Winding Paths

Enhancing Realism of Maintenance Optimization Under Time-Varying Costs

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Author:
Catalin Gheorghiu (SN: 504458)

Supervisor:
Prof. Dr. Ir. Rommert R. Dekker

Second Assessor:
Dr. Wilco van den Heuvel

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Erasmus University Rotterdam
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Abstract

This thesis' parent paper, Schouten et al. (2022), finds that accounting for time-varying downtime costs can lead to significant savings when planning maintenance for wind turbines. After replicating these results, I extend the core models in several directions to investigate whether the savings are stable in more complex, practically relevant scenarios. Using CPLEX to find optimal age-based maintenance schedules in Markov Decision Process models, I find that savings up to 6.1% can be achieved depending on the energy price and the available maintenance techniques, while neither limits on the time spent in a failure state nor constrained flows of maintenance opportunities have a negative effect on the achieved savings.

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1 Introduction

Between a global pandemic, proxy wars that threaten energy security and talk of policing bodily autonomy, Western democracies may find it enticing to block out the terrifying long-term repercussions of hastening climate change through human activity. Coal is making a resurgence on both sides of the Atlantic Ocean (International Energy Agency, 2021), emphasizing the importance of sustained efforts to promote and improve renewable energy infrastructure. While solar power is currently the fastest growing sector among renewables, wind power remains at the core of many countries' transition plans from emission-heavy energy thanks to its long-term efficiency, only a few percentage points in overall popularity behind the giant that is hydropower (Eurostat, 2022).

The long-term cannot be taken for granted, however, and good management of maintenance resources is a core pillar supporting it. In spite of their higher costs, offshore windmill farms are attractive over the onshore counterpart due to the synergy between high wind speeds at sea and the occupation of otherwise barren space. It follows naturally that efforts to make this technology more popular should be foremost aimed at cost reduction. The impact of reducing costs is both material and environmental, as the savings invite further investments into expanding the renewable energy sector, which in turn reduces the appeal of cheaper, emission-heavy fossil fuels.

This paper will add to the analysis of operational and maintenance costs of offshore wind turbines undertaken by its parent work, "Maintenance optimization for a single wind turbine component under time-varying costs" (Schouten et al., 2022). Building on the policy-generating models that allow for preventive and corrective maintenance to replace a single component with known (statistical) life-cycle function, I want to relax key assumptions on the availability of maintenance opportunities, their effectiveness, and the economic conditions in order to answer to central research question:

Are savings achieved by maintenance policies mindful of time-varying costs reduced when subjected to more realistic conditions?

On one hand, weaker assumptions may lead the previously identified cost-savings policies to become infeasible or no longer achieve cost reductions at all. On the other hand, as the parent paper itself proves, making the characteristics of the model more nuanced can lead to finding a better-tailored, more effective optimal solution. To better ground the complex research question into reality, the following set of sub-questions chunks the problem into corroborating pieces:

1. *Does replacing a constant energy price with historical data lead to higher cost savings?*

2. *How fast do PM executions and the total cost increase as less CM is allowed?*

3. *If imperfect maintenance is an option, will it be employed to achieve lower costs?*

4. *Does limited availability of maintenance operations significantly reduce savings?*

The social relevance of cost analysis in less abstract conditions stems from attractiveness. A more thorough look at how different factors change the performance of maintenance policies helps bridge the gap between what the theoretical realm can lead policymakers to believe and what real world impact they can expect to see. The more stable savings remain in the face of realistic conditions, the better one can budget for the investments and build confidence into the reliability and effectiveness of future investments in the expanding wind power sector, with a strong positive impact on both the climate and national energy security.

As the authors of the parent work point out, incorporating time-varying costs into maintenance optimization is scientifically relevant for maintenance literature as a whole by means of better aligning models with the real behaviour of the wind speed that drives gains when the turbine is operational and losses during downtime. The wide scope of sensitivity analysis I will perform creates a flexible framework to adapt the model formulations to different conditions of economic and logistic nature, such as price movements, partial repair opportunities and delays in maintenance and component availability. Future research can seek to refine stochastic elements and cost functions to better suit real-life behaviour, all towards the same goal of model fidelity that would allow exploring more complex - and also more effective - policies by means of computation power.

This work is structured slightly atypically in order to both present the replication process of select results from the parent paper and organically discuss the extensions. Section 2 contains a literature review of maintenance optimization papers that increasingly restricts the field of interest until the framework of Schouten et al. (2022) is reached. Section 3 lays down the foundation needed to implement the three models at the core of both this paper and the aforementioned original work. Section 4 deals with the entire replication process - data, methodology, results and discussion - while section 5 contains the complete analysis of each extension, treated in separate subsections. Section 6 brings the paper together with a holistic discussion of all results and their interplay whose goal is to bridge the gap between the economic and logistic nature of the thesis. Finally, section 7 contains concluding remarks, including limitations and suggestions for further research.

2 Literature Review

The maintenance of offshore wind turbines, by nature of its specificity, encompasses several layers of scientific research. Maintenance optimization can be most intuitively understood as the trunk from which the subproblems branch out; the field has seen vast amounts of research and has been reviewed numerous times, with new techniques commonly appended. As de Jonge and Scarf (2020) indicates, the field has been expanding increasingly fast over the past three decades, often building on previous works such as McCall (1965), Dekker (1996) and Wang (2002). Approaches such as the Bayesian probabilistic networks found in Apeland and Aven (2000) and Markov decision processes found in Ossai et al. (2016) have become classics in the literature thanks to versatility and ease of interpretation.

More recent papers, however, primarily use machine learning techniques such as the neural network employed by Tian et al. (2011), the supervised random forests of Patil et al. (2018) or unsupervised reinforcement learning as seen in Koprinkova-Hristova (2013). Independent of modelling strategy, the maintenance itself is either corrective (CM), which deals with unpredictable failures, or preventive (PM), which is performed while the system is still operational in order to postpone failure. Articles such as De Carlo and Arleo (2017) futher distinguish perfect repairs - often equivalent with replacing the component - from minimal and imperfect repairs; only the former is relevant for the analysis of Schouten et al. (2022).

Focusing on the maintenance of wind turbines specifically does not leave the research field any less crowded. Both reviewers and individual authors often state a very good reason for this: according to Shafiee and Sørensen (2019) for instance, operational and maintenance costs constitute a substantial part of the life-cycle total of wind turbines, accounting for up to 30% of the energy generation cost. The aforementioned literature review offers several criteria by which windmill maintenance models can be classified, including system configuration, planning horizon, optimality criterion and solution technique. As Schouten et al. (2022) concur, a striking imbalance exists between the large volume of research pertaining to onshore wind farms and the comparatively scarce foray into the challenges posed by offshore farms.

While the most obvious such challenge is the transport logistics of maintenance teams, which Besnard et al. (2012) addresses, Schouten et al. (2022) joins the ongoing efforts to incorporate more realistic conditions in the windmill operation models in order to facilitate the discovery of more efficient maintenance schedules that take advantage of previously ignored system characteristics. With the implementation and improvement of sensors to provide updated diagnostics of various windmill components, condition-based maintenance algorithms have been more popular than the classic time-based counterpart in recent history; Ciang et al. (2008) provides a valuable introduction to this strand of literature.

Nevertheless, time-based maintenance using only some distribution to model the expected lifetime of a component stays relevant primarily through versatility. This is reflected by the large spectrum of policies developed in this setting, beginning with the baseline age-replacement and block-replacement policies (ARP and BRP, respectively) introduced by Proschan and Barlow (1967) and their later adaptations, most notably the modified block replacement policy (MBRP) of Berg and Epstein (1976). Later additions include the failure limit policy (Bergman, 1978), sequential PM (Nakagawa, 1988) and reference time policy (Muth, 1977). All of the works cited previously join the mainstream literature strand that only considers constant maintenance costs, and while works such as Truong Ba et al. (2018) employ time-varying costs in an opportunistic maintenance setting, this paper follows up on Schouten et al. (2022) in the unique investigation of adapting the policies for time-varying cost due to fluctuating wind speeds between seasons.

3 Theoretical Framework

In order to quantify the impact of seasonally dependent costs, the maintenance problem's underlying system must keep track of both the component's age and the passage of time itself. Electing to discretize time for the sake of more planning practicality, a Markov decision process (MDP) becomes highly suitable thanks to its ability to organise the different states by period and age simultaneously. Furthermore, as the age of the component is essentially treated as a random variable following a known statistical distribution, the MDP can incorporate the varying failure probability at different ages into the transition probabilities between states. A final, distinctly valuable advantage of the MDP is the possibility to incorporate the maintenance team's actions as decisive elements of the transition between states, thus modelling the effect of maintenance on the age (and consequently failure probability) of the component.

A crucial element of the MDP that necessitates further elaboration is the state transitioning mechanism: the previously mentioned random variable describing the component's lifetime. Schouten et al. (2022) prove the necessity of an assumption made by most papers in the field: the component's failure rate increases with age. In order to model all-positive ages beginning at 0 in addition to that, the Weibull distribution is the parent paper's choice and a popular choice in the field overall, as showcased in Tian et al. (2011) and Love and Guo (1996). Let X then denote the windmill gearbox's lifetime with $X \sim Weibull(\alpha, \beta)$ where $\beta = 2$ is henceforth assumed for a more simple, linearly-increasing failure rate, and the shape parameter α is closely related to the average lifetime.

