

- 1.3 Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

- (a) $x_a(t) = 3 \cos(5t + \pi/6)$
- (b) $x(n) = 3 \cos(5n + \pi/6)$
- (c) $x(n) = 2 \exp[j(n/6 - \pi)]$
- (d) $x(n) = \cos(n/8) \cos(\pi n/8)$
- (e) $x(n) = \cos(\pi n/2) - \sin(\pi n/8) + 3 \cos(\pi n/4 + \pi/3)$

- 1.4 (a) Show that the fundamental period N_p of the signals

$$s_k(n) = e^{j2\pi kn/N}, \quad k = 0, 1, 2, \dots$$

is given by $N_p = N/\text{GCD}(k, N)$, where GCD is the greatest common divisor of k and N .

- (b) What is the fundamental period of this set for $N = 7$?
- (c) What is it for $N = 16$?

- 1.5 Consider the following analog sinusoidal signal:

$$x_a(t) = 3 \sin(100\pi t)$$

- (a) Sketch the signal $x_a(t)$ for $0 \leq t \leq 30$ ms.
 - (b) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = 1/F_s$, and show that it is periodic.
 - (c) Compute the sample values in one period of $x(n)$. Sketch $x(n)$ on the same diagram with $x_a(t)$. What is the period of the discrete-time signal in milliseconds?
 - (d) Can you find a sampling rate F_s such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?
- 1.6 A continuous-time sinusoid $x_a(t)$ with fundamental period $T_p = 1/F_0$ is sampled at a rate $F_s = 1/T$ to produce a discrete-time sinusoid $x(n) = x_a(nT)$.
- (a) Show that $x(n)$ is periodic if $T/T_p = k/N$ (i.e., T/T_p is a rational number).
 - (b) If $x(n)$ is periodic, what is its fundamental period T_p in seconds?
 - (c) Explain the statement: $x(n)$ is periodic if its fundamental period T_p , in seconds, is equal to an integer number of periods of $x_a(t)$.
- 1.7 An analog signal contains frequencies up to 10 kHz.
- (a) What range of sampling frequencies allows exact reconstruction of this signal from its samples?
 - (b) Suppose that we sample this signal with a sampling frequency $F_s = 8$ kHz. Examine what happens to the frequency $F_1 = 5$ kHz.
 - (c) Repeat part (b) for a frequency $F_2 = 9$ kHz.

- 1.8 An analog electrocardiogram (ECG) signal contains useful frequencies up to 100 Hz.
- (a) What is the Nyquist rate for this signal?
 - (b) Suppose that we sample this signal at a rate of 250 samples/s. What is the highest frequency that can be represented uniquely at this sampling rate?

- 1.9 An analog signal $x_a(t) = \sin(480\pi t) + 3 \sin(720\pi t)$ is sampled 600 times per second.
- (a) Determine the Nyquist sampling rate for $x_a(t)$.
 - (b) Determine the folding frequency.

- (c) What are the frequencies, in radians, in the resulting discrete time signal $x(n)$?
- (d) If $x(n)$ is passed through an ideal D/A converter, what is the reconstructed signal $y_a(t)$?

- 1.10 A digital communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 3 \cos 600\pi t + 2 \cos 1800\pi t$$

The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

- (a) What are the sampling frequency and the folding frequency?
- (b) What is the Nyquist rate for the signal $x_a(t)$?
- (c) What are the frequencies in the resulting discrete-time signal $x(n)$?
- (d) What is the resolution Δ ?

- 1.11 Consider the simple signal processing system shown in Fig. P1.11. The sampling periods of the A/D and D/A converters are $T = 5$ ms and $T' = 1$ ms, respectively. Determine the output $y_a(t)$ of the system, if the input is

$$x_a(t) = 3 \cos 100\pi t + 2 \sin 250\pi t \quad (t \text{ in seconds})$$

The postfilter removes any frequency component above $F_s/2$.

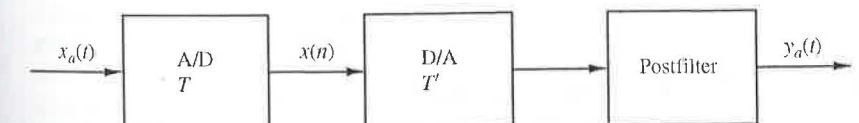


Figure P1.11

- 1.12 (a) Derive the expression for the discrete-time signal $x(n)$ in Example 1.4.2 using the periodicity properties of sinusoidal functions.
 - (b) What is the analog signal we can obtain from $x(n)$ if in the reconstruction process we assume that $F_s = 10$ kHz?
- 1.13 The discrete-time signal $x(n) = 6.35 \cos(\pi/10)n$ is quantized with a resolution (a) $\Delta = 0.1$ or (b) $\Delta = 0.02$. How many bits are required in the A/D converter in each case?