Sampling and Aliasing

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Background

When sampling to convert a continuous-time (or analog) signal to a digital form for computer processing and storage, the primary issue is aliasing and the sampling strategy necessary to avoid aliasing of frequency components.

The objective of our presentation is to understand the Sampling Theorem which states that the sampling rate must be greater than twice the highest frequency contained in the analog signal. Frequency content is taken to mean the spectral content of a signal when represented as a sum of sinusoids.

We present the signal reconstruction of a D-to-A converter from a practical point of view as a generalization of interpolation.

Lab P-8: Digital Images: A/D and D/A

A. Digital Images

An image can be represented as a function x(t1, t2) . (t2): the horizontal length and (t1) is vertical length of two continuous variable coordinates of a point in space.
I.For monochrome images (called grayscale): The function will be a scalar function of the two spatial variables. **II. For color images :** The function will be a vector-valued function of the two variables. Ex: RGB needs three values at each spatial location.

For this lab,we will consider only sampled still images for the gray-scale images.

- These images will be represented as a two-dimensional array of numbers of the form :

$x[m,n] = x(mT_1, nT_2) \qquad 1 \le m \le M, \text{ and } 1 \le n \le N$

- T1 : Sample spacing in the horizontal direction
- T2 : Sample spacing in the vertical direction
- Typical M & N values:256 or 512.Ex.:a 512x512 image

✓In MATLAB we represent an image as a matrix, so it would consist of M rows and N columns. The matrix entry at *(m,n)* is the sample value *x[m,n]* — called a pixel.

✓ An important property of light images (photographs) to note: Their values are always nonnegative and finite in magnitude: $0 \le x[m,n] \le X_{\max} < \infty$. It's because they formed by measuring the intensity of reflected or emitted light. The values of x[m,n] have to be scaled relative to a maximum value Xmax when stored in a computer or displayed. With 8-bit integers, the maximum value, Xmax = 2^8 -1 = 255, and there will be 2^8 = 256 different gray levels for the display.

B. Displaying Images

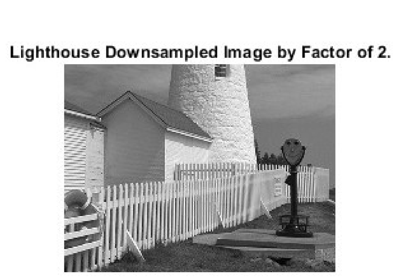
The correct display of an image on a gray-scale monitor can be tricky.

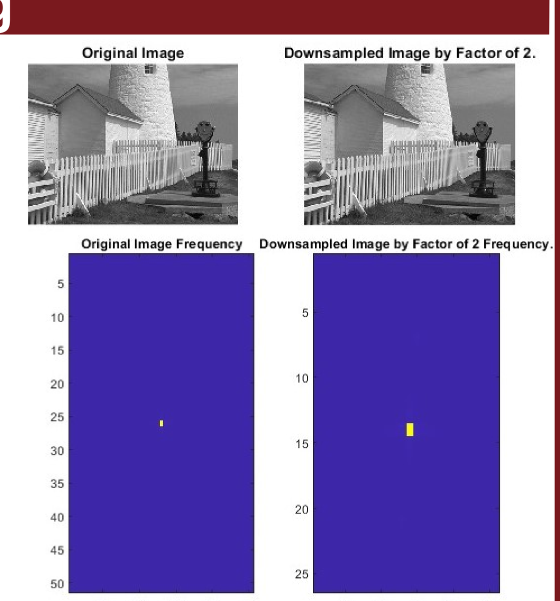
- Filtering may introduce negative values, especially if differencing is used (Ex:a high-pass filter). All image values must be nonnegative to display.
- The default format for most gray-scale displays is 8bits. Hence, the pixel values x[m, n] in the image must be converted to integers in the range 0 =< [m, n] =< 255 = 2^8 - 1.
- Matlab's built-in "imshow " function handles the color map and the "true" size of an image.
- We'll do a "grayscale display" where all three primary colors (red, green and blue, or RGB) are used equally and creates a "gray map." In this lab, we'll do a linear color mapping as the non-linear color mappings would introduce an extra level of complication.
- If the image values lie outside the range [0,255] or the scaled image occupies only a small portion of the range, the image may have poor quality. The following function represents the linear mappings/scaling: $x_s[m,n] = \mu x[m,n] + \beta$. The scaling constants μ and β can be derived from the min and max values of the image. Hence, the pixel values are computed via :

$$x_s[m,n] = \left\lfloor 255.999 \left(\frac{x[m,n] - x_{\min}}{x_{\max} - x_{\min}} \right) \right\rfloor$$

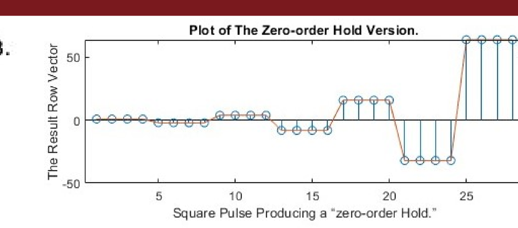
where $\lfloor x \rfloor$ is the floor function, i.e., the greatest integer less than or equal to x.

