Algebras for a Functor

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Motivation

Modelling inductive types.

F-Algebras

category C, endofunctor $F \colon C \to C$

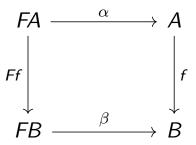
F-Algebras

category C, endofunctor $F \colon C \to C$

$$FA \longrightarrow A$$

F-Algebras

category C, endofunctor $F \colon C \to C$



Initial Objects

Such an object I, that for every object X, there exist a **unique** morphism $I \rightarrow X$.

Lambek Lemma

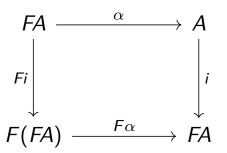
Lemma (Lambek)

If $I = (A, \alpha)$ is an initial algebra, then A is isomorphic to FA via α .

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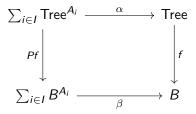


Polynomial Functor

- project only defines them on **Sets**
- $PX = \sum_{i \in I} X^{A_i}$, for $A: I \to \mathbf{Set}$
- natural numbers from PX = 1 + X

Initial algebra for polynomial functors

- Tree has a constructor Node $\sum_{i \in I} \text{Tree}^{A_i}$
- initial object is the F-algebra of the Tree



Problems in Implementation

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```
= λ f g → record {
   F-Algebra-Morphism.f f o F-Algebra-Morphism.f g ;
commutes = glue {! 4!} (F-Algebra-Morphism.commutes f)
(F-Algebra-Morphism.commutes g)
```

Future work

- implement presentation of natural numbers with F-algebras
- implement presentation of lists with F-algebras
- generalize polynomial functors
- generalize existance of initial algebras