# Algebras for a Functor

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### Motivation

Modelling inductive types.

# F-Algebras

category C, endofunctor  $F \colon C \to C$ 

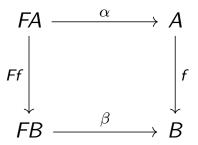
### F-Algebras

category C, endofunctor  $F \colon C \to C$ 

$$FA \xrightarrow{\alpha} A$$

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### Initial Objects

Such an object I, that for every object X, there exist a **unique** morphism  $I \rightarrow X$ .

### Lambek Lemma

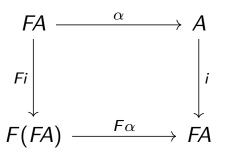
### Lemma (Lambek)

If  $I = (A, \alpha)$  is an initial algebra, then A is isomorphic to FA via  $\alpha$ .

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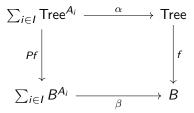


# Polynomial Functor

- project only defines them on **Sets**
- $PX = \sum_{i \in I} X^{A_i}$ , for  $A: I \to \mathbf{Set}$
- natural numbers from PX = 1 + X

# Initial algebra for polynomial functors

- Tree has a constructor Node  $\sum_{i \in I} \text{Tree}^{A_i}$
- initial object is the F-algebra of the Tree



#### Future work

- implement presentation of natural numbers with F-algebras
- implement presentation of lists with F-algebras
- generalize polynomial functors
- generalize existance of initial algebras

## Problems in Implementation

### Problems in Implementation

```
; _o_ = λ f g → record {

f = F-Algebra-Morphism.f f ∘ F-Algebra-Morphism.f g;

commutes = glue {! 4!} (F-Algebra-Morphism.commutes f)

(F-Algebra-Morphism.commutes g) }
```

#### Sources

- Awodey, Steve (2010). *Category Theory*. 2nd. USA: Oxford University Press, Inc. ISBN: 0199237182.
- nLab authors (May 2022). initial algebra of an endofunctor. URL:

http://ncatlab.org/nlab/show/initial%20algebra%20of%20an%20endofunctor.