Assignment 2 ECE 5530

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- ## Problem 2.7

1) Static or dynamic: if the present output depends on present input the system is static or could be memoryless so it is dynamic.

2) Linear or Non-Linear: if the system satisfies the principles of superposition & scaling. otherwise the system is non-linear

3) Time variant or time varying: a time invariant if its behavior & characteristics do not change over time so, a time shift in the input signal results an identical time shift in the output, otherwise is called time-variant

4) Causal or non-casual: if present output depends on past outputs, past inputs, and present inputs is causal.
if present output depends on future inputs & outputs is non-causal

5) Stable or unstable: if every bounded input produces a bounded output (BIBO)

Examine the following systems

(a) $y(n) = \cos[x(n)]$: as operation is performed on $x(n)$ it is non linear. as output depends only in present input it is static. behavior doesn't change over time is time invariant.

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$: the system is dynamic because the present output depends on past & future values of input (ex.) $y(4) = \sum_{k=-\infty}^{5} x(k)$

- for input $x_1(n)$

$$y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$$

- for $x_2(n)$

$$y_2(n) = \sum_{k=-\infty}^{n+1} x_2(k)$$

$$x_3(n) = ax_1(n) + bx_2(n)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} x_3(k)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} ax_1(k) + bx_2(k)$$

$$y_3(n) = ay_1(n) + by_2(n)$$

So, it is linear

- $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

for $x_1(n) \Rightarrow y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$

for $y_1(n)$

$$y_1(n-n_0) = \sum_{k=-\infty}^{n+1-n_0} x_1(k)$$

for $x_2(n) = x_1(n-n_0)$ for $x_1(n-n_0)$

$$y_2(n) = \sum_{k=-\infty}^{n+1} x_2(k)$$

$$= \sum_{k=-\infty}^{n-n_0+1} x_1(k-n_0)$$

$$= \sum_{k=-\infty}^{n+1-n_0} x_1(k-n_0)$$

$$y_2(n) = y_1(n-n_0)$$

System is time-invariant

- $y(n) = \sum_{k=-\infty}^{n+1} x(k)$ let $n=0 \Rightarrow y(0) = \cdots x(-\infty) + \cdots x(0) + x(1)$

system is non-casual!

- let $x(n)$ is bounded then the output $y(n)$ is also bounded

system is stable.

ⓒ $y(n) = x(n)\cos(w_0 n)$:

- $y(n)$ depends only on the present input values

So system is static.

- let $x_1(n) \Rightarrow y_1(n) = x_1(n)\cos(w_0 n)$

let $x_2(n) \Rightarrow y_2(n) = x_2(n)\cos(w_0 n)$

let $x_3(n) = a x_1(n) + b x_2(n)$

$$y_3(n) = x_3(n)\cos(w_0 n)$$

$$= [a x_1(n) + b x_2(n)]\cos(w_0 n)$$

system is linear

- $y(n)$ depends on the present input values so, it is causal.
- let $x(n)$ is bounded, $y(n)$ will be bounded so, it is stable

(d) $y(n) = x(-n+2)$

- present output depends on past & future values = dynamic
- Superposition & scaling principle are met = linear
- let $x_2(n) = x_1(n-n_0) \Rightarrow y_2(n) = x_2(-n+2)$

$$y_1(n-n_0) = x_1(-(n-n_0)+2) \qquad y_2(n) = x_1(-n-n_0+2)$$
$$= x_1(-n+n_0+2) \neq$$

system is time-varying

- present output depends on future inputs & outputs = non causal
- if $x(n)$ is bounded, then $x(-n+2)$ also bounded = stable

(e) $y(n) = \text{Trun}[x(n)]$

- it doesnt depend on the time index n = static
- Non linear due to truncation
- time - invariant
- It's based on the present of the input sequence = Causal
- Truncation doesnt lead to unbounded values = Stable

