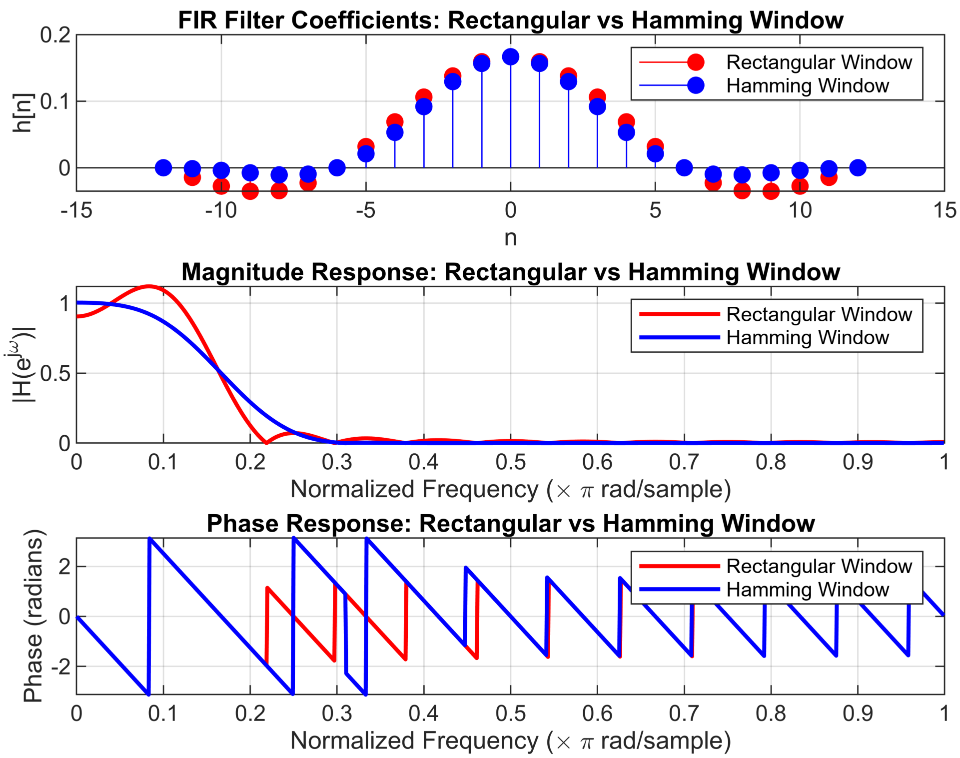
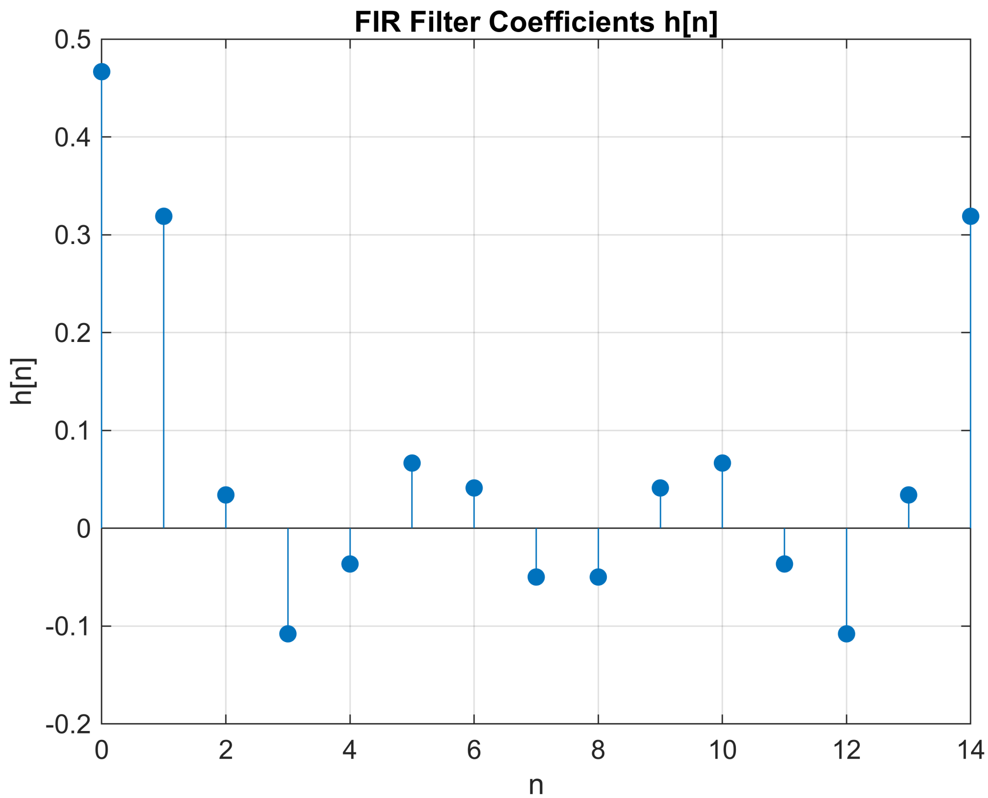
```matlab
% Assignment 6
% Problem 10.1 a to c. Use MATLAB (30 points).
M = 25;  % Number of taps
wc = pi / 6;  % Cut-off frequency

% filter coefficients using the window method
n = -(M-1)/2 : (M-1)/2;  % symmetric around zero

% Ideal impulse response of a lowpass filter with cutoff wc
hd = (wc/pi) * sinc((wc/pi) * n);

% 1. Rectangular Window Method
w_rect = ones(1, M);  % Rectangular window (all ones)
h_rect = hd .* w_rect;  % FIR filter coefficients using rectangular window

% 2. Hamming Window Method
w_hamming = hamming_windown(M);  % Call the custom Hamming function
h_hamming = hd .* w_hamming;  % FIR filter coefficients using Hamming window

% Plot the filter coefficients
figure;
subplot(3, 1, 1);
stem(n, h_rect, 'filled', 'r');
hold on;
stem(n, h_hamming, 'filled', 'b');
title('FIR Filter Coefficients: Rectangular vs Hamming Window');
xlabel('n');
ylabel('h[n]');
legend('Rectangular Window', 'Hamming Window');
grid on;

% Plot the magnitude and phase response for both filters
[H_rect, omega] = freqz(h_rect, 1, 1024);  % frequency response for rectangular
window
[H_hamming, ~] = freqz(h_hamming, 1, 1024);  % frequency response for Hamming window

% Plot the magnitude response
subplot(3, 1, 2);
plot(omega/pi, abs(H_rect), 'r', 'LineWidth', 1.5);
hold on;
plot(omega/pi, abs(H_hamming), 'b', 'LineWidth', 1.5);
title('Magnitude Response: Rectangular vs Hamming Window');
