

# Assignment 2 ECE 5530

Name: SKYLAR STOCKHAM  
 KYLE GAYLYEV  
 EDDIE FRANCO

UID: U1379136

## • Problem 2.7

- 1) Static or dynamic: if the present output depends on present input the system is static or could be memoryless so it is dynamic.
- 2) Linear or Non-Linear: if the system satisfies the principles of superposition & scaling. otherwise the system is non-linear.
- 3) Time variant or time varying: a time invariant if its behavior & characteristics do not change over time so, a time shift in the input signal results an identical time shift in the output, otherwise is called time-variant
- 4) Causal or non-causal: if present output depends on past outputs, past inputs, and present inputs is causal.  
 if present output depends on future inputs & outputs is non-causal
- 5) Stable or unstable: if every bounded input produces a bounded output (BIBO)

## Examine the following systems

a)  $y(n) = \cos[x(n)]$ : as operation is performed on  $x(n)$  it is non linear. as output depends only in present input it is static. behavior doesn't change over time is time invariant.

b)  $y(n) = \sum_{k=-\infty}^{n+1} x(k)$ : the system is dynamic because the present output depends on past & future values of input (ex.)  $y(4) = \sum_{k=-\infty}^5 x(k)$

• for input  $x_1(n)$

$$y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$$

• for  $x_2(n)$

$$y_2(n) = \sum_{k=-\infty}^{n+1} x_2(k)$$

$$x_3(n) = a x_1(n) + b x_2(n)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} x_3(k)$$

$$y_3(n) = \sum_{k=-\infty}^{n+1} a x_1(k) + b x_2(k)$$

$$y_3(n) = a y_1(n) + b y_2(n)$$

So, it is linear

$$\bullet \quad y(n) = \sum_{k=-\infty}^{n+1} x(k)$$

for  $x_1(n) \Rightarrow y_1(n) = \sum_{k=-\infty}^{n+1} x_1(k)$

for  $x_2(n) = x_1(n-n_0)$  for  $x_1(n-n_0)$

for  $y_1(n)$

$$y_1(n-n_0) = \sum_{k=-\infty}^{n+1-n_0} x_1(k)$$

$$y_2(n) = y_1(n-n_0)$$

System is time-invariant

$$\begin{aligned} y_2(n) &= \sum_{k=-\infty}^{n+1} x_2(k) \\ &= \sum_{k=-\infty}^{n-n_0+1} x_1(k-n_0) \\ &= \sum_{k=-\infty}^{n+1-n_0} x_1(k-n_0) \end{aligned}$$

$$\bullet \quad y(n) = \sum_{k=-\infty}^{n+1} x(k) \quad \text{let } n=0 \Leftrightarrow y(0) = \dots x(-\infty) + \dots x(0) + x_1$$

System is non-causal!

- let  $x(n)$  is bounded then the output  $y(n)$  is also bounded  
system is stable.

(c)  $y(n) = x(n) \cos(\omega_0 n)$  :

- $y(n)$  depends only on the present input values

So system is static.

- let  $x_1(n) \Rightarrow y_1(n) = x_1(n) \cos(\omega_0 n)$

let  $x_2(n) \Rightarrow y_2(n) = x_2(n) \cos(\omega_0 n)$

let  $x_3(n) = ax_1(n) + bx_2(n)$

$$y_3(n) = x_3(n) \cos(\omega_0 n)$$

$$= [ax_1(n) + bx_2(n)] \cos(\omega_0 n)$$

System is linear

- $y(n)$  depends on the present input values so, it is causal.

- let  $x(n)$  is bounded,  $y(n)$  will be bounded so, it is stable

(a)  $y(n) = x(-n+2)$

- present output depends on past & future values = dynamic

- superposition & scaling principle are met = linear

- let  $x_2(n) = x_1(n-n_0) \Rightarrow y_2(n) = x_2(-n+2)$

$$y_1(n-n_0) = x_1(-(n-n_0)+2) \quad y_2(n) = x_1(-n-n_0+2)$$

$$= x_1(-n+n_0+2) \neq$$

system is time-varying

- present output depends on future inputs & outputs = non causal
- if  $x(n)$  is bounded, then  $x(-n+2)$  also bounded = stable

