B.Sc. Engg. / HD CSE 7th Semester (75) **14 September 2021 (Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

FINAL SEMESTER EXAMINATION **WINTER SEMESTER: 2020-2021**

COURSE NUMBER: MATH 4741 **FULL MARKS: 75**

COURSE TITLE: MATHEMATICAL ANALYSIS **DURATION: 1 Hour 30 Minutes**

Programmable calculators are not allowed. Do not write anything on the question paper.

There are **3(three)** questions. Answer all **3 (three)** of them.

Marks of each question and corresponding CO and PO are written in brackets.

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1. a) Consider a two-server system in which customers arrive at a Poisson rate λ at server 1. After being served by server 1 they then join the queue in front of server 2. Suppose there is infinite waiting space at both servers. Each server serves one customer at a time with server i taking an exponential time with rate μi for a service, i = 1, 2. 15 CO3 PO2

   I. Calculate the probability that there are n customers at server 1 and 2
   II. Verify that the number of customers at server 1 is independent of the number at server 2
   III. Calculate the average time a customer spends in the system

   b) Students arriving at IUT student registration center must have their registration materials processed by an operator seated at a computer terminal. The system's design calls for 5 operators to be on duty, each operator performing an identical service. Students arrive according to a Poisson process at an average rate of 120 per hour. Each operator can process 30 students per hour with service time exponentially distributed. 10 CO2 CO3 PO2

   I. Calculate what fraction of the time that there are no students in the registration center?
   II. If the waiting room inside the IUT registration center will comfortably accommodate 4 students, what percentage of time will there be students lined up outside the building?
   III. How long is the line of students waiting to register?
   IV. On the average, can we expect a student arriving 3 minutes before closing time to make it just before closing time? (He or She is able to register)

2. a) ROBI, one of the largest mobile network operators in Bangladesh has 15 million customers and would like to earn additional profits by launching a smartphone upgrade program. The company's marketing team has created 6 different campaigns, each of which offers customers specific features, promotions, or discounts to try to entice them to upgrade. Though the company knows how much profit it will make per sale for each advertising campaign, it does not know how effective each campaign will be. So, they have reached out to the IUT CSE department students. 15 C04 P03

   Mathematically analyze an AI system that can maximize the company's profit for its smartphone upgrade program.

   b) Suppose that it costs **cμ**$/hour to provide service at a rate μ. Suppose also that we incur a gross profit of **A**$ for each customer served. If the system has a capacity N, what service rate μ maximizes our total profit? Let, N = 2, λ = 1, A = 20, c = 1. 10 CO2 PO3

3. a) In a sequence of independent flips of a biased coin (probability of a head is .65), let N denote the number of flips until there is a run of three consecutive heads. Find 10

I. $P(N \leq 8)$ CO2

II. $P(N = 8)$ PO2

b) In MDP our goal is to identify the optimal policy that maximizes the expected reward. State the steps (Bellman Equation) employed to calculate the maximum expected utility /reward. 10

CO3

PO3

c) In CMDP, there are multiple decision-makers, and the transition probabilities and rewards depend on the actions of all or a subset of all decision-makers. Identify 2 disadvantages of MDP for such cases. 05

CO2

PO2

GOOD LUCK

Equations

Single Channel – Poisson / Exponential Model (M/M/1)

$$\rho = \frac{\lambda}{\mu} \qquad \rho < 1$$

$$L = \frac{\lambda}{\mu - \lambda} \qquad L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W = \frac{1}{\mu - \lambda} \qquad W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$P_n = \rho^n (1 - \rho) \qquad P(n > N) = \left(\frac{\lambda}{\mu}\right)^{N+1}$$

Multiple Server, Poisson / Exponential Model (M/M/C)

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \cdot \frac{1}{1 - \frac{\rho}{c}}} \qquad \rho = \frac{\lambda}{\mu}$$

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0 & 0 \le n \le c \\ \frac{\rho^n}{c^{n-c} c!} P_0 & n \ge c \end{cases} \qquad \frac{\rho}{c} < 1$$

$$L_q = \frac{\rho^{c+1}}{c!\,c} \frac{1}{\left(1 - \frac{\rho}{c}\right)^2} P_0$$

Single Channel , Poisson Arrivals , Arbitrary Service Time (M/G/1)

$$\rho = \frac{\lambda}{\mu} \qquad \rho < 1$$

$$P_n = \rho^n(1 - \rho)$$

$$L = \frac{\lambda}{\mu} + \frac{\lambda^2 V(t) + (\rho)^2}{2(1 - \rho)}$$

$$L_q = \frac{\lambda^2 V(t) + (\rho)^2}{2(1 - \rho)}$$

$$W_s = \frac{L_s}{\lambda}$$

$$W_q = \frac{L_q}{\lambda}$$

Poisson Arrival And Service Rate , Infinite Number of Servers (M/M/ ∞)

$$\rho = \frac{\lambda}{\mu} \qquad \text{(No Restriction)}$$

$$P_n = \frac{\rho^n}{n!} e^{-\rho} \qquad \text{(Poisson Distributed)}$$

$$L = \rho$$

$$W = \frac{1}{\mu}$$

$$L_q = W_q = 0$$

Single Channel – Poisson / Exponential Model (M/M/1) Finite Queue

$$P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{m+1}}; & \rho \neq 1 \\ \dfrac{1}{m+1}; & \rho = 1 \end{cases}$$

$$\lambda_e = \lambda(1 - P_m)$$

$$P_n = \begin{cases} \rho^n P_0; & \rho \neq 1 \\ \dfrac{1}{m+1}; & \rho = 1 \end{cases}$$

$$L = \frac{\rho}{1-\rho} - \frac{(m+1)\rho^{m+1}}{1-\rho^{m+1}} \qquad \rho \neq 1$$

$$L = \frac{m}{2} \qquad \rho = 1$$