ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2019-2020**

DURATION: 1 Hour 30 Minutes **FULL MARKS: 75**

Math 4641: Numerical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **4 (four)** questions. Answer any **3 (three)** of them.

Figures in the right margin indicate marks.

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1. a) When approximating any mathematical model using numerical methods, how can you use relative approximate errors to minimize the error? Explain your answer with the help of Tailor's Remainder Theorem. 5

   b) What do you understand by truncation error and round off error. Explain both of them with the McLauren Series expansion of $e^{1.3}$ Your error should be calculated up to four significant digits. 8

   c) Given that *f(3) =6, f'(3) = 8, f''(3)+1=12,* $f'''(3)=f(3)+4$ and that all other higher order derivatives of *f(x)* are zero at x = 3, and assuming the function and all its derivatives exist and are continuous between x =3 and x = 4.5. Find out the value of f (4.3). 12

2. a) Explain the difference between interpolation and regression with appropriate example. 4

   b) Solve the following non-linear equation within the range [-1, 0] and [0, 1] with at least 3 iterations. 14

   $$f(x)=230x^4+18x^3+9x^2-221x-9$$

   In each of the iterations, calculate the relative approximate error. Your answer should be correct up to 4th digit.

   c) Why do we need to use Spline interpolation over Lagrange interpolation for higher order approximation? Explain with appropriate logic. Use figure if necessary. 7

3. a) In order to find out the values of *3n* number of unknowns, you need *3n* number of equations in Quadratic Spline Method. How can you get necessary *3n* number of simultaneous equations from *(n-1)* data points in Quadratic Spline method of interpolation? 12

   b) The following table lists the average weight of a type of larvae at times in the first 28 days after birth. 13

Day	0	6	10	13	17	20	28
Weight (mg)	6.67	17.33	42.67	37.33	30.10	29.31	28.74

   Find the average weight of the larvae species on 15th day after birth with the help of **Third order Lagrange** interpolation method.

4. a) Explain why we minimize the sum of square of the residuals instead of absolute value of the residuals in regression with sound mathematical reasoning. 6

   b) The following table contains the approximated number of COVID-19 cases worldwide. Fit the following data into appropriate regression model and predict the approximate number of COVID-19 cases in the month of November and December 2020. 13

Month ->	March	April	May	June	July	Aug	Sep
COVID cases (in millions)	0.01	0.5	1.2	1.8	2.8	5.0	6.1

   c) Which method is better between Newton-Raphson and Secant method for finding out the root of a non-linear equation? Justify your answer with appropriate reason. 6

Some Necessary Formula

Taylor Theorem:

$$f(x+h)=f(x)+f'(x)h+\frac{f''(x)}{2!}h^2+\frac{f'''(x)}{3!}h^3+\cdots$$

Lagrange Interpolation:

$$f_n(x)=\sum_{i=0}^{n}L_i(x)f(x_i)$$

$$L_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_i-x_j}$$

Linear Regression:

$$a_1=\frac{n\sum_{i=1}^{n}x_iy_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}$$

$$a_0=\frac{\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_iy_i}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}$$

Growth Model (1):

$$y=\frac{a}{1+be^{-cx}}$$

Polynomial Model:

$$y=a_0+a_1x+a_2x^2+\cdots\cdots+a_mx^m,\ m<n$$

Exponential Model:

$$y=ae^{bx}$$

Growth Model (2)

$$y=\frac{ax}{b+x}$$