(b)  $y(n) = \text{Trunc}[x(n)]$

- it doesn't depend on the time index  $n$  = static

- Non linear due to truncation

- time-invariant

- It's based on the present of the input sequence = Causal

- Truncation doesn't lead to unbounded values = Stable

### Problem 2.11

$$x_1(n) = \left\{ -1, \frac{2}{r}, 1 \right\}$$

$$x_2(n) = \left\{ 1, -\frac{1}{r}, -1 \right\} \Rightarrow x_1(n) + x_2(n) = \left\{ 0, 1, 0 \right\} = \delta(n)$$

$$x_3(n) = \left\{ 0, \frac{1}{r}, 1 \right\}$$

$$= \delta(n) + \delta(n-1)$$

$$x_3(n) = x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1)$$

$$y_3(n) = \begin{cases} 1, 2, 1 \\ \uparrow \end{cases}$$

$$= \delta(n) + 2\delta(n-1) + \delta(n-2)$$

So,

$$y_4(n) = y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$$

$$= \begin{cases} 1, 2, -1, 0, 1 \\ \uparrow \end{cases} + \begin{cases} -1, 1, 0, 2 \\ \uparrow \end{cases} + \begin{cases} 1, 2, -1, 0, 1 \\ \uparrow \end{cases} + \begin{cases} -1, 1, 0, 2 \\ \uparrow \end{cases}$$

$$= \begin{cases} 3, 2, 1, 3, 1 \\ \uparrow \end{cases}$$

So

$$y_4(n) \neq x_3(n)$$

then

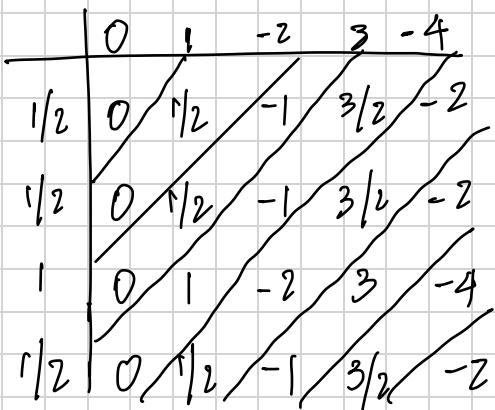
$$x_1(n) + x_2(n) + x_1(n-1) + x_2(n-1) \neq y_1(n) + y_2(n) + y_1(n-1) + y_2(n-1)$$

So, the system is invariant

Problem 2.1b      b3 & b11

Compute convolution  $y(n) = x(n) * h(n)$  of the following signals & check the correctness of the results by using the test in (a)

③  $x(n) = \{0, 1, -2, 3, -4\}$      $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$



$$y(n) = \left\{ 0, \frac{1}{2}, \frac{1}{2}-1, 1-\frac{1}{2}+\frac{3}{2}, \frac{1}{2}-2+\frac{3}{2}-2, -1+3-2, \right.$$

$$\left. \frac{3}{2}-4, -2 \right\}$$

$$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$$

$$\sum x(n) = -2$$

$$\sum y(n) = -5$$

$$\sum h(k) = 5/2$$

- $\sum y = \sum x \sum n$

$$-5 = -2 \times (5/2)$$

$$-5 = -5$$

$$11) x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$h(n) = \left(\frac{1}{4}\right)^n u(n)$$

$$y(n) = \sum_{k=0}^n h(k) x(n-k)$$

$$= \sum_{k=0}^n \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^n \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^n \left(\frac{1}{2}\right)^k$$

$$= 0.5^n \left[ 1 + 0.5 + 0.5^2 + \dots + (n+1) \right] u(n)$$

$$= 0.5^n \left[ \frac{1 - 0.5^{n+1}}{1 - 0.5} \right] u(n)$$

$$= 0.5^n \left[ \frac{1 - 0.5^{n+1}}{0.5} \right] u(n)$$

$$y(n) = 0.5^n (2 - 0.5^n) u(n)$$

$$\cdot \sum x(n) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{1 - \frac{1}{2}} = 2$$

$$\cdot \sum y(n) = \sum_{n=0}^{\infty} (0.5)^n (2 - 0.5^n)$$

$$= \sum_{n=0}^{\infty} 2(0.5)^n - 0.25^n$$

$$= 2 \left(\frac{1}{1-0.5}\right) - \left(\frac{1}{1-0.25}\right)$$

$$= 2(2) - 4/3 = 8/3$$

$$\cdot \sum h(n) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \Rightarrow \frac{1}{1 - \frac{1}{4}} = 4/3$$

