Linear Model

Least Square Method

- Finds the line of best fit for a dataset, providing a visual demonstration of the relationship between the data points.

- The differences between the actual and estimated function values on the training examples are called residuals

$$\epsilon_i = f(x_i) - \hat{f}(x_i).$$

- The *least-squares method*, consists in finding ˆ *f* such that $\sum_{i=1}^{n} \epsilon_i^2$ is minimised

Linear Regression

- **Simple:**

$$y = b_0 + b_1 * x$$

- **Multiple:**

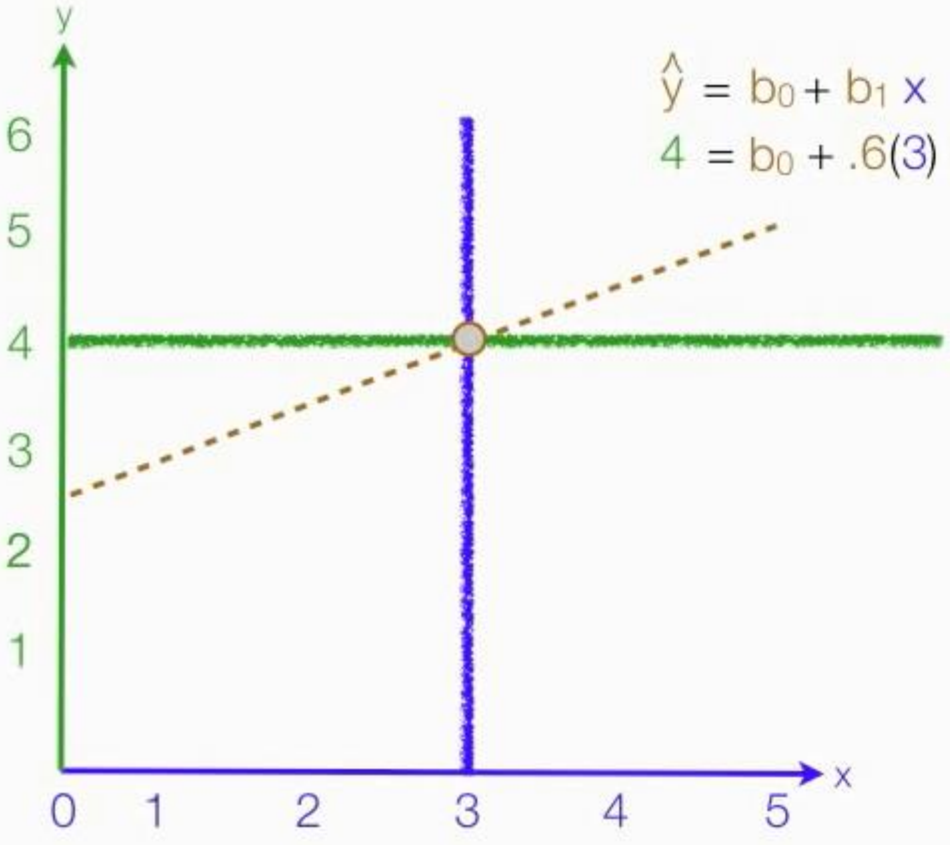
$$y = b_0 + b_1 * x_1 + \dots + b_n * x_n$$

Ex: Least Square method using Univariate Regression

x	y	x – x mean	y - y mean	$(x – x\ mean)^2$	(x – x mean). (y – y mean)
1	2	-2	-2	4	4
2	4	-1	0	1	0
3	5	0	1	0	0
4	4	1	0	1	0
5	5	2	1	4	2

