Thus far, we've mostly worked with assets that pay annually.

What if it pays more frequently?

Take a simple case: your savings account

Suppose your savings account has an annual interest rate of 2% & you have \$1,000 in your account.

If it compounds annually (pays interest once a year) You will have

$$\underbrace{\$1000}_{\text{principal}} + \underbrace{\$1000 \times 0.02}_{\text{interest payment}}$$

$$= \$1020$$

After 1 year

But suppose your bank makes payments more frequently.

Suppose they pay every 6 months

After 6 months, they pay half of the annual interest:

After 6 months you have

$$\$1000 + \$1000 \times \frac{0.02}{2}$$

$$= \$1010$$

After another 6 months, they make another interest payment, but now the principal is higher (\$1010) because they already made 1 payment

$$\$1010 + \$1010 \times \frac{0.02}{2} = \boxed{\$1020.10}$$

So with twice-a-year compounding you finish the year with <u>more</u> than you would with annual compounding.

<u>In general</u> If a loan compounds n-times per year, the total payment will be

$$\left(1+\frac{i}{n}\right)^{N\times n} \times \text{Principal}$$

$N \times n$ = total number of compoundings after N-years

<u>NOTE</u>: This is <u>not</u> a fixed-payment loan. Each payment is larger than the last.

How does this this work for a <u>fixed-payment</u> loan?

For annual payments, we have our usual formula:

$$P = \frac{F_y}{1+i} + \frac{F_y}{(1+i)^2} + \ldots + \frac{F_y}{(1+i)^N}$$

Which we can simplify as

$$P = \frac{F_y}{i}\left[1 - \frac{1}{(1+i)^N}\right]$$

F_y = yearly payment

N = # of years

or equivalently,

$$F_y = \frac{Pi}{1-\left(\frac{1}{1+i}\right)^N}$$

But suppose we had a <u>monthly</u> payment then we have

F_M = monthly payment

$$P = \frac{F_M}{1+\frac{i}{12}} + \frac{F_M}{(1+i/12)^2} + \dots + \frac{F_M}{(1+\frac{i}{12})^{12\times N}}$$

or equivalently

$$P = \frac{F_M}{i/12}\left[1 - \frac{1}{\left[1+\left(\frac{i}{12}\right)\right]^{12\times N}}\right]$$

$$F_M = \frac{P\left(\frac{i}{12}\right)}{1-\left(\frac{1}{1+\frac{i}{12}}\right)^{12\times N}}$$

<u>CHECK YOUR UNDERSTANDING:</u>

1) Consider 2 fixed payment loans with the same principal

   ① Has annual fixed payments

   ② Has twice-annual fixed payments of half of the payments for loan ①.

   Which has the higher effective interest rate?
   (Try it with P=1000, N=20 years
   annual payment = \$100
   twice-annual payment = \$50)

2) Consider 2 fixed payment loans with the same principal and interest rate, but one pays annually & the other twice annually. Which has higher <u>total</u> payments each year.

3) Write a formula for YTM for a perpetuity that costs P today and pays F every month.