$$\text{So, } \sum_y = \sum_x \sum_h$$

$$\frac{8}{3} = 2 \left(\frac{4}{3}\right)$$

$$\frac{8}{3} = \frac{8}{3}$$

## Problem 2.24

$$y(n) = ny(n-1) + x(n) \quad n \geq 0$$

(a)  $x_1(n) \rightarrow y_1(n) = ny(n-1) + x_1(n)$

$$x_2(n) \rightarrow y_2(n) = ny(n-1) + x_2(n)$$

$$x_3(n) = x_1(n) + x_2(n) \rightarrow y_3(n) = y_1(n) + y_2(n) \Rightarrow ny_3(n-1) + x_3(n)$$

$$y_3(n) = ny_3(n-1) + x_3(n)$$

$$= n[y_1(n-1) + y_2(n-1)] + [x_1(n) + x_2(n)]$$

$$= ny_1(n-1) + ny_2(n-1) + x_1(n) + x_2(n)$$

$$= y_1(n) + y_2(n)$$

$$y_3(n) = y_1(n) + y_2(n) \quad \checkmark$$

(b)  $x_1(n) \rightarrow y_1(n) = ny_1(n-1) + x_1(n)$

$$x_2(n) = kx_1(n) \rightarrow y_2(n) = ky_1(n)$$

$$y_2(n) = ny_2(n-1) + x_2(n)$$

$$= n[ky_1(n-1)] + [kx_1(n)]$$

$$= k(ny_1(n-1) + x_1(n))$$

$$y_2(n) = k y_1(n) = \text{homogeneous} = \text{linear}$$

(c)  $x(n) \rightarrow y(n)$

$$x(n-n_0) \rightarrow y(n-n_0) = ny(n-n_0-1) + x(n-n_0)$$

shifted:

$$y(n-n_0) = (n-n_0)y_2(n-n_0-1) + x_2(n-n_0)$$

not the same = time-varying

$$\textcircled{d} \quad y(n) = ny(n-1) + x(n) \quad n \geq 0$$

$$x(n) = \mu(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is bounded

$n=0$	$n=2$	
$y(0) = 1$	$y(2) = 5$	$y(n) = \{1, 2, 5, 16, \dots\}$
$n=1$	$n=3$	not bounded
$y(1) = 2$	$y(3) = 6$	

The system is BIBO.

• 2.27

$$y(n) = \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n) \quad x(n) = 2^n \mu(n)$$

$$y_p(n) = k 2^n \mu(n)$$

$$k 2^n \mu(n) = \frac{1}{6}k 2^{n-1} \mu(n-1) - \frac{1}{6}k 2^{n-2} \mu(n-2) + 2^n \mu(n)$$

$$n=2$$

$$k 2^2 = \frac{5}{6}k 2^1 - \frac{1}{6}k 2^0 + 4$$

$$k = \frac{8}{5}$$

$$y_p(n) = \frac{8}{5} 2^n \mu(n)$$

Homogeneous sol.

$$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \quad \lambda = \frac{1}{2}, \frac{1}{3}$$

$$y_h(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \left(\frac{1}{3}\right)^n$$

$$y_n(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \left(\frac{1}{3}\right)^n + \left(\frac{8}{5}\right) 2^n \mu(n)$$

$$y(0) = C_1 + C_2 + 8/5$$

$$y(-1) = y(-2) = 0$$

$$y(1) = 1/2 C_1 + 1/3 C_2 + 16/5$$

$$y(0) = 5/6 y(-1) - 1/6 y(-2) + 1$$

$$y(0) = 1$$

$$y(1) = 5/6 y(0) - 1/6 y(-1) + 2$$

$$y(1) = \frac{17}{6}$$

$$C_1 + C_2 + 8/5 = 1$$

$$C_1 = 1$$

$$1/2 C_1 + 1/3 C_2 + 16/5 = \frac{17}{6} \quad C_2 = 2/5$$

So,

$$y(n) = -\left(\frac{1}{2}\right)^n \mu(n) + \frac{2}{5} \left(\frac{1}{3}\right)^n \mu(n) + \left(\frac{8}{5}\right) 2^n \mu(n)$$

