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# The Master-Slave Paradigm with Heterogeneous Processors

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**Heterogeneous implementation** Slave processors have different computation speeds.

**Applications** Any Monte Carlo simulation:

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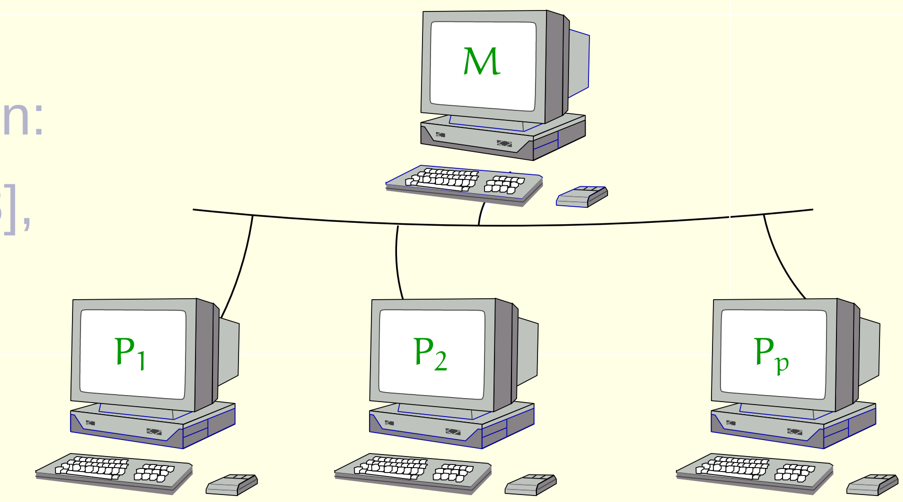

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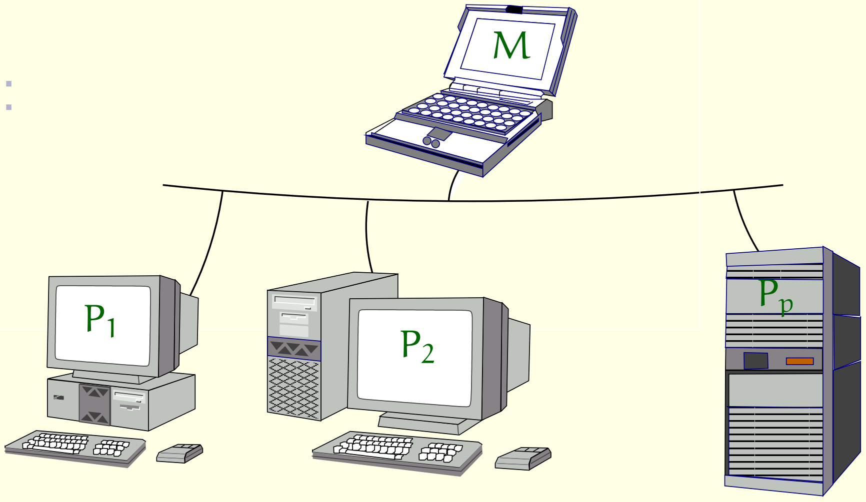

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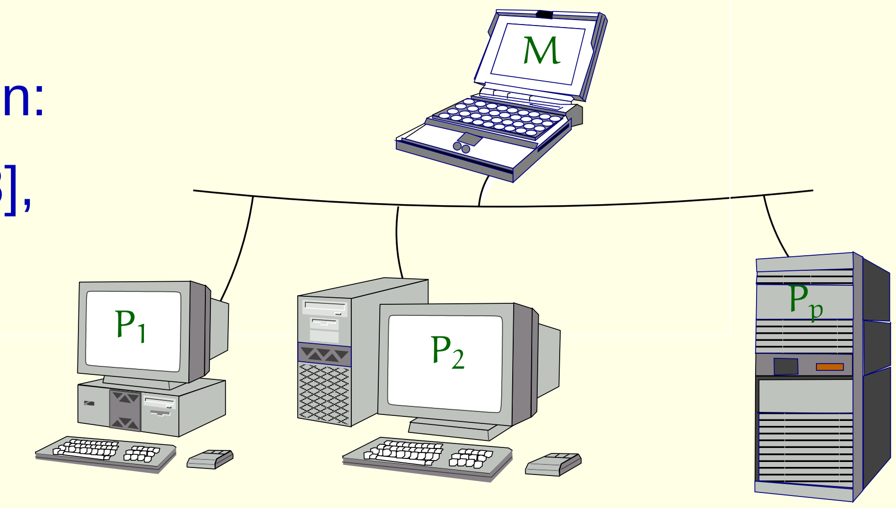