While the Weibull distribution is continuous, I follow the parent paper's technique for discretizing it, hence $P(X = x) = F(x) - F(x-1)$ for time period $x \in \mathbb{N}$, where the Weibull cumulative density function is given by $F(x) = P(X \leq x) = 1 - exp[-(\frac{x}{\alpha})^2]$. The lifetime probability distribution serves to guide the transition between states of the system. To define these states, let $\mathcal{I}_1 = \{1, 2, ..., N\}$ describe the discretized N periods of a year, where for instance $N = 52$ if partitioning is done by week and $N = 12$ if it is done by month. Assuming there is an age $M \in \mathbb{N}$ beyond which it is deemed unsafe for the component to continue functioning and replacement is thus performed, let $\mathcal{I}_2 = \{0, 1, ..., M\}$ describe the possible ages of the component (in periods); a newly-installed component is assumed to have age 0.

For the baseline models, both PM and CM are assumed to be perfect repairs - equivalent to a component replacement - with no limit on how often maintenance can be performed, therefore let the state-progressing action $a \in \{0, 1\}$ denote doing nothing and repairing the gearbox, respectively. Note that under limitless maintenance possibility, once a failure occurs it is never profitable to delay repairs and thus CM is immediately performed. The state-dependent action space can now be defined as per equation 1.

$$\mathcal{A}(i_1, i_2) = \begin{cases} \{1\}, & \text{if } i_2 \in \{0, M\} \\ \{0, 1\}, & \text{if } i_2 \notin \{0, M\} \end{cases} \tag{1}$$

Using the previously defined random lifetime X of the gearbox, let $p_{i_2} = P(X = i_2 | X \geq i_2)$ indicate the failure probability at age i_2, regardless of period. It is important to use the conditional probability as the probability of a gearbox working for i_2 periods when it is brand new is markedly different from when the gearbox has already successfully functioned for some time. When the component operates normally, both the period and age simultaneously advance by 1. Should a failure occur, age is set to 0 while the period is incremented normally, using the modulo operator to ensure the transition happens correctly across different years. Equation set 2 describes the transition probabilities formally.

$$\pi_{(i_1,i_2)(j_1,j_2)}(0) = \begin{cases} 1 - p_{i_2} & \text{for } j_1 = (i_1+1) \bmod N,\ j_2 = i_2 + 1,\ i_2 \notin \{0, M\} \\ p_{i_2} & \text{for } j_1 = (i_1+1) \bmod N,\ j_2 = 0,\ i_2 \notin \{0, M\} \\ 0 & \text{otherwise} \end{cases} \tag{2a}$$

$$\pi_{(i_1,i_2)(j_1,j_2)}(1) = \begin{cases} 1 - p_1 & \text{for } j_1 = (i_1+1) \bmod N,\ j_2 = 1 \\ p_1 & \text{for } j_1 = (i_1+1) \bmod N,\ j_2 = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2b}$$

The actions through which states progress have different costs: while it is assumed that the planned nature of PM allows for the required materials and manpower to be available in the period when maintenance occurs, CM is only ever performed reactively and a certain lead time to the maintenance being performed is imposed, which translates to additional windmill downtime and thus larger costs compared to PM. Doing nothing is, of course, free. The aforementioned downtime costs depend on the variable wind speed, the non-linear power conversion function of the windmill, and the wider economy price of the energy generated by the windmill. Their interaction will be explained in more depth within an extension, but for the purpose of the base problem, Schouten et al. (2022) directly assigns values to the yearly average cost rates such that the costs of CM are sufficiently larger than PM to satisfy necessary conditions for the existence of finite optimal policies.

Denoting these average costs as $\overline{c}_p$ and $\overline{c}_f$ for PM and CM respectively, period costs can now be individually defined to incorporate deviations from the mean that are given by the period's average wind speeds. Under the informed assumption that wind speeds are generally lower in summer and higher in winter, the parent paper uses a cosine function to mirror this behaviour such that the minimum value is achieved in July and the maximum value is achieved in January. Equation set 3 defines the PM and CM costs in an arbitrary period i_1, regardless of component age, where Δ allows for amplifying (or dampening) the impact of seasonal wind speeds on cost and ϕ is a constant that shifts the function's peak and slump as desired.

$$c_p(i_1) = \overline{c}_p + \Delta_p cos\left(\frac{2\pi i_1}{N} + \phi\right) \tag{3a}$$

$$c_f(i_1) = \overline{c}_f + \Delta_f cos\left(\frac{2\pi i_1}{N} + \phi\right) \tag{3b}$$

The transition probabilities between system states and time-varying costs open the way to the ultimate objective of the paper - assessing long-term average costs under the three periodic policies considered, to be described in detail and have their underlying models formally displayed in the next section.

4 Replication

4.1 Data & Methodology

While the parent paper uses several different datasets and sources to flesh out the real-life example of a wind turbine gearbox, this replication process strictly concerns the "Numerical results" section, which simply makes assumptions on the values of different parameters in order to observe the theoretical effectiveness of the model. Consequently, there is no data to be mentioned or gathered, and the exact parameter values utilised are left for the results section. The lion's share of this subsection is thus devoted to explaining each of the three optimization problems from which different maintenance policies are derived. Each formulation is directly transcribed from the parent paper, with the exception of two additional sets of constraints being inserted into the p-MBRP formulation in order to prevent the policy from reducing to p-ARP. I elaborate further on these constraints when the p-MBRP model is presented in formulation (6).

Across all three policies, the transition probabilities used correspond to the definitions in equation set 2 and the cost parameters used correspond to the definitions in equation set 3. The first one, dubbed the periodic age replacement policy (p-ARP), assigns a critical age from the set $\mathcal{I}_2$ to each period such that PM is performed in a given period if the component's age is at or above the critical age. The underlying LP of p-ARP is shown in formulation (4). Decision variables $x_{i,a}$ can be interpreted as the long-term probability of the Markov decision process being in state $i = (i_1, i_2)$ when action $a \in \mathcal{A}(i)$ is chosen.

Constraints (4b) and (4c) are necessary to emulate this interpretation, ensuring that the total probability of entering a state from other states is equal to the probability of being in that state (essentially equating inflow and outflow across actions) and that all periods are equally likely to be visited, respectively. Additionally, as probabilities should not be negative, constraint (4d) is introduced. Lastly, the objective function computes the monthly long-term average costs: using the set $\mathcal{I}^b = \mathcal{I}_1 \times \{0\}$ to define breakage states that require CM, the probabilities of residing in states where maintenance is performed are weighed by the respective cost of either PM or CM, depending on the component's age in the corresponding state.

$$
\begin{aligned}
\underset{\text{(p-ARP)}}{\min} \quad & \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1)x_{i,1} && && \text{(4a)}\\
\text{s.t.} \quad & \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0, && \forall i=(i_1,i_2)\in\mathcal{I}, && \text{(4b)}\\
& \sum_{i_2\in\mathcal{I}_2}\sum_{a\in\mathcal{A}(i_1,i_2)} x_{(i_1,i_2),a} = \frac{1}{N}, && \forall i_1\in\mathcal{I}_1, && \text{(4c)}\\
& x_{i,a}\geq 0, && \forall i=(i_1,i_2)\in\mathcal{I},\ a\in\mathcal{A}(i) && \text{(4d)}
\end{aligned}
$$

The second policy, dubbed the periodic block replacement policy (p-BRP), splits a repeatable time horizon of m years into blocks of time separated by PM executions. For instance, in a two-year horizon, the policy can state that PM must be done in months 4, 10, 15 and 18; such two-year maintenance plans would then be chained indefinitely, and while the amount of time between maintenance executions is not fixed, the size of the blocks remains constant over all such future two-year horizons. The policy-generating optimization problem is shown in formulation (5). This model strongly relies on the skeleton of p-ARP, as the objective function, transitional balance constraint and probability non-negativity constraint are all directly imported.

As the theoretical framework section suggests, the set of period is extended to $\mathcal{I}=\{1,2,...,mN\}$ to allow for a policy spanning m years. The transition probabilities, cost parameters, constraints and decision variables seen in the formulation below all incorporate this modification; for instance, constraints (5e) now equate the long-term residence probabilities across all mN periods. The binary decision variables y_{i_1} are equal to 1 only if PM is performed in period i_1. If PM is performed in a given period, (5c) ensures that it is not possible to do nothing in the respective period, regardless of the component's age. Similarly, (5d) makes it impossible to act in a given period if it is decided that PM will not be performed.

$$
\begin{aligned}
\underset{\text{(p-BRP)}}{\min} \quad & \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1)x_{i,1} && && \text{(5a)}\\
\text{s.t.} \quad & \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0, && \forall i=(i_1,i_2)\in\mathcal{I}, && \text{(5b)}\\
& x_{i,0}+y_{i_1}\leq 1, && \forall i=(i_1,i_2)\in\mathcal{I}:\ i_2>0, && \text{(5c)}\\
& x_{i,1}-y_{i_1}\leq 0, && \forall i=(i_1,i_2)\in\mathcal{I}:\ i_2>0, && \text{(5d)}\\
& \sum_{i_2\in\mathcal{I}_2}\sum_{a\in\mathcal{A}(i_1,i_2)} x_{(i_1,i_2),a} = \frac{1}{mN}, && \forall i_1\in\mathcal{I}_1, && \text{(5e)}\\
& x_{i,a}\geq 0, && \forall i=(i_1,i_2)\in\mathcal{I},\ a\in\mathcal{A}(i), && \text{(5f)}\\
& y_{i_1}\in\mathbb{B}, && \forall i_1\in\mathcal{I}_1 && \text{(5g)}
\end{aligned}
$$

The last policy is introduced as the periodic modified block replacement policy (p-MBRP) in Berg and Epstein (1976) and it blends the ARP into the framework of BRP: PM is performed in a given period dictated by the BRP only if the component's age exceeds a critical period-dependent threshold, similar to the p-ARP. Formulation (6) is the policy-generating LP for the p-MBRP. In the same way p-BRP built on p-ARP, formulation (4) is now used as the skeleton, maintaining the updated definition of the set $\mathcal{I}_1$ to accommodate m year horizons. Two new sets of decision variables are introduced: binary variables z_i with value 1 only when PM is performed for age i_2 in period i_1, and positive integer variables t_{i_1} denoting the critical age at or beyond which PM is performed in period i_1. Constraints (6d) and (6e) are the ones mentioned at the beginning of the section, mistakenly omitted from the parent paper; they are simply the adaptation of constraints (5c) and (5d) to the new policy, not allowing other actions than what was assigned in the first block for the respective state (i_1, i_2) in all future blocks.

