Name of the Program: B. Sc. (CSE) **Date: 10 September, 2021**
Semester: 5th Sem. **Time: 2:30 PM – 4:00 PM**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF MECHANICAL AND PRODUCTION ENGINEERING(MPE)

Semester Final Examination **Winter Semester:** 2020 - 21
Course Number: Math 4541 **Full Marks:** 75
Course Title: Multivariable Calculus and Complex Variables **Time:** 1.5 Hours

Answer all **three** questions. The symbols have their usual meanings. The examination is **Online**. Marks of each question and corresponding CO and PO are written in the brackets. You may not use your books, notes, or any programmable calculator and cell phone on this exam.

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1. **(a)** What exactly is the distinction between a horizontal trace and a level curve? Discuss the connection between them. **(5) (CO2)**
(5) (PO1)

(b) Is it possible for the level curves of two distinct functions to intersect? Explain **(15)**

(c) Sketch the domains of:

$(i)\ f(x,y)=\sqrt{9-x^2-y}$ $(ii)\ g(x,y,z)=x\sqrt{y}+\ln(z-1)$

What are the ranges of these functions?

2. **(a)** What is the value of $f(2,3)$ if $f(x,y)$ is continuous at (2, 3) and $f(2,y)=y^3$ for $y\neq 3$? **(5) (CO1/CO2) (PO1)**

(b) Examine: $\lim\limits_{x\to(0,0)} \dfrac{x^2}{x^2+y^2}$ numerically and then prove that the limit does not exist. **(8)**

(c) (i) In your explanation, explain why it is not essential to utilize the Quotient Rule to compute $\dfrac{\partial}{\partial x}\left(\dfrac{x+y}{y+1}\right)$. Should the Quotient Rule be applied to calculate $\dfrac{\partial}{\partial y}\left(\dfrac{x+y}{y+1}\right)$? **(5)**

(ii) Find an equation of the tangent plane to the graph of f $f(x,y)=xy^3+x^2$ at $(2,-2,f(2,-2))$. **(7)**

3. **(a)** Describe the two main geometric properties of the gradient ∇f. **(3) (CO3) (PO2)**

(b) The altitude of a mountain at (x, y) is $f(x,y)=2500+100\left(x+y^2\right)e^{-0.3y^2}$ where x, y are in units of 100 m.

(i) Find the directional derivative of f at *P(-1, -1)* in the direction of unit vector $\boldsymbol{u}$ making an angle of $\theta = \frac{\pi}{4}$ with the gradient is shown in Fig. 3(b) **(8)**

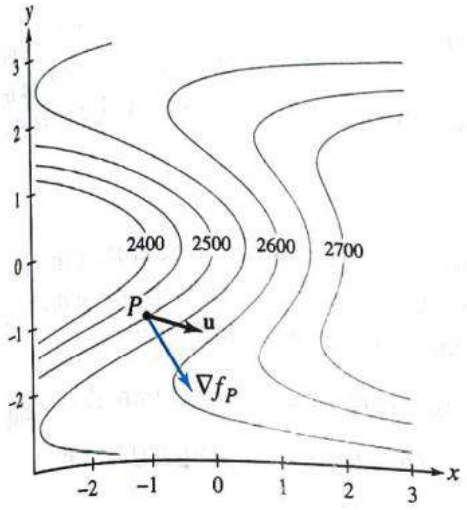

Fig. 3(b)

(ii) What is the interpretation of this derivative? **(4)**

(c) Find the point on the plane $\frac{x}{2} + \frac{y}{4} + \frac{z}{4} = 1$ closest to the origin in R^3. **(10)**