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- Pool of independent tasks to be processed by the $p$ slaves.
- All tasks are of same-size, i.e. they represent the same amount of processing.
- Tasks are considered to be atomic (execution cannot be preempted once initiated).
- Communication through a shared medium, that can be accessed only in exclusive mode.

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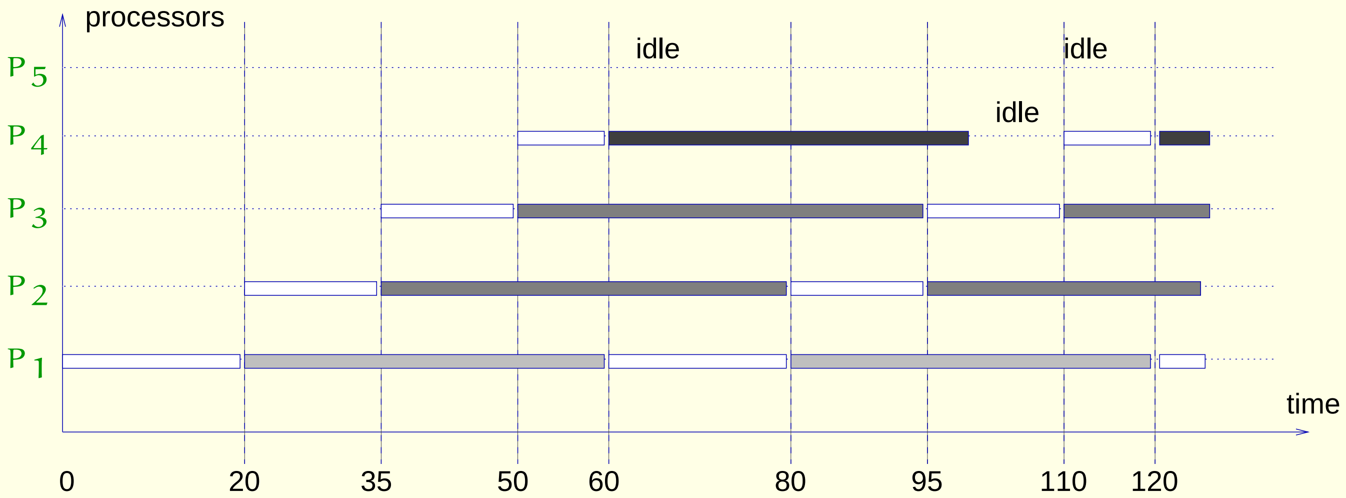

# Heterogeneous platform : Optimization problems

Processors are heterogeneous: slave $P_i$ requires $t_i$ units of time to process a single task. Each $P_i$ will execute $c_i$ tasks (where $c_i$ is to be determined) from the pool.

**MinTime($C$)** Given a total number of tasks $C$, determine the best allocation of tasks to slaves, i.e. the allocation $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$ s.t. $\sum_{i=1}^{p} c_i = C$ and which minimizes the total execution time.

**MaxTasks($T$)** Given a time bound $T$, determine the best allocation of tasks to slaves, i.e. the allocation $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$ s.t. all processors complete their execution within $T$ units of time and $\sum_{i=1}^{p} c_i = C$ is maximized.

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# Without any communication cost

Assume first that there is no communication cost at all. It is not difficult to solve both previous problems using a greedy algorithm.

The solution of problem **MaxTasks($T$)** is straightforward: we let $c_i = \left\lfloor \frac{T}{t_i} \right\rfloor$ for all $i$, $1 \leq i \leq p$, which obviously defines the optimal solution.

