Quantifying the dynamics of chaotic ecological networks

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Using a generalized Lotka-Volterra (gLV) framework, theory predicts that ecosystems with a large number of many interacting species become less stable with increasing interaction complexity and diversity. But this fails to explain the persistence of highly diverse ecosystems observed in nature, and typically attributes fluctuations in ecosystems only to external perturbations.

In this work, I reproduce and extend the model from Roy *et al.*[1] to characterize the chaos emergent in complex, spatially structured ecosystems. Using a gLV system of multiple separated patches coupled with migration, I investigate how this spatial structure leads to chaotic behavior and how this behavior changes with the complexity of interactions. In the spatially structured case, local populations change due purely to migration, and not external drivers, lead to highly diverse ecosystems with fluctuations but without global extinctions. I calculate the leading characteristic Lyapunov exponents and corresponding Kaplan-Yorke dimension, which serves as a measure of the system's attractor dimensionality, and thus the degrees of freedom necessary for the system to be conservative.

I will extend the work by computing the Lyapunov spectrum and attractor dimension for systems with increasing inter-species interaction strengths, to analyze how the complexity of interactions changes the chaotic behavior of the system. This will provide a quantitative measure of how ecological complexity and dynamical chaos are coupled. Understanding how ecosystems can persist in a chaotic state gives insight into the fluctuations and complex interactions observed in nature, and can lead to better models for understanding and conserving diversity.

I. INTRODUCTION

Ecosystems observed around the globe have remarkable diversity, but how these communities remain stable to perturbations, both natural and manmade, is still not fully understood. Classical stability analyses, dating back to Robert May's work in 1972[2] using the generalized Lotka-Volterra model, show that as species diversity and complexity of interactions increase, ecosystems are less likely to be stable. Accordingly, unstable ecosystems lead to extinction until stability is reached at a lower dimension. But this framework does not explain the high diversity observed in many communities, and emperical tests to validate these claims of stability are difficult to do with high accuracy and large enough samples, so more theoretical work is needed in the area.

In this work, I reproduce and extend work done by Roy *et al.*[1] They demonstrated that in ecosystems of many species connected by complex interactions, persistent chaotic fluctuations from migration alone can maintain higher species diversity than a linearly stable equilibrium. This work, however, does not characterize the chaos emergent in these systems.

Using the same generalized Lokta-Volterra model of metacomminities presented in[1], I calculate the first 50 characteristic Lyapunov exponents and the Kaplan-Yorke dimension of the resulting chaotic attractor. The Lyapunov exponents of the system characterize how fast a small change in initial conditions lead to large divergence from the original trajectory. The spectrum of Lyapunov exponents is used to find the Kaplan-Yorke attractor dimension: the effective attractor dimension of the attractor emergent in this system. The Kaplan-Yorke dimension is useful to link the dynamical stability to the complexity of the ecosystem. I aim to characterize how these measures change as the strength of interactions in the ecosystem increase.

In stable communities, it is shown that increasing interaction strength destabilizes large communities[3], and understanding how this translates into the chaotic, highly diverse case will lead to a better understanding of the highly diverse communities observed in nature.

II. KEY FIGURE

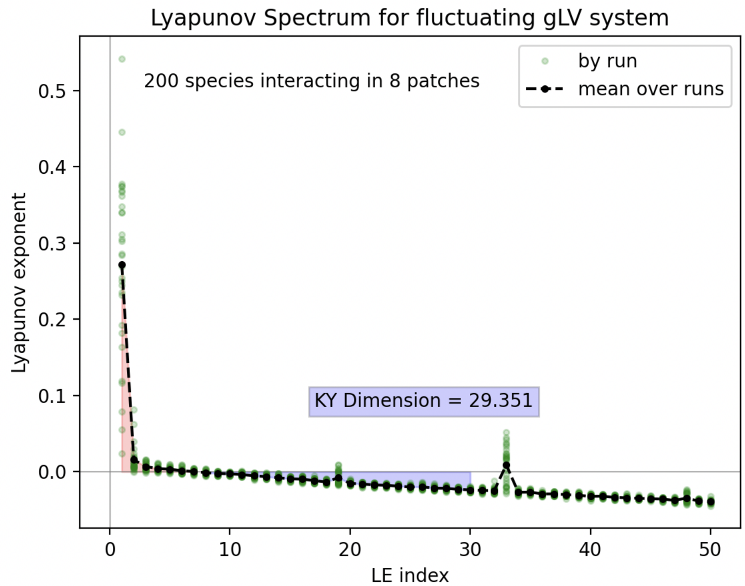

FIG. 1. The spectrum of characteristic Lyapunov Exponents for the interacting system in[1]. One manifestation of interaction matrix A was used with 30 sets of random initial conditions. This system includes 200 species, each having a population across 8 patches, for a total system dimension of 1600. The system was evolved for 2500 time steps past initial extinctions, then the first 50 Lyapunov exponents were computed in the next 500 time steps. The Kaplan-Yorke dimension is calculated as $D_{KY} = 29.351$ using the mean spectrum across all 30 sets of initial conditions (shown in black), and the accumulation of positive (red) and negative (blue) Lyapunov exponents up to the Kalplan-Yorke dimension is shaded. This is much lower than the total system dimension, indicating that despite chaotic behavior, the system retains some stability.

III. AI TOOL USAGE

ChatGPT was used for bug fixes in my Lyapunov Spectrum code when I was first refining it on known systems (Lorenz attractor) and to make my simulation of the high-dimension system more efficient with large matrix math when first replicating[1]. The framework of the code and all writing here is my own.

[1]F. Roy, M. Barbier, G. Biroli, and G. Bunin, "Complex interactions can create persistent fluctuations in high-diversity ecosystems," PLOS Computational Biology **16**, 1–14 (2020).
[2]R. M. May, "Will a large complex system be stable," Nature **283**, 413–414 (1972).
[3]S. Allesina and S. Tang, "Stability criteria for complex ecosystems," Nature **483**, 205–208 (2012).