$$
\begin{aligned}
\underset{\text{(p-MBRP)}}{\min} \quad & \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}^b} c_f(i_1)x_{i,1} && && \text{(6a)}\\
\text{s.t.} \quad & \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0, && \forall i=(i_1,i_2)\in\mathcal{I}, && \text{(6b)}\\
& \sum_{i_2\in\mathcal{I}_2}\sum_{a\in\mathcal{A}(i_1,i_2)} x_{(i_1,i_2),a} = \frac{1}{mN}, && \forall i_1\in\mathcal{I}_1, && \text{(6c)}\\
& x_{i,0} + z_i \le 1, && \forall i=(i_1,i_2)\in\mathcal{I}:\ i_2>0, && \text{(6d)}\\
& x_{i,1} - z_i \le 0, && \forall i=(i_1,i_2)\in\mathcal{I}:\ i_2>0, && \text{(6e)}\\
& z_{i_1,i_2} - y_{i_1} \le 0, && \forall (i_1,i_2)\in\mathcal{I}, && \text{(6f)}\\
& z_{i_1,i_2} - z_{i_1,j_2} \le 0, && \forall i\in\mathcal{I},\ j_2\in\mathcal{I}_2:\ i_2<j_2, && \text{(6g)}\\
& t_{i_1} + (j_1+mN)y_{j_1} \le mN + i_1, && \forall i_1,j_1\in\mathcal{I}_1:\ j_1<i_1, && \text{(6h)}\\
& t_{i_1} + j_1 y_{j_1} \le mN + i_1, && \forall i_1,j_1\in\mathcal{I}_1:\ j_1>i_1, && \text{(6i)}\\
& M(y_{i_1} - z_{i_1,i_2}) - t_{i_1} \le M - 1 - i_2, && \forall (i_1,i_2)\in\mathcal{I}, && \text{(6j)}\\
& M z_{i_1,i_2} + t_{i_1} \le M + i_2, && \forall (i_1,i_2)\in\mathcal{I}, && \text{(6k)}\\
& x_{i,a} \ge 0, && \forall i=(i_1,i_2)\in\mathcal{I},\ a\in\mathcal{A}(i), && \text{(6l)}\\
& z_i \in \mathbb{B}, && \forall i\in\mathcal{I}, && \text{(6m)}\\
& y_{i_1} \in \mathbb{B}, && \forall i_1\in\mathcal{I}_1, && \text{(6n)}\\
& t_{i_1} \in \mathbb{N}^+, && \forall i_1\in\mathcal{I}_1 && \text{(6o)}
\end{aligned}
$$

Constraint (6f) ensures that PM cannot be scheduled for any critical age through z_i if p-BRP does not maintain in period i_1 through y_{i_1}. Constraint (6g) ensures that if maintenance is scheduled for critical age i_2, it is also scheduled for all greater ages as well in the respective period. Constraints (6h) and (6i) are only active in periods when maintenance is scheduled, ensuring that the critical maintenance age is at least as large as the number of periods since the last scheduled PM. Lastly, constraints (6j) and (6k) linearise the relationship between z_i, y_{i_1} and t_{i_1}: for all ages at or above the period's age threshold, z_i should have the same value as y_{i_1}. Otherwise, $z_i = 0$ should be forced since the critical maintenance age is not reached. Formal proofs for the existence of unique optimal finite-horizon policies are presented in detail for all three maintenance strategies in Schouten et al. (2022), under a minimal set of necessary and sufficient conditions.

Regardless of model, the aforementioned work describes how the maintenance policy itself is derived the same way from the optimal solution $x^*_{i,a}$. Such a policy, or "strategy", is defined as a complete sequence R of actions, one for each system state in $\mathcal{I}$. In order to form the optimal strategy R^*, the first step is to assign $R^*(i) = a$ if $x^*_{i,a} > 0$. The actions performed in unvisited states (i.e. with long-term residence probability 0) do not matter in terms of the policy's performance, therefore in the second step, for an arbitrary state $j \in \mathcal{I}$ without an action already assigned, it is sufficient to assign any action a under which a non-zero transition probability exists from j to a state whose action was already decided. Performing the second step recursively until every state has an assigned action completes the optimal maintenance strategy.

4.2 Results

The specific results that I aim to replicate are presented in tables 2 and 3 of the parent paper. The focus falls on comparing the constant cost case ($\Delta = 0$) to the case with the most impactful seasonal fluctuation ($\Delta = 50\%$) under two different possible lifetime distributions of the gearbox and a fixed set of cost-related parameters. More precisely, the yearly average costs of PM and CM are assumed to be $\bar{c}_p = 10$ and $\bar{c}_f = 50$ respectively, and the year is split into $N = 12$ periods that match the calendar months, with $i_1 = 1$ marking January and $i_1 = 12$ marking December. In order to attain the lowest costs in summer and highest in winter, correction term $\phi = \frac{-2\pi}{12}$ is used. All cost parameters come together in the code to define the period dependent costs from equation set 3, leading to $c_p(i_1) = 10$ and $c_f(i_1) = 50\,\forall i_1 \in \mathcal{I}_1$ in the constant cost case and equation set 7 in the highly fluctuating case.

$$c_p(i_1) = 10\left(1 + 0.5cos\left(\frac{2\pi(i_1 - 1)}{12}\right)\right) \tag{7a}$$

$$c_f(i_1) = 50\left(1 + 0.5cos\left(\frac{2\pi(i_1 - 1)}{12}\right)\right) \tag{7b}$$

The different structures of the gearbox lifetime distribution are the distinctive element between the two relevant result tables of Schouten et al. (2022), and their primary effect is on the transition probabilities between the MDP's states. Concretely, the first table uses a 1-year horizon, implying $m = 1$ and $\alpha = 12$ months, while the second table uses a 3-year horizon implying $m = 3$ and $\alpha = 36$ months. As the shape parameter of the Weibull lifetime distribution, α works together with the fixed scale parameter $\beta = 2$ to pre-determine the probabilities that the system fails at each age $i_2 \in \mathcal{I}_2$ in the code. As all p_{i_2} of equation set 2 are known, the transition probabilities are also determined, and CPLEX is used to solve the LP formulation shown in the methodology section. The full commented code, while too large to be included in the appendix, is available as an archive on e-mail demand at `504458cg@student.eur.nl`.

Table 1 merges all the relevant results to be replicated from the parent paper. I obtain identical values to the decimal for costs and the same corresponding policies across all parameters, hence the additional table for my replication results is redundant. "CPU" values give my own compilation times on an SSD, 16 GB RAM and a 14-core 2.3 GHz processor for the sake of thorough documentation. These too are insignificantly different from the results of Schouten et al. (2022). The footnotes describe the optimal policies in the constant cost case; since costs do not differ between months, it is redundant to specify PM periods or ages since any option is valid as long as the correct intervals are maintained. Of note is the fact that formulations (4), (5) and (6) yield average monthly costs in the objective value; in order to obtain the values in the table, the raw output of CPLEX must be multiplied by 12. The precision of the replication process lends credibility to the extension results that elaborate on this core framework.

Table 1. *Selected yearly costs (in thousands of €), savings relative to constant costs, maintenance months and ages from Schouten et al. (2022).*

Δ		0%		50%	
Horizon		$m = 1$	$m = 3$	$m = 1$ ($\alpha = 12$)	$m = 3$ ($\alpha = 36$)
p-ARP	costs	40.098	13.530	37.635	9.900
	savings	-	-	6.14%	26.83%
	CPU	<1s	<1s	<1s	<1s
p-BRP	costs	41.501	14.173	38.466	10.072
	savings	-	-	7.31%	28.94%
	PM periods	-	-	7,10	7,19,31
	CPU	<1s	2.24s	<1s	1.56s
p-MBRP	costs	40.311	13.622	37.773	9.900
	savings	-	-	6.30%	27.32%
	PM periods	-	-	6,10	7,19,31
	PM ages	-	-	5,3	7,7,7
	CPU	<1s	17.77s	<1s	2.41s

* Optimal age for $\Delta = 0$ is $t^* = 6$ months when $m = 1$ and 19 months for $m = 3$.
** Optimal block for $\Delta = 0$ is $T^* = 6$ months when $m = 1$ and 18 months for $m = 3$.
*** Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (4, 6)$ for $m = 1$, $(11, 18)$ for $m = 3$.

5 Extension

5.1 Time-Varying Energy Prices

The starting point for all extensions is the realistic offshore wind turbine example in section 4.3 of Schouten (2019), which also serves as the basis for the gearbox example from section 6 of the parent paper. First, following up on the suggestion for further research offered by Schouten et al. (2022), dynamic energy prices replace the constant electricity price of €0.06/kWh for the computation of downtime costs. Using monthly prices to mirror the structure of the problem, this is largely a matter of converting the period-dependent power output function, which in turn depends on wind speed and turbine model.

Following the parent paper, data from the Royal Netherlands Meteorological Institute (2018) is used for wind speed estimates, and data from the Netherlands Commission for Environmental Assessment (NCEA) (2016) is scaled through the scheme in Schouten (2019) to approximate the power output of a Vestas V164 9.5MW wind turbine in the North Sea. The specifics of the power output function are left for the methodology section. As energy prices rise, the downtime costs increase and maintenance is less attractive. This may lower the level of savings identified by the parent paper if energy prices increase in summer or decrease in winter, however neither is likely as energy demand is usually higher in winter and solar power drastically increases energy availability in summer; this demand imbalance is likely to translate to lower prices in summer, hence savings ought to increase under a non-constant energy price overall.