$$b_0 = 2.2$$
$$b_1 = .6$$
$$\hat{y} = 2.2 + .6x$$

$$\hat{y} = b_0 + b_1 x$$
$$4 = b_0 + .6(3)$$

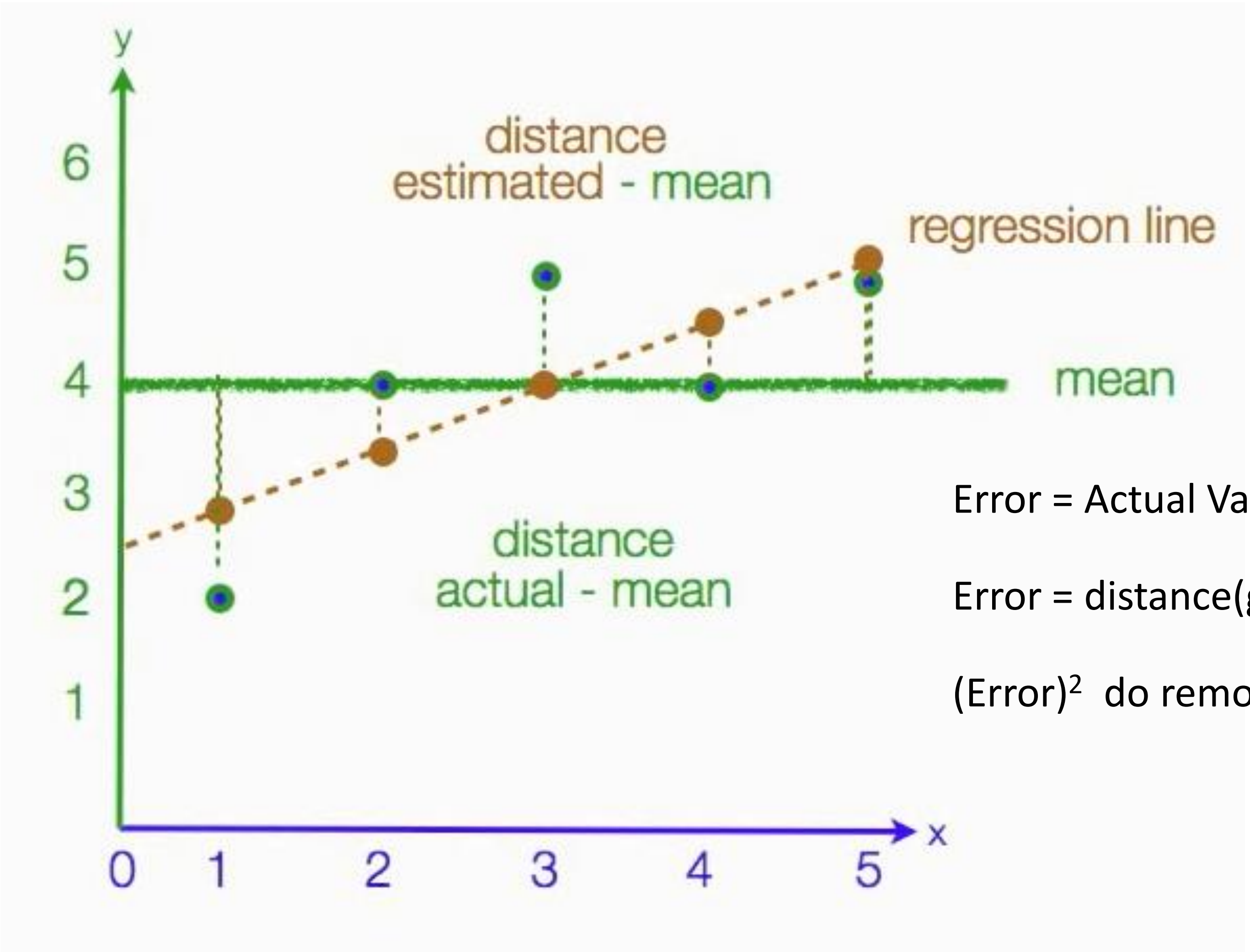


x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	2	-2	-2	4	4
2	4	-1	0	1	0
3	5	0	1	0	0
4	4	1	0	1	0
5	5	2	1	4	2
				10	6

mean 3 4

$$4 = b_0 + .6(3)$$
$$4 = b_0 + \cancel{1.8}$$
$$-1.8 \qquad \cancel{-1.8}$$
$$2.2 = b_0$$

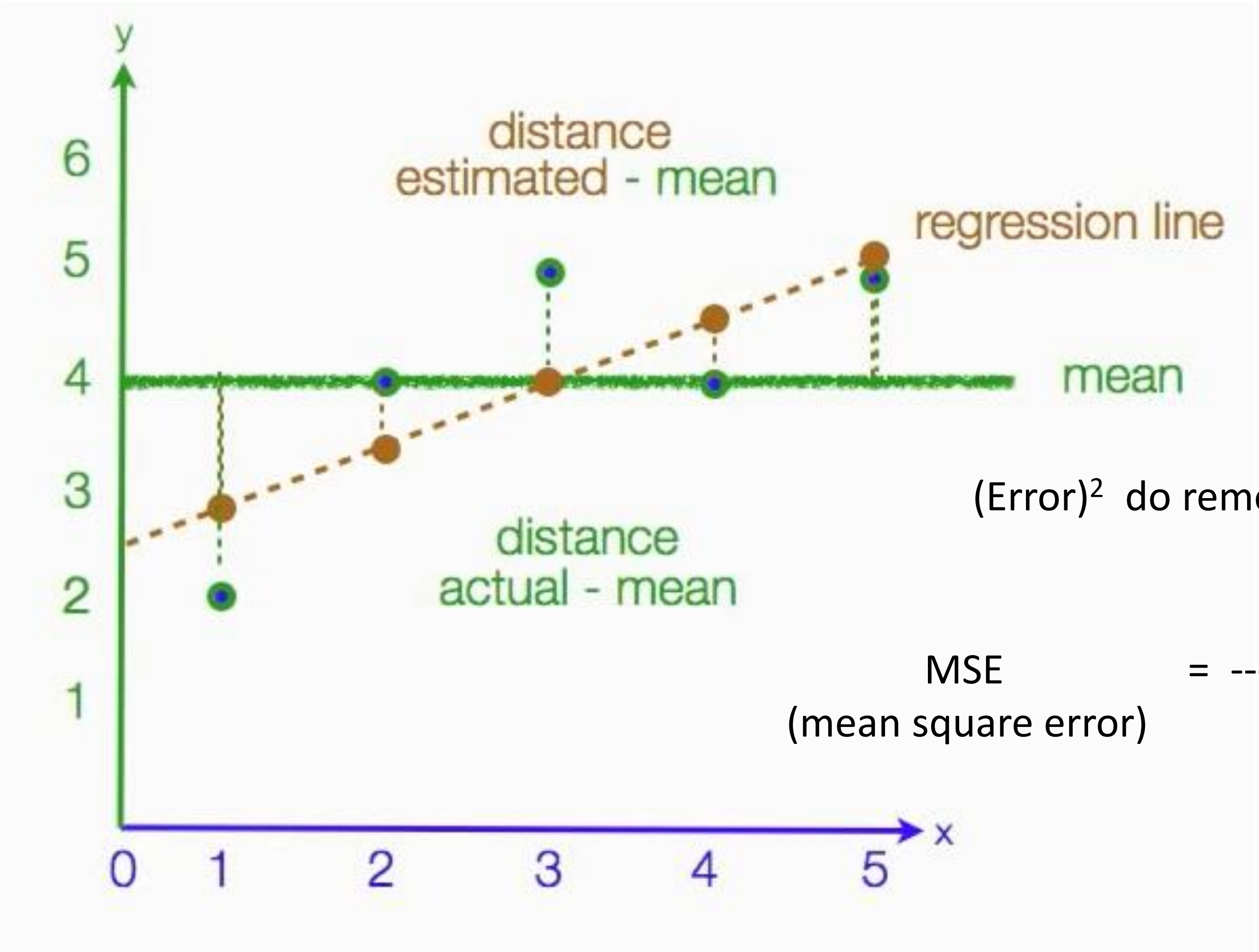
$$b_1 = \frac{6}{10} = .6 = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(x - \bar{x})^2}$$



Error = Actual Value – Predicted value

Error = distance(green dot – brown dot)

