Sampling and Aliasing

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Background

When sampling to convert a continuous-time (or analog) signal to a digital form for computer processing and storage, the primary issue is aliasing and the sampling strategy necessary to avoid aliasing of frequency components.

The objective of our presentation is to understand the Sampling Theorem which states that the sampling rate must be greater than twice the highest frequency contained in the analog signal. Frequency content is taken to mean the spectral content of a signal when represented as a sum of sinusoids.

We present the signal reconstruction of a D-to-A converter from a practical point of view as a generalization of interpolation.

Lab P-8: Digital Images: A/D and D/A

A. Digital Images

An image can be represented as a function x(t1, t2) . (t2): the horizontal length and (t1) is vertical length of two continuous variable coordinates of a point in space.

I.For monochrome images (called grayscale): The function will be a scalar function of the two spatial variables. **II. For color images :** The function will be a vector-valued function of the two variables. Ex: RGB needs three values at each spatial location.

For this lab,we will consider only sampled still images for the gray-scale images.

- These images will be represented as a two-dimensional array of numbers of the form :

$x[m,n] = x(mT_1, nT_2) \qquad 1 \le m \le M, \text{ and } 1 \le n \le N$

- T1 : Sample spacing in the horizontal direction
- T2 : Sample spacing in the vertical direction
- Typical M & N values:256 or 512.Ex.:a 512x512 image

✓In MATLAB we represent an image as a matrix, so it would consist of M rows and N columns. The matrix entry at *(m,n)* is the sample value *x[m,n]* — called a pixel.

✓ An important property of light images (photographs) to note: Their values are always nonnegative and finite in magnitude $0 \le x[m,n] \le X_{\max} < \infty$. It's because they formed by measuring the intensity of reflected or emitted light. The values of x[m,n] have to be scaled relative to a maximum value Xmax when stored in a computer or displayed. With 8-bit integers, the maximum value, Xmax = 2^8 -1 = 255, and there will be 2^8 = 256 different gray levels for the display.

B. Displaying Images

The correct display of an image on a gray-scale monitor can be tricky.

- Filtering may introduce negative values, especially if differencing is used (Ex:a high-pass filter). All image values must be nonnegative to display.
- The default format for most gray-scale displays is 8bits. Hence, the pixel values x[m, n] in the image must be converted to integers in the range 0 =< [m, n] =< 255 = 2^8 - 1.
- Matlab's built-in "imshow " function handles the color map and the "true" size of an image.
- We'll do a "grayscale display" where all three primary colors (red, green and blue, or RGB) are used equally and creates a "gray map." In this lab, we'll do a linear color mapping as the non-linear color mappings would introduce an extra level of complication.
- If the image values lie outside the range [0,255] or the scaled image occupies only a small portion of the range, the image may have poor quality. The following function represents the linear mappings/scaling: $x_s[m,n] = \mu x[m,n] + \beta$. The scaling constants μ and β can be derived from the min and max values of the image. Hence, the pixel values are computed via :

$$x_s[m,n] = \left\lfloor 255.999\left(\frac{x[m,n]-x_{\min}}{x_{\max}-x_{\min}}\right)\right\rfloor$$

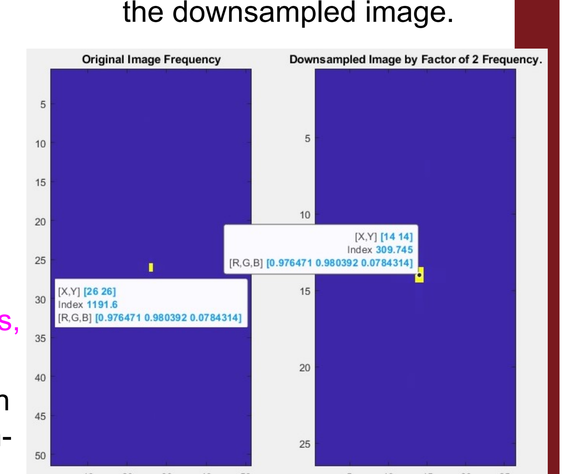
where $\lfloor x \rfloor$ is the floor function, i.e., the greatest integer less than or equal to x.

