Algebras for a Functor

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Motivation

F-Algebras

category C, endofunctor $F \colon C \to C$
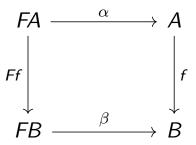
F-Algebras

category C, endofunctor $F : C \to C$

$$FA \xrightarrow{\ \ \alpha\ \ } A$$

category C, endofunctor $F \colon C \to C$

$$
\begin{array}{ccc}
FA & \xrightarrow{\;\;\alpha\;\;} & A \\
\Big\downarrow{\scriptstyle Ff} & & \Big\downarrow{\scriptstyle f} \\
FB & \xrightarrow{\;\;\beta\;\;} & B
\end{array}
$$

Initial Objects

Such an object I, that for every object X,
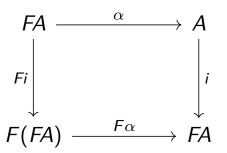there exist a **unique** morphism $I \to X$.

Lambek Lemma

Lemma (Lambek)

If $I = (A, \alpha)$ is an initial algebra, then A is isomorphic to FA via α.

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If $I = (A, \alpha)$ is an initial algebra, then A is isomorphic to FA via α.

$$
\begin{array}{ccc}
FA & \xrightarrow{\ \alpha\ } & A \\
\ \downarrow{\scriptstyle Fi} & & \ \downarrow{\scriptstyle i} \\
F(FA) & \xrightarrow{\ F\alpha\ } & FA
\end{array}
$$

Problems in Implementation

Future work