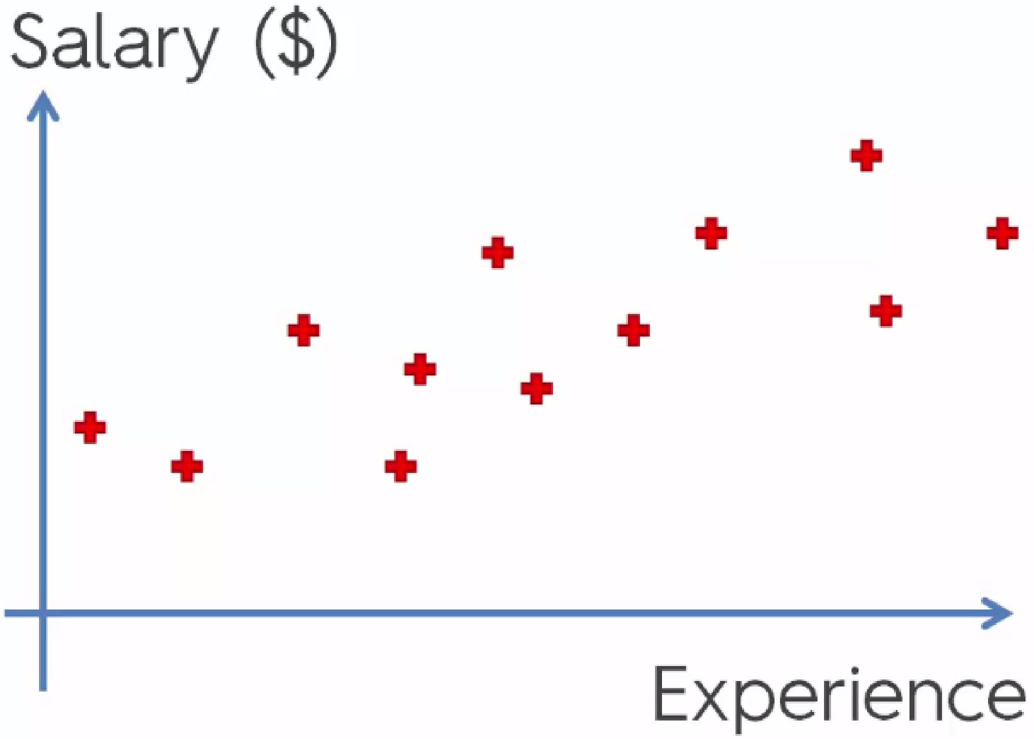
$(\text{Error})^2$ do remove negative distances



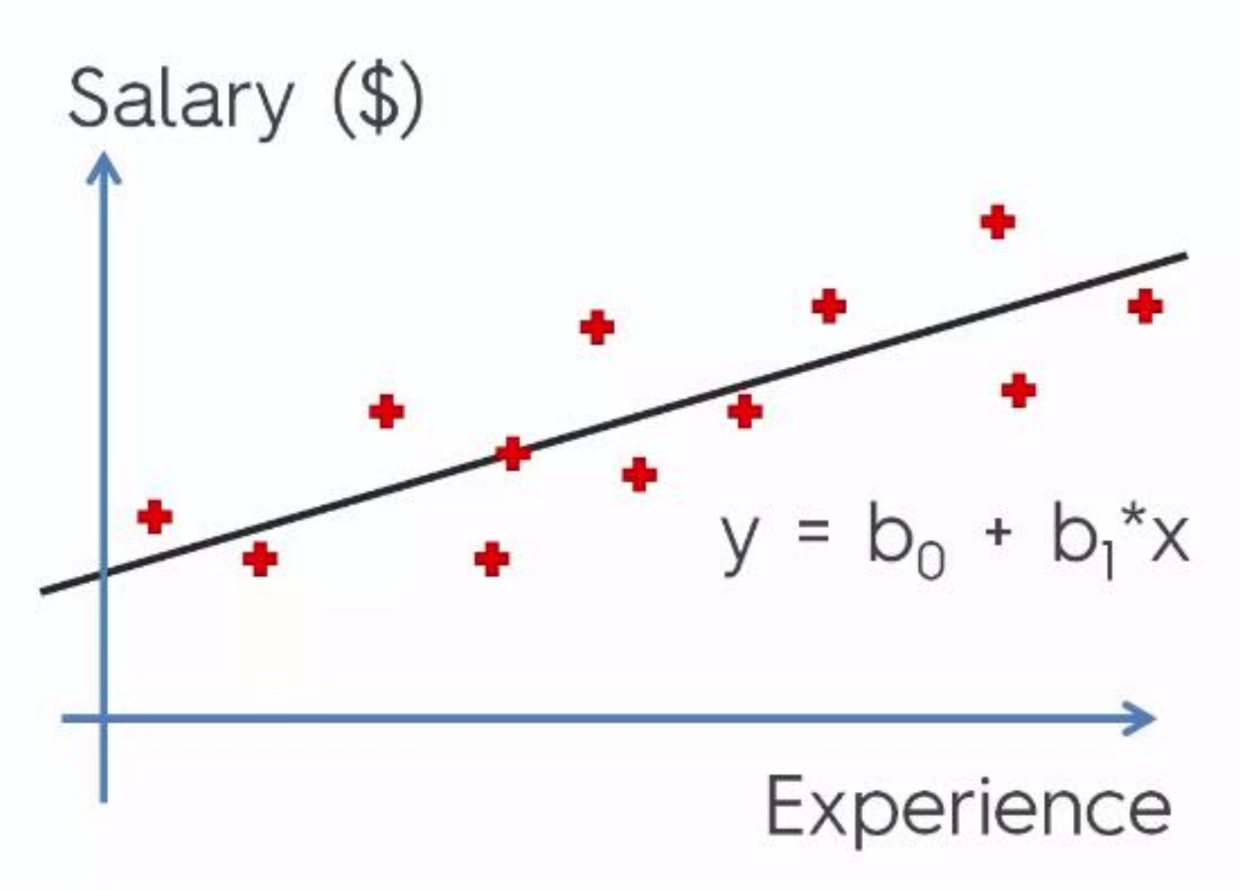
$(\text{Error})^2$ do remove negative distances

$$\underset{\text{(mean square error)}}{\text{MSE}} = \frac{\text{Sum of all Squared Errors (SE)}}{\text{Total number of errors}}$$