1. Down-Sampling

Lighthouse Original Image.

Lighthouse Downsampled Image by Factor of 2.

- Low sampling freq.rate on the downsampled image.

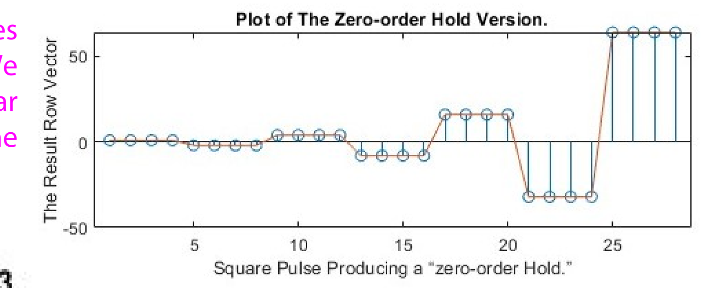


- Less smooth and more distorted
- Some part of the fence, roof shingles, the edges of the windows, doors and the lighthouse itself appear blurred.

Problem: The downsampled image is unable to completely capture the high frequency information in the original image. Nyquist rate violation: The sampling rate must to be >> than x2 the max. freq. in the original image.

2. Reconstruction of Images

- When sampled, we can fill in the missing samples by interpolation. It's like D-to-A converter. We could use a "square pulse" or a "triangular pulse" or other pulse shapes for the reconstruction.
- For the reconstruction experiment, we'll use :



The below image shows how zero-order hold interpolation is used to fill in the missing spots.

The Interpolation Factor = 4

Lighthouse Downsampled Image by Factor of 3.

The goal: Reconstruct the original size of 256x256.

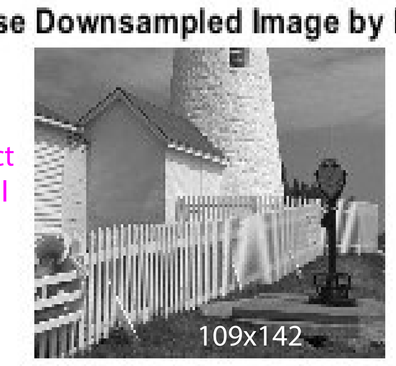
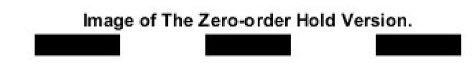


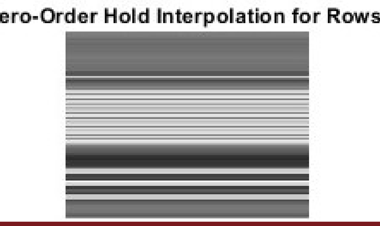
Image of The Zero-order Hold Version.



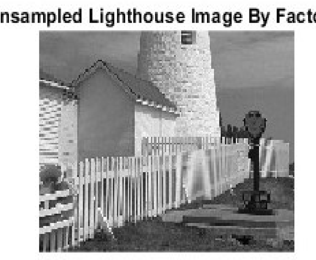
The simplest interpolation is a square pulse reconstruction which produces "zero-order" hold.

Interpolation_factor = length(nn)/L;

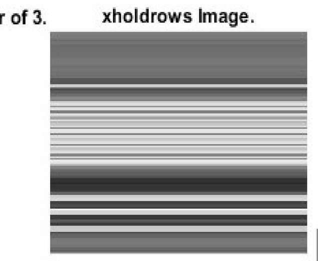
Zero-Order Hold Interpolation for Rows.



Downsampled Lighthouse Image By Factor of 3.



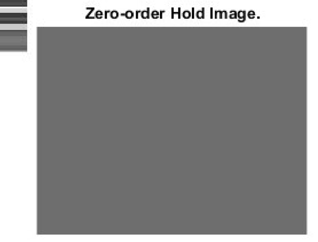
xholdrows Image.



Original Lighthouse Image.