xlabel('Normalized Frequency (\times \pi rad/sample)');
ylabel('|H(e^{j\omega})|');
legend('Rectangular Window', 'Hamming Window');
grid on;

% Plot the phase response
subplot(3, 1, 3);
plot(omega/pi, angle(H_rect), 'r', 'LineWidth', 1.5);
```

```
hold on;
plot(omega/pi, angle(H_hamming), 'b', 'LineWidth', 1.5);
title('Phase Response: Rectangular vs Hamming Window');
xlabel('Normalized Frequency (\times \pi rad/sample)');
ylabel('Phase (radians)');
legend('Rectangular Window', 'Hamming Window');
grid on;
```




```
% Hamming window function
function w = hamming_window(M)
    % Hamming window of length M
    n = 0:M-1;  % Sample indices
    w = 0.54 - 0.46 * cos(2 * pi * n / (M - 1));
end

% Problem 10.6. Use MATLAB (30 points).
% parameters
L = 15;  % Length


%  H[k] = 1 for k = 0, 1, 2, 3 and H[k] = 0 for k = 4, 5, 6, 7
H = [1 1 1 1 0 0 0 0 0 0 0 1 1 1 1]; % Length is 15

% IDFT to get h[n]
h = ifft(H, 'symmetric');  % Using 'symmetric' to ensure real-valued output
```

```
% Plot the filter coefficients
t = 0:L-1;  % Time indices
figure;
stem(t, h, 'filled');
title('FIR Filter Coefficients h[n]');
xlabel('n');
ylabel('h[n]');
grid on;
```




```
% Display the filter coefficients in the command window
disp('Filter Coefficients h[n]:');
```

```
Filter Coefficients h[n]:
```

```
disp(h');
```

```
    0.4667
    0.3189
    0.0341
   -0.1079
   -0.0365
    0.0667
    0.0412
   -0.0498
   -0.0498
    0.0412
    0.0667
   -0.0365
   -0.1079
```

```
0.0341
0.3189
```

```