With an initial scattering of data

# Problem statement

Formulation of this problem is taken from Andonie et al. [ACGG98] (implementation of distributed backpropagation neural networks on heterogeneous networks of workstations).

- each slave must receive some data file from the master processor
- each slave must receive the same amount of data $\rightsquigarrow$ each reception costs $t_{\text{com}}$ units of time
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# A new optimization problem

What are we looking for?

- A permutation $\sigma$ which determines the ordering of the messages from the host: the host sends data to slave $P_i$ at time $\sigma(i)t_{\text{com}}$
- The number of tasks each slave is going to process $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$

**MaxTasks1($T$):** Given a time bound $T$, determine the best allocation of tasks to slaves, i.e. a permutation $\sigma$ and an allocation $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$ s.t. all processors complete their execution within $T$ units of time and the total number of tasks is maximized:

$$\max \left( \sum_{i=1}^{p} c_i \;\middle|\; \sigma,\ \forall i \in [1,p] : \boxed{\sigma(i)t_{\text{com}} + c_i t_i \leq T} \right)$$

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# Solution from Andonie et al. [ACGG98]

They restrict the search to allocations where the fastest processors start computing first.

They use a dynamic programming algorithm to solve the optimization problem **MinTime($C$)**.

With our setting for problem **MaxTasks1($T$)**, this amount to sort the cycle-times as $t_1 \leq t_2 \leq \ldots \leq t_p$ and to let $\sigma(i) = i$ for $1 \leq i \leq p$.

The intuition is that fastest processors execute tasks more rapidly than the others, hence they should work longer.

The intuition is misleading in some cases:
$t_1 = 5$, $t_2 = 9$ and $t_{\text{com}} = 1$.

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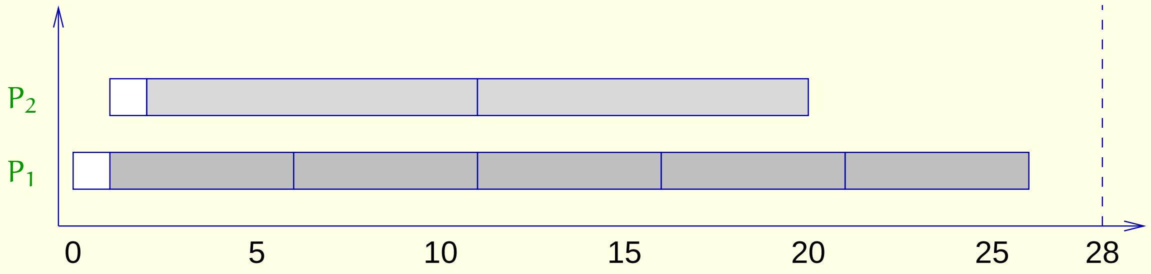


$\sigma = \mathrm{Id}$

7 tasks

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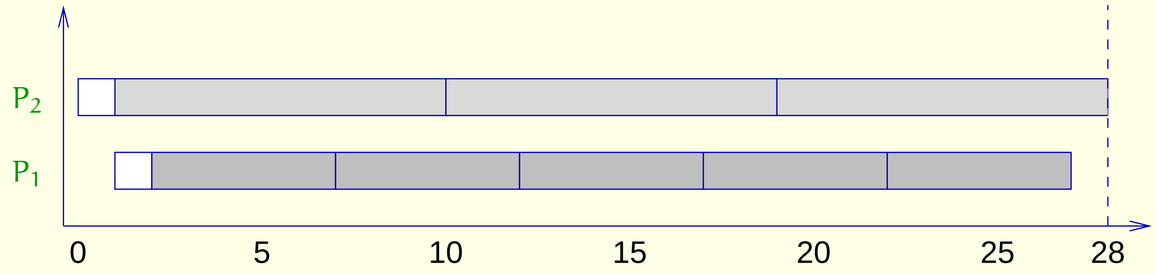