Zero-order Hold vs Linear Interpolation Difference:

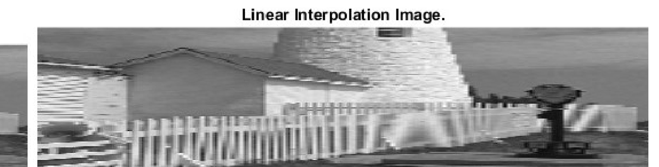
Zero-order Hold Image.



Linear interpolation can be done in MATLAB using the interp1 function

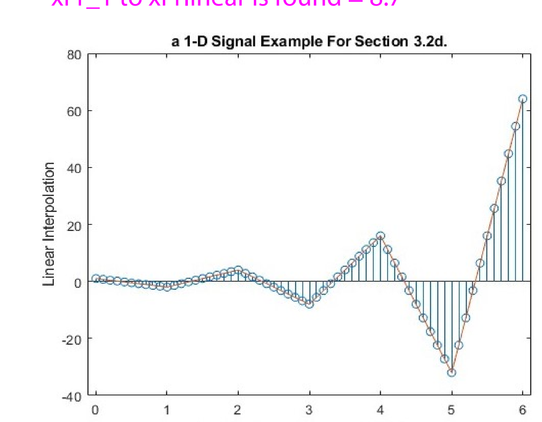
xxlinear (Linear Interpolation) Image.

Linear Interpolation Image.



Linear interpolation with triangle signal.

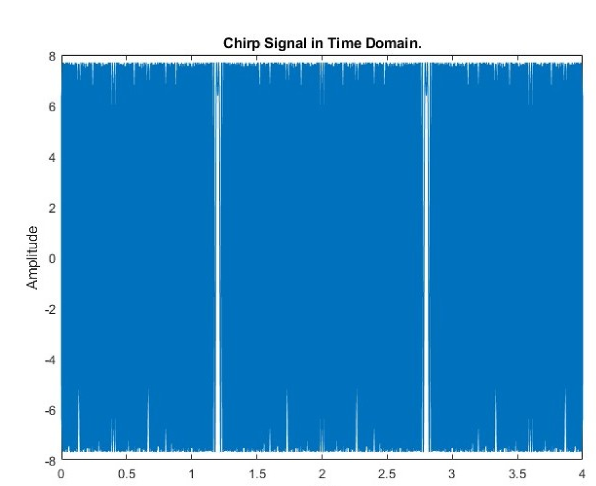
The Interpolation Factor Converting xr1_1 to xr1linear is found = 8.7

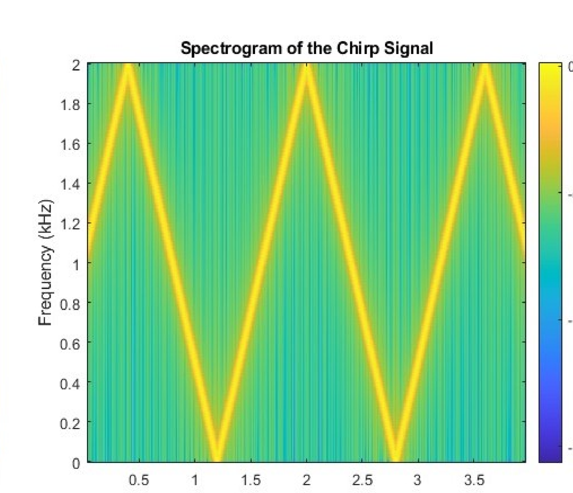


Lab S-8: Spectrograms: Harmonic Lines & Chirp Aliasing

Analyze the Effects of Aliasing on a Chirp signal

Aliasing is represented in the spectrogram, with mirrored or folded frequency components As the frequency of the chirp exceeds 2000Hz,
It is folded back into the lower frequency range
aliasing theorem, which states that frequencies above the Nyquist limit appear at incorrect, lower frequencies in the sampled signal
DFT, which is used in calculating the spectrogram, treats frequency content as periodic



