Assignment 1

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1. Problem 1.3 (25 points)

- Periodic only if $x(n+N) = x(n)$ for all n.
- the smallest value of N = fundamental period.

- for sin with frequency f_0 to be periodic, we should have

$$\cos[2\pi f_0(N+n)+\theta] = \cos(2\pi f_0 n+\theta)$$

or
$$\begin{cases} 2\pi f_0 N = 2k\pi & \text{intiger } k \text{ shoud exist.} \\ f_0 = \dfrac{k}{N} \end{cases}$$

$\boxed{2\pi F = 5}$

a) $x_a(t) = 3\cos(\underbrace{5}_{2\pi F}t + \underbrace{\pi/6}_{\theta})$

$\Downarrow$ sample $F_s = 1/T$

$$x(n) = x_a(nT) = \cancel{3\cos(5n+\pi/6)} = 3\cos(2\pi F n T + \theta) =$$

$$= 3\cos(2\pi n \frac{F}{F_s} + \theta)$$

$$x(n) = 3\cos(2\pi f n + \theta) \quad ; \quad f = \frac{F}{F_s} \text{ normalized frequncy}$$

$$x(n) = 3\cos(2\pi \cdot \frac{5}{2\pi F_s} n + \theta)$$

Since $F_s = \dfrac{1}{T_p}$; $2\pi F_s = 5 \Rightarrow F_s = \dfrac{5}{2\pi}$

periodic with period $\Rightarrow$ $\boxed{T_p = \frac{1}{F_s} = \frac{2\pi}{5}}$

(b) $x(n) = 3\cos(\underbrace{5}_{\omega}n + \pi/6)$

$\boxed{X(n) = A\cos(\omega n + \theta)}$ amplitude freq. phase

$2\pi f = 5$

$\boxed{f = \frac{5}{2\pi}}$ $\boxed{\text{non-periodic}}$

$\omega = 2\pi f.$

f must be rational number for $x(n)$ to be periodic

(c) $x(n) = 2\exp[j(n/6 - \pi)]$ → convert it to cosine.

Euler's Identity:

$$x(n) = 2\left[e^{j\frac{n}{6}} \cdot e^{-j\pi}\right] = 2\left[\left(\cos\frac{n}{6} + j\sin\frac{n}{6}\right)\cdot\right.$$

$$\left.\cdot\left(\underbrace{\cos\pi}_{-1} - \cancel{j\sin\pi}^{\,0}\right)\right] = -2\left[\cos\frac{n}{6} + j\sin\frac{n}{6}\right] =$$

$$= -2\left[e^{j\frac{n}{6}}\right]$$;

$S_k(n) = e^{j2\pi k f_0 n}$

$N = \frac{1}{f_0}$; should be periodic with $\frac{N}{k}$

$$\cancel{j}2\pi k f_0 \cancel{n} = \cancel{j}\frac{\cancel{n}}{6}$$ ($f_0 \to \frac{1}{N}$)

$2\pi k(6) = N$

$\boxed{\frac{N}{k} = 12\pi}$ $\boxed{f_0 = \frac{1}{N} = \frac{1}{12\pi}}$ $\boxed{\text{non-periodic}}$

d) $x(n) = \underbrace{\cos\frac{n}{8}}_{\#1} \cdot \underbrace{\cos\left(\frac{\pi n}{8}\right)}_{\#2}$

#1 $w = 2\pi f \Rightarrow 2\pi f = \frac{1}{8}$; * $\boxed{f = \frac{1}{16\pi}}$; therefore, $\cos\frac{1}{8}$ is * $\boxed{\text{non-periodic}}$

#2 $2\pi f = \frac{\pi}{8} \Rightarrow$ * $\boxed{f = \frac{1}{16} \quad \text{periodic}}$

their product:

$$x(n) = \left(\frac{e^{j\frac{n}{8}} + e^{-j\frac{n}{8}}}{2}\right) \cdot \left(\frac{e^{j\frac{\pi n}{8}} + e^{-j\frac{\pi n}{8}}}{2}\right) =$$

$$= \frac{e^{j\left(\frac{n}{8}+\frac{\pi n}{8}\right)} + e^{j\left(\frac{n}{8}-\frac{\pi n}{8}\right)} + e^{j\left(\frac{\pi n}{8}-\frac{n}{8}\right)} + e^{-j\left(\frac{\pi n}{8}+\frac{n}{8}\right)}}{4}$$

$$f_1 \cdot f_2 = \frac{1}{16} \cdot \frac{1}{16\pi} = \frac{1}{256\pi}$$

* $\boxed{\text{their product is non-periodic}}$

$\boxed{\text{product of 1 periodic and 1 non-periodic, makes it non-periodic}}$

e) $x(n) = \underbrace{\cos\left(\frac{\pi n}{2}\right)}_{\#1} - \underbrace{\sin\left(\frac{\pi n}{8}\right)}_{\#2} + 3\underbrace{\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right)}_{\#3}$