```

Assignment 6

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3. Problem 9.6

Determine a_1, a_2 & c_1 & c_0 in terms of b_1 & b_2 so that the two systems are equivalent

from system 1: the structure is in parallel

So $H(z) = H_1(z) + H_2(z)$

$$H_1(z) = \frac{1}{1-b_1 z^{-1}} \qquad H_2(z) = \frac{1}{1-b_2 z^{-1}}$$

$$H(z) = \frac{1}{1-b_1 z^{-1}} + \frac{1}{1-b_2 z^{-1}} = \frac{2 - b_1 z^{-1} - b_2 z^{-1}}{(1-b_1 z^{-1})(1-b_2 z^{-1})}$$

$$H(z) = \frac{2 + (-(b_1+b_2) z^{-1})}{(1-b_1 z^{-1})(1-b_2 z^{-1})}$$

from system 2: it look like is a cascade structure

$H'(z) = H_1'(z) \cdot H_2'(z)$

$$H_1'(z) = \frac{1}{1-a_1 z^{-1}} \qquad H_2'(z) = \frac{c_0 + c_1 z^{-1}}{1-a_2 z^{-1}}$$

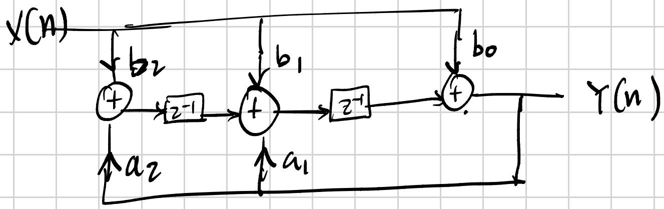
$$H'(z) = \left(\frac{1}{1-a_1 z^{-1}}\right)\left(\frac{c_0 + c_1 z^{-1}}{1-a_2 z^{-1}}\right)$$

2nd System   1st System

$$H'(z) = \frac{c_0 + c_1 z^{-1}}{(1-a_1 z^{-1})(1-a_2 z^{-1})} \qquad H(z) = \frac{2 + (-(b_1+b_2) z^{-1})}{(1-b_1 z^{-1})(1-b_2 z^{-1})}$$

$c_0 = 2$; $c_1 = -(b_1+b_2)$; $a_1 = b_1$; $a_2 = b_2$

4. Problem 9.7



(a) Determine system function

So, diff. equation for the filter

$$y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

$$Y(z) = a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$

$$Y(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$

$$Y(z)\left(1 - a_1 z^{-1} - a_2 z^{-2}\right) = X(z)\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = H(z)$$

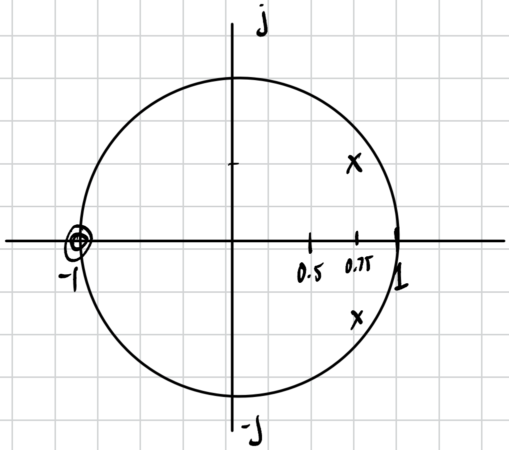
(b) 1.

$b_0 = b_2 = 1$, $b_1 = 2$, $a_1 = 1.5$, $a_2 = -0.9$

$$H(z) = \frac{1 + 2z^{-1} + 1z^{-2}}{1 - 1.5z^{-1} + 0.9z^{-2}} = \frac{z^2}{z^2} \Rightarrow \frac{z^2 + 2z + 1}{z^2 - 1.5z + 0.9}$$

$$H(z) = \frac{(z+1)^2}{z - (0.75 \pm j0.581)}$$

zeros $(z) = -1$
poles $(z) = 0.75 \pm j0.581$



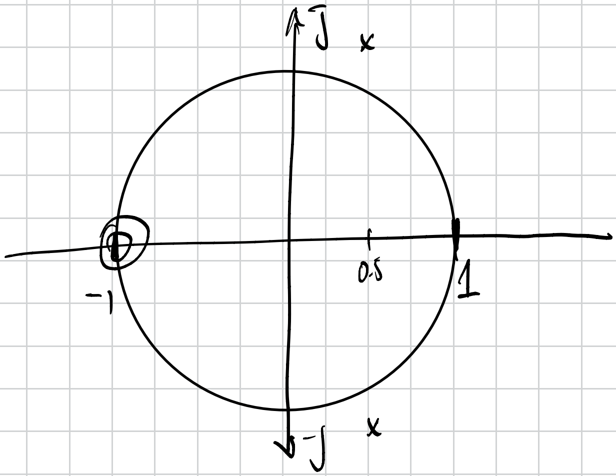
poles are inside of unit circle = stable

② $H(z) = \frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+2z^{-2}} = \frac{z^2}{z^2} \Rightarrow \frac{z^2+2z+1}{z^2-z+2}$

$H(z) = \frac{(z+1)^2}{z-(0.5 \pm j1.322)}$

zeros $(z) = -1$

poles $(z) = 0.5 \pm j1.322$



poles are out of unit circle = Unstable.