Problem 2.11

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\}$$
$$x_2(n) = \{1, -\underset{\uparrow}{1}, -1\}$$
$$\Rightarrow \quad x_1(n) + x_2(n) = \{0, 1, 0\} = \delta(n)$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\}$$
$$= \delta(n) + \delta(n-1)$$

$$x_3(n) = x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1)$$

$y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$

$= \delta(n) + 2\delta(n-1) + \delta(n-2)$

So,

$y_4(n) = y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$

$= \{1, \underset{\wedge}{2}, -1, 0, 1\} + \{-1, \underset{\wedge}{1}, 0, 2\} + \{\underset{\wedge}{1}, 2, -1, 0, 1\} + \{\underset{\wedge}{-1}, 1, 0, 2\}$

$= \{\underset{\wedge}{3}, 2, 1, 3, 1\}$

So

$y_4(n) \neq x_3(n)$

then

$x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1) \neq y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$

So, the system is invariant

Problem 2.16 b3 & b11

Compute convolution $y(n) = x(n) * h(n)$ of the following signals & check the correctness of the results by using the test in (a)

(3) $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$y(n) = \{0, \frac{1}{2}, \frac{1}{2} - 1, 1 - 1 + 3/2, 1/2 - 2 + 3/2 - 2, -1 + 3 - 2, 3/2 - 4, -2\}$

$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$

$\Sigma x(n) = -2$

$\Sigma y(n) = -5$

$\Sigma h(k) = 5/2$

- $\Sigma_y = \Sigma x \, \Sigma h$

$-5 = -\cancel{2}(5/\cancel{2})$

$-5 = -5$

11 $x(n) = \left(\frac{1}{2}\right)^n \mu(n)$

$h(n) = \left(\frac{1}{4}\right)^n \mu(n)$

$$y(n) = \sum_{k=0}^{n} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} (1/4)^k (1/2)^{n-k}$$

$$= (1/2)^n \sum_{k=0}^{n} (1/4)^k (1/2)^{-k}$$

$$= (1/2)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

$$= 0.5^n \left[1 + 0.5 + 0.5^2 + \cdots + (n+1)\right] \mu(n)$$

$$= 0.5^n \left[\frac{1 - 0.5^{n+1}}{1 - 0.5}\right] \mu(n)$$

$$= 0.5^n \left[\frac{1 - 0.5^{n+1}}{0.5}\right] \mu(n)$$

$$y(n) = 0.5^n \left(2 - 0.5^n\right) \mu(n)$$

- $\sum x(n) = \sum_{n=0}^{\infty} (1/2)^n \Rightarrow \frac{1}{1 - 1/2} = 2$
- $\sum y(n) = \sum_{n=0}^{\infty} (0.5)^n \left(2 - 0.5^n\right)$

$$= \sum_{n=0}^{\infty} 2(0.5)^n - 0.25^n$$

$$= 2\left(\frac{1}{1-0.5}\right) - \left(\frac{1}{1-0.25}\right)$$

$$= 2(2) - 4/3 = 8/3$$

- $\sum h(n) = \sum_{n=0}^{\infty} (1/4)^n \Rightarrow \frac{1}{1 - 1/4} = 4/3$

So, $\sum_y = \sum_x \sum_h$

$$\frac{8}{3} = 2\left(\frac{4}{3}\right)$$

$$\frac{8}{3} = \frac{8}{3}$$

Problem 2.24

$$y(n) = ny(n-1) + x(n) \qquad n \geq 0$$

(a) $x_1(n) \rightarrow y_1(n) = ny(n-1) + x_1(n)$

$x_2(n) \rightarrow y_2(n) = ny(n-1) + x_2(n)$

$x_3(n) = x_1(n) + x_2(n) \rightarrow y_3(n) = y_1(n) + y_2(n) \Rightarrow ny_3(n-1) + x_3(n)$

$$
\begin{aligned}
y_3(n) &= ny_3(n-1) + x_3(n) \\
&= n[y_1(n-1) + y_2(n-1)] + [x_1(n) + x_2(n)] \\
&= ny_1(n-1) + ny_2(n-1) + x_1(n) + x_2(n) \\
&= y_1(n) + y_2(n)
\end{aligned}
$$