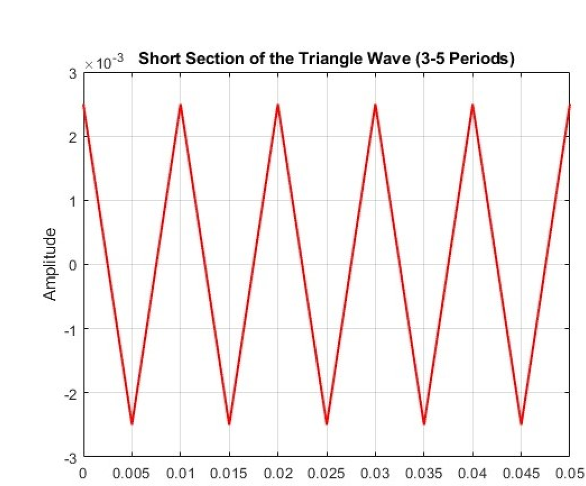


Spectrogram of Periodic Signal

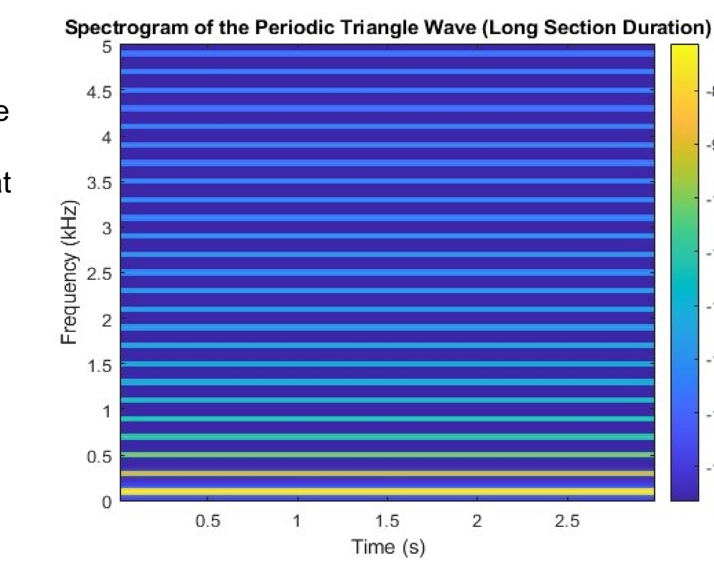
the triangle wave contains only odd harmonics of the fundamental frequency. The even harmonics are absent
The amplitude of each harmonic decreases as the harmonic number increases

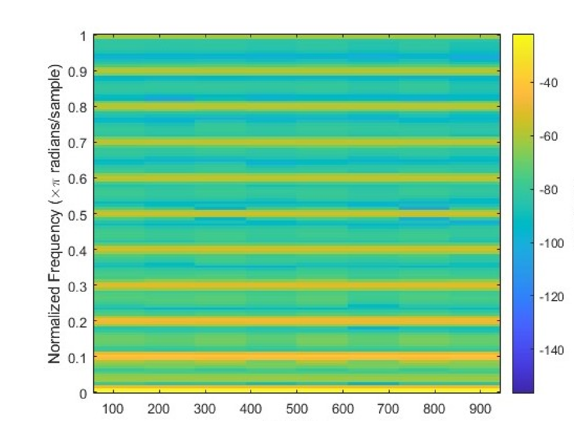
Time scaling of the signal affects the frequency of the harmonics but does not change the Fourier coefficients. This means that the same harmonic structure is preserved, although the signal oscillates faster or slower depending on the scaling factor
Higher harmonics contribute less to the signal, leading to a smoother shape

