Gravitational N-Body Simulations : resumé

The problem takes into account N particles, hence the N-body, it aims to calculate the force applied to a particle i during its interaction with other particles under the influence of gravitation.

The problem is a set of non-linear second order differential equations relating the acceleration of particles with the position of all particles in the system. Only up to two bodies a unique solution exists if initial position and velocities are specified. More than two bodies require numerical integration. We come across multiple problems such as: singularities, non linear nature of the system,computational complexity, the choice of timestamps and softening etc. This article explains multiple different methods used to tackle these problems. A system of N particles interacting with total mass of M and reference dimension R can be in equilibrium or dynamic equilibrium and be collisionless or collisional.The most appropriate technique to use depends on the timescale and collisionality of the problem.

The boltzmann equation can be used for both collisionless and collisional systems, in the latter the equation has a collision operator on its right side. The velocity moments of the Boltzmann Equation define a set of equations known as the Jeans Equations, Therefore the numerical algorithms developed to follow the dynamics of N-body systems find a wide application also in the context of fluid-dynamics.

The development of fast and efficient algorithms and regularization techniques and progress in hardware coupled with parallelization techniques helped the field to bloom and be used in various areas.

Direct methods: highest accuracy, huge complexity.

Tree codes: it’s a fast general integrator for collisionless systems. It comes with time complexity of O(Nlog(N)) with the price of accuracy, it introduces some (small) errors.

Fast Multipole Methods: The tree code doesn’t take advantage of the fact that nearby particles will be subject to a similar acceleration due to distant group of particles . the FMM method uses this idea. It is discussed that this method reduces Complexity from O(Nlog(N)) to O(N) but the exact scaling has been debated.

Particle-Mesh Codes:This method achieves O(N) complexity in the number of particles and O(Ng log(Ng)) in the number of grid cells,less time complexity comes with a cost on accuracy.

Adaptive Mesh Refinement Method: The grid elements are concentrated where a higher resolution is needed. Unfortunately time complexity is not given in this article but from the previous method we can deduce that complexity increases over the high resolution areas.

Self Consistent field Methods: This method guarantees at fixed computational resources a higher accuracy than the tree code and the particle mesh algorithms, provided that the set of basis function is appropriately selected. This limits the scalability of the method.

PM-Tree Codes: couples a mean field description on large scales with a direct, softened treatment of the gravitational interactions on short distances. If large numbers of particles cluster around, it will slow down the computation to O(N^2). Using adaptive meshes in these regions of high density the complexity of this problem can be decreased to O(Nlog(N)).

Celestial Mechanics Codes: These methods are used to study small N (<20) systems.This method requires extremely high precision. Numerical methods are based on the local coordinate systems to minimize rounding errors.

Mean Field Methods: It’s an alternative to particle based N-body methods.

Grid based solvers for Collisionless Boltzmann Equation: It takes advantage of iterative methods such as successive over-relaxation and conjugate gradient methods, thus it requires to solve a highly dimensional non-linear system of partial differential equations. The cost of this method comes with the large amount of memory needed.

Fokker Planck and Monte Carlo Methods:These methods solve the collisional Boltzmann equation starting from given distribution. The complexity of Monte Carlo codes is linear with the number of particles.

Beyond Newton: In presence of strong gravitational fields this problem can not be based on newtonian physics and must take into account a general relativity framework.

Specialized hardwares to solve bottlenecks such as compute of gravitational force.