Assignment 2 ECE 5530

Name: SKYLAR STOCKHAM
KYLE GAYLIYEV
EDDIE FRANCO

• Problem 2.7

1) Static or dynamic: if the present output depends on present input the system is static or could be memoryless so if is dynamic.

2) Linear or Non-Linear: if the system satisfies the principles of superposition & scaling. otherwise the system is non-linear

3) Time variant or time varying: a time invariant if its behavior & characteristics do not change over time so, a time shift in the input signal results an identical time shift in the output, otherwise is called time-variant

4) Causal or non-casual: if present output depends on past outputs, past inputs, and present inputs is causal.
if present output depends on future inputs & outputs is non-causal

5) Stable or unstable: if every bounded input produces a bounded output (BIBO)

Examine the following systems

ⓐ $y(n) = \cos[x(n)]$: as operation is performed on $x(n)$ it is non linear. as output depends only in present input it is static. behavior doesn't change over time is time invariant. =>Depends on past and present so it's Causal. =>BIBO stable.

ⓑ $y(n) = \sum_{k=-\infty}^{n+1} x(k)$: • the system is dynamic because the present output depends on past & future values of input ex. $y(4) = \sum_{k=-\infty}^{5} x(k)$

• for input $x_1(n)$

$$y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$$

• for $x_2(n)$

$$y_2(n) = \sum_{k=-\infty}^{n+1} x_2(k)$$

$$x_3(n) = ax_1(n) + bx_2(n)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} x_3(k)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} ax_1(k) + bx_2(k)$$

$$y_3(n) = ay_1(n) + by_2(n)$$

So, it is linear

- $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

for $x_1(n) \Rightarrow y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$

for $y_1(n)$

$$y_1(n-n_0) = \sum_{k=-\infty}^{n+1-n_0} x_1(k)$$

for $x_2(n) = x_1(n-n_0)$ for $x_1(n-n_0)$

$$y_2(n) = \sum_{k=-\infty}^{n+1} x_2(k)$$

$$= \sum_{k=-\infty}^{n-n_0+1} x_1(k-n_0)$$

$$= \sum_{k=-\infty}^{n+1-n_0} x_1(k-n_0)$$

$$y_2(n) = y_1(n-n_0)$$

System is time-invariant

- $y(n) = \sum_{k=-\infty}^{n+1} x(k)$ let $n=0 \Rightarrow y(0) = \cdots x(-\infty) + \cdots x(0) + x(1)$

system is non-casual!

- $y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$ For the bounded input $x(k) = u(k)$, output $\boxed{y(n) \rightarrow \infty \text{ so } n \rightarrow \infty}$

The system is unstable

ⓒ $y(n) = x(n)\cos(w_0 n)$:

- $y(n)$ depends only on the present input values So system is $\boxed{\text{static.}}$

*timevariant because behavior/characteristic of the function changes over time:time shift in the input signal doesn't result in identical signal in the output.

- let $x_1(n) \Rightarrow y_1(n) = x_1(n)\cos(w_0 n)$

let $x_2(n) \Rightarrow y_2(n) = x_2(n)\cos(w_0 n)$

let $x_3(n) = ax_1(n) + bx_2(n)$

$$y_3(n) = x_3(n)\cos(w_0 n)$$

$$= [ax_1(n) + bx_2(n)]\cos(w_0 n)$$

system is $\boxed{\text{linear}}$

- $y(n)$ depends on the present input values so, it is $\boxed{\text{causal.}}$
- let $x(n)$ is bounded, $y(n)$ will be bounded so, it is $\boxed{\text{stable}}$

(d) $y(n) = x(-n+2)$

- present output depends on past & future values = $\boxed{\text{dynamic}}$
- Superposition & scaling principle are met = $\boxed{\text{linear}}$
- let $x_2(n) = x_1(n-n_0) \Rightarrow y_2(n) = x_2(-n+2)$

$$y_1(n-n_0) = x_1(-(n-n_0)+2) \qquad y_2(n) = x_1(-n-n_0+2)$$
$$= x_1(-n+n_0+2)$$

time-invariant

- present output depends on future inputs & outputs = $\boxed{\text{non causal}}$
- if $x(n)$ is bounded, then $x(-n+2)$ also bounded = $\boxed{\text{stable}}$