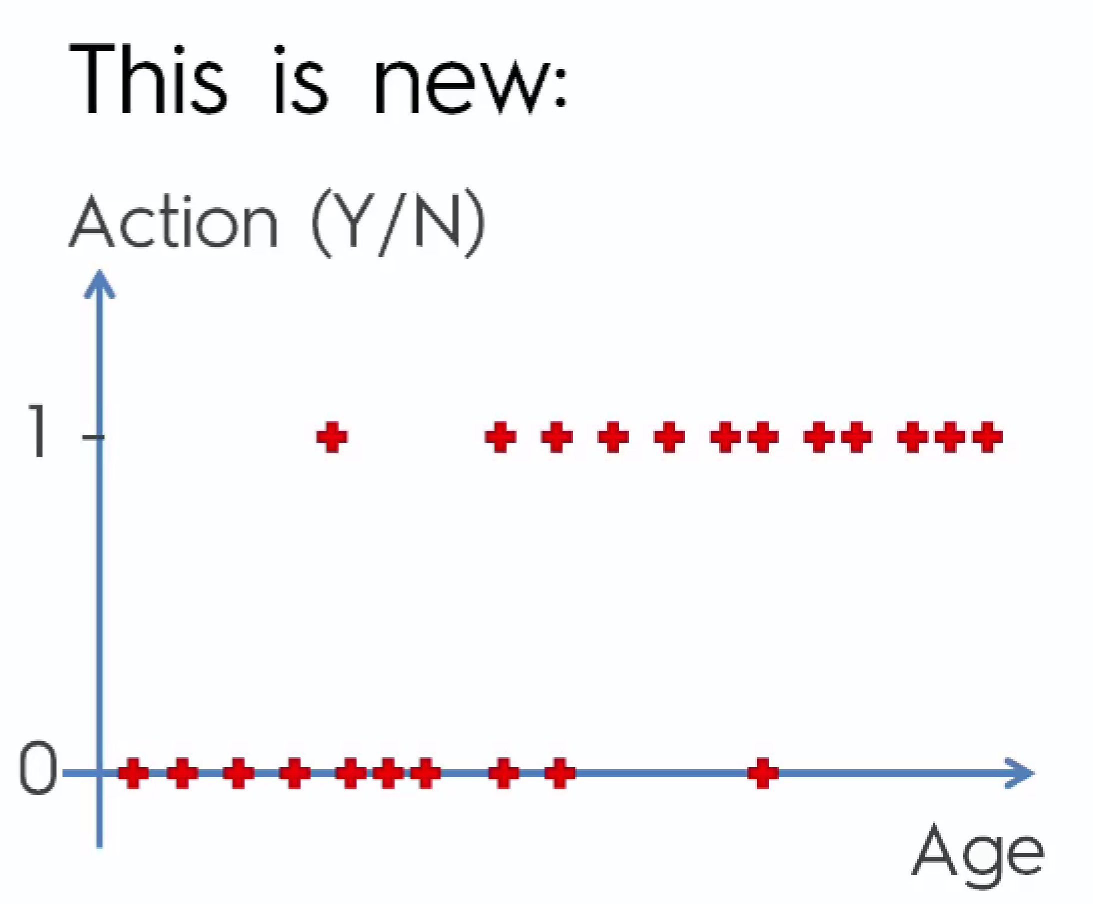
We know this:



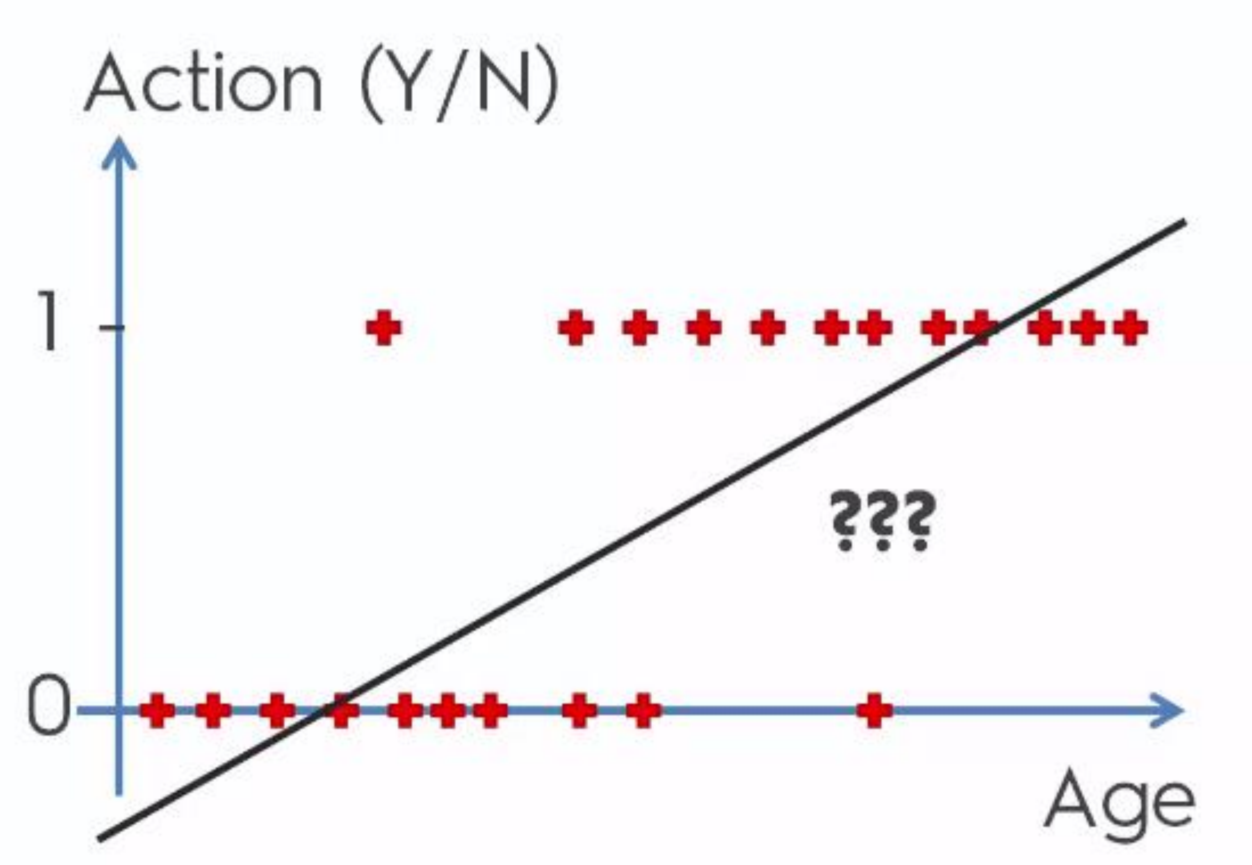
We can solve this using Linear Regression