2-31

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

for impulse

So,

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$y_p(n) = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0 \Rightarrow \lambda(\lambda-4) + 1(\lambda-4) = 0$$

$$\Rightarrow (\lambda+1)(\lambda-4) = 0 \quad \lambda = 4, -1$$

$$y_n(n) = C_1 4^n + C_2 (-1)^n$$

for  $n=0$

$$y(0) - 3y(-1) - 4y(-2) = x(0) + 2x(-1)$$

$$y(0) - 0 - 0 = 1 + 0$$

$$y(0) = 1$$

for  $n=1$

$$y(1) - 3y(0) - 4y(-1) = x(1) + 2(0)$$

$$y(1) - 3y(0) = 2$$

$$y(1) = 5$$

So

$$C_1 + C_2 = 1$$

$$C_1 = 6/5$$

$$4C_1 - C_2 = 6$$

$$C_2 = -1/5$$

$$h(n) = \left[ \left( \frac{6}{5} \right) 4^n - \left( \frac{1}{5} \right) (-1)^n \right] \mu(n)$$

$$x(0) = 1$$

$$y(-1) = y(-2) = 0$$

2.46

$$\begin{aligned}
 \textcircled{a} \quad & 2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5) / 2 \\
 & y(n) + 0.5y(n-1) - 2y(n-3) = 0.5x(n) + 1.5x(n-5) \\
 & y(n) = -0.5y(n-1) + 2y(n-3) + 0.5x(n) + 1.5x(n-5)
 \end{aligned}$$

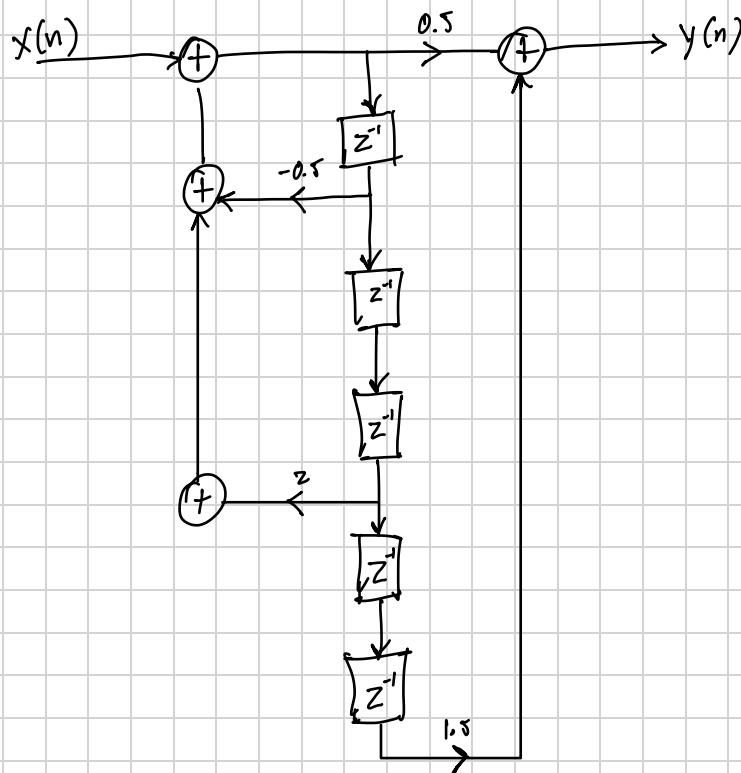
$$y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$$