⑤ Problem 9.9 part a

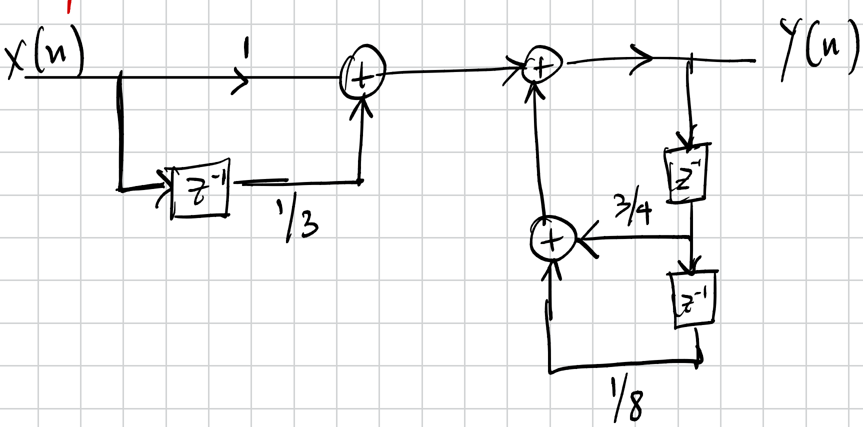
$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

$Y(z) = \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) + X(z) + \frac{1}{3}z^{-1}X(z)$

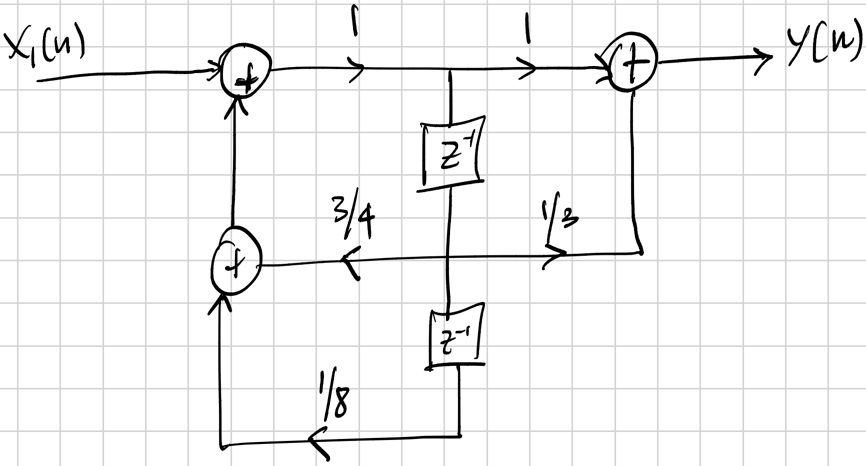
$Y(z)\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{2}Y(z) = X(z)\left(1+\frac{1}{3}z^{-1}\right)\right.$

$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{2}}$

Direct form

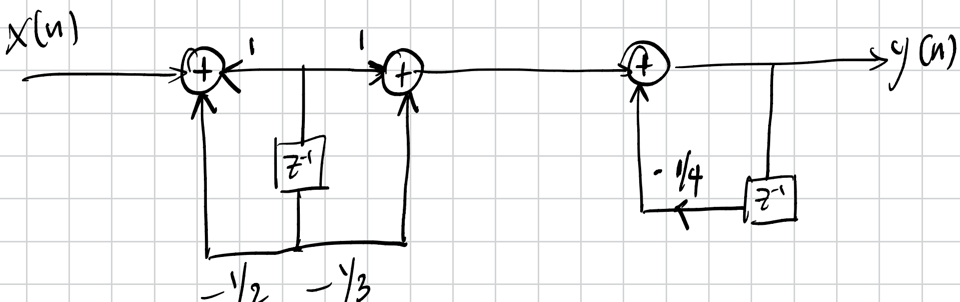


Direct form II



Cascade

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \Rightarrow \left(\frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}\right)\left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right)$$



Parallel

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1 + \frac{1}{3}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} = \frac{10/3}{1 - \frac{1}{2}z^{-1}} + \frac{-7/3}{1 - \frac{1}{4}z^{-1}}$$