Sampling and Aliasing

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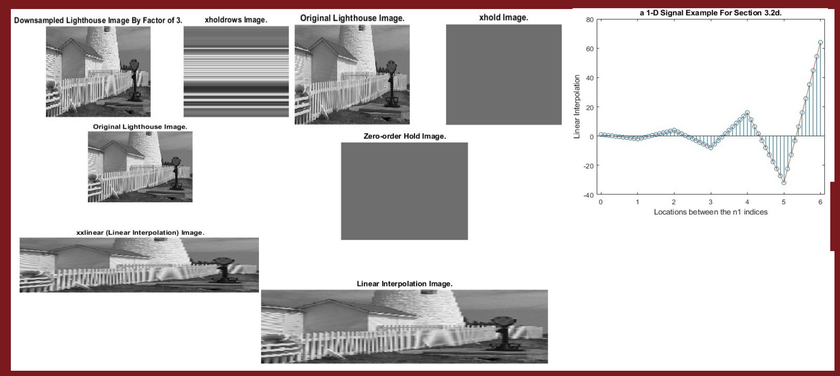
Background

When sampling to convert a continuous-time (or analog) signal to a digital form for computer processing and storage, the primary issue is aliasing and the sampling strategy necessary to avoid aliasing of frequency components.

The objective of our presentation is to understand the Sampling Theorem which states that the sampling rate must be greater than twice the highest frequency contained in the analog signal. Frequency content is taken to mean the spectral content of a signal when represented as a sum of sinusoids.

We present the signal reconstruction of a D-to-A converter from a practical point of view as a generalization of interpolation.



Result:
The harmonic analysis of a triangle wave provides insights into its frequency domain representation the contribution of different harmonics to the waveform, the absence of even harmonics, and how the amplitudes decrease with increasing frequency. This information helps explain why the triangle wave has a specific shape, as its harmonics add up in a particular way to create the characteristic linear rise and fall of the waveform.

Digital Images: A/D and D/A

An image can be represented as a function x(t1, t2) . (t2): the horizontal length and (t1) is vertical length of two continuous variable coordinates of a point in space.
I.For monochrome images (called grayscale): The function will be a scalar function of the two spatial variables. **II. For color images :** The function will be a vector-valued function of the two variables. Ex: RGB needs three values at each spatial location.

I. Gray-scale Images : We will consider only sampled still images.

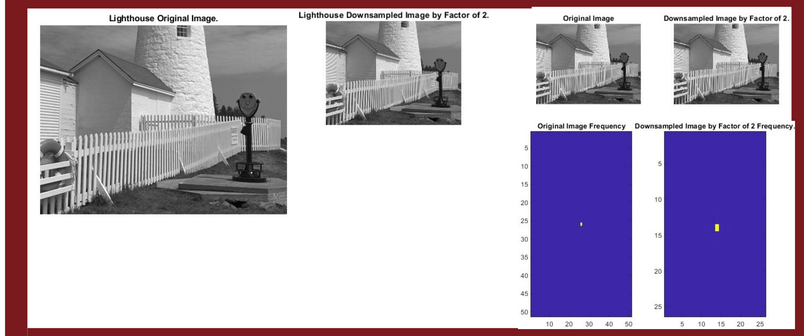
- These images will be represented as a two-dimensional array of numbers of the form :

$x[m,n] = x(mT_1, nT_2) \qquad 1 \le m \le M, \text{ and } 1 \le n \le N$

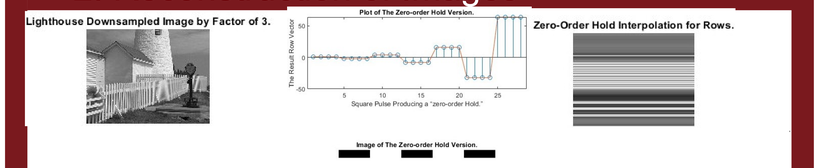
  - T1 : Sample spacing in the horizontal direction
  - T2 : Sample spacing in the vertical direction
  - Typical M & N values:256 or 512.Ex.:a 512x512 image

✓ In MATLAB we represent an image as a matrix, so it would consist of M rows and N columns.

1. Down-Sampling



2. Reconstruction of Images

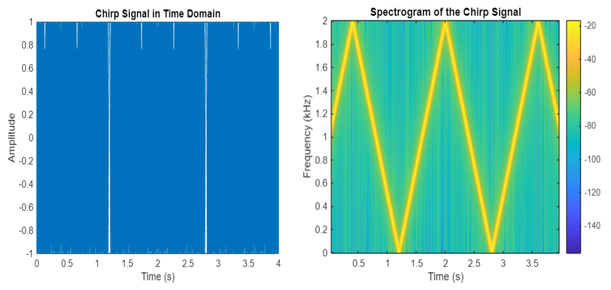


Spectrograms: Harmonic Lines & Chirp Aliasing

Aliasing in Chirp Signals
Synthesize a chirp signal with a linearly varying frequency, selecting parameters such that aliasing occurs when the frequency exceeds the Nyquist limit.

The spectrogram will show aliasing as the frequency of the chirp exceeds the Nyquist 2000 Hz, resulting in mirrored frequency components

DFT, which is used in calculating the spectrogram, treats frequency content as periodic. frequency axis is considered to be periodic, with the range extending from 0 to f_s, but practically folding at f_s/2



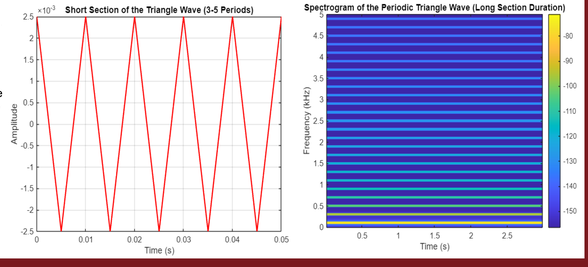
Periodic Triangle Wave and Analyze its Harmonic Spectrum

Triangle wave contains only odd harmonics of the fundamental frequency. The even harmonics are absent.

$$a_k = \begin{cases} \frac{-2}{\pi^2 k^2} & \text{for } k \text{ odd} \\ 0 & \text{for } k \text{ even} \end{cases}$$

Coefficient ratios show that the amplitude of each harmonic decreases significantly as the harmonic number (k) increases, higher harmonics contribute less to the signal, leading to a smoother shape.

Time scaling of the triangle wave does not change the Fourier coefficients. The coefficients are only dependent on the shape of the waveform and not on how stretched or compressed it is.



Summary

References

1. James H. McClellan, Ronald W. Schafer. 4. Sampling and Aliasing, dspfirst.gatech.edu/chapters/04samplin/overview.html. Accessed 2 Dec. 2024.
2. Proakis, John G., and Dimitris G. Manolakis. Digital Signal Processing. Prentice Hall, 2006.