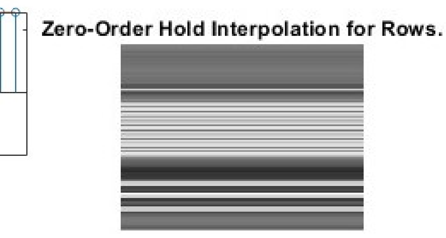
1. Down-Sampling

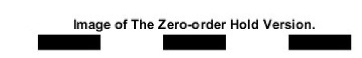


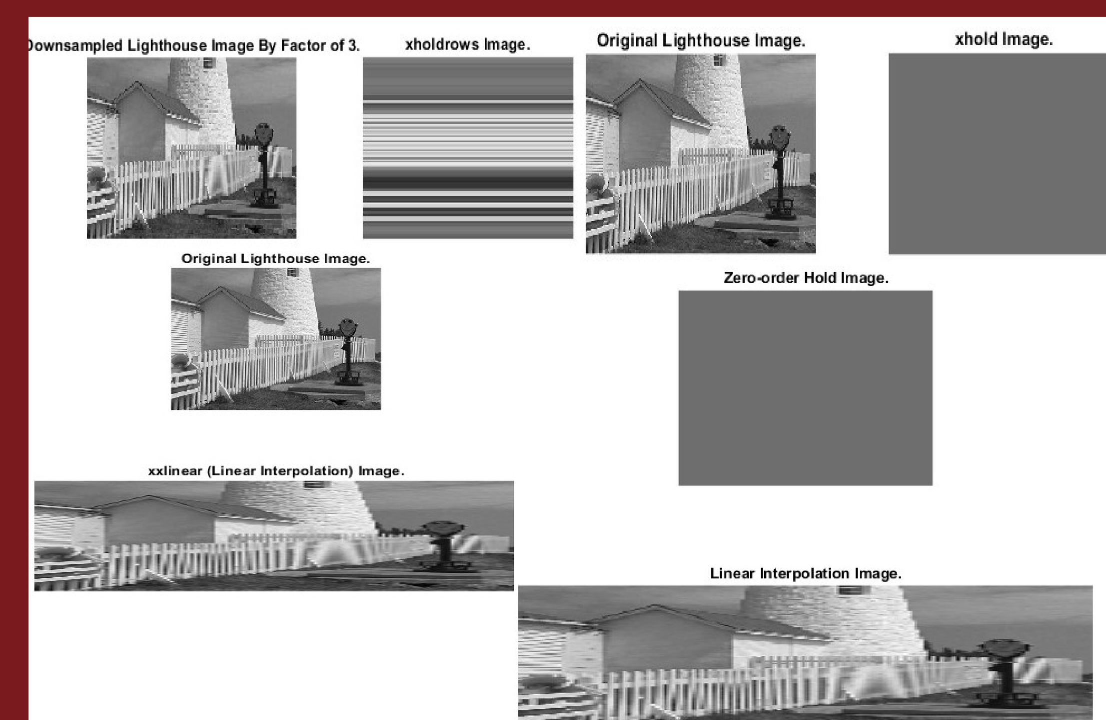


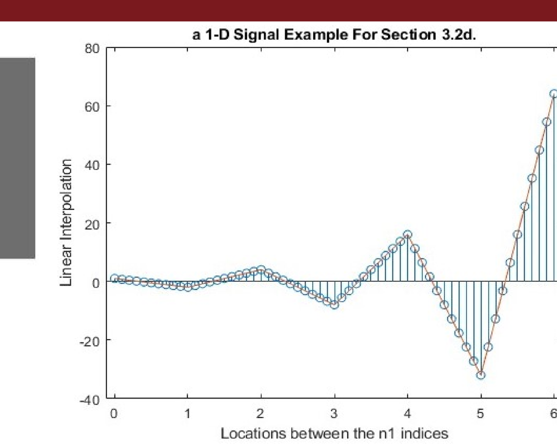
2. Reconstruction of Images











Lab S-8: Spectrograms: Harmonic Lines & Chirp Aliasing

Summary

References

1. James H. McClellan, Ronald W. Schafer. 4. Sampling and Aliasing, dspfirst.gatech.edu/chapters/04samplin/overview.html. Accessed 2 Dec. 2024.
2. Proakis, John G., and Dimitris G. Manolakis. Digital Signal Processing. Prentice Hall, 2006.