$y_3(n) = y_1(n) + y_2(n)$ ✓

(b) $x_1(n) \rightarrow y_1(n) = ny_1(n-1) + x_1(n)$

$x_2(n) = Kx_1(n) \rightarrow y_2(n) = Ky_1(n)$

$$
\begin{aligned}
y_2(n) &= ny_2(n-1) + x_2(n) \\
&= n[Ky_1(n-1)] + [Kx_1(n)] \\
&= K(ny_1(n-1) + x_1(n))
\end{aligned}
$$

$y_2(n) = Ky_1(n)$ = homogeneous = linear

(c) $x(n) \rightarrow y(n)$

$x(n-n_0) \rightarrow y(n-n_0) = ny(n-n_0-1) + x(n-n_0)$

shifted:

$y(n-n_0) = (n-n_0)y_2(n-n_0-1) + x_2(n-n_0)$

not the same = time-varying

(d) $y(n) = ny(n-1) + x(n) \qquad n \geq 0$

$x(n) = \mu(n)$

$$= \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$ is bounded

(e) $n=0$

$y(0) = 1$

$n=1$

$y(1) = 2$

$n=2$

$y(2) = 5$

$n=3$

$y(3) = 6$

$y(n) = \{1, 2, 5, 16 \cdots\}$

not bounded

the system is BIBO.

2.27

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \qquad x(n) = 2^n \mu(n)$$

$$y_p(n) = K 2^n \mu(n)$$

$$K 2^n \mu(n) = \frac{5}{6} K 2^n \mu(n-1) - \frac{1}{6} K 2^{n-2} \mu(n-2) + 2^n \mu(n)$$

$n=2$

$$K 2^2 = \frac{5}{6} 2K - \frac{K}{6} + 4$$

$$K = \frac{8}{5}$$

$$y_p(n) = \frac{8}{5} 2^n \mu(n)$$

Homogeneous sol.

$$\lambda^n - \frac{5}{6} \lambda^{n-1} + \frac{1}{6} \lambda^{n-2} = 0$$

$$\lambda^2 - \frac{5}{6} \lambda + \frac{1}{6} = 0 \qquad \lambda = \frac{1}{2}, \frac{1}{3}$$

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

$$y_n(n) = c_1(1/2)^n + c_2(1/3)^n + \left(\frac{8}{5}\right)2^n \mu(n)$$

$$y(0) = c_1 + c_2 + 8/5 \qquad y(-1) = y(-2) = 0$$

$$y(1) = 1/2\, c_1 + 1/3\, c_2 + 16/5$$

$$y(0) = 5/6\, y(-1) - \frac{1}{6} y(-2) + 1$$

$$y(0) = 1$$

$$y(1) = 5/6\, y(0) - \frac{1}{6} y(-1) + 2$$

$$y(1) = \frac{17}{6}$$

$$c_1 + c_2 + 8/5 = 1 \qquad c_1 = 1$$

$$1/2\, c_1 + 1/3\, c_2 + \frac{16}{5} = \frac{17}{6} \qquad c_2 = 2/5$$

So,

$$y(n) = -(1/2)^n \mu(n) + \frac{2}{5}\left(1/3\right)^n \mu(n) + \left(\frac{8}{5}\right)2^n \mu(n)$$

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$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

for impulse

$y_p(n) = 0$

So,

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$\lambda^2 - 4\lambda + \lambda - 4 = 0 \Rightarrow \lambda(\lambda-4) + 1(\lambda-4) = 0$