5.1.1 Data & Methodology

As I transition from using the assumed average cost values of Schouten et al. (2022) to more complex cost functions, a deep dive into how they are obtained is in order. There are three practical components to the period-dependent cost: the manpower, the materials, and the lost output during the maintenance downtime. Following the parent paper, I use the analysis of Tian et al. (2011) to set manpower and material costs at 148.2 thousand euros for PM and 592.8 thousand euros for CM on a gearbox.

Still mimicking the process, data from Papatzimos et al. (2018) is used to estimate the time required for maintenance at 10 days, with an additional lead time of 30 days needed for CM. The downtime is translated to cost through the power output function of the wind turbine, previously described in the theoretical framework, which in turn makes use of wind speed estimates from the Royal Netherlands Meteorological Institute (2018). Figure 1 is a copy of the function's graph, originally found in Schouten (2019), where a cosine fit is superimposed to justify using this approximation for the rest of the analysis.

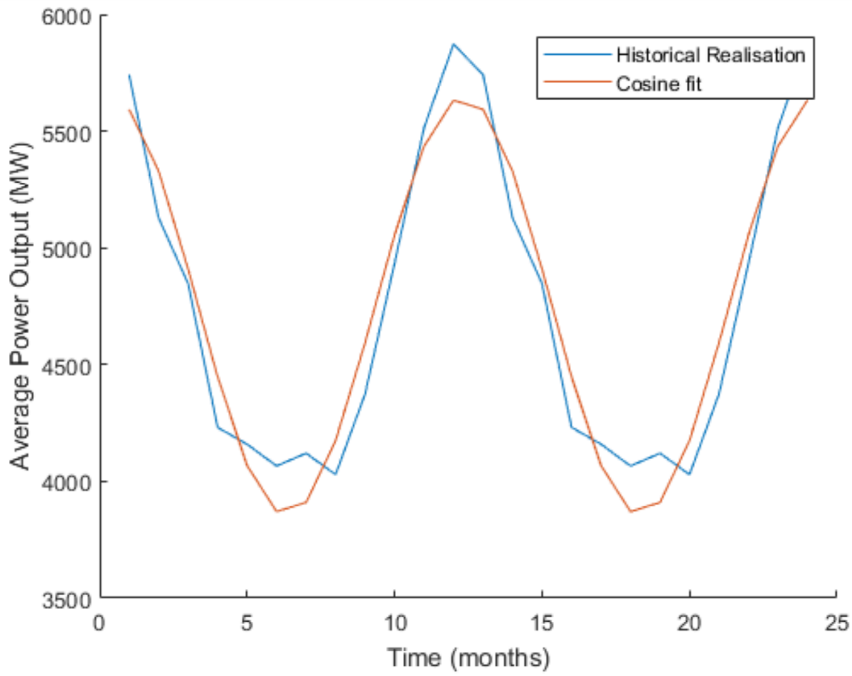

Figure 1: Seasonal pattern of the average power output of a Vestas V164-9.5 MW wind turbine in IJmuiden, as seen in Schouten (2019).

In equation 8 I use the same underlying expression of the power output function as Schouten (2019), scaled up by 24 to measured kilowatts per day. Letting m_{i_1} be the energy price in each month $i_1 \in \mathcal{I}_1$, a different daily downtime cost can be derived as $C_{mean}(i_1) = m_{i_1} P(i_1)$. This cost can then be multiplied by ten to obtain the downtime cost of the 10 days needed for PM, and by forty to obtain the cost of the 40 days needed for CM. These can then be incorporated directly into the LP formulations.

In the contrasting constant cost case, the average of all period costs is used for every individual period. The fact that extending the time horizon to three years has no effect on p-ARP is noteworthy, since this particular policy is only influenced by the lifetime distribution. The monthly prices used for this case are averages of the three years, hence if the shape parameter is set to 36 months, the price in January is the average between those of January 2019, January 2020 and January 2021. This way, some of the additional information from the full three year series is incorporated in the analysis.

$$P(i_1) = 114024 + 21480\ cos\left(\frac{2\pi i_1}{12} - 0.178\right) \tag{8}$$

The data on energy prices comes from the Centraal Bureau voor de Statistiek (CBS) (2022). Expressed in euro per kWh, average consumer prices from January 2019 to December 2021 are shown in figure 2; these values include VAT, but exclude the ODE and Energy tax. The effectiveness of policies mindful of time-varying costs will be investigated over an infinite time horizon - making full use of the MDP's steady state - under two different cycle lengths: one-year cycles for 2019, 2020 and 2021 individually, as well as a three-year cycle that brings them all together. This makes for a total of four instances, each with a different set of prices that repeat indefinitely, for each of the three policy-generating LP formulations.

The length of the price cycle is equal to the length of the component's life cycle in every instance, similar to how the number of years m that covers the periods in $\mathcal{I}_1$ is equal to the shape parameter α in the results of the parent paper. The one-year cycles lean more towards scientific value over practicality; rarely seen in practice, I study them to see whether the short lifespan makes the optimal solution more sensitive to small differences in price across months.

On the other hand, the three-year cycle tends to smooth out the effect of extrinsic shocks in the economy, and because there is no apparent reason why any specific year should be used instead of the other two, most of the following sections will build upon the three-year instance. Given the apparent price surge toward the end of 2021 for instance, the results of this year will likely be markedly different from the other two, yet the impact on the three-year horizon analysis may not be as significant.

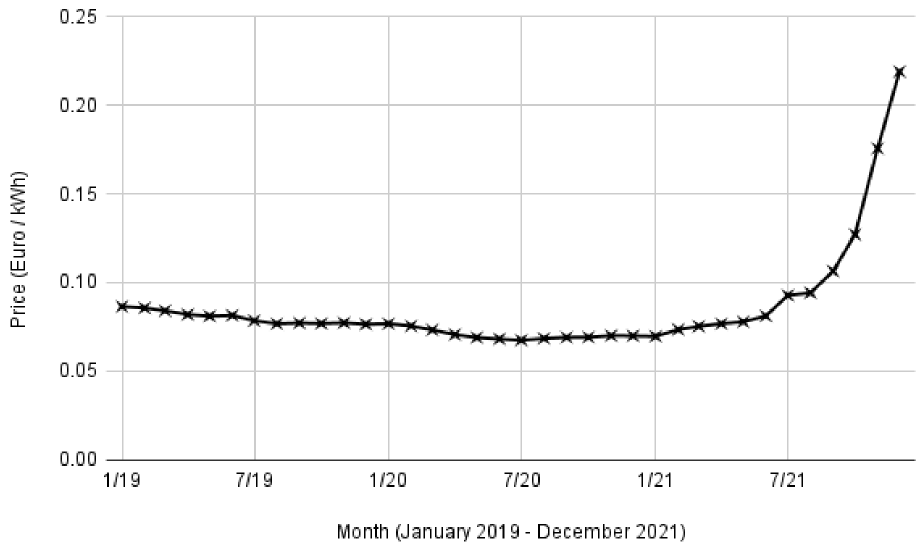

Figure 2: Average consumer electricity prices from January 2019 to December 2021, all taxes except VAT excluded.

5.1.2 Results & Discussion

Combining the manpower, material and downtime costs for each type of maintenance yields the cost functions in equation set 9, expressed in thousands of euro (hence the division of the power output by 1000). The different runs of the optimization model vary in one aspect other than the set of prices: I use $\alpha = 12$ months when using the prices from a single year and $\alpha = 36$ months when using all prices at once, thus intuitively equating the component's expected lifetime to the experiment's time horizon in the same manner as Schouten et al. (2022).

$$c_p(i_1) = 148.2 + 10 \cdot m_{i_1} P(i_1)/1000 \tag{9a}$$

$$c_f(i_1) = 592.8 + 40 \cdot m_{i_1} P(i_1)/1000 \tag{9b}$$

These costs are directly inserted in formulations (4), (5) and (6) to obtain the results in table 2. Given that the fixed component and downtime cost are much larger than before, it comes as no surprise that objective values are much higher across the board compared to the replication results, even if the average energy price is not a lot higher than €0.06/kWh. The long-term average cost of every policy reflects the apparent price movements in figure 2 as the 2019 value falls between 2020 and 2021. Moreover, the general balance between policies remains in place, with the objective value found under p-MBRP falling between p-ARP and p-BRP in every single year, and the time-varying cost model always scores better than constant-cost with savings between 0.3% and 6.1%.

The highest savings are consistently achieved in 2021, likely the result of concentrating maintenance in the first half of the year in order to avoid the price surge visible in the second half. One last important observation concerns the 3-year cycle results in the last column: while the difference between the varying-cost and constant-cost models varies between policies, it is consistently much closer to what is seen in 2019 and 2020 than 2021; the previous suspicion that considering the entire horizon would smooth out the impact of the price jump at the end of 2021 is thus confirmed.

Table 2. *Yearly average cost (in thousands of euro) using variable monthly energy prices. Savings compared to the respective constant-cost result in brackets, expressed in percentage points.*

Policy	Result	2019	2020	2021	All Three
p-ARP	cost (savings)	833.471(0.362)	796.313(0.298)	908.616(4.430)	281.011(3.628)
p-BRP	cost (savings)	880.321(0.663)	841.068(0.599)	946.822(6.166)	296.749(5.226)
	months	6,8,12	6,12	1,2,8,10	8,29
p-MBRP	cost (savings)	843.395(0.599)	805.745(0.541)	913.368(5.371)	282.610(4.474)
	(month, age)	(6,5),(12,5)	(6,5),(12,5)	(2,5),(8,4)	(8,12),(29,13)

Whether the optimal solution avoids maintaining in the months with significantly higher energy prices can be determined based off the critical ages and maintenance months of the different policies. To this end, figure 3 shows the critical age distribution of the four varying-cost p-ARP models and the two constant-cost models, covering a 1-year and 3-year lifetime respectively. Age M is the maximum tolerated, maintaining in M is equivalent to never performing PM in the respective month.