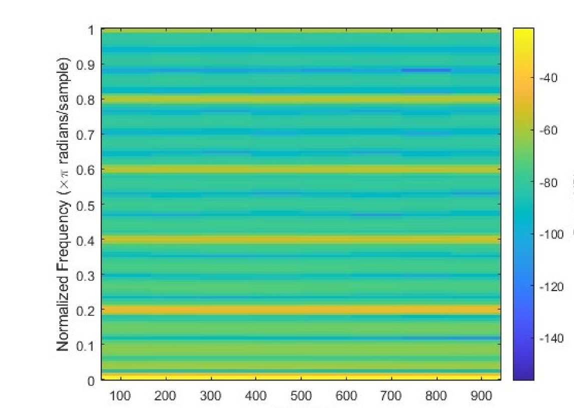
$$a_k = \begin{cases} \dfrac{-2}{\pi^2 k^2} & \text{for } k \text{ odd} \\ 0 & \text{for } k \text{ even} \end{cases}$$

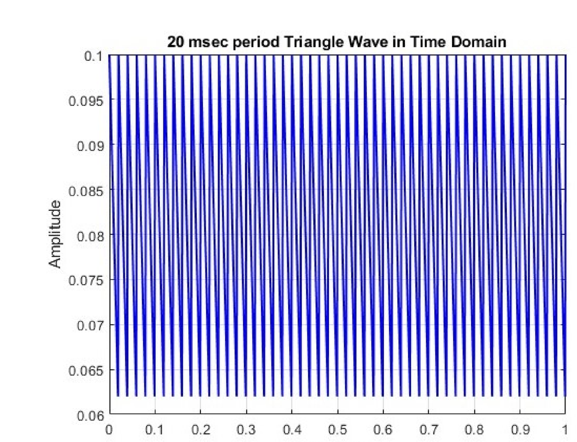


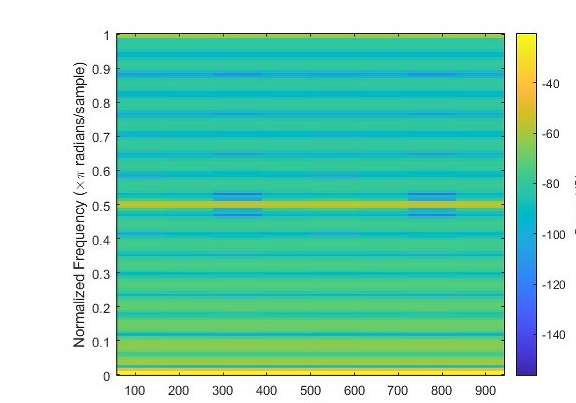
- The spectrogram of the periodic triangle wave you provided shows multiple horizontal lines at different frequencies. These lines represent the harmonics of the triangle wave.
- The triangle wave is a periodic signal, which means that it has a fundamental frequency and a series of harmonics.
- Distinct horizontal lines at different frequency values, each corresponding to an odd harmonic of the fundamental frequency
- The amplitude of each harmonic of a triangle wave decreases with increasing harmonic order
- Horizontal lines indicate that the signal contains stable frequency components over time, which is typical for a periodic waveform











Summary

- In summary, in the image, aliasing appears visually as jagged edges or stair-step patterns particularly along sharp transitions between colors or high-contrast areas. In a downsampled image aliasing is visually represented as a distorted / jagged pattern where high spatial frequencies are present. The aliasing effect occurs when sampling rate is less than twice the maximum frequency in the original. It's called *Nyquist rate*.
- Comparing the quality of the linear interpolation vs zero-order hold result is the linear interpolation image is more closely matches the original lighthouse image. Although the reconstruction procedure can eliminate some aliasing from the down sampled lighthouse image, it doesn't solve it completely. It's possible to obtain closely to the original image if the sampling frequency is high.
- In the spectrogram, aliasing occurs when once the chirp frequency exceeds 2kHz because of the mirrored frequency components. It folds back into the frequency range below 2kHz.
- The dB difference depends only on the k indices because the amplitude of each harmonic ak is inversely proportional.

References

1. James H. McClellan, Ronald W. Schafer. 4. Sampling and Aliasing, dspfirst.gatech.edu/chapters/04samplin/overview.html. Accessed 2 Dec. 2024.
2. Proakis, John G., and Dimitris G. Manolakis. Digital Signal Processing. Prentice Hall, 2006.