$\Rightarrow (\lambda+1)(\lambda-4) = 0 \qquad \lambda = 4, -1$

$y_h(n) = C_1 4^n + C_2(-1)^n$

$x(0) = 1$

$y(-1) = y(-2) = 0$

for $n=0$

$y(0) - 3y(-1) - 4y(-2) = x(0) + 2x(-1)$

$y(0) - 0 - 0 = 1 + 0$

$y(0) = 1$

for $n=1$

$y(1) - 3y(0) - 4y(-2) = x(1) + 2(0)$

$y(1) - 3y(0) = 2$

$y(1) = 5$

So

$C_1 + C_2 = 1 \qquad C_1 = 6/5$

$4C_1 - C_2 = 6 \qquad C_2 = -1/5$

$h(n) = \left[(6/5)4^n - (1/5)(-1)^n\right] u(n)$

2.46

(a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$ $/2$

$y(n) + 0.5y(n-1) - 2y(n-3) = 0.5x(n) + 1.5x(n-5)$

$y(n) = -0.5y(n-1) + 2y(n-3) + 0.5x(n) + 1.5x(n-5)$

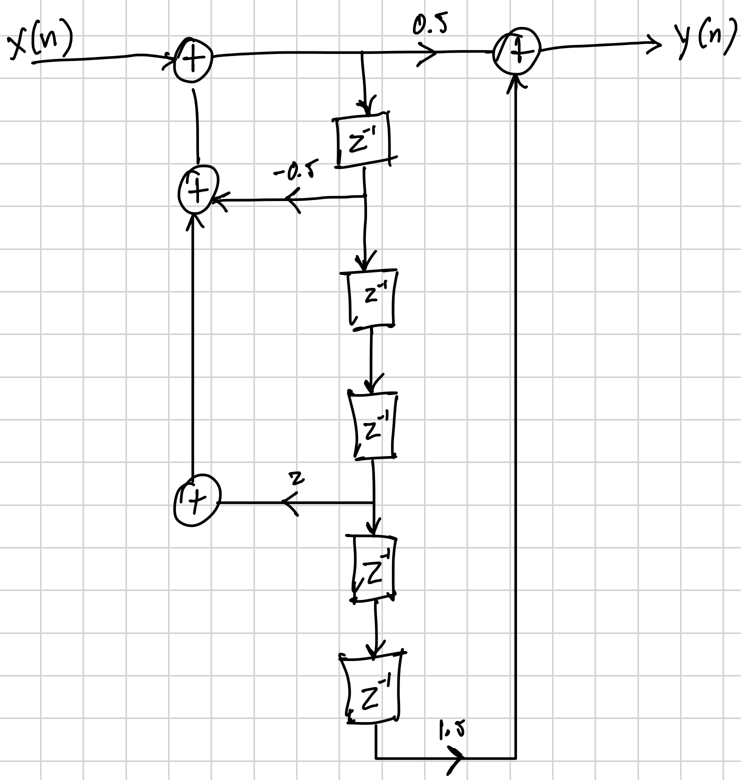
$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