#1. $2\pi f = \frac{\pi}{2} \Rightarrow \boxed{f = \frac{1}{4}}$

$N_p = \frac{1}{f} \Rightarrow$ * $\boxed{N_p = 4 \quad \text{periodic}}$

#2 $2\pi f = \frac{\pi}{8} \Rightarrow f = \frac{1}{16}$; * $\boxed{N_p = \frac{1}{f} = 16 \quad \text{periodic}}$

conversion:

$$\boxed{-\sin\left(\frac{\pi n}{8}\right) = +\cos\left(\frac{\pi n}{8} + 90^\circ\right)}$$

(#3). $2\pi f = \frac{\pi}{4} \Rightarrow f = \frac{1}{8}$; $N_p = \frac{1}{f}$; $\boxed{N_p = 8}$ * $\boxed{\text{periodic}}$

Hence, $x(n)$ is periodic with period $\boxed{N_p = 16}$ The least common multiple of 4, 8, 16.

PROBLEM 1.7 (25 points)

a) A.S. freq. up to 10 kHz.

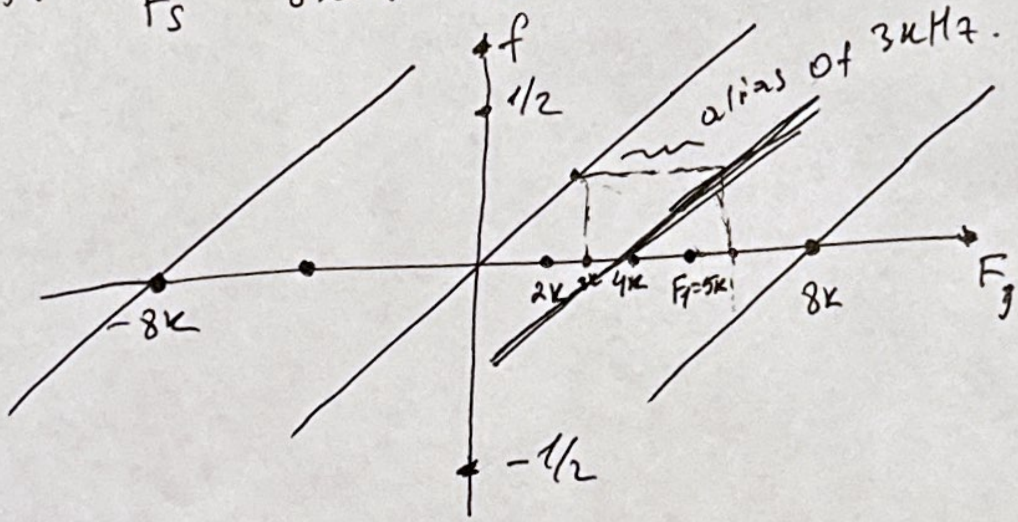
sampling freq.

$F_{max} = 10\,kHz$. the rule of thumb is to have $F_s > 2F_{max}$ ← Nyquist Rate

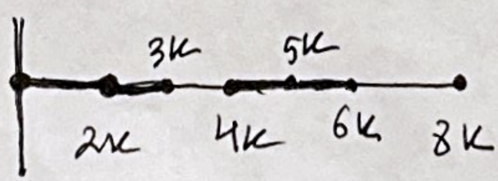
So, $F_s > 2(10\,kHz) \geq 20\,kHz$ ← it should be minimum of 20 kHz sampling rate.

(b) $F_s = 8\,kHz$, examine $F_1 = 5\,kHz$

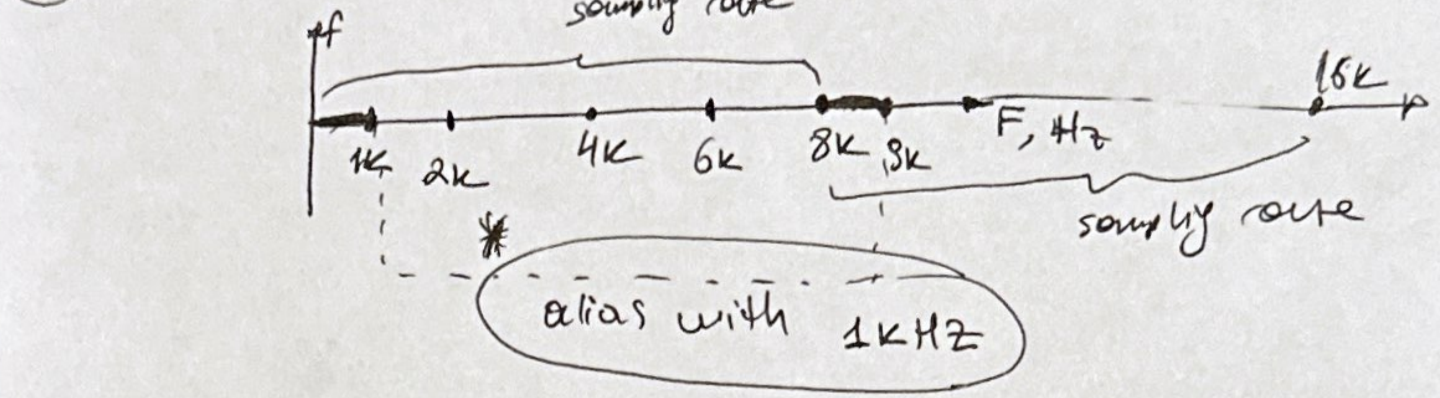
$$f_1 = \frac{F_1}{F_s} = \frac{5\,kHz}{8\,kHz} = \frac{5}{8}$$



