Historical Indian Ocean Temperature Change

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1 Method

Our goal is to extract the decadal signal of water-mass change from the historical temperature observations. The historical-to-WOA temperature difference is computed by interpolating the World Ocean Atlas values onto the historical data locations. The historical-to-WOA temperature difference profile is computed by a least-squares method that accounts for contamination by measurement error and signals that are not representative of the decadal-mean, basinwide-average temperature. The contamination is assumed to have three parts: (1) transient effects such as isopycnal heave due to internal waves or mesoscale eddies, (2) irregular spatial sampling of each basin, and (3) measurement or calibration error of the thermometer. The expected size of (1) is taken from the WGHC error estimates and depends upon location. Following (*1*), the variance due to (2) is assumed to be 20% that of (1), and the standard error due to (3) is assumed equal to 0.14°C (*2*).

1.1 Vertical profile of temperature difference

The temperature difference between the historical observations and the corresponding World Ocean Atlas (WOA) value is,

$$\Delta T(r_i) = T(r_i, t_w) - T(r_i, t_h), \tag{1}$$

where $T(r_i, t_h)$ is the ith *historical* temperature observation at location, r_i, and time, t_h, and $T(r_i, t_w)$ is the WOA temperature at the same location. The observations are combined into a vector,

$$\mathbf{\Delta T} = \begin{pmatrix} \Delta T(r_1) \\ \Delta T(r_2) \\ \vdots \\ \Delta T(r_M) \end{pmatrix}. \tag{2}$$

Temperature changes at a given pressure are assumed equivalent to potential temperature changes.

1.2 Basinwide-average temperature profiles

Our goal is to extract the decadal signal of water-mass change from the historical temperature observations

$$\overline{\mathbf{\Delta T}} = \begin{pmatrix} \overline{\Delta\theta(z_1)} \\ \overline{\Delta\theta(z_2)} \\ \vdots \\ \overline{\Delta\theta(z)_K} \end{pmatrix}, \tag{3}$$

where we have defined a grid of K depths. Given knowledge of the basinwide averages, one can make a prediction for each WOA$-$historical temperature difference,

$$\mathbf{\Delta T} = \mathbf{V}\overline{\mathbf{\Delta T}} + \mathbf{q}, \tag{4}$$

where $\mathbf{V}$ maps the basinwide mean onto the observational point by noting the basin of the observations and vertical linear interpolation, $\mathbf{q}$ is contamination by measurement error and signals that are not representative of the decadal-mean, basinwide-average temperature. The contamination is decomposed into three parts,

$$\mathbf{q} = \mathbf{n}_T + \mathbf{n}_S + \mathbf{n}_M, \tag{5}$$

where $\mathbf{n}_T$ is contamination by transient effects such as isopycnal heave due to internal waves or mesoscale eddies, $\mathbf{n}_S$ is due to the irregular spatial sampling of each basin, and $\mathbf{n}_M$ is measurement or calibration error of the thermometer. Note that no depth correction is made here, and temperature differences may be biased toward warming.

The expected size of $\mathbf{n}_T$ is related to the energy in the interannual and higher-frequency bands. We use estimates from the WOCE Global Hydrographic Climatology (*3*) to quantify this error and its spatial pattern. Errors that primarily reflect an uncertainty due to a representativity error were previously estimated in this climatology, where the magnitude of interannual temperature variability is $1.6°$C at the surface, decreasing to $0.8°$C below the mixed layer, and $0.02°$C at 3000 meters depth. Inherent in their mapping is a horizontal lengthscale of $L_{xy}^T = 450$ km. This corresponds to a vertical lengthscale of $L_z^T = 450$ meters when applying an aspect ratio based upon mean depth and lateral extent of the ocean. Their mapping is the degree of error necessary to place the non-synoptic cruises of a 10-year time interval into a coherent picture. Estimated errors are similar to those of (*4*), who also note that the spatial scales increase as the temporal scales increase. Above 1300 meters depth, the aliased variability is typically larger than the measurement error described below.