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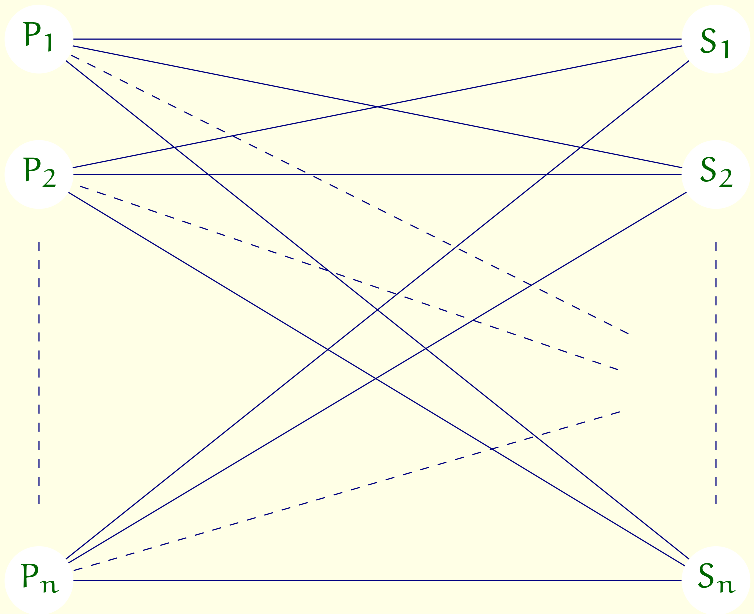
# Optimal solution




Bipartite graph for **MaxTasks1($T$).**

$$w(P_i, S_j) = \begin{cases} \text{maximum number of tasks} \\ \text{that } P_i \text{ can execute if } \sigma(i) = j \end{cases}$$
$$= \left\lfloor \frac{T - j t_{\text{com}}}{t_i} \right\rfloor$$

Solving **MaxTasks1($T$)** reduces to finding the maximum weighted matching in the bipartite graph.

**Proposition 1.** *The optimal solution to the* **MaxTasks1($T$)** *problem with initial messages can be found in time of order $O(p^3)$ with $p$ processors using some classical algorithm ([GM84, Wes96]) to find the maximum weighted matching in a bipartite graph*

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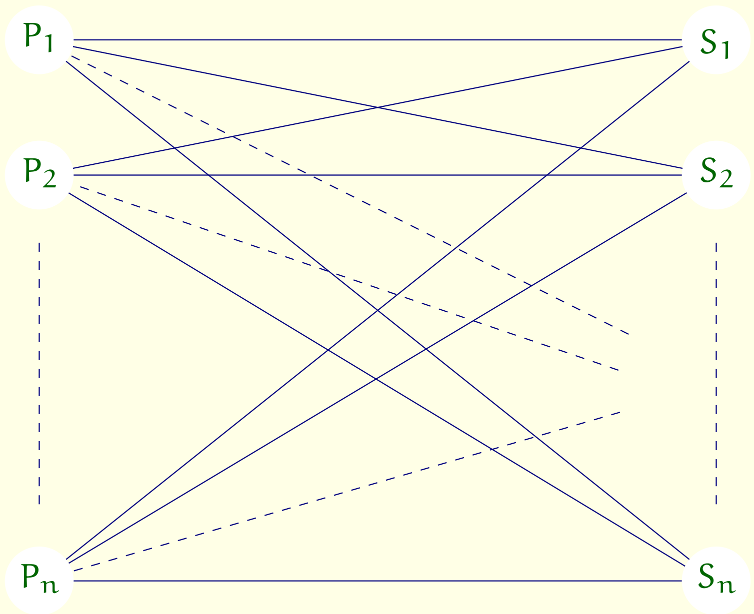