$F_{fold} = \frac{F_s}{2} = \frac{8\,kHz}{2} = 4\,kHz \Rightarrow$ 5kHz will alias with 3kHz



(c) $F_2 = 9\text{K Hz}$



PROBLEM 1.9 (25 points)

* use sampling rate of 600 Hz, not Nyquist rate for (b) (d)

$$x_a(t) = \sin(\underbrace{480\pi t}_{2\pi F_1 t}) + 3\sin(\underbrace{720\pi t}_{2\pi F_2 t})$$

$F_s = 600\text{ Hz}.$

a) $F_1 = 240\text{ Hz}$

$F_2 = \frac{720}{2} = 360\text{ Hz}$ } $F_{max} = F_2 = 360\text{ Hz}$

Nyquist Rate $= F_N = 2F_{max} = 360\text{ Hz}(2) = \boxed{720\text{ Hz}}$ *

(b) Folding frequency $= F_{fold} = \frac{F_s}{2} = \frac{600\text{Hz}}{2} = \boxed{300\text{ Hz}}$

(c) $x(n) = x_a(nT) = x_a(n/F_s) = \sin\left(\frac{480\pi t}{600}\right) +$

$+ 3\sin\left(\frac{720\pi t}{600}\right) = \sin\left(\frac{4\pi n}{5}\right) - 3\sin\left(\frac{4\pi n}{5}\right) =$

$= -2\sin\left(\frac{4\pi n}{5}\right)$; Hence, * $\boxed{w = \frac{4\pi}{5}}$

(d) $y_a(t) =$ Reconstructed signal $= x(F_s t) = -2\sin\left(\frac{4\pi}{5} \cdot 600\right) =$

$y_a(t)$ $\boxed{= -2\sin(480\pi t)}$ *

$\boxed{\text{PROBLEM 1.10 (25 points)}}$

Clarification: Found sampling rate in part (a), use that in part (c) and (d)

$$\boxed{x_a(t) = 3\cos \underbrace{600\pi t}_{2\pi F_1 t} + 2\cos \underbrace{1800\pi t}_{2\pi F_2 t}}$$

* quantized into 1024 different voltage levels.
* operates at 10,000 bits/s

a) sampling freq., $F_s = ?$
$F_{Fold} = ?$

$\#$ of bits/sample $= \log_2 1024 = 10$

*

$$F_s = \frac{10,000 \text{ bits/s}}{10 \text{ bits/sample}} = \boxed{1000 \text{ samples/s}}$$

*

$$F_{fold} = \frac{F_s}{2} = \frac{1000 \text{ (samples/s)}}{2} = \underline{\underline{500 \text{ Hz}}}$$

(b) $\cancel{2\pi} F_1 \cancel{t} = 600 \cancel{\pi t} \Rightarrow F_1 = 300 \text{ Hz}$

$\cancel{2\pi} F_2 \cancel{t} = 1800 \cancel{\pi t} \Rightarrow \boxed{F_2 = 900 \text{ Hz}}$ highest: $F_{max} = F_2$

*

$$F_N = 2F_{max} = 2(900 \text{ Hz}) = \boxed{1800 \text{ Hz}}$$

(c) Find frequencies in resulting - descrete time.

$$f_1 = \frac{F_1}{F_s} = \frac{300 \text{ Hz}}{1000} = \underline{\underline{0.3}}$$

$$f_2 = \frac{F_2}{F_s} = \frac{900}{1000} = \underline{\underline{0.9}}$$

$F_{fold} = \frac{500}{1000} = \underline{0.5}$

*

$f_2 = 0.9 > 0.5 \Rightarrow \boxed{f_2 = 0.1}$

therfore:

*

$$\boxed{x(n) = 3\cos[(2\pi)(0.3)n] + 2\cos[(2\pi)(0.1)n]}$$

(d) Resolution $\Delta = ?$

$$\Delta = \frac{X_{max} - X_{min}}{m-1} = \frac{5-(-5)}{1024-1} = \frac{10}{1023}$$