(e) $y(n) = \text{Trun}[x(n)]$

- it doesnt depend on the time index n = $\boxed{\text{static}}$
- $\boxed{\text{Non linear}}$ due to truncation
- $\boxed{\text{time-invariant}}$
- It's based on the present of the input sequence = $\boxed{\text{Causal}}$
- Truncation doesnt lead to unbounded values = $\boxed{\text{Stable}}$

Problem 2.11

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\}$$
$$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\}$$

$$\Rightarrow \quad x_1(n) + x_2(n) = \{0, 1, 0\} = \delta(n)$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\}$$
$$= \delta(n) + \delta(n-1)$$

$$x_3(n) = x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1)$$

$$y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$
$$= \delta(n) + 2\delta(n-1) + \delta(n-2)$$

So,

$$y_4(n) = y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$$
$$= \{1, \underset{\wedge}{2}, -1, 0, 1\} + \{-1, \underset{\wedge}{1}, 0, 2\} + \{\underset{\wedge}{1}, 2, -1, 0, 1\} + \{\underset{\wedge}{-1}, 1, 0, 2\}$$
$$= \{\underset{\wedge}{3}, 2, 1, 3, 1\}$$

So

$$y_4(n) \neq x_3(n)$$

then

$$x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1) \neq y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$$

So, the system is **time-variant.**

Problem 2.16 b3 & b11

Compute convolution $y(n) = x(n) * h(n)$ of the following signals & check the correctness of the results by using the test in (a)

(3) $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$$y(n) = \{0, \tfrac{1}{2}, \tfrac{1}{2} - 1, 1 - 1 + 3/2, 1/2 - 2 + 3/2 - 2, -1 + 3 - 2, 3/2 - 4, -2\}$$

$$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$$

$$\Sigma x(n) = -2$$

$$\Sigma y(n) = -5$$

$$\Sigma h(k) = 5/2$$

- $\Sigma_y = \Sigma x \, \Sigma h$

$$-5 = -\cancel{2}(5/\cancel{2})$$

$$-5 = -5$$

(11) $x(n) = \left(\frac{1}{2}\right)^n \mu(n)$

$h(n) = \left(\frac{1}{4}\right)^n \mu(n)$

$$y(n) = \sum_{k=0}^{n} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} (1/4)^k (1/2)^{n-k}$$

$$= (1/2)^n \sum_{k=0}^{n} (1/4)^k (1/2)^{-k}$$

$$= (1/2)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

$$= 0.5^n \left[1 + 0.5 + 0.5^2 + \cdots + (n+1)\right] \mu(n)$$

$$= 0.5^n \left[\frac{1 - 0.5^{n+1}}{1 - 0.5}\right] \mu(n)$$

$$= 0.5^n \left[\frac{1 - 0.5^{n+1}}{0.5}\right] \mu(n)$$

$$y(n) = 0.5^n \left(2 - 0.5^n\right) \mu(n)$$

- $\sum x(n) = \sum_{n=0}^{\infty} (1/2)^n \Rightarrow \frac{1}{1 - 1/2} = 2$

- $\sum y(n) = \sum_{n=0}^{\infty} (0.5)^n \left(2 - 0.5^n\right)$

$$= \sum_{n=0}^{\infty} 2(0.5)^n - 0.25^n$$

$$= 2\left(\frac{1}{1-0.5}\right) - \left(\frac{1}{1-0.25}\right)$$

$$= 2(2) - 4/3 = 8/3$$

- $\sum h(n) = \sum_{n=0}^{\infty} (1/4)^n \Rightarrow \frac{1}{1 - 1/4} = 4/3$

So, $\sum_y = \sum_x \sum_h$

$$\frac{8}{3} = 2\left(\frac{4}{3}\right)$$

$$\frac{8}{3} = \frac{8}{3}$$

Problem 2.24

$$y(n) = ny(n-1) + x(n) \qquad n \geq 0$$

ⓐ $x_1(n) \to y_1(n) = ny(n-1) + x_1(n)$

$x_2(n) \to y_2(n) = ny(n-1) + x_2(n)$

$x_3(n) = x_1(n) + x_2(n) \to y_3(n) = y_1(n) + y_2(n) \Rightarrow ny_3(n-1) + x_3(n)$

$$\begin{aligned} y_3(n) &= ny_3(n-1) + x_3(n) \\ &= n[y_1(n-1) + y_2(n-1)] + [x_1(n) + x_2(n)] \\ &= ny_1(n-1) + ny_2(n-1) + x_1(n) + x_2(n) \\ &= y_1(n) + y_2(n) \end{aligned}$$

