B.Sc. Engg. CSE 6th Semester (82) **January 11, 2021**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL **SUMMER SEMESTER, 2019-2020**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

Math 4641: Numerical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 3 **(three)** questions. Answer **all** of them. Figures in the right margin indicate marks.

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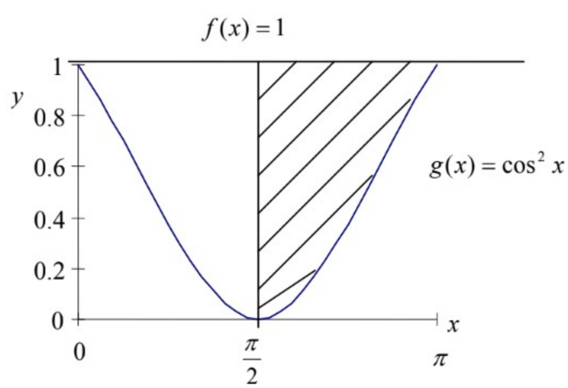
1. a) With the help of Taylor's series, show that the error in the central difference method for first derivative is less than both forward and backward difference method. 12

   b) The upward velocity of a rocket is given at three different times in the following table. 13

Time, t (s)	Velocity, v (m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as $v(t)=a_1t^2+a_2t+a_3$ within the range $5\le t\le 12$. Find the values of a_1, a_2 and a_3 with the help of Naive Gaussian Elimination method. Also find the acceleration of the rocket at $t=10$.

2. a) Find the area of the shaded region between the given functions $f(x)$ and $g(x)$ within the limits $[\pi/2, \pi]$ given in the following figure: 12



   b) The distance covered by a rocket in meters from $t=8s$ to $t=30s$ is given by the following formula. 13

$$x=\int_{8}^{30}\left(2000\ln\left[\frac{140000}{140000-2100t}\right]-9.8t\right)dt$$

   i. Use Simpson's $\frac{1}{3}$ rule to find the approximate value of x.

   ii. Find the true error. Find the absolute integral using calculator.

   iii. Find the absolute relative true error $|\epsilon_t|$

3. a) Find the ***exact area*** of the region between the parabola $y=x^2$ and the X-axis on the interval *[0,b]*. Use the Riemann's sum method. 15

b) Derive the Simpson's 1/3 rule from second order polynomial equation. 10

Appendix (Necessary Formula):

Derivatives

$f(x)$	$f'(x)$
$x^n, n \neq 0$	nx^{n-1}
$kx^n, n \neq 0$	knx^{n-1}
$sin(x)$	$cos(x)$
$cos(x)$	$-sin(x)$
$tan(x)$	$sec^2(x)$
$sinh(x)$	$cosh(x)$
$cosh(x)$	$sinh(x)$
$tanh(x)$	$1-tanh^2(x)$
$sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$tan^{-1}(x)$	$\frac{1}{1+x^2}$
$csc(x)$	$-csc(x)cot(x)$
$sec(x)$	$sec(x)tan(x)$
$cot(x)$	$-csc^2(x)$
$csch(x)$	$-coth(x)csch(x)$
$sech(x)$	$-tanh(x)sech(x)$

$coth(x)$	$1-coth^2(x)$
$csc^{-1}(x)$	$-\frac{\lvert x\rvert}{x^2\sqrt{x^2-1}}$
$sec^{-1}(x)$	$\frac{\lvert x\rvert}{x^2\sqrt{x^2-1}}$
$cot^{-1}(x)$	$\frac{-1}{1+x^2}$
a^x	$ln(a)a^x$
$ln(x)$	$\frac{1}{x}$
$log_a(x)$	$\frac{1}{xln(a)}$
e^x	e^x

Maclaurin's series: $f(x) = f(0) + x.f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$

Taylor's series: $f(x+h) = f(x) + h.f'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \ldots$

Table 1 A brief table of integrals

$\int dx = x + C$	$\int \sin x dx = -\cos x + C$
$\int a\, f(x)dx = a\int f(x)dx + C$	$\int \cos x dx = \sin x + C$
$\int [u(x) \pm v(x)]dx = \int u(x)dx \pm \int v(x)dx + C$	$\int \tan x dx = -\ln\lvert\cos x\rvert + C = \ln\lvert\sec x\rvert + C$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int \sec(ax)dx = \frac{1}{a}\ln\lvert\sec(ax) + \tan(ax)\rvert + C$
$\int u\, dv = uv - \int v\, du + C$	$\int \cot x dx = -\ln\lvert\csc x\rvert + C = \ln\lvert\sin x\rvert + C$
$\int \frac{dx}{ax+b} = \frac{1}{a}\ln\lvert ax+b\rvert + C$	$\int \sec^2 ax dx = \frac{1}{a}\tan(ax) + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \sec(x)\tan(x)dx = \sec(x) + C$
$\int e^{ax} dx = \frac{e^{ax}}{a} + C$	$\int \csc(x)\cot(x)dx = -\csc(x) + C$