

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER EXAMINATION
DURATION: 1 Hour 30 Minutes

WINTER SEMESTER, 2014-2015
FULL MARKS: 75

Math 4107: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.
 There are **4 (four)** questions. Answer any **3 (three)** of them.
 Figures in the right margin indicate marks.

- a) If transformation equation of $ax^2 + 2hxy + by^2 = 0$ is $a'x^2 + 2h'xy + b'y^2 = 0$ when the direction of axes is turned through an angle θ without change of origin, then show that $a + b$ and $ab - h^2$ are invariant i.e. $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$. 7
- b) Find the condition that the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight line. 8
- c) Show that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$. Also show that the distance between them is $2\sqrt{\frac{(g^2 - ac)}{a(a+b)}}$. 10
- a) If two straight lines represented by the equation $x^2(\tan^2 \phi + \cos^2 \phi) - 2xy \tan \phi + y^2 \sin^2 \phi = 0$ makes angles α and β with the x -axis respectively, then show that $\tan \alpha - \tan \beta = 2$. 7
- b) If one of the lines $ax^2 + 2hxy + by^2 = 0$ is perpendicular to one of the lines $a'x^2 + 2h'xy + b'y^2 = 0$, prove that $(aa' - bb')^2 + 4(a'h + bh')(ah' + b'h) = 0$. 9
- c) Prove that the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ always represent a circle for all values of f, g and c ; find its centre and radius. Also discuss real circle, imaginary circle and point circle. 9
- a) Define pole and polar with sketch graph. 5
- b) Find the equation of the circle inscribed in the triangle formed by the lines $2x - 3y + 21 = 0$, $3x - 2y - 6 = 0$, $2x + 3y + 9 = 0$. 10
- c) Define limiting points of a co-axial system of circles. Find the co-ordinates of limiting points of the co-axial system determined by the circles $x^2 + y^2 + 2x - 6y = 0$ and $2x^2 + 2y^2 + 10y + 5 = 0$. 10
- a) The direction cosines of two straight lines are given by the relations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$. Prove that the straight lines will be perpendicular if $a^2(v+w) + b^2(w+u) + c^2(u+v) = 0$ and parallel if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$. 12
- b) A point p moves on a fixed plane, the plane through p perpendicular to op meets the axes in A, B, C respectively. If the planes are drawn through A, B, C parallel to the co-ordinate planes, then show that the locus of the point intersection is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}$. 13