Two important results stand out. First, the optimal solution in the 2019 one-year price cycle setting never maintains in July. This result is surprising as downtime costs are at their lowest in July, which implies that the price fluctuations have an overall stronger impact. This difference between 2019 and 2020 may be caused by the fact that in 2019, the energy price in July is higher than all subsequent months, while in 2020 it is the lowest of the whole year. Small price fluctuations can thus indeed impact the optimal solution significantly in one-year price cycle settings.

Second, it is strictly the summer months that see any maintenance when the longer time horizon is used and maintenance is never performed in the last three months of 2021 when the corresponding one-year price cycle is used, supporting my previous claim on the effect of the price increase. The results of p-BRP and p-MBRP further corroborate the findings, seeing how maintenance is performed in May at the latest when the prices over all 3 years are used and the last repairs under the 2021 prices are done in August under p-MBRP.

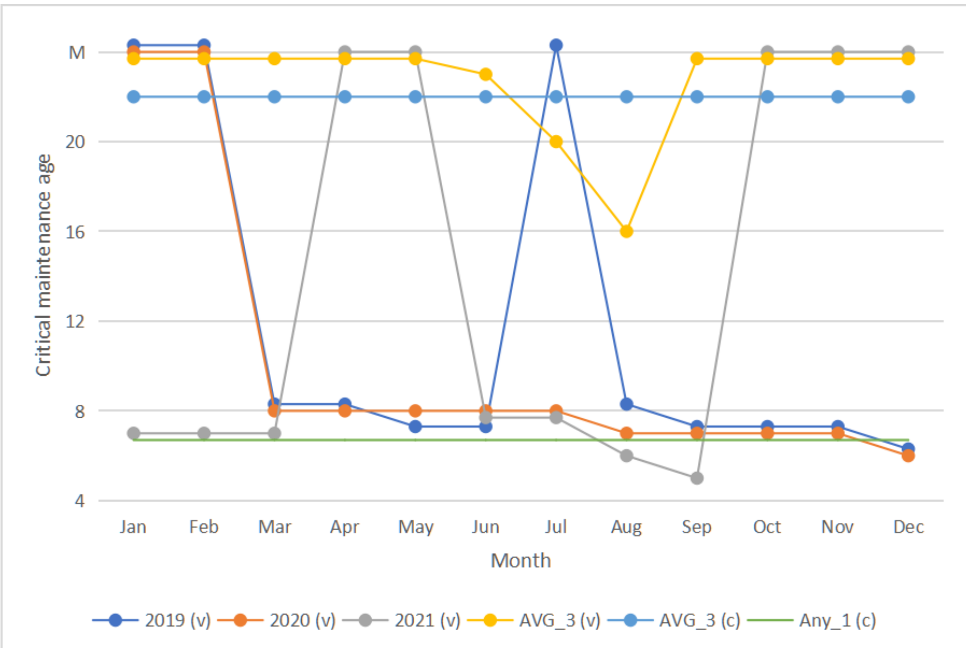

Figure 3: p-ARP optimal solutions for the three single-year price vectors (2019, 2020, 2021) and the single three-year one (AVG_3), under time-varying (v) and constant cost (c) models. All one-year cycle instances have the same constant cost optimal solution (Any_1).

5.2 Limit of Failure Frequency

Without modifying the MDP's transition probabilities or action space just yet, the second sub-question requires that some user-defined limit is placed on the long-term probability of the component reaching age 0 under no maintenance action, which is identified as the failure state requiring CM. As the limit is lowered, the optimizer would likely undertake PM at higher frequencies, and to the extent that these extra maintenance executions can be concentrated around the lower-cost periods in summer, the time-varying cost model may achieve greater savings than before. Studying just how fast costs ramp up and PM executions become more frequent as the tolerance for CM decreases is valuable if there is a yearly limit on work hours allocated to emergency response maintenance and other similar contexts.

At this point, it is important to clarify the comparative: it is the constant-cost model being compared to the time-varying cost model for each different version of the extended policies. Mirroring the technique of the parent paper, the value of the constant cost is equal to the average over all the periods in the problem instance. While it is virtually guaranteed that forcing more PM to prevent the system from needing CM will be significantly more costly than the baseline model for instance, the issue is secondary in importance to the thesis' primary objective of evaluating the impact of accounting for time-varying costs.

5.2.1 Methodology

Since it is assumed that CM is performed whenever the gearbox malfunctions, limiting the amount of CM that can be performed is equivalent to limiting the amount of time the system can spend broken. In order to allow the imposition of such limits, the constraint in equation 10 can be added to formulations (4), (5) and (6), directly limiting the long-run fraction of time spent in the undesired states through the manually set constant L. Lower values of L ought to force the inclusion of more PM in the optimal policy. As previously stated, the price 3-year horizon with the full set of prices is used for p-BRP and p-MBRP, while $\alpha = 36$ months and the average monthly prices are used for p-ARP.

$$\sum_{a \in \mathcal{A}_{(i_1,i_2)}} \sum_{i_1 \in \mathcal{I}_1} x_{(i_1,0),a} \leq L \tag{10}$$

Preliminary analysis would indicate that the system spends a rather low fraction of time in the breakage state to begin with - setting the CM limit above 3% long-term residence probability fails to influence the optimal solution in any way. As a result, each of the three optimization models is run for a range of limiting values L between 0.1% and 2.5%, with 0.3% intervals, with the same primary goal of comparing the time-varying cost setting to constant costs.

5.2.2 Results & Discussion

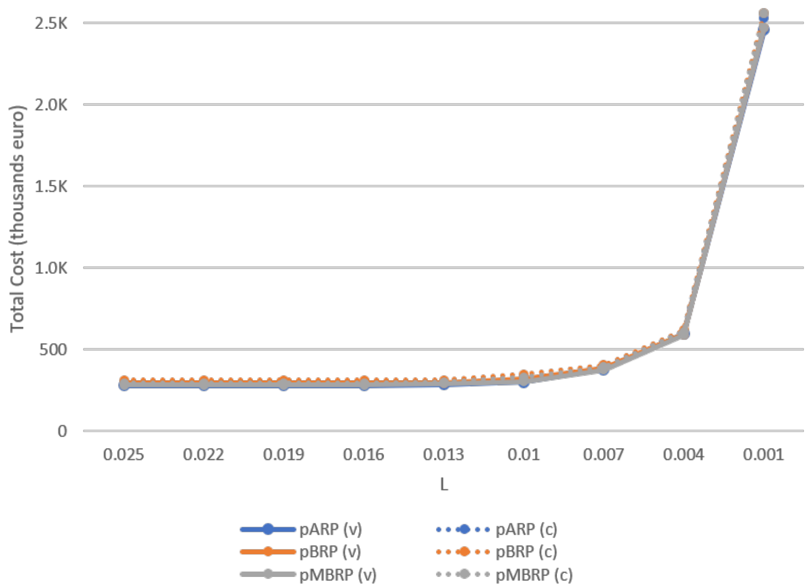

Figure 4: Objective values of the three different policies under increasingly tight CM limits.

The evolution of the objective values as L decreases is shown in figure 4, where each policy gets a different colour and constant-cost results are differentiated with broken lines from the varying-cost counterparts. The decreasing direction of the limit highlights the fact that for the first four limit points, the objective value remains virtually unchanged, indicating that the long-term fraction of time the system is broken is normally close to 1.5%. Let this value - as well as its corresponding cost - be the benchmark moving forward. Under this point, it is apparent that the average costs rise increasingly fast for all three policies, peaking with a roughly 54% increase when the limit drops to 0.4% and a 450% increase when the limit finally drops to 0.1%. Rigor demands that I indicate the splitting value of the p-MBRP average cost at this lowest CM limit, as the problem becomes too large for CPLEX to solve effectively - the gap between the best result and best bound is 1.11%.

Whatever the true objective value may be, it falls in the same patterns as the rest of the results, even though the close values make visual identification difficult: p-MBRP achieves long-run costs between those of p-ARP and p-BRP, with the varying-cost model outperforming constant cost as savings fluctuate between 3.6% and 5.8% across policies. These differences between policies matter less for the extension's objective than identifying the so-called "breaking point" where limiting CM becomes extremely harmful. Since the models are based on the 3-year price set - which achieved the 3.6% savings that now constitute the minimum amount - I conclude that accounting for time-varying costs is indeed more impactful when CM is limited, likely as a direct result of PM being forced to happen more often.

To verify this increased PM frequency, the critical ages assigned by p-ARP to every month are shown in figure 5 for three different levels of the CM limit: 0.1%, 1% and 1.9%. The last limit can be taken as a benchmark for the other two since it produces the same set of ages one would obtain without any limits. Not only does the critical age drop overall when the limit decreases, but maintenance starts being performed in more periods, for instance May and April when moving to $L = 1\%$. At the lowest limit, it becomes apparent that the enormous costs are caused by the PM scheduled every period, regardless of component age, except for December. Varying-cost models never schedule maintenance in December because of the comparatively higher energy price, even under the smoothing effect of averaging this price over the three years. Following this analysis, it is clear that the time spent in breakage states can be limited to about 0.7% of total time without incurring any severe cost increases, as the elbow in figure 5 shows.