$y_3(n) = y_1(n) + y_2(n)$ ✓

ⓑ $x_1(n) \to y_1(n) = ny_1(n-1) + x_1(n)$

$x_2(n) = kx_1(n) \to y_2(n) = ky_1(n)$

$$\begin{aligned} y_2(n) &= ny_2(n-1) + x_2(n) \\ &= n[ky_1(n-1)] + [kx_1(n)] \\ &= k(ny_1(n-1) + x_1(n)) \end{aligned}$$

$y_2(n) = ky_1(n)$ = homogeneous = linear

ⓒ $x(n) \to y(n)$

$x(n-n_0) \to y(n-n_0) = ny(n-n_0-1) + x(n-n_0)$

shifted:

$y(n-n_0) = (n-n_0)y_2(n-n_0-1) + x_2(n-n_0)$

not the same = time-varying/**time variant**

(d) $y(n) = ny(n-1) + x(n) \quad n \geq 0$

$x(n) = \mu(n)$

$$= \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{is bounded}$$

(e) $n=0$

$y(0) = 1$

$n=1$

$y(1) = 2$

$n=2$

$y(2) = 5$

$n=3$

$y(3) = 6$

$y(n) = \{1, 2, 5, 16 \cdots\}$

$\boxed{\text{not bounded}}$

the system is BIBO. unstable.

2.27

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n) \qquad x(n) = 2^n\mu(n)$$

$$y_p(n) = K2^n\mu(n)$$

$$K2^n\mu(n) = \frac{5}{6}K2^n\mu(n-1) - \frac{1}{6}K2^{n-2}\mu(n-2) + 2^n\mu(n)$$

$n=2$

$$K2^2 = \frac{5}{6}2K - \frac{K}{6} + 4$$

$$K = \frac{8}{5}$$

$$y_p(n) = \frac{8}{5}2^n\mu(n)$$

Homogeneous sol.

$$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \qquad \lambda = \frac{1}{2}, \frac{1}{3}$$

$$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n$$

$$y_n(n) = C_1(1/2)^n + C_2(1/3)^n + \left(\frac{8}{5}\right)2^n\mu(n)$$

$y(0) = C_1 + C_2 + 8/5$ $\qquad$ $y(-1) = y(-2) = 0$

$y(1) = 1/2\,C_1 + 1/3\,C_2 + 16/5$

$$y(0) = 5/6\,y(-1) - \frac{1}{6}y(-2) + 1$$

$$y(0) = 1$$

$$y(1) = 5/6\,y(0) - \frac{1}{6}y(-1) + 2$$

$$y(1) = \frac{17}{6}$$

$$C_1 + C_2 + 8/5 = 1$$

$$1/2\,C_1 + 1/3\,C_2 + \frac{16}{5} = \frac{17}{6}$$

$$\boxed{\begin{aligned} C_1 &= -1 \\ C_2 &= 2/5 \end{aligned}}$$

So,

$$\boxed{y(n) = -(1/2)^n\mu(n) + \frac{2}{5}(1/3)^n\mu(n) + \left(\frac{8}{5}\right)2^n\mu(n)}$$

2-31

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

for impulse

$y_p(n) = 0$

So,

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0 \Rightarrow \lambda(\lambda-4) + 1(\lambda-4) = 0$$

