1.3 Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

(a) $x_a(t) = 3\cos(5t + \pi/6)$

(b) $x(n) = 3\cos(5n + \pi/6)$

(c) $x(n) = 2\exp[j(n/6 - \pi)]$

(d) $x(n) = \cos(n/8)\cos(\pi n/8)$

(e) $x(n) = \cos(\pi n/2) - \sin(\pi n/8) + 3\cos(\pi n/4 + \pi/3)$

1.4 **(a)** Show that the fundamental period N_p of the signals

$$s_k(n) = e^{j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots$$

is given by $N_p = N/\text{GCD}(k, N)$, where GCD is the greatest common divisor of k and N.

(b) What is the fundamental period of this set for $N = 7$?

(c) What is it for $N = 16$?

1.5 Consider the following analog sinusoidal signal:

$$x_a(t) = 3\sin(100\pi t)$$

(a) Sketch the signal $x_a(t)$ for $0 \le t \le 30$ ms.

(b) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = 1/F_s$, and show that it is periodic.

(c) Compute the sample values in one period of $x(n)$. Sketch $x(n)$ on the same diagram with $x_a(t)$. What is the period of the discrete-time signal in milliseconds?

(d) Can you find a sampling rate F_s such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?

1.6 A continuous-time sinusoid $x_a(t)$ with fundamental period $T_p = 1/F_0$ is sampled at a rate $F_s = 1/T$ to produce a discrete-time sinusoid $x(n) = x_a(nT)$.

(a) Show that $x(n)$ is periodic if $T/T_p = k/N$ (i.e., T/T_p is a rational number).

(b) If $x(n)$ is periodic, what is its fundamental period T_p in seconds?

(c) Explain the statement: $x(n)$ is periodic if its fundamental period T_p, in seconds, is equal to an integer number of periods of $x_a(t)$.

1.7 An analog signal contains frequencies up to 10 kHz.

(a) What range of sampling frequencies allows exact reconstruction of this signal from its samples?

(b) Suppose that we sample this signal with a sampling frequency $F_s = 8$ kHz. Examine what happens to the frequency $F_1 = 5$ kHz.

(c) Repeat part (b) for a frequency $F_2 = 9$ kHz.

1.8 An analog electrocardiogram (ECG) signal contains useful frequencies up to 100 Hz.

(a) What is the Nyquist rate for this signal?

(b) Suppose that we sample this signal at a rate of 250 samples/s. What is the highest frequency that can be represented uniquely at this sampling rate?

1.9 An analog signal $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled 600 times per second.

(a) Determine the Nyquist sampling rate for $x_a(t)$.

(b) Determine the folding frequency.

(c) What are the frequencies, in radians, in the resulting discrete time signal $x(n)$?

(d) If $x(n)$ is passed through an ideal D/A converter, what is the reconstructed signal $y_a(t)$?

1.10 A digital communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

(a) What are the sampling frequency and the folding frequency?

(b) What is the Nyquist rate for the signal $x_a(t)$?

(c) What are the frequencies in the resulting discrete-time signal $x(n)$?

(d) What is the resolution Δ?

1.11 Consider the simple signal processing system shown in Fig. P1.11. The sampling periods of the A/D and D/A converters are $T = 5$ ms and $T' = 1$ ms, respectively. Determine the output $y_a(t)$ of the system, if the input is

$$x_a(t) = 3\cos 100\pi t + 2\sin 250\pi t \qquad (t \text{ in seconds})$$

The postfilter removes any frequency component above $F_s/2$.

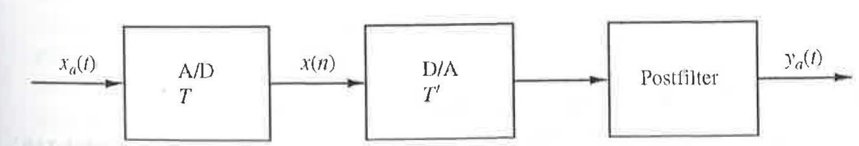

Figure P1.11

1.12 **(a)** Derive the expression for the discrete-time signal $x(n)$ in Example 1.4.2 using the periodicity properties of sinusoidal functions.

(b) What is the analog signal we can obtain from $x(n)$ if in the reconstruction process we assume that $F_s = 10$ kHz?

1.13 The discrete-time signal $x(n) = 6.35\cos(\pi/10)n$ is quantized with a resolution (a) $\Delta = 0.1$ or (b) $\Delta = 0.02$. How many bits are required in the A/D converter in each case?