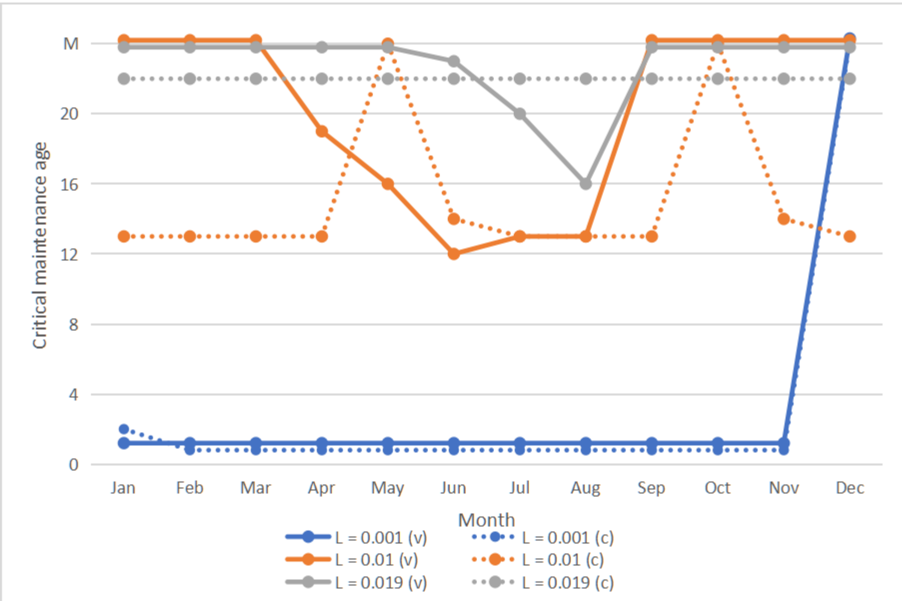

Figure 5: p-ARP optimal policies under three different CM limits, expressed as a set of critical ages over the months of the year. Time-varying models are traced in solid lines, while the constant-cost models' lines are broken.

5.3 Imperfect Preventive Maintenance

Imperfect maintenance, as defined in De Carlo and Arleo (2017) and many other works in the field of maintenance optimization, prolongs the life of the component by a shorter amount compared to a complete replacement that is equivalent to perfect maintenance, usually for a lower price. The other, less common definition of imperfect maintenance as a repair procedure that fails and leaves the component broken in the next period does not fall under the scope of this paper. Since CM is meant as a response to the component breaking down, it is intuitive to reserve imperfect maintenance as an option solely for PM and keep the replacement action of corrective maintenance.

The possibility to essentially set the gearbox's age back by some proportion of its age for the same proportion of the price of a regular, replacement-based PM may further enhance cost savings if it is used to cheaply go through high-cost periods more safely, especially once the component reaches higher ages. Alternatively, perfect PM can be planned before winter to minimize the potential downtime, while lower-cost imperfect PM can be undertaken in summer to cheaply prolong the gearbox's life through the low-wind period.

5.3.1 Methodology

Let then action $a = 2$ define imperfect PM (henceforth referred to as iPM). If this action is performed in state $i = (i_1, i_2) \in \mathcal{I}$, the component's age should decrease by ρi_2 for the cost of $\rho c_p(i_1)$, where ρ is a manually-set parameter with values between 0 and 1. If $\rho = 0$ the operation reduces to doing nothing ($a = 0$), and if $\rho = 1$ a regular PM is performed instead ($a = 1$). Equation 11 redefines the action space to include iPM and equation 12 defines the transition probabilities under the newly-introduced action. Note that, in the latter, notation $[i]$ expresses the rounding of i to the nearest integer, the most convenient approach to dealing with rational ages. To the extent of my tests, the results do not change significantly when the age is always rounded either up or down.

$$\mathcal{A}(i_1, i_2) = \begin{cases} \{1\}, & \text{if } i_2 \in \{0, M\} \\ \{0, 1, 2\}, & \text{if } i_2 \notin \{0, M\} \end{cases} \tag{11}$$

$$\pi_{(i_1,i_2)(j_1,j_2)}(2) = \begin{cases} 1 - p_{[(1-\rho)i_2]+1} & \text{for } j_1 = (i_1 + 1) \bmod N,\ j_2 = [(1-\rho)i_2] + 1,\ i_2 \notin \{0, M\} \\ p_{[(1-\rho)i_2]+1} & \text{for } j_1 = (i_1 + 1) \bmod N,\ j_2 = 0,\ i_2 \notin \{0, M\} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Across all three optimization models, the objective function is the first element that must be modified once a new set of decision variables $x_{i,2}$ is introduced: since iPM can only be performed outside of breakage states, adding $\sum_{i \in \mathcal{I} \setminus \mathcal{I}^b} \rho c_p(i_1) x_{i,2}$ to the already-established objective is sufficient. There are no further modifications needed for p-ARP; on the other hand, p-BRP and p-MBRP require a good deal more and constitute the subject of what remains of this subsection.

Equation set 13 contains the additional constraints for the p-BRP formulation. Standard PM and imperfect PM are scheduled in separate, independent blocks, interacting only insofar as no two actions can happen simultaneously in one period. The newly-introduced binary v_{i_1} variables take value 1 if iPM is scheduled in period i_1 and function exactly like the y_{i_1} variables in their supportive role. Their introduction warrants not only a set of constraints mirroring (5c) and (5d), but also a set of constraints that would not allow iPM to be performed in periods when PM was scheduled and vice versa.

$$x_{i,2} + y_{i_1} \leq 1 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : \; i_2 > 0 \tag{13a}$$

$$x_{i,0} + v_{i_1} \leq 1 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : \; i_2 > 0 \tag{13b}$$

$$x_{i,1} + v_{i_1} \leq 1 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : \; i_2 > 0 \tag{13c}$$

$$x_{i,2} - v_{i_1} \leq 0 \qquad \forall i = (i_1, i_2) \in \mathcal{I} : \; i_2 > 0 \tag{13d}$$

The additional variables and constraints for the p-MBRP formulation can be fully defined based on formulation (6) and the previously presented new constraint set. In a similar manner to how the variables v_{i_1} were introduced, it is necessary to create variable sets $w_{i_1,i_2} \in \mathbb{B}$ and $r_{i_1} \in \mathbb{N}$ to mirror the function of z_{i_1,i_2} and t_{i_1} for the long-term residence time of the system in iPM states. Furthermore, similar constraints to (6f) through (6k) must be included in the formulation to link the new variables together, with w_{i_1,i_2} replacing z_{i_1,i_2} and r_{i_1} replacing t_{i_1} everywhere. Finally, four constraints mirroring set 13 that replace y_{i_1} and v_{i_1} by z_{i_1,i_2} and w_{i_1,i_2} respectively must be added to complement constraints (6d) and (6e). For the sake of space economy, full formulations of the iPM-augmented p-BRP and p-MBRP will only be presented in the appendix.

5.3.2 Results & Discussion

Table 3 contains results for five levels of the iPM impact parameter ρ, from 0.1 to 0.9, in four different settings: one for each of the three policies over a 1-year horizon with $\alpha = 12$ and an additional p-MBRP setting with a 3-year horizon and $\alpha = 36$. The 3-year cycle saw no iPM executions whatsoever under the p-ARP and p-BRP models. With the goal of investigating the impact of the end-year price surge, 2021 was the year of choice for prices in the smaller cycle. What is more, under p-ARP, iPM is only performed when $\rho = 0.1$ and varying costs are considered, always at age 5 - likely taking advantage of the rounding used in the transition chain formulation. As this variation completely disappears when rounding down is used instead of rounding to the nearest integer, the p-ARP ages are hardly worth discussing further.

Table 3. *Yearly average costs (in thousands of euros) for the 1-year cycle setting of all three policies and 3-year cycle setting of p-MBRP under the imperfect maintenance model.*

Policy	Cost Model	$\rho = 0.1$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.9$
p-ARP	Varying	895.569	908.616	908.616	908.616	908.616
	Constant	948.864	948.864	948.864	948.864	948.864
p-BRP	Varying	931.647	946.822	946.822	943.211	925.491
	Constant	996.475	1005.205	1005.205	1005.205	988.455
p-MBRP (1-year)	Varying	912.379	912.944	913.251	913.016	902.445
	Constant	961.413	962.084	962.288	962.355	957.661
p-MBRP (3-year)	Varying	282.592	282.610	282.598	282.602	282.527
	Constant	295.120	295.141	295.141	295.137	295.110

If table 2 is used as a benchmark, it is immediately apparent that aside from a select few exceptions, the objective values obtained by the iPM models are very close to the results of dynamic prices alone, even though some iPM is part of the optimal solution in all p-MBRP settings and over half the p-BRP settings. The aforementioned exceptions are the objective values under $\rho = 0.1$ and $\rho = 0.9$ for p-BRP and p-MBRP in the 1-year setting, where both the varying-cost and constant-cost models achieve savings between 1.2% and 2.3%. Seeing as all of these savings are achieved in the presence of scheduled iPM, there may be some merit to the technique.

On the other hand, it seems likely that $\rho = 0.1$ achieves savings by maintaining consecutively for a few months at the beginning of the year in order to take advantage of the lower price. From a practical standpoint, this is would probably be too time-consuming and labor-intensive a maintenance schedule to uphold; the combination of one period standard PM and two to three periods iPM seen under $\rho = 0.9$ are nevertheless convincing enough to refrain from brushing off the benefits of this extension entirely. Albeit small, the savings achieved purely thanks to imperfect maintenance may play a more significant role in the upcoming final model expansion, when combined with some limitation on as-good-as-now maintenance.

5.4 Interactions With Limited Maintenance Availability

The main goal of this final extension is to relax the assumption that maintenance is available in infinite supply. Be it due to a shortage of components or transportation means, a maintenance opportunity may only become available after a certain number of periods, with the possibility of saving opportunities for a later time in an inventory-like setting. Should the system advance to a state where PM was scheduled or a breakage occurs without any repair means in stock, the maintenance team is forced to do nothing until an opportunity arises, at which point the corresponding repairs can be done.