$$a_1 = 0.5$$

$$a_3 = -2$$

$$b_0 = 0.5$$

$$b_5 = 1.5$$



$$\textcircled{b} \quad y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$$

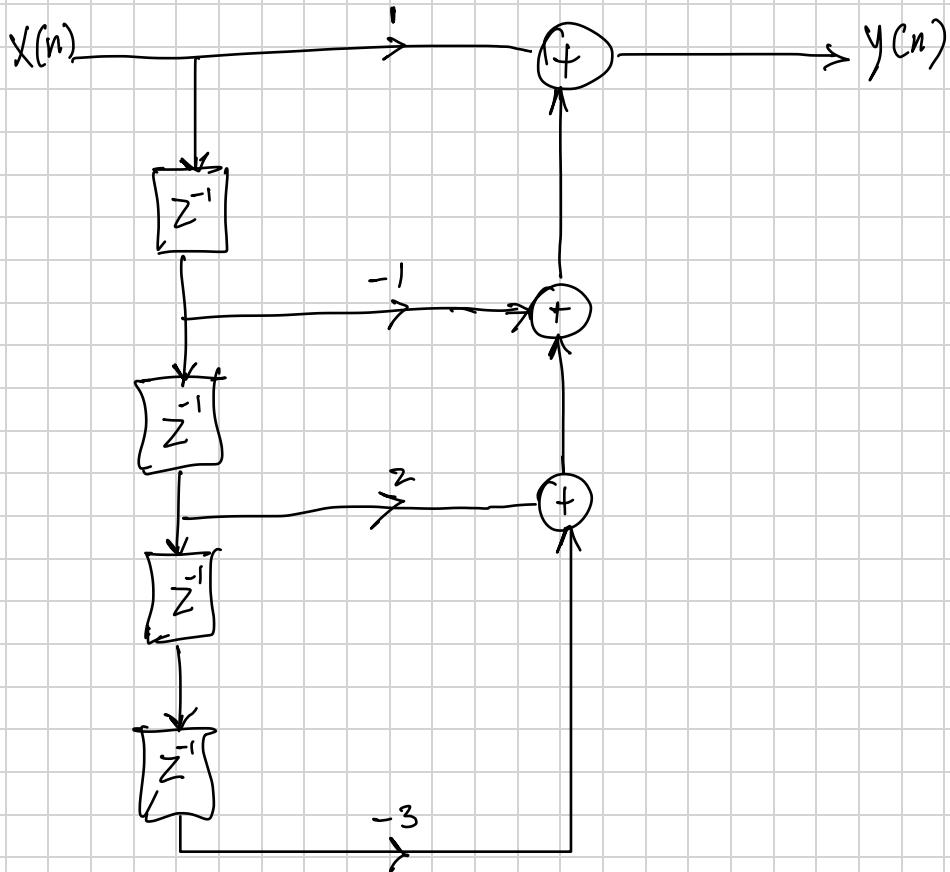
$$y(n) = \sum_{k=0}^M b_k x(n-k)$$

$$b_0 = 1$$

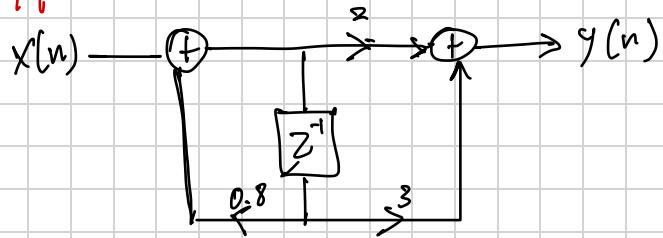
$$b_1 = -1$$

$$b_2 = 2$$

$$b_4 = -3$$



2.49



a)

$$y(n) = 2x(n) + 3x(n-1) + 0.8y(n-1)$$

$$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

System relax:  $n=0$

$$y(0) - 0.8y(-1) = 2\delta(0) + 3\delta(-1)$$

$$y(0) = 2$$

$$y(n) - 0.8y(n-1) = 0$$

$$\lambda^n - 0.8\lambda^{n-1} = 0$$

$$\lambda^{n-1}(\lambda - 0.8) = 0 \Rightarrow \lambda = 0.8$$

$$y_h(n) = C\lambda^n$$

$$y_h(n) = C 0.8^n$$

$$n=0$$

$$y_h(0) = C(0.8)^0$$

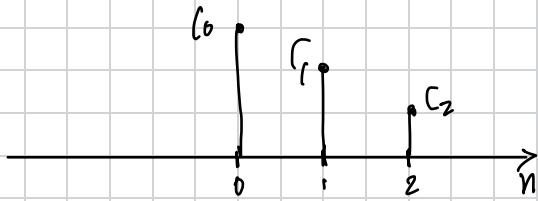
$$\boxed{2 = C}$$

$$y_h(n) = 2(0.8)^n$$

2.52

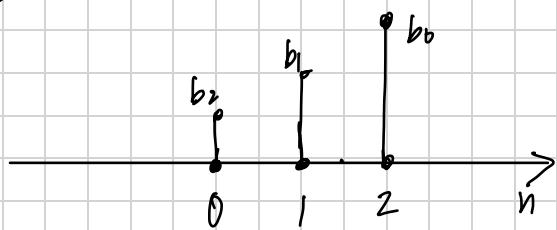
$$h_1(n) = c_0 \delta(n) + c_1 \delta(n-1) + c_2 \delta(n-2)$$

$$c_0 > c_1 > c_2$$



$$h_2(n) = b_2 \delta(n) + b_1 \delta(n-1) + b_0 \delta(n-2)$$

$$b_2 < b_1 < b_0$$



$$h_3(n) = a_0 \delta(n) + (a_1 + a_0 a_2) \delta(n-1) + a_1 a_2 \delta(n-2)$$

$$a_0 = 1, a_1 = 2, a_2 = 3$$