$a_1 = 0.5$

$a_3 = -2$

$b_0 = 0.5$

$b_5 = 1.5$



(b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

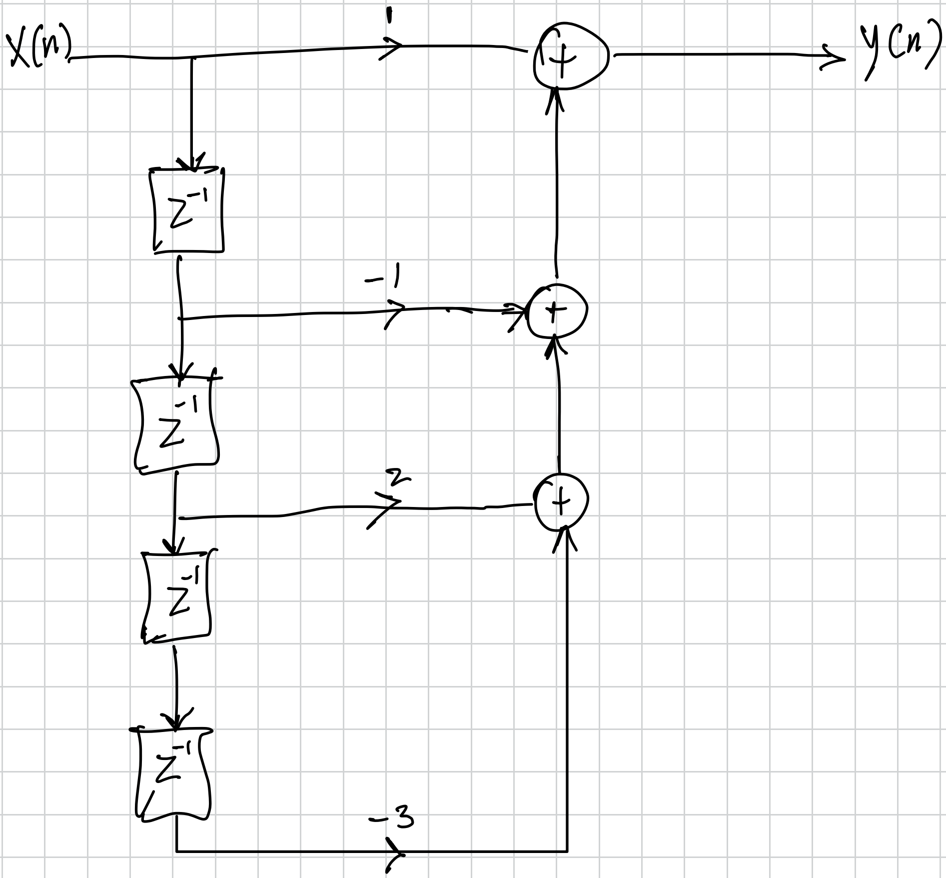
$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$$

$b_0 = 1$

$b_1 = -1$

$b_2 = 2$

$b_4 = -3$



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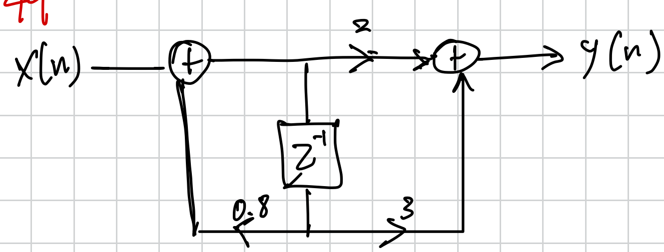

(a)

$$y(n) = 2xn + 3x(n-1) + 0.8y(n-1)$$

$$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

system relax: $n=0$

$$y(0) - 0.8y(-1) = 2\delta(0) + 3\delta(-1)$$

$$y(0) = 2$$

$$y(n) - 0.8y(n-1) = 0$$

$$\lambda^{n} - 0.8\lambda^{n-1} = 0$$

$$\lambda^{n-1}(\lambda - 0.8 = 0 \Rightarrow \lambda = 0.8$$

$$y_h(n) = C\lambda^{n}$$

$$y_h(n) = C\,0.8^{n}$$

$n=0$

$$h(0) = C(0.8)^{0}$$

$$\boxed{2 = C}$$

$$y_h(n) = 2(0.8)^{n}$$

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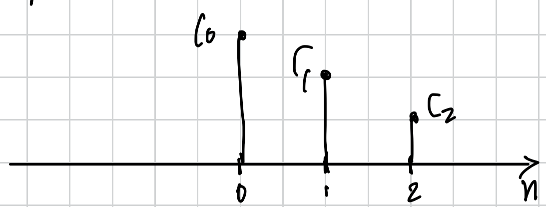
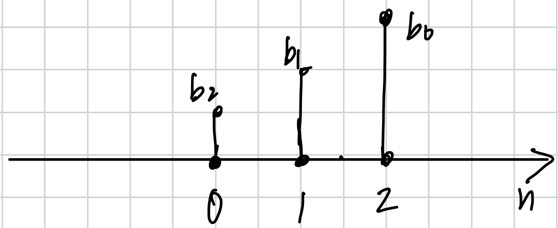
$h_1(n) = c_0\,\delta(n) + c_1\,\delta(n-1) + c_2\,\delta(n-2)$ $\qquad c_0 > c_1 > c_2$



$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2)$ $\qquad b_2 < b_1 < b_0$



$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2)$

$a_0 = 1,\ a_1 = 2,\ a_2 = 3$