This problem becomes more interesting as the number of periods between opportunities increases, likely forcing the optimizer to schedule fewer maintenance executions or set higher age thresholds in order to deal with the shortage of repair means. While overall long-term costs are bound to increase with fewer maintenance opportunities, significant savings may be achieved if the optimal solution focuses on low-cost periods. Moreover, the cost increase may be further dampened if imperfect maintenance is weaved into the model, allowing the system to function longer without needing to use a gearbox. For all of these reasons, the workhorse of this extension is a nested model that combines CM limits, imperfect maintenance and gearbox shortages to observe their interaction.

5.4.1 Methodology

Let there be an interval $\gamma \leq N$ defining how many periods pass before a new gearbox becomes available (only an example, gearbox will often be used interchangeably with maintenance opportunity throughout the section). In order to keep track of the stock, let the variable $h \in \mathbb{N}$ define how many gearboxes are available in the Markov decision process in any state, with a maximum of $H = 6$. Scale parameter α is set at 12 months in order for multiple PM executions to be scheduled in a year and therefore make the impact of a shortage more easily observable. If previous optimal policies serve as any indication, a maximum of six maintenance opportunities should not be prohibitive.

The action space, the transition probabilities and the costs must once again be modified, building on the previous extensions. Starting with the former, state variable h is integrated in the definition with the primary purpose of not allowing any maintenance when no opportunities are in stock, as seen in equation 14. Note that it should not be possible to delay repairs if a scheduled PM comes up or the system breaks as long as there is at least one gearbox in stock.

$$\mathcal{A}(i_1, i_2, h) = \begin{cases} \{1\}, & \text{if } i_2 \in \{0, M\},\ h > 0 \\ \{0, 1, 2\}, & \text{if } i_2 \notin \{0, M\},\ h > 0 \\ \{0\}, & \text{if } h = 0 \end{cases} \tag{14}$$

Secondly, since it is possible to pass through a breakage state without repairing, downtime costs should be separately applied to the corresponding periods. Assuming 30 days of inactivity under the daily downtime costs described in the dynamic price extension, a complete description of costs depending on state and action is given in equation 15. The aforementioned downtime cost when $a = 0$ and $i_2 = 0$ will henceforth be denoted $c_d(i_1)$ for convenience.

$$c_{(i_1, i_2, h, a)} = \begin{cases} 0, & \text{if } a = 0,\ i_2 \neq 0 \\ 30\ m_{i_1} P(i_1) = c_d(i_1), & \text{if } a = 0,\ i_2 = 0 \\ c_p(i_1), & \text{if } a = 1,\ i_2 \neq 0 \\ c_f(i_1), & \text{if } a = 1,\ i_2 = 0 \\ \rho c_p(i_1), & \text{if } a = 2,\ i_2 \neq 0 \end{cases} \tag{15}$$

Lastly, the transition probabilities must model how a new gearbox is made available in certain periods (thus incrementing h) and how a gearbox is used after a replacement maintenance action ($a = 1$) is performed (thus decrementing h). The indicator function determining availability makes use of the newly-defined set $G = \{i_1 \in \mathcal{I}_1 \mid i_1 \bmod \gamma = 1\}$ containing periods when a new gearbox arrives; the first period is by definition always in the set to prevent the model softlocking. A complete definition for the transition between arbitrary states (i_1, i_2, h) and (j_1, j_2, k) under all three actions is presented in equation set 16.

$$\pi_{(i_1,i_2,h)(j_1,j_2,k)}(0) = \begin{cases} 1-p_{i_2} & \text{for } j_{1,2} = i_{1,2}+1,\ k = h + I_{j_1 \in G} \\ p_{i_1,i_2} & \text{for } j_1 = i_1+1,\ j_2 = 0,\ k = h + I_{j_1 \in G} \\ 0 & \text{otherwise} \end{cases} \tag{16a}$$

$$\pi_{(i_1,i_2,h)(j_1,j_2,k)}(1) = \begin{cases} 1-p_1 & \text{for } j_1 = i_1+1,\ j_2 = 1,\ k = h-1 + I_{j_1 \in G} \\ p_{i_1,1} & \text{for } j_1 = i_1+1,\ j_2 = 0,\ k = h-1 + I_{j_1 \in G} \\ 0 & \text{otherwise} \end{cases} \tag{16b}$$

$$\pi_{(i_1,i_2,h)(j_1,j_2,k)}(2) = \begin{cases} 1-p_{[(1-\rho)i_2]+1} & \text{for } j_1 = i_1+1,\ j_2 = [(1-\rho)i_2]+1,\ k = h + I_{j_1 \in G} \\ p_{[(1-\rho)i_2]+1} & \text{for } j_1 = i_1+1,\ j_2 = 0,\ k = h + I_{j_1 \in G} \\ 0 & \text{otherwise} \end{cases} \tag{16c}$$

The model formulations - or formulation, to be precise - will conclude this subsection. p-ARP is the only baseline model to be woven into this extension as not being able to repair when the polict demands poses an issue for the other two. Limited maintenance defeats the whole simplicity advantage of the block-based policies if repairs are performed as soon as a gearbox becomes available, and if maintenance is deferred to the next scheduled PM, downtime costs would add up tremendously. The final optimization model - built on formulation (4) and all extensions thus far - is formally presented in formulation (17).

$$\min \quad \sum_{i \in \mathcal{I} \setminus \mathcal{I}^b} \left(\sum_{h>0} c_p(i_1) x_{i,h,1} + \sum_{h} \rho c_p(i_1) x_{i,h,2} \right) + \sum_{i \in \mathcal{I}^b} \left(\sum_{h>0} c_f(i_1) x_{i,h,1} + c_d(i_1) x_{i_1,0,0,0} \right) \tag{17a}$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{A}(i)} x_{i,h,a} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}(j)} \sum_{0 \le k \le H} \pi_{(j,k)(i,h)}(a) x_{j,k,a} = 0, \qquad \forall i = (i_1,i_2) \in \mathcal{I},\ 0 \le h \le H, \tag{17b}$$

$$\sum_{i_2 \in \mathcal{I}_2} \sum_{a \in \mathcal{A}(i_1,i_2)} \sum_{0 \le h \le H} x_{i_1,i_2,h,a} = \frac{1}{N}, \quad \forall i_1 \in \mathcal{I}_1, \tag{17c}$$

$$x_{i,0,1} = 0, \qquad \forall i \in \mathcal{I}, \tag{17d}$$

$$\sum_{i_1 \in \mathcal{I}_1} \sum_{h \le H} x_{i_1,0,h,1} \le L, \quad , \tag{17e}$$

$$x_{i,h,a} \ge 0, \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A}(i),\ h \le H \tag{17f}$$

From constraint (17f) it is clear that the decision variable set has been expanded to cover the additional iPM action and include the count of available maintenance opportunities in the MDP state space. Constraints (17b) and (17c) have had their form modified from formulation (4) only to include these additional variables - they still serve their old functions, namely to equate the probabilistic inflow and outflow of each state and ensure that the system spends an equal amount of time in all periods, respectively. Constraint (17d) has been added to enforce that no as-good-as-new maintenance may be performed when no gearboxes are available, and constraint (17e) is directly adapted from the CM limit constraint in order to study its interaction with the iPM and limited maintenance availability. Lastly, the objective function has expanded considerably compared to the p-ARP original formulation, now including the total costs of iPM and total downtime costs incurred when the system is broken with no maintenance opportunity available to correct it.

5.4.2 Results & Discussion

I hereby present the interactions between extensions through a diverse range of values for the three relevant control parameters: CM time proportion limiter L, iPM scaling factor ρ and maintenance opportunity interval γ. A key nuance of the problem is of course the extent to which the "gearbox stock" impacts maintenance decisions. When maintenance is unavailable because of a gearbox shortage, an age-based opportunity maintenance problem similar to the subject of Sherwin (1999) is formed. As long as at least one gearbox is available though, the problem loses this layer of complexity and behaves as if there was no limit on maintenance executions.

Table 4 shows 25 objective values, each generated using $L = 0.03$ and the specified combination of ρ and γ, in a time-varying cost setting. It is immediately obvious that the gearbox arrival interval hardly ever influences the result, while the iPM scaling factor only significantly brings down long-term average cost when it is closest to one - likely because that is when it most easily acts as a replacement-free PM for periods where no gearboxes are available. The magnitude of the total cost may seem high overall, but not only is this expected given that using $\alpha = 12$ months yielded similar results in the earlier dynamic prices extension, but all settings actually achieve savings of at least 2% compared to their constant-cost counterparts. Moreover, the savings can be even higher if one considers the constant-cost result for $\rho = \gamma = 1$ and $L = 0.03$ of 956.619 thousand euro.

Table 4. *Yearly average cost (in thousands of euro) for the fully extended p-ARP using time-varying costs and the specified parameters. Savings compared to the respective constant-cost result in brackets, expressed in percentage points.*

L=0.03		γ 2	3	4	5	6
	0.1	936.393(2.161)	936.393(2.161)	936.393(2.161)	936.393(2.161)	936.393(2.161)
	0.3	933.094(2.522)	933.094(2.522)	933.094(2.522)	933.177(2.513)	933.13(2.518)
ρ	0.5	934.878(2.326)	934.887(2.325)	934.878(2.326)	934.894(2.324)	934.894(2.324)
	0.7	936.393(2.161)	936.393(2.161)	936.393(2.161)	936.393(2.161)	936.393(2.161)
	0.9	878.333(2.072)	878.334(2.072)	878.334(2.072)	878.334(2.072)	878.334(2.072)

To contextualize the impact of lower pressure to maintain, the previous analysis is repeated for the higher CM limit of $L = 0.06$ in table 5. It is once again apparent that variation in γ does not directly impact the long-run average cost, however the influence of ρ appears more streamlined. In table 4, the objective values decrease by a small amount, then rise back up and plummet at the end, whereas the more mild conditions in 5 give the objective values a smoother bell curve shape as ρ increases. The lowest overall costs are now achieved at the lower extreme of the iPM scaling factor, yet the highest savings overall compared to the constant-cost model are achieved in the more costly setting of $\rho = 0.9$.