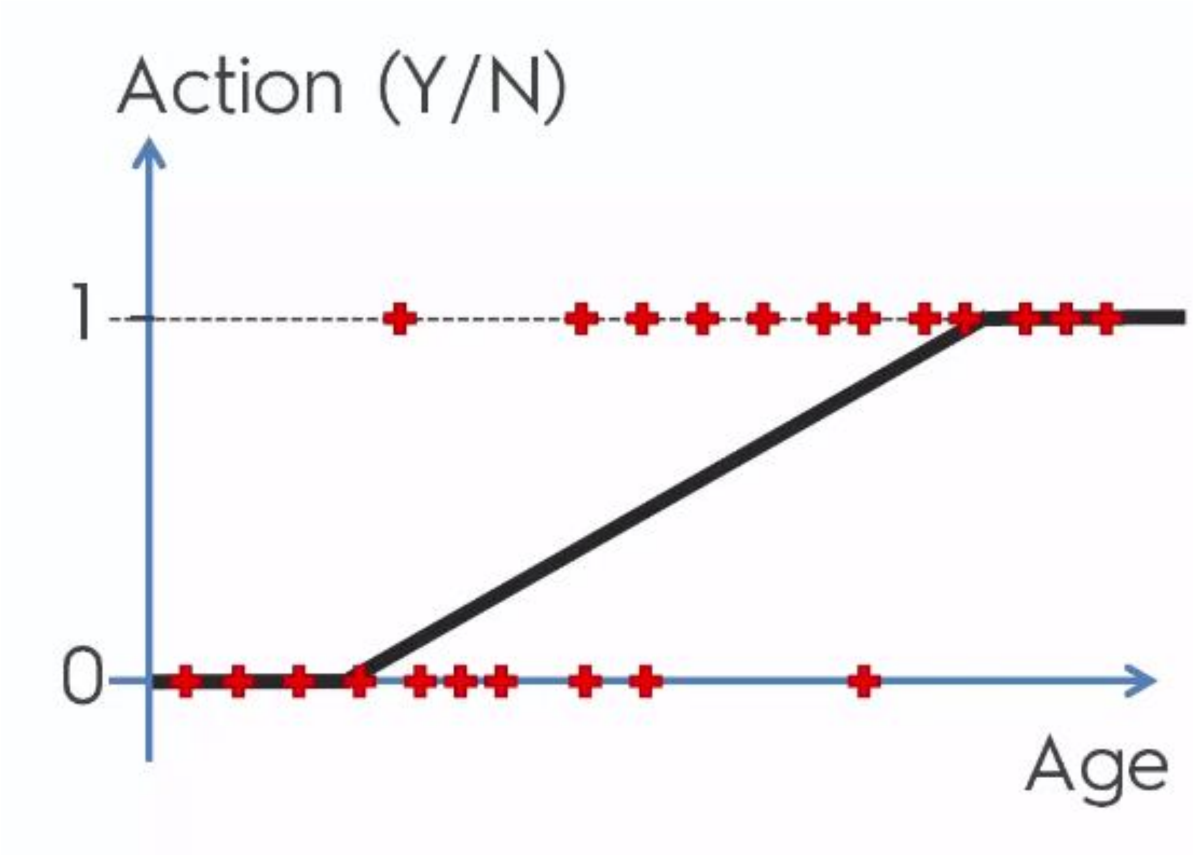
But what about this?