$$\Rightarrow (\lambda+1)(\lambda-4) = 0 \qquad \lambda = 4, -1$$

$$y_h(n) = C_1 4^n + C_2 (-1)^n$$

$x(0) = 1$

$y(-1) = y(-2) = 0$

for $n=0$

$$y(0) - 3y(-1) - 4y(-2) = x(0) + 2x(-1)$$

$$y(0) - 0 - 0 = 1 + 0$$

$$y(0) = 1$$

for $n=1$

$$y(1) - 3y(0) - 4y(-2) = x(1) + 2(0)$$

$$y(1) - 3y(0) = 2$$

$$y(1) = 5$$

So

$$C_1 + C_2 = 1$$

$$4C_1 - C_2 = 6$$

$$\boxed{\begin{aligned} C_1 &= 6/5 \\ C_2 &= -1/5 \end{aligned}}$$

$$\boxed{h(n) = \left[(6/5)4^n - (1/5)(-1)^n\right] u(n)}$$

2.46

(a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$ $/2$

$y(n) + 0.5y(n-1) - 2y(n-3) = 0.5x(n) + 1.5x(n-5)$

$y(n) = -0.5\,y(n-1) + 2y(n-3) + 0.5x(n) + 1.5x(n-5)$

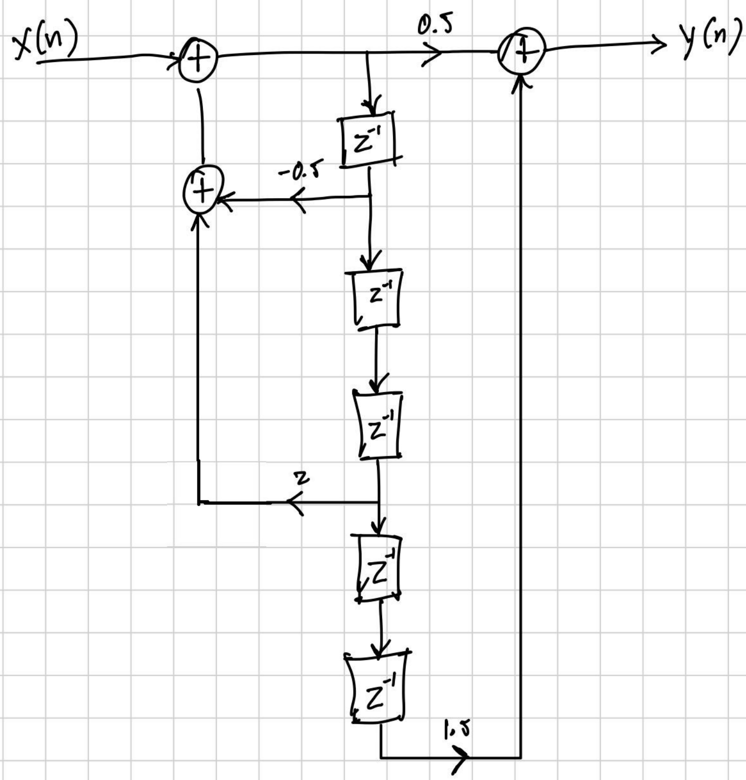
$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$a_1 = 0.5$

$a_3 = -2$

$b_0 = 0.5$

$b_5 = 1.5$



(b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

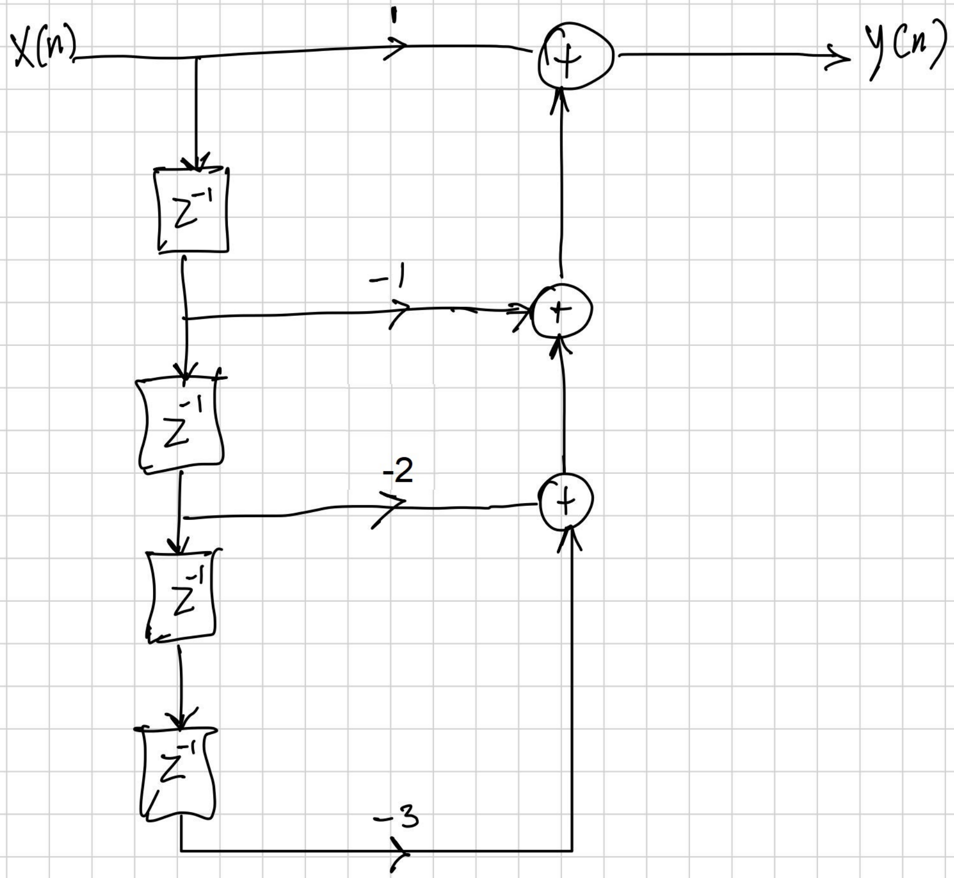
$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$$

