B.Sc. Engg. CSE 7th Semester

2 December 2022

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION | **WINTER SEMESTER, 2021-2022**

DURATION: 3 HOURS | **FULL MARKS: 150**

Math 4741: Mathematical Analysis

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer all 6 (six) questions. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

---

1. a) Bonus-malus system is based on penalizing insurance holders who are responsible for one or more claims by a premium surcharge, and awarding insurance holders with a premium discount if they have few claims. 15 (CO1) (PO1)

If we suppose that the number of yearly claims made by a particular policyholder is a Poisson random variable with a mean of 0.4, then what is the average annual premium paid by an insured?

		Next State if				
State	Annual Premium ($)	0 Claim	1 Claim	2 Claims	3 Claims	>= 4 Claims
1	100	1	2	2	2	3
2	270	1	2	2	3	3
3	450	2	2	3	3	3

b) i. If 3% of electronic units manufactured by a company are defective, find the probability that in a sample of 200 units, less than 2 bulbs are defective. 3 (CO1) (PO1)

ii. An insurance company insures a large number of homes. The insured value, $\$X$, of a randomly selected home is assumed to follow the given distribution with the probability density function: 3 (CO1) (PO1)

$$f(x) = \begin{cases} \frac{3}{x^4} & x > 1, \\ 0 & otherwise \end{cases}$$

Given that a randomly selected home is insured for at least \$1.5, calculate the probability that it is insured for less than \$2.

iii. Let X be a continuous random variable with PDF given by: 4 (CO1) (PO1)

$$f(x) = \frac{1}{2} e^{-|x|}, \text{ for all } x \in \mathbb{R}$$

If $Y = X^2$, find the CDF of Y and represent it as a graph.

2. a) i. Marvin is arguing with his mom about buying a \$150 light. His mom wants him to be safe, but Marvin thinks that lights are too expensive. Based on traffic data, Marvin makes the table below which shows the time, probability, and cost of three different types of accidents if Marvin does not have the light. If Marvin purchases the light, it will not matter in the morning or at dusk, but it will prevent a night accident from occurring. Assume that Marvin can get in at most one accident and that Marvin's table is accurate. Compare the expected total cost of accidents and lights in the cases that Marvin buys the light and in the case that he does not buy the light. If Marvin wants the best payoff in the long run, what should he do? 10 (CO2) (PO2)

Time of accident	Cost (\$)	Probability (%)
Morning	2000	10
Dusk	4000	15
Night	2000	20

ii. After COVID peaked, millions of Americans quit their jobs to take advantage of a tight labor market, a phenomenon being called the great resignation. Suppose Dave is unsure whether he should take part in the great resignation or not. Dave is currently working for \$40,000 a year. He thinks that if he quits his job and works for himself that there is a 50% chance he could earn \$20,000 in their first year, a 30% chance he could earn \$60,000, and a 10% chance he would earn \$0. There is a 10% chance he falls sick, in which case he will need to spend \$10,000 in medical fees in one year. If he needs \$20,000 annually for his expenses, should he quit his job? 5 (CO3) (PO3)

b) i. Suppose variable X has a binomial distribution with parameters n= 6 and p= 0.5. Show that $X = 3$ is the most likely outcome over a large number of trials. 2 (CO1) (PO1)

ii. What would the most likely outcome over a large number of trials be, if X has a geometric distribution? 2 (CO1) (PO1)

c) i. Suppose you order a pizza from your favorite pizzeria at 7:00 pm, knowing that the time it takes for your pizza to be ready is uniformly distributed between 7:00 pm and 7:30 pm.
What is the probability that you will have to wait longer than 10 minutes for your pizza? 3 (CO1) (PO1)

ii. If at 7:15pm, the pizza has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes? 3 (CO1) (PO1)

3. a) Explain the following with examples:
i. Memoryless property 2 (CO1) (PO1)

ii. Transient states 2 (CO1) (PO1)

b) Consider a rover that operates on a slope and uses solar panels to recharge. It can be in one of three states: high, medium and low on the slope. If it spins its wheels, it climbs the slope in each time step (from low to medium or from medium to high) or 5 (CO2) (PO2) stays high. If it does not spin its wheels, it slides down the slope in each time step (from high to medium or from medium to low) or stays low. Spinning its wheels uses one unit of energy per time step. Being high or medium on the slope gains three units of energy per time step via the solar panels, while being low on the slope does not gain any energy per time step. The robot wants to gain as much energy as possible. Represent this problem as a Markov Decision Process graphically.

c) Suppose that a production process changes states in accordance with the Markov chain given below S={1, 2, 3} and that states 1 and 3 are considered acceptable and the remaining state is unacceptable.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

i. What is the rate of breakdowns (rate of change from acceptable to unacceptable states) in this process? 10 (CO1) (PO1)

ii. What is the average length of time that the process is in acceptable states? 3 (CO1) (PO1)

iii. What is the average length of time that the process is in unacceptable states? 3 (CO1) (PO1)

4. a) Poker is a classic game of chance, in which each player is dealt a hand of five cards. The player with the best five-card hand wins. For three of a kind, the hand will look like {x,x,x,y,z} for some distinct values. What is the probability of getting a hand with three of a kind? 6 (CO1) (PO1)

b) i. Explain the Gambler's Ruin problem with a state transition diagram. 5 (CO1) (PO1)

ii. Let P_i denote the probability that, starting with i, the gambler's fortune will eventually reach N. Derive an equation for P_i. 10 (CO2) (PO2)

iii. Suppose Dan and Mary decide to flip pennies; the loser gives their penny to the winner. Mary has a probability 0.55 of winning on each flip. If Mary starts with 7 pennies and Dan with 19, what is the probability that Dan will take all of Mary's pennines? 4 (CO1) (PO1)

5. a) i. Suppose that whether or not it is sunny today depends on previous weather conditions through the last two days. If it has been sunny for the past two days, then it will be sunny tomorrow with probability 0.7; if it is sunny today but not yesterday, then it will be sunny tomorrow with probability 0.5; if it was sunny yesterday but not today, then it will be sunny tomorrow with probability 0.4; if it was not sunny in the past two days, then it will be sunny tomorrow with probability 0.2. Denote the weather conditions as a Markov chain and create a state transition matrix. 6 (CO2) (PO2)

ii. Given that it was sunny on Sunday and Monday, what is the probability that it will not be sunny on Wednesday? 9 (CO2) (PO2)

b) A certain bus provides service in two zones of a city. Fares picked up in zone A will have destinations in zone A with probability 0.7 or in zone B with probability 0.3. Fares picked up in zone B will have destinations in zone A with probability 0.8 or in zone B with probability 0.2. The expected profit for a trip entirely in zone A is 5; for a trip entirely in zone B is 9; and for a trip that involves both zones is 14. Find the bus's average profit per trip. 10 (CO4) (PO3)

6. a) An urn always contains 2 balls which may be colored red or blue. At each stage a ball is randomly chosen and then replaced by a new ball. There is a probability of 0.8 that it is of the same color, and a probability of 0.2 that it is of the opposite color as the ball it replaces. If initially both balls are red, find the probability that the fifth ball selected is red. 6 (CO2) (PO2)

b) The Hardy-Weinberg Law is a principle stating that the variation in a population will remain constant from one generation to the next in the absence of disturbing factors. Design a statistical method by which this law can be proven. 9 (CO4) (PO3)

c) The number of traffic accidents on successive days are independent Poisson random variables with mean 2.

i. Find the probability that 3 of the next 5 days have two accidents. 5 (CO1) (PO1)

ii. Find the probability that there are a total of 6 accidents over the next 2 days. 5 (CO1) (PO1)