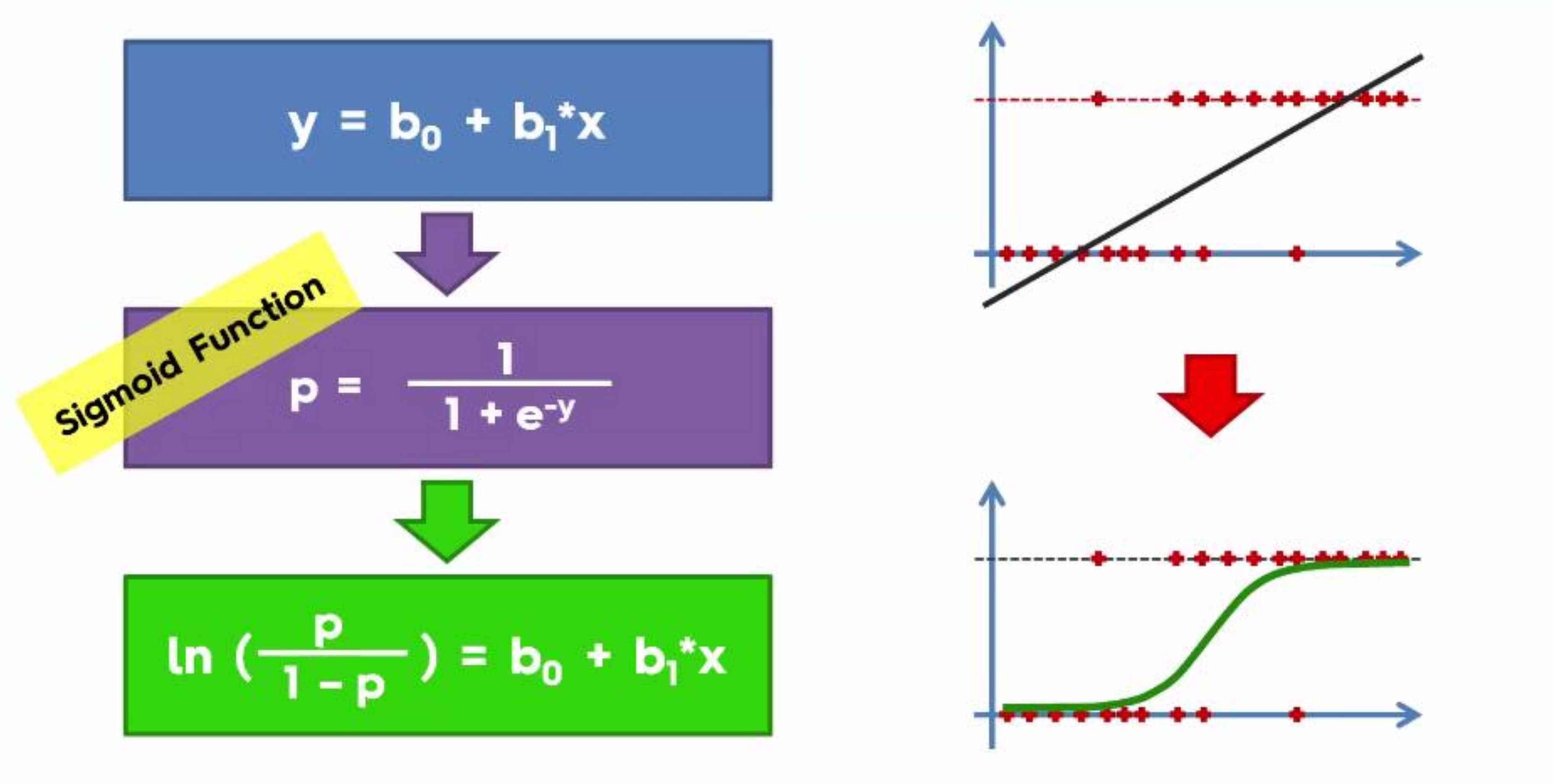
Not giving the target



We want something like

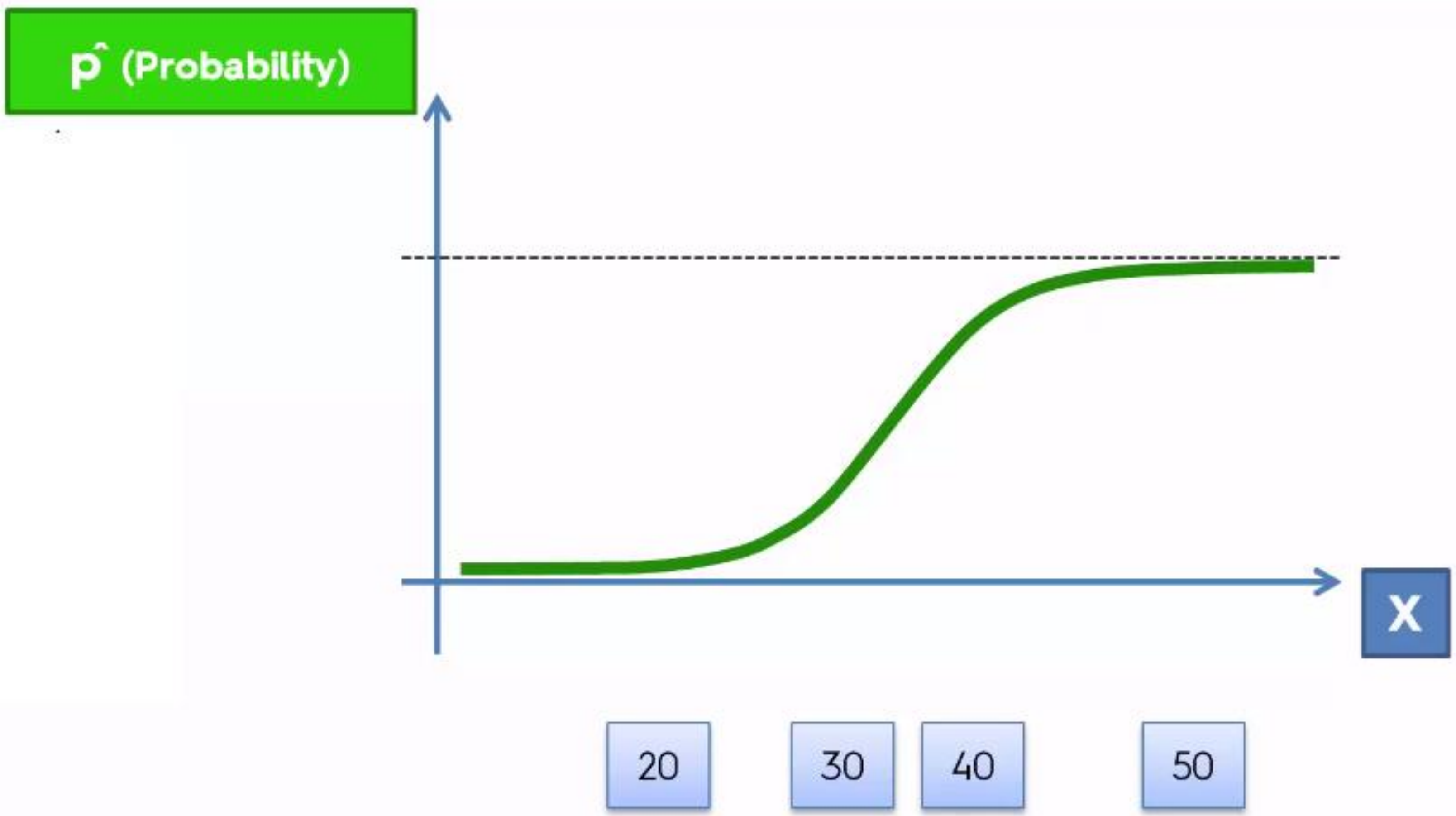


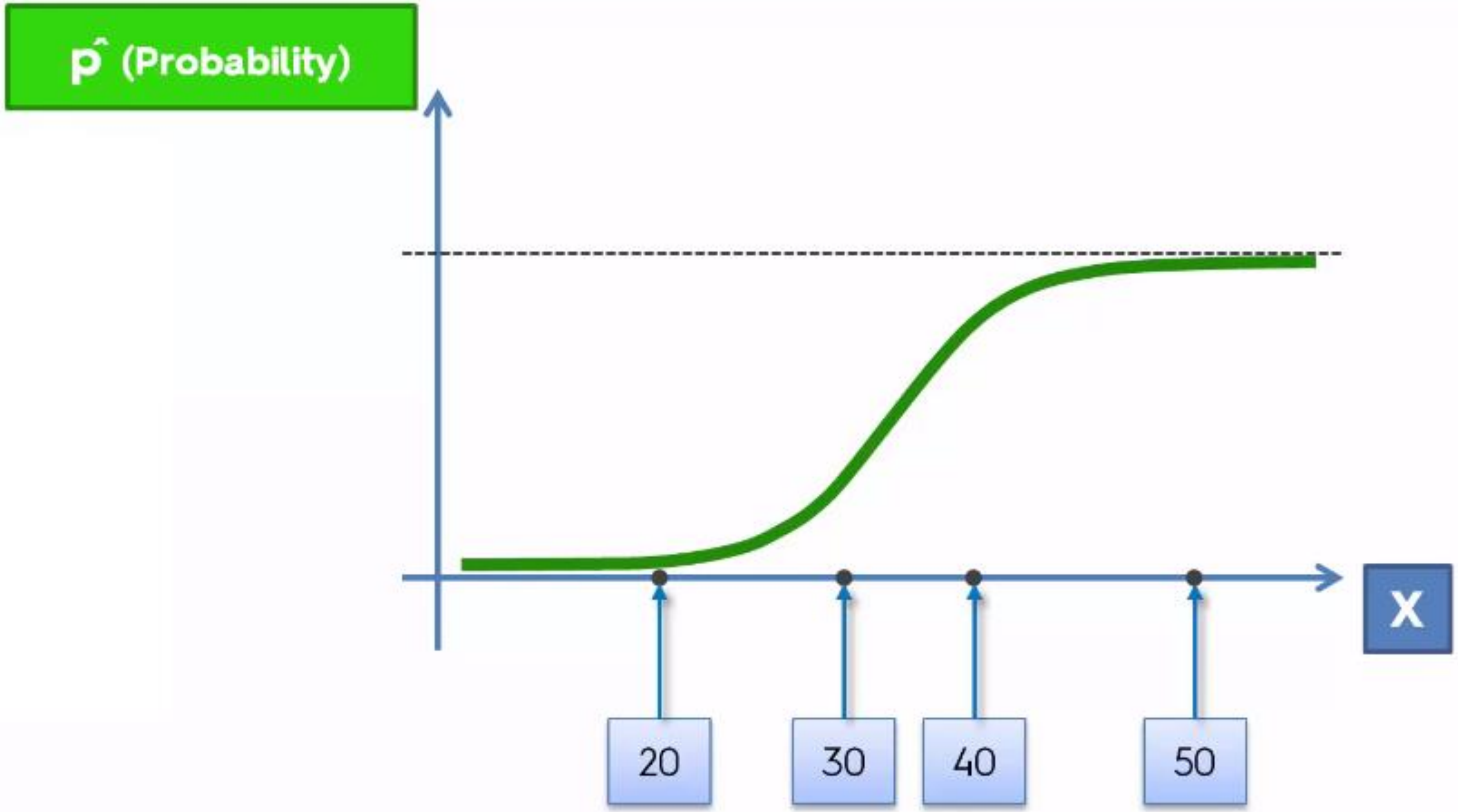
How to get it?