Next we describe the second moment matrix of temporal contamination, $\mathbf{R}_{TT} =< \mathbf{n}_T(\mathbf{n}_T)^T >$. Note that $\mathbf{n}_T$ depends on the difference of contamination during the two time periods, $n_T(r_i) = \eta_T(r_i, t_w) - \eta_T(r_i, t_h)$, where $\eta_T(r, t)$ is the difference between temperature at a given time and the decadal average. The WGHC statistics give the error covariance for $\eta_T(r, t_w)$ not $n_T(r)$.

This covariance matrix is reconstructed by first creating a correlation matrix,

$$\mathbf{R}_\rho = \begin{pmatrix} \rho(0) & \rho(\delta) & \rho(2\delta) & \dots & \\ \rho(\delta) & \rho(0) & \rho(\delta) & \dots & \\ \rho(2\delta) & \rho(\delta) & \rho(0) & \dots & \\ \vdots & \vdots & \vdots & \ddots & \\ & & & & \rho(0) \end{pmatrix}, \tag{6}$$

where the autocorrelation function, $\rho(\delta)$, is given by a Gaussian with a horizontal lengthscale of 450 km and a vertical lengthscale of 450 meters. We derive the covariance matrix by pre- and post-multiplying the correlation matrix, $\mathbf{R}_{\eta\eta} = \boldsymbol{\sigma}_\eta \boldsymbol{\sigma}_\eta^T \circ \mathbf{R}_\rho$, where $\boldsymbol{\sigma}_\eta$ is the vector of the standard deviation of the WGHC interannual variability and $\circ$ is the Hadamard product. Here the time interval of the historical cruises is about 20 years, or twice as long as the WOCE era. Due to the red spectrum of ocean varabilility, the potential for aliased variability over this longer time interval is increased. We roughly approximate this extra energy with $T_{ratio} = 2$. Both the modern and historical intervals have variability and are assumed to be statistically independent, and thus, $\mathbf{R}_{TT} = 2T_{ratio}\mathbf{R}_{\eta\eta}$.

We assume that the variance due to spatial water-mass variability, i.e., $\mathbf{R}_{SS} =< \mathbf{n}_S(\mathbf{n}_S)^T >$, has a magnitude that is 20% that of the temporal variability as the local water-mass variability on interannual scales is dwarfed by heaving motions (*1*). The relevant parameter is $S_{ratio} = 0.2$. These water-mass variations are assumed to have a larger spatial scale ($L_{xy}^S = 2000$ km horizontally, $L_z^S = 1$ km vertically), as seen in an evaluation of water-mass fractions on an isobaric surface (*5*). Accounting for this spatial variability has the potential to increase the final error of our estimates by taking into account biases that may occur due to the specific expedition tracks. Finally, we assume that the measurement covariance, R_{MM}, is a matrix with the diagonal equal to the observational uncertainty, $\sigma_{obs} = 0.14$°C, squared (*2*).

We solve for the basinwide-average temperature profiles using a weighted and tapered least-squares formulation that minimizes,

$$J = \mathbf{q}^T \mathbf{R}_{qq}^{-1} \mathbf{q} + \mathbf{m}^T \mathbf{S}^{-1} \mathbf{m}, \tag{7}$$

where $\mathbf{R}_{qq}$ reflects the combined effect of the three types of errors (i.e., $\mathbf{R}_{qq} = \mathbf{R}_{TT} + \mathbf{R}_{SS} + \mathbf{R}_{MM}$). This least-squares weighting is chosen such that the solution coincides with the maximum likelihood estimate (assuming that the prior statistics are normally distributed and appropriately defined). Only a weak prior assumption, reflected in the weighting matrix, $\mathbf{S}$, is placed on the solution, namely that the correlation lengthscale is $L_z^{AVG} = 500$ m in the vertical, the variance is on the order of $(\sigma_S = 1°\text{C})^2$, and the expected value is $< \overline{\mathbf{\Delta T}} >= 0$. The least-squares estimate is then,

$$\tilde{\overline{\mathbf{\Delta T}}} = (\mathbf{V}^T \mathbf{R}_{qq}^{-1} \mathbf{V} + \mathbf{S}^{-1})^{-1} \mathbf{V}^T \mathbf{R}_{qq}^{-1} \mathbf{\Delta T}. \tag{8}$$

The error covariance of the estimate is,

$$\mathbf{C}_{\tilde{\Delta T}} = (\mathbf{V}^T \mathbf{R}_{qq}^{-1} \mathbf{V} + \mathbf{S}^{-1})^{-1}, \tag{9}$$

where the standard error is $\boldsymbol{\sigma}_{\tilde{\Delta T}} = \sqrt{\text{diag}(\mathbf{C}_{\tilde{\Delta T}})}$. This method also recovers the off-diagonal terms that correspond to the correlated errors among different parts of the basinwide-average.

