xmgrace to produce the following plot, where we plotted both our solution and the exact solution to the differential equation:

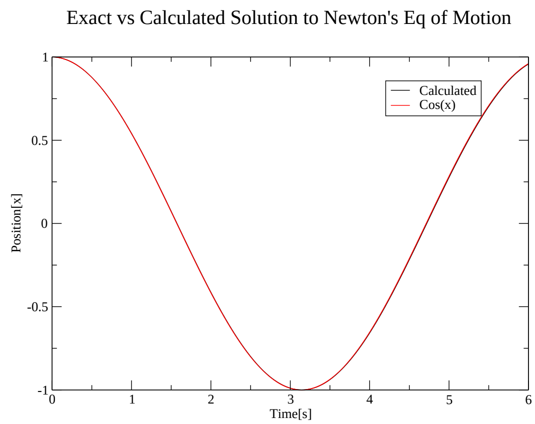

Figure 1: The exact solution plotted over the calculated solution with initial conditions of $x_i = m = k = \omega = 1$ and $v_i = \phi = 0$.

Upon first glance, we could say that our solution is very good since it overlaps with the exact solution. But in order to provide better analysis, we produce a plot of their difference in position at each unit time:

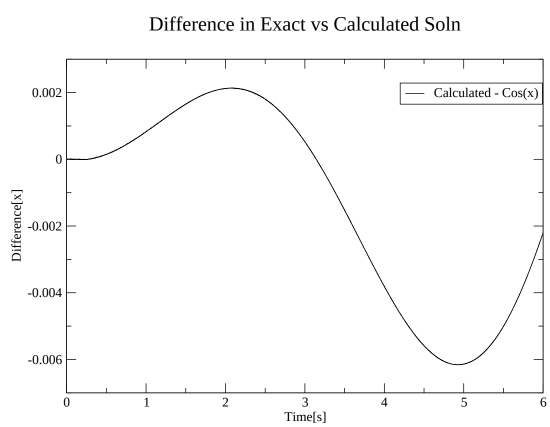

Figure 2: The difference in the exact and approximate solution.

Using the difference plot, we realize that the difference in our solution to the exact solution is of very small magnitude when time is small. However, the difference does increase as time increases.

Conclusion:

We conclude that for small intervals, our program is a reliable one, but if we wanted to run our program for a much larger time interval, we would need to be cautious of the accuracy at those large time values. Therefore, we would need to update our program to produce results that don't deviate from the exact solution at higher time values or we would need to consider a different approach to the problem.