$b_0 = 1$

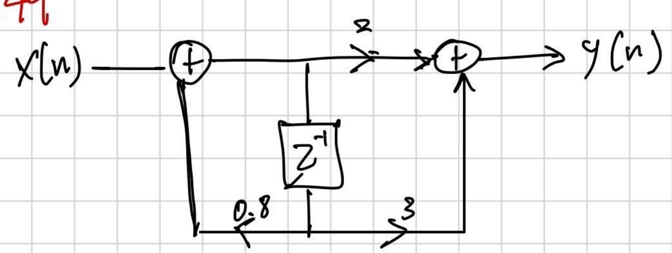
$b_1 = -1$

$b_2 = 2$

$b_4 = -3$



2.49



(a)

$$y(n) = 2xn + 3x(n-1) + 0.8y(n-1)$$

$$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

System relax: $n=0$

$$y(0) - 0.8y(-1) = 2\delta(0) + 3\delta(-1)$$

$$y(0) = 1$$

$$y(n) - 0.8y(n-1) = 0$$

$$\lambda^n - 0.8\lambda^{n-1} = 0$$

$$\boxed{\lambda^{n-1}(\lambda - 0.8) = 0 \Rightarrow \lambda = 0.8}$$

$$y_h(n) = C\lambda^n$$

$$\boxed{y_h(n) = C\, 0.8^n}$$

$n=0$

$$h(0) = C(0.8)^0$$

$$\boxed{1 = C}$$

$$y_h(n) = c(0.8)^n$$

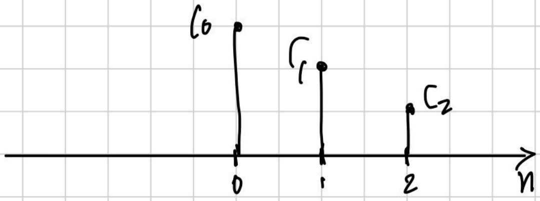
response of the system: y(n) - 0.8 y(n-1) = x(n).

x(n)=δ(n). y(0)=1, c=1, therefore, the impulse response of the original system is:

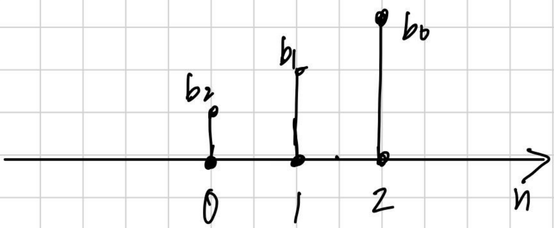
h(n) = 2(0.8)^n u(n) + 3(0.8)^(n-1) u(n-1) = 2*δ(n) + 4.6* (0.8)^(n-1) u(n-1).

2.52

$h_1(n) = C_0\,\delta(n) + C_1\,\delta(n-1) + C_2\,\delta(n-2)$ $\qquad C_0 > C_1 > C_2$



$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2)$ $\qquad b_2 < b_1 < b_0$



$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2)$

$a_0 = 1,\ a_1 = 2,\ a_2 = 3$