

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION

SUMMER SEMESTER, 2020-2021

DURATION: 1 HOUR 30 MINUTES

FULL MARKS: 75

CSE 4803: Graph Theory**Programmable calculators are not allowed. Do not write anything on the question paper.**Answer **all 3 (three)** questions. Marks of each question and corresponding CO and PO are written in the right margin.

1. a) A graph has 6 vertices and 10 edges. Must it be planar? Must it be non-planar? Justify your answer. 6
(CO2)
(PO2)
- b) Determine with explanation whether the following statements are true or false: 6
 - i. Any graph homeomorphic to an Eulerian graph must also be Eulerian. (CO1)
 - ii. Any graph homeomorphic to a Hamiltonian graph must also be Hamiltonian. (PO1)
- c) Let T be a tree. Each vertex of T has either degree a or degree b . Among the vertices of T , 9 vertices have degree a , while 92 vertices have degree b . Calculate a and b . 5
(CO1)
(PO1)
- d) The Petersen graph is one of the most popular graphs in Graph Theory. It is an undirected graph with ten vertices and fifteen edges, commonly drawn as a pentagram within a pentagon, with corresponding vertices attached to each other. It has many unusual mathematical properties, and has frequently been used as a counterexample to conjectures in Graph Theory. One drawing of the Petersen graph is given in Figure 1. 3x2
(CO2)
(PO2)

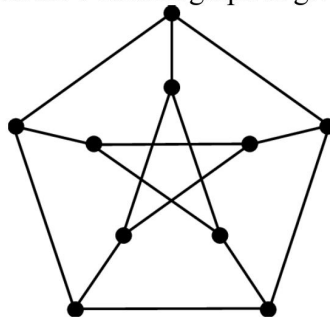


Figure 1: The Petersen Graph

Answer the followings with proper justification:

- i. Is the Petersen graph Eulerian?
 - ii. Is it Hamiltonian?
 - iii. Is it bipartite?
2. a) The graph H has a node set $\{a, b, c, d, e\}$ and nine edges, consisting of all possible edges except (c, e) . 6
(CO2)
(PO2)
 - i. Are all graphs with five nodes and nine edges isomorphic to H ? Justify your answer. (PO2)
 - ii. Is H a planar graph? Prove your answer.
 - b) Construct the labelled tree which has the Prüfer code $(6, 7, 6, 4, 3)$. 8
(CO1)
(PO1)
 - c) Let T be a tree, all of whose vertices have degree either 1 or 3. Such a tree is called trivalent and often occurs in evolutionary biology. If T has n leaves, show that it has $n - 2$ vertices of degree 3. 5
(CO1)
(PO1)

- d) Woody and Buzz were instructed by their boss Mr. Andy to design a model of a communication network. Woody is the more industrious of the two and started working early. He built a model with n nodes and it has weighted links where the weight of each link is either 1 or 2. Buzz, the lazy one, started working on the project deadline. He copied the model of Woody, but to make it unique he deleted all the links with weight 2 keeping only the weight 1 links. When Mr. Andy checked the models, he found a Hamiltonian circuit in Buzz's model.

What is the weight of a minimum spanning tree of Woody's model? Justify your answer.

3. Reunion Game:

Class of 2011 of Imperial University of Texas (IUT) arranged a reunion to celebrate 10 years of their graduation. To make this occasion more enjoyable, they arranged some childhood puzzle games after the dinner. To their surprise, it was found that many of the games remained unsolved. None of them could complete them despite being professional engineers. Even the organizers failed to justify the reason. As many of the puzzles can be formulated into graph theory, we, the graph theory enthusiasts, should have a look at these problems.

- a) **Round 1:** Each of the participants were given a piece of paper and a pencil. They were supposed to draw 9 rectangles in the plane such that each of them intersects either 1 or 3 other rectangles. Is it possible? If yes, draw a possible solution. If not, justify. 5
(CO2)
(PO2)
- b) **Round 2:** "There was once a mighty king *Ozymandias* with his five sons. He used to rule a very big empire. His sons were the most ferocious fighters known to people. The king was worried that his sons may start a war against each other for the throne after his death, eventually destroying the kingdom. To solve this problem, he made a will. In his will he stated that after his death the sons should divide the kingdom into five regions so that the boundary of each region should have a frontier line in common with each of the other four regions." 7
(CO2)
(PO2)
- Participants, provided with a map of the kingdom, were asked to divide it according to the will. Is there any possible solution for this problem? If yes, draw a possible solution. If not, justify.
- c) **Round 3:** Participants were provided with 10 equal rectangular pieces of paper, say, the standard 3"×5" cards. Each of the cards came with one unique word written on it. Words were - **HEN, HUT, WIT, SAW, CAR, CUB, MOB, DIM, RED, SON**. The task was to arrange the cards in a rectangular closed chain in the manner of dominoes (The chain is shown in Figure 2). In the chain, any two adjacent words must share a common letter. 15
(CO2,
CO3)
(PO2,
PO3)

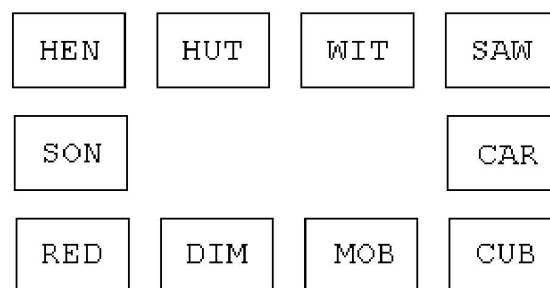


Figure 2: A possible arrangement of cards

Figure 2 may serve to demonstrate a possible arrangement. However, note that it is not a valid configuration. The words SON and RED, although adjacent, do not have a letter in common.

Is it possible to solve this problem? If yes, draw a valid arrangement. If not, justify.