B.Sc. Engg. CSE 7th Semester

30 September 2022 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **WINTER SEMESTER, 2021-2022**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 75**

Math 4741: Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all** **3 (three)** questions. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

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1. a) i. With a brief explanation, illustrate the Josephus Problem with a diagram. 5 (CO1) (PO1)

   ii. When sending messages over a network, there is a chance that the bits will be corrupted. A Hamming code allows for a 4 bit code to be encoded as 7 bits, with the advantage that if 0 or 1 bit(s) are corrupted, then the message can be perfectly reconstructed. 5 (CO1) (PO1)

   You are working on the Voyager space mission and the probability of any bit being lost in space is 0.1. Determine the improvement in reliability when using a Hamming code with 7 bits instead of 4 bits.

   b) Suppose that an airplane engine will fail, when in flight, with probability $(1-p)$ independent of all engines. Again, the airplane will make a successful flight if at least 50 percent of its engines remain operative. Determine the value of p for which a four-engine plane preferable to a two-engine plane. 7 (CO2) (PO2)

   c) Suppose that the amount of time that a light bulb works before burning itself out is exponentially distributed with a mean of 10 hours. Suppose, that Dobby enters a room in which a light bulb is burning. If he desires to work for 5 hours, find out the probability that he will be able to complete his work without the bulb burning out. 8 (CO3) (PO2)

2. a) A box contains three coins: two regular coins with head on one side and tail on the other side and one fake coin with heads on both sides.

   i. You pick a coin at random and toss it. Find out the probability that it lands heads up. 2 (CO1) (PO1)

   ii. You pick a coin at random and toss it, and get heads. Find out the probability that it is the two-headed coin. 2 (CO1) (PO1)

   b) i. Assume that a rare disease infects 1 out of every 10000 people in a population. There is a good, but not perfect,test for this disease: if a person has the disease, the test comes back positive 98% of the time. However, 7% of uninfected people also test positive. Given that someone just tested positive, determine their chances of having this disease. 5 (CO2) (PO2)

   ii. An individual uses the following gambling system in Las Vegas. He bets \$1 that the roulette wheel will come up red. If he wins, he quits. If he loses then he makes the same bet a second time only this time he bets \$2; and then regardless of the outcome, quits. Assuming that he has a probability of 0.5 of winning each bet, find out the probability that he goes home a winner. 5 (CO3) (PO2)

c) The traffic authority is facing low levels of traffic at GG Tower junction road. Late at night, at an average 5 automobiles per hour pass through the junction.

i. Find the probability that no one passes in a given minute. 3 (CO3) (PO2)

ii. Determine the expected number passing in two minutes. 4 (CO3) (PO2)

iii. Find the probability that this expected number actually passes through in a given two minute period. 4 (CO3) (PO2)

3\. a) In a certain species of rats, black dominates over brown. Suppose that a black rat with two black parents has a brown sibling.

i. Find out the probability that this rat is a pure black rat as opposed to being a hybrid with one black and one brown gene. 3 (CO1) (PO1)

ii. Suppose that when the black rat is mated with a brown rat, all five of their offspring are black. Now, find out the probability that the rat is a pure black rat. 5 (CO2) (PO2)

b) In a sequence of independent flips of a biased coin (probability of a head is 0.65), let N denote the number of flips until there is a run of three consecutive heads. Find

i. $P(N \leq 8)$ 5 (CO2) (PO2)

ii. $P(N = 8)$ 5 (CO2) (PO2)

c) Borsha tosses a coin repeatedly. The coin is unfair and $P(H)=p$. The game ends the first time that two consecutive heads (HH) or two consecutive tails (TT) are observed. She wins if HH is observed and loses if TT is observed. For example, if the outcome is HTHTT, she loses. On the other hand, if the outcome is THTHTHH, she wins.
Find the probability that she wins 7 (CO2) (PO2)