1.3 Ocean heat content change

Ocean heat content change, $\Delta\mathcal{H}$, is a linear function of the temperature change and can be written as an inner vector product:

$$\Delta\mathcal{H} = \mathbf{h}^T \tilde{\overline{\mathbf{\Delta T}}}, \tag{10}$$

where $\mathbf{h}$ is a vector containing coefficients related to ocean heat capacity, seawater density, the representative area of the Indian Ocean, and the integration of temperature change over the vertical dimension. Here we integrate to a depth of $z\star = 700$ m so that we obtain heat content change from the sea surface to this depth. The Indian Ocean area is assumed to be equal to 15% of the global ocean area at all depths (ignoring the hypsometric effect).

The error covariance of $\Delta\mathcal{H}$ is an outer product,

$$\mathbf{C}_H = < (\mathbf{h}^T \tilde{\overline{\mathbf{\Delta T}}})(\mathbf{h}^T \tilde{\overline{\mathbf{\Delta T}}})^T >, \tag{11}$$

where $<>$ refers to the expected value. $\mathbf{C}_H$ is a scalar like $\Delta\mathcal{H}$. Rearranging this equation, we obtain,

$$\mathbf{C}_H = \mathbf{h}^T \mathbf{C}_{\tilde{\Delta T}} \mathbf{h}, \tag{12}$$

where $\mathbf{C}_{\tilde{\Delta T}}$ is known from the calculation of the previous section. The standard error of the heat content change is the square root of $\mathbf{C}_H$.

1.4 Historical thermometer corrections

The historical observations are corrected for the compressibility of the thermometers (*6*). Subsequent to the cruise, a bias of no more than 0.04°C per the equivalent of a kilometer of depth was estimated in pressure-tank experiments (*6*). In order to guard against biases that would predispose our analysis toward deep-Pacific cooling, temperature is adjusted to be 0.04°C cooler per kilometer of depth, in keeping with previous analyses (*2*). With this adjustment, the amount of Pacific cooling is diminished between the historical and recent eras.

We correct for pressure-compression effects altering the mercury level in the max-min thermometers used in the *historical* expedition. Subsequent to the cruise, a bias of no more than 0.04°C per the equivalent of a kilometer of depth was estimated in pressure-tank experiments (*6*). In order to guard against biases that would predispose our analysis toward deep-Pacific cooling, we adjust temperature to be 0.04°C cooler per kilometer of depth, in keeping with previous analyses (*2*). With this adjustment, the amount of Pacific cooling is diminished between the historical and WOA eras.

There are other issues related to the depth of the observation. In abyssal locations with temperature inversions, the max-min thermometer will lead to a cold bias that may obscure trends in temperature since the historical expeditions were completed. For this reason, points could be (but are not) eliminated from the analysis.

Results are labeled with ‘ tait‘ when pressure corrections are applied.

References and Notes

1. R. X. Huang, *Climate Dynamics* **45**, 3563 (2015).
2. D. Roemmich, W. J. Gould, J. Gilson, *Nature Climate Change* **2**, 425 (2012).
3. V. Gouretski, K. Koltermann, WOCE Global Hydrographic Climatology, *Tech. Rep. 35*, Berichte des Bundesamtes für Seeschifffahrt und Hydrographie (2004).
4. C. Wortham, C. Wunsch, *Journal of Physical Oceanography* **44**, 944 (2014).
5. G. Gebbie, P. Huybers, *J. Phys. Oceanogr.* **40**, 1710 (2010).
6. P. Tait, The pressure errors of the Challenger thermometers, Narr. Chall. Exp. Vol. II, Appendix A, *Tech. rep.*, HM Stationary Office (1882).