The most important aspect to recognize in both tables is how they corroborate the findings of the iPM extension. The possibility of prolonging the component's life by marginally less than standard PM without using a spare ($\rho = 0.9$) and the possibility of essentially halting aging for a period with very little cost ($\rho = 0.1$) both hold more value to the optimization model than the middle-of-the-pack options.

Table 5. *Yearly average cost (in thousands of euro) for the fully extended p-ARP using time-varying costs and the specified parameters. Savings compared to the respective constant-cost result in brackets, expressed in percentage points.*

L=0.06		γ 2	3	4	5	6
	0.1	837.22(2.146)	837.22(2.146)	837.219(2.145)	837.22(2.146)	837.22(2.146)
	0.3	849.554(1.388)	849.554(1.389)	849.554(1.388)	849.554(1.389)	849.554(1.389)
ρ	0.5	849.554(1.388)	849.554(1.389)	849.554(1.388)	849.554(1.389)	849.554(1.389)
	0.7	849.554(1.388)	849.554(1.389)	849.554(1.388)	849.554(1.389)	849.554(1.389)
	0.9	839.405(2.566)	839.405(2.566)	839.405(2.566)	839.405(2.566)	839.405(2.566)

While variation in γ may not directly change the objective value, it is possible that the underlying policies respond better. The four parts of figure 6 trace the critical maintenance ages of four selected combinations of ρ and γ. In addition to the information provided by the caption, note that iPM schedules are marked by broken lines to contrast the solid lines of standard PM. Furthermore, varying-cost schedules are shown in gray, constant-cost schedules are shown in blue, and the benchmark setting - most notably the PM one, since $\rho = 1$ is used and iPM is thus absent - is shown in orange.

Comparing the leftmost two amounts to fixing $\rho = 0.3$ and looking at the policy differences between $\gamma = 3$ and $\gamma = 5$. Even though the objective values in table 4 barely differed, the policies seem to tell another story. The most striking distinction can be made between the constant-cost iPM policies: much less is performed when gearboxes are more sparse, likely the product of standard PM being done a month younger on average; furthermore, the already-low ages created by the CM limit can be easily understood to be setbacks for iPM as breakage chances are quite low under such PM frequency. The same phenomenon can be acutely picked up when comparing the rightmost two, indicating that gearbox shortages push harder against iPM the more expensive the latter gets. Horizontal comparisons are much less interesting, as they reflect the trend previously discussed with the objective value table.

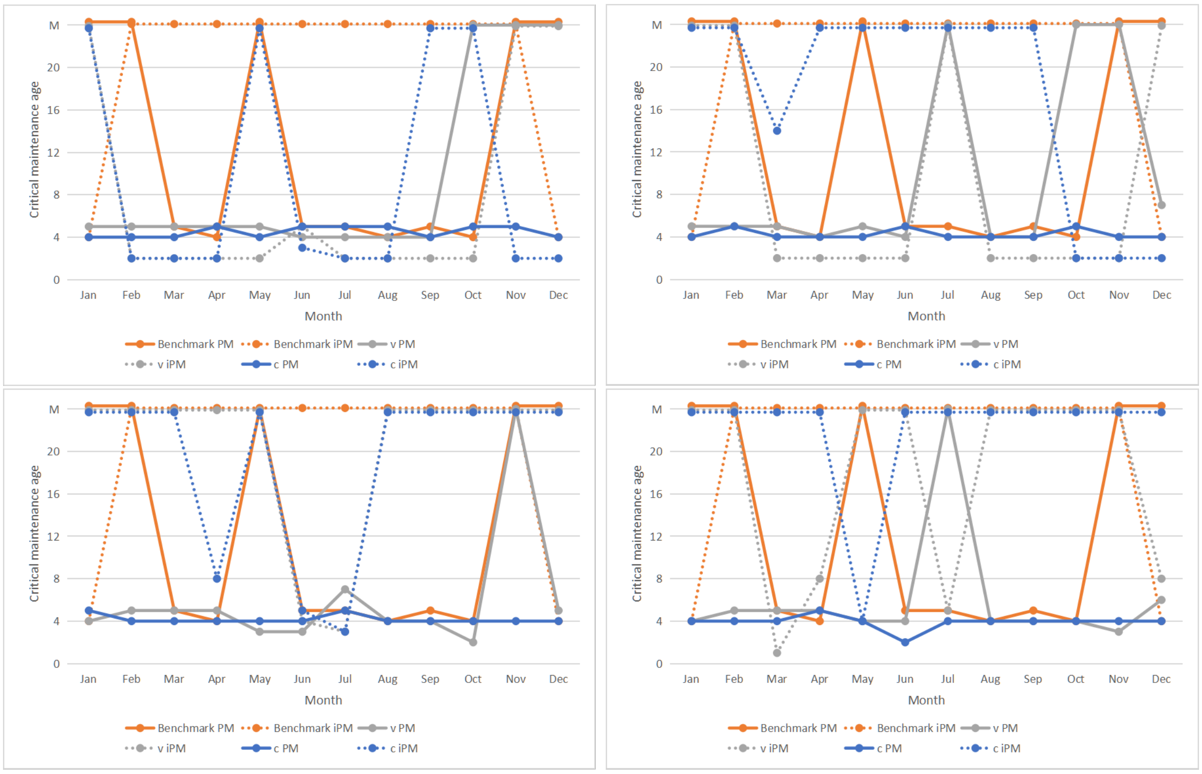

Figure 6: Extended p-ARP critical maintenance ages under $L = 0.03$, $\alpha = 12$ months and different pairs (ρ, γ). From the top left, clockwise: (0.3, 3), (0.7, 3), (0.7, 5), (0.3, 5). Constant-cost setting for $\gamma = \rho = 1$ used as benchmark.

6 Conclusion

Given the scarcity of additions to Schouten et al. (2022) in the theoretical framework and the success of the replication process, this final section is bound to focus on the extension. Following the structure already laid out by the subsections of my extension, I shall concisely answer the sub-questions previously presented in the introduction to eventually tackle the main research question: are the savings identified by the parent paper stable in more complex settings one may encounter in practice?

For the most part, it is apparent that assuming energy prices to be constant at €0.06/kWh is not significantly divorced from the variable component of the consumer electricity price paid by Dutch households. This is clearly observable through the 2019 and 2020 prices from figure 2. The key finding when historical, varying prices are used is that savings increase further if economic shocks like the one at the end of 2021 can be predicted, since the optimizer can avoid maintaining at that time. Moreover, even small price deviations turned out to potentially outweigh small deviations in wind speed, with maintenance in a higher-speed period being justifiable through the lower consumer price. Achieving these savings, however, is conditional on notoriously-difficult-to-obtain accurate price predictions, hence further research should be devoted to refining prediction algorithms and investigating whether the savings stand when the maintenance schedule cannot be tailored to the exact economic conditions of a certain period.

There are many reasons why an interest in limiting how much time a wind turbine is broken would exist. Regardless of motivation, the second extension subsection shows that setting this limit above 1.5% of the total time the turbine is expected to ever function is completely inconsequential. Should one aim lower, I have shown that costs increase slowly up to the breakpoint of 0.7%, after which the increase accelerates dramatically and the limit would likely be deemed infeasible. With well-scheduled preventive maintenance, it is thus possible to guarantee very low failure rates, and accounting for time-varying maintenance costs continues to provide significant savings when the additional PM operations can be scheduled favourably.

The possibility to mend some of the wear that a component suffers over time would be highly valued in practical applications, making the study of its interaction with the PM and CM presented in Schouten et al. (2022) worthwhile. In the basic implementation where this imperfect PM - or iPM - sets the component's age back by less than PM for a proportionally lower cost, however, savings turned out to be marginal. Only the most cheap and ineffective iPM version on one hand, and the most expensive and similar to PM version on the other, managed to find niche uses that would increase savings. Overall though, most forms of iPM that may be considered "reasonable" in practice do not find any use in cost reduction. How fast savings increase if the proportion of maintenance cost is even lower than the proportion of age it shaves off may merit further research.

The final extension subsection deepens the complexity of the problem by several layers, nesting together varying energy price, CM limits and iPM alongside the possibility to limit the inflow of spare components. In the end, savings were not affected by any reasonable interval between the availability of two maintenance operations. Even if only two component replacements are allowed in a year, it turns out that maintenance can be scheduled so that costs remain largely unchanged, likely due to the component's expected lifetime being significantly longer than the availability interval. Very impactful such intervals would lose all sense of practicality; as they get closer to one maintenance operation or less throughout the gearbox's expected lifetime, resources would surely be better invested in shrinking this interval - for replacement-based PM to not be wasteful among other reasons - rather than scheduling maintenance around the unrealistic shortage.

Considering all of the facts, maintenance policies mindful of time-varying costs consistently achieve savings up to 6.1% depending on economic conditions and the available type of imperfect PM, with neither controlled component inventories nor limits on how much the system is allowed to spend in a failure state reducing these savings in any practical setting. To the extent that wind speeds and energy prices can be predicted well in advance, it is thus safe to conclude that scheduling maintenance in low-cost periods stands to benefit both economies and the environment as the renewable energy sector expands, even when assumptions are relaxed and the model's complexity approaches realistic settings.

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