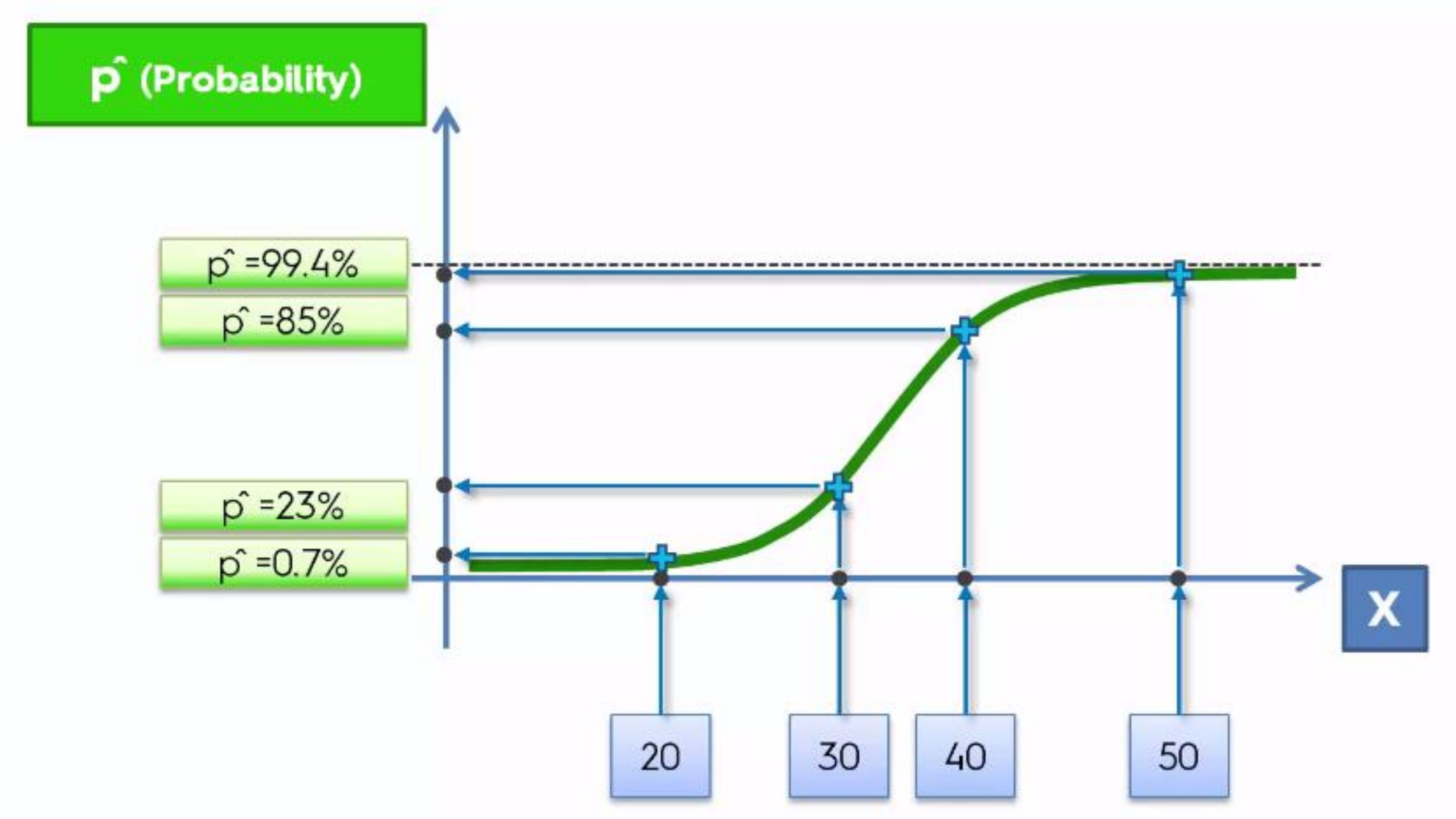
$y = b_0 + b_1{*}x$
Sigmoid Function
$p = \frac{1}{1 + e^{-y}}$
$\ln\left(\frac{p}{1-p}\right) = b_0 + b_1{*}x$

