

**ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)****ORGANISATION OF ISLAMIC COOPERATION (OIC)****Department of Computer Science and Engineering (CSE)****MID SEMESTER EXAMINATION****WINTER SEMESTER, 2020-2021****DURATION: 1 Hour 30 Minutes****FULL MARKS: 75****MATH 4741: Mathematical Analysis****Programmable calculators are not allowed. Do not write anything on the question paper.**There are **4(four)** questions. Answer all **3 (three)** of them.

Figures in the right margin indicate marks.

1. a) **Uganda Metro Rail** has finally started being operational and in the first week of service it was recorded that some people are using the Metro while others are still commuting by Bus. Suppose that the chance of a certain **Sakif** using Metro tomorrow depends on his previous days travel only through whether or not he used Metro or Bus and not on past travels. If he uses Metro today, then he will use Metro tomorrow with probability **0.85** and if he does not use Metro today, then he will use Metro tomorrow with probability **0.40**. **7**  
**CO2**  
**PO1**
- Calculate** the probability that Sakif will use **Metro Six days from today** given he traveled by **Metro** today.
- b) **Shoumik** is interested to determine the proportion of society that has an upper-class or lower-class occupation. Assume that transitions between social classes of the successive generations in a family can be regarded as transitions of a Markov chain and the occupation of a child depends only on his or her parent's occupation. **4+7+7**  
**CO2**  
**CO3**  
**PO2**
- Suppose that the child of the **A-class** worker will attain an A-, B-, or C-class occupation with respective probabilities **0.52, 0.45, 0.03**. The child of a **B-class** worker will attain an A-, B-, or C-class occupation with respective probabilities **0.05, 0.60, 0.35**. The child of a **C-class** worker will attain an upper-, middle-, or lower-class occupation with respective probabilities **0.01, 0.34, 0.65**.
- Calculate** the Limiting Probabilities.
  - In the long run**, what proportion of time is the process in each of the three states?
  - Calculate** the probability of the following sequence of social classes **{A-class > C-class > C-class > B-class > C-class}** in the successive generations of Shoumik's family.
2. a) Let the Markov chain consisting of the states 0, 1, 2, 3, 4 have the transition probability matrix **P**. **Determine** which states are transient and which are recurrent. **7**  
**CO2**  
**PO1**

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

- b) A simple example of a Hidden Markov Model (HMM) is predicting the weather (hidden) based on the type of Clothes that someone wears (observed). We can use Viterbi algorithm to compute the most probable weather sequence (hidden) for a particular clothes (observed) sequence. 11  
C02  
P02

For any scenario of your own choice, Justify “**There are often many state sequences that can produce the same particular output sequence, but with different probabilities.**”

- c) Suppose **Shofi** and **Afridi** decide to flip pennies, the one coming closest to the wall wins. Afridi being the better player, has a probability 0.55 of winning on each flip. If Afridi starts with four pennies and Shofi with ten, **what is the probability that Afridi will wipe Shofi out?** 7  
CO2  
PO1
3. a) Suppose that an airplane engine will fail, when in flight, with probability  $1 - p$  independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what value of  $p$  is a four-engine plane preferable to a two-engine plane? 7  
CO1  
PO1
- b) The traffic authority is facing peak traffic congestion at **GG** Tower junction road. During office time at an average rate of 555 automobiles per hour passes through the junction. 11  
CO3  
PO3
- Find the probability that none passes in a given minute.
  - What is the expected number passing in two minutes?
  - Find the probability that this expected number actually pass through in a given two-minute period.
- c) Suppose that the amount of time that a light bulb works before burning itself out is exponentially distributed with mean ten hours. Suppose that **Nahian** enters a room in which a light bulb is burning. If he desires to work for five hours, then what is the probability that he will be able to complete his work without the bulb burning out? 7  
CO3  
PO3
4. a) **Sourov** is attending PHD program at Purdue Computer Science, Since he is from Electrical background he along with similar students’ needs to take and pass a Programming course to start his degree and as per requirements students must take the course during the first quarter that they enroll. If they do not pass the course they take it again in the second semester. If they fail twice, they are not permitted to retake it again, and so would be unable to continue their PHD at Computer Science. 7  
CO1  
PO3

Students can be in one of 4 states {P,E,R,F}: **P**-from CS background or already passed the course, **E**-first time enrolled in the course, **R**- failed once and retaking the course or **F**-failed twice and cannot retake. Data shows **65%** of students taking the class for the first time pass and **72%** of students taking the class for the second time pass.

**Identify the absorbing states and Find the probability of being absorbed eventually in each of the absorbing states.**

- b) At all times, an urn contains  $N$  balls—some white balls and some black balls. At each stage, a coin having probability  $p$ ,  $0 < p < 1$ , of landing heads is flipped. If heads appears, then a ball is chosen at random from the urn and is replaced by a white ball; if tails appears, then a ball is chosen from the urn and is replaced by a black ball. Let  $X_n$  denote the number of white balls in the urn after the  $n$ th stage. 4+7+7  
CO3  
PO2

- I. Is  $X_n$  a Markov chain? If so, **explain** why.
- II. Illustrate that if the chain is positive recurrent then the long-run proportions are the unique solution of the equations.
- III. If  $p = 1$ , what is the **expected time** until there are only white balls in the urn if initially there are  $i$  white and  $N - i$  black?

GOOD LUCK