Expectation theory assumes longer-term assets are perfect substitutes for a sequence of short-term assets.

Thus long-term interest rates should reflect average of 1-year rates?

Why?

Consider two 2-year holding strategies on an investment of $100

Strategy 1: Buy a single 2-year bond & hold to maturity

Strategy 2: Buy a 1-year bond this year, hold to maturity, then buy another 1-year bond at next year's rate.

Under Expectations Theory, these should have equal return.

$$\Rightarrow \underbrace{(1+i_{t2})^2}_{\text{Today's 2-year rate}} 1\cancel{00} = \underbrace{(1+i_{t1})}_{\text{Today's 1-year rate}}\underbrace{(1+i_{(t+1)1})}_{\text{Next year's 1-year rate}}1\cancel{00}$$

$$\Rightarrow 1+i_{t2} = \underbrace{\sqrt{(1+i_{t1})(1+i_{(t+1)1})}}$$

(ASIDE: This is called the geometric average)

Equivalently, if we wanted a formula for next-year's expected 1-year rate given today's 1-year & 2-year rate we can rearrange to get

$$\boxed{i_{(t+1)1} = \frac{(1+i_{t2})^2}{1+i_{t1}} - 1}$$

(FORMULA FROM LECTURE)

How do we reconcile this with the long-term interest rates being the average of short-term rates like as stated in section & in the textbook?

Expand the polynomials: $(1+i_{t2})^2 = (1+i_{t1})(1+i_{(t+1)1})$

$$\Rightarrow \cancel{1} + 2i_{t2} + i_{t2}^2 = \cancel{1} + i_{t1} + i_{t2} + i_{t1} i_{(t+1)1}$$

$$\Rightarrow 2i_{t2} + i_{t2}^2 = i_{t1} + i_{t2} + i_{t1} i_{(t+1)1}$$

Now, remember that interest rates are typically close to zero. E.g. 5% interest $\Leftrightarrow$ $i = 0.05$

So i^2 is typically <u>very</u> small

$$i = 0.05 \Rightarrow i^2 = 0.0025$$

So we can neglect the i_{t2}^2 & $i_{t1} i_{(t+1)1}$ terms above.

$$\Rightarrow 2i_{t2} \simeq i_{t1} + i_{t2}$$

approximately equals

$$\Rightarrow \boxed{i_{t2} \approx \frac{i_{t1} + i_{t2}}{2}}$$

Which is the formula from the textbook & the one I covered in section.