

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

WINTER SEMESTER EXAMINATION

TIME: 1 Hour 30 Minutes

WINTER SEMESTER, 2013-2014

FULL MARKS: 75

MATH 4107: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 4 (four) questions. Answer any 3 (three) of them.
Figures in the right margin indicate marks.

a) Discuss the continuity and differentiability of the function

$$f(x) = \begin{cases} x^2 + ax + b, & x \leq 1 \\ x + 1, & 1 < x \leq 3 \\ x^2 + ax + b, & x > 3 \end{cases}$$

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at the points $x = 1$ and $x = 3$.

b) Evaluate the following limit:

$$\lim_{x \rightarrow \infty} \left[\frac{e^x}{x^4} \right]$$

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c) Describe L' Hospital's theorem and evaluate:

$$\lim_{x \rightarrow 0} (\sin x)^{\tan x}$$

8

a) State and prove Leibnitz's theorem.

b) If $y = a \cos(\ln x) + b \sin(\ln x)$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$.

c) If $y = \sin^4 x \cos^5 x$, find the n -th derivative of y by De-Moivre's theorem.

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a) State Rolle's theorem.

b) Verify Rolle's theorem for the function $f(x) = 2x^3 + 2x^2 - 10x + 6$.

c) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that $xu_x + yu_y + zu_z = 0$.

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a) If $V = \sin^{-1} \left[\frac{x^2 + y^2}{x + y} \right]$, show that $xV_x + yV_y = \tan V$.

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b) If $u = F(y - z, z - x, x - y)$, prove that $\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} + \frac{\delta u}{\delta z} = 0$.

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c) Define stationary point and critical point. Find the maximum and minimum value of the function by first derivative test of the function $f(x) = x^{2/3}(x^2 - 8)$.