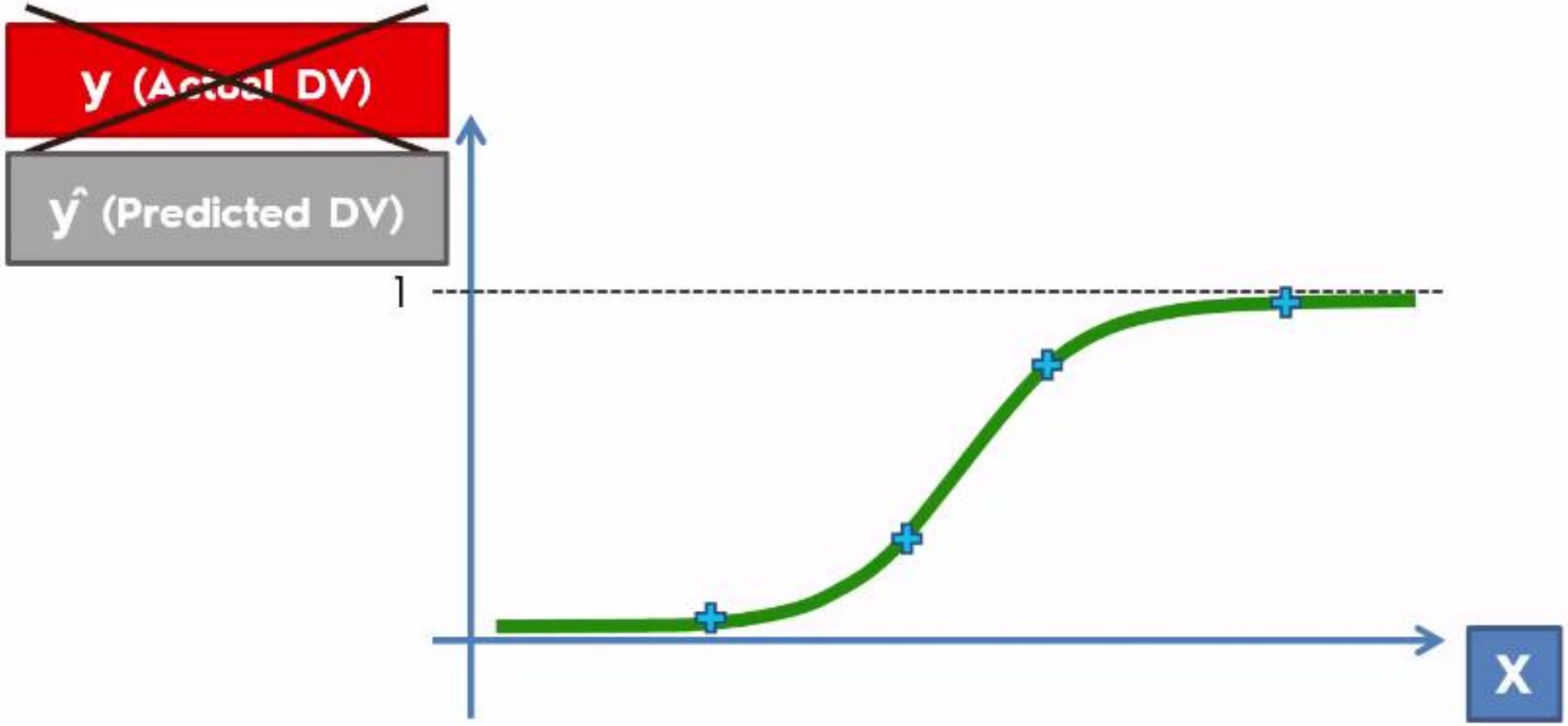
Lets say some probabilities are given



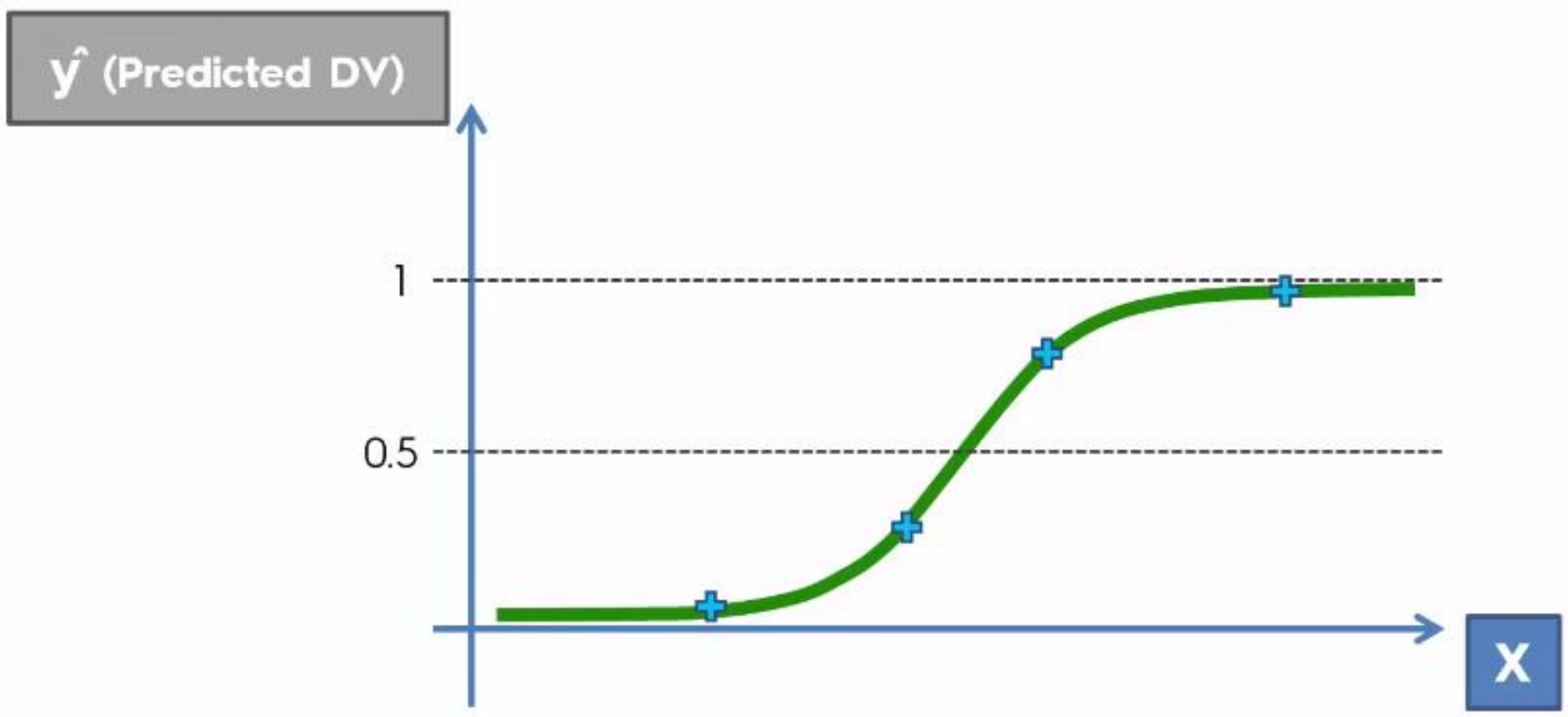

p̂ (Probability)
X
20
30
40
50

X vs Y can be drawn like this




~~y (Actual DV)~~
ŷ (Predicted DV)
1
x

Draw threshold of 50% or 0.5



So, below threshold labelled 0 and above threshold labelled 1