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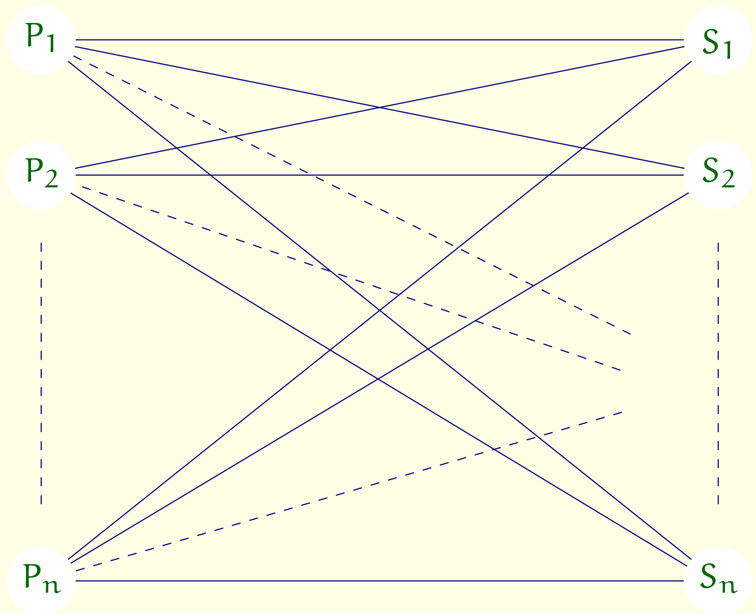


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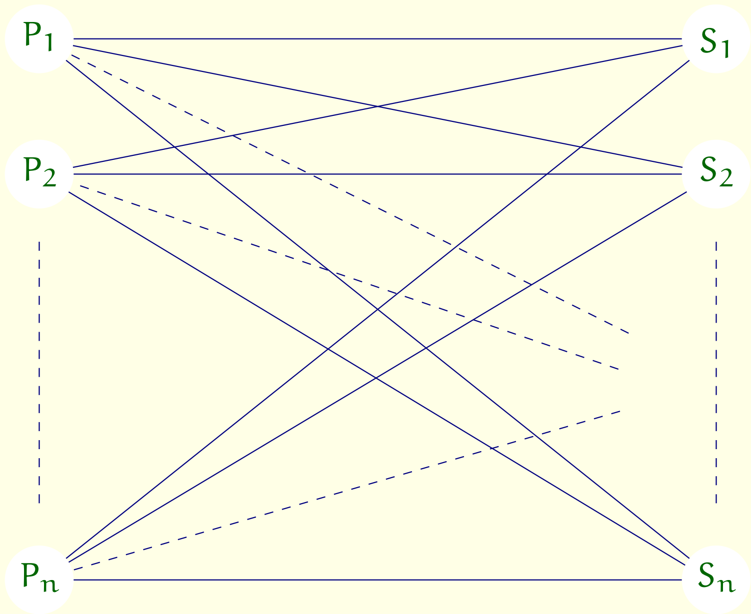


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With initial and final communications

# Problem statement

Some results have been produced by the slaves or the master need some feedback on the computation:

- each slave must receive some data file from the master processor and send some results back after the processing $\rightsquigarrow$ two communication costs: $t^1_{\text{com}}$ and $t^2_{\text{com}}$
- slave $P_i$ process one task in $t_i$ units of time

What are we looking for?

- A first permutation $\sigma_1$ which determines the ordering of the initial messages from the host: the host sends data to slave $P_i$ at time $\sigma_1(i)t^1_{\text{com}}$
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# A new optimization problem

**MaxTasks2($T$):** Given a time bound $T$, determine the best allocation of tasks to slaves, i.e. two permutations $\sigma_1$ and $\sigma_2$, and an allocation $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$ s.t. all processors complete their execution within $T$ units of time and the total number of tasks is maximized:

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- The solution to the MaxTasks2(T) problem with initial and final messages turns out to be surprisingly difficult.
- We do not know of any polynomial algorithm for the general case $\rightsquigarrow$ efficient guaranteed approximation using matching techniques.

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- We do not know of any polynomial algorithm for the general case $\leadsto$ efficient guaranteed approximation using matching techniques.

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**MaxTasks2($T$):** Given a time bound $T$, determine the best allocation of tasks to slaves, i.e. two permutations $\sigma_1$ and $\sigma_2$, and an allocation $\mathcal{C} = \{c_1, c_2, \ldots, c_p\}$ s.t. all processors complete their execution within $T$ units of time and the total number of tasks is maximized:

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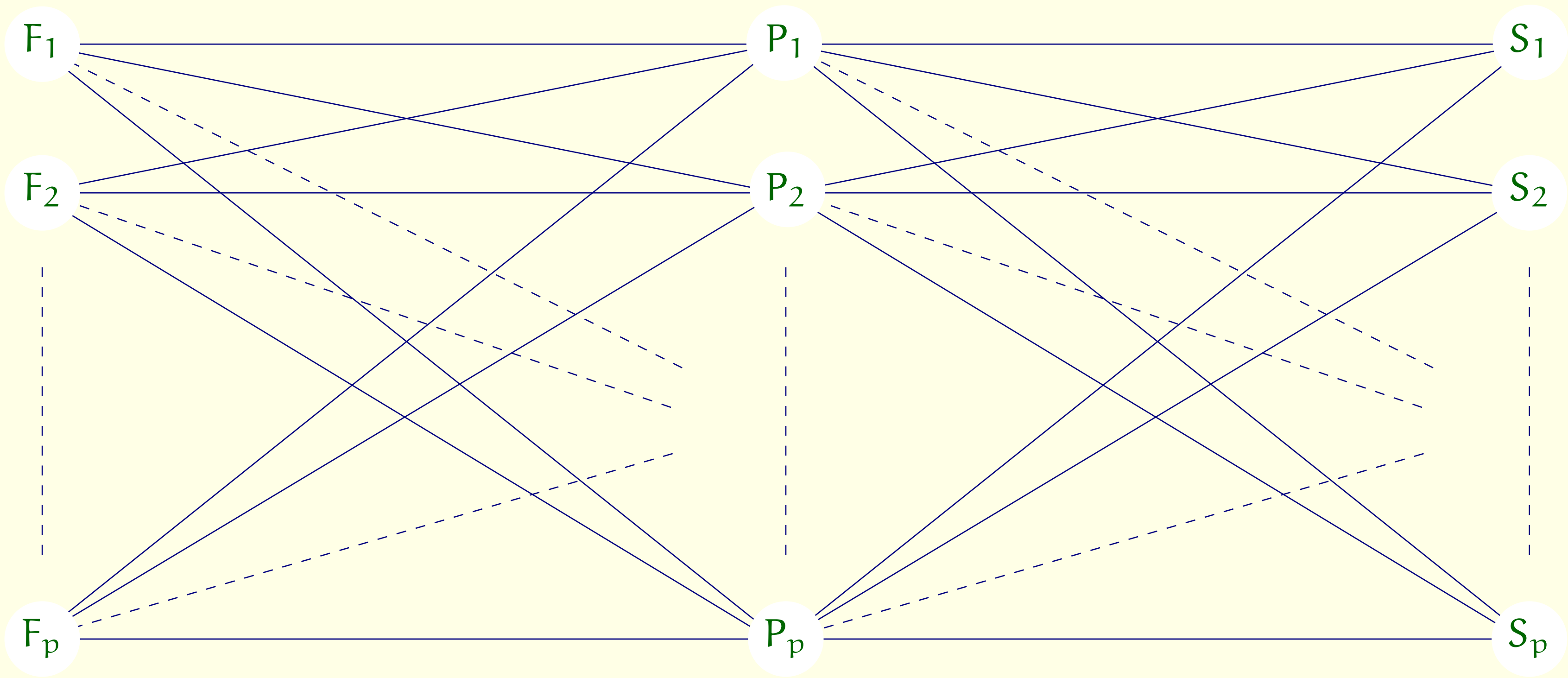
# 2-factor in a Tripartite graph




Tripartite graph with initial and final communications.

$F_i$'s correspond to $\sigma_1$ and $S_i$'s correspond to $\sigma_2$. Rather than a matching, we extract a $2$-factor from the graph. The complexity of extracting $2$-factor from the graph with $3p$ vertices is of order $O(p^3)$ again.

# Edge weights approximation

The problem is that edge weights cannot be determined fully accurately; the inequality $\sigma_1(i)t^1_{\text{com}} + c_i t_i + \sigma_2(i)t^2_{\text{com}} \leq T$ translates into

$$c_i \leq \left\lfloor \frac{T - \sigma_1(i)t^1_{\text{com}} - \sigma_2(i)t^2_{\text{com}}}{t_i} \right\rfloor,$$

and we need to know both $\sigma_1(i)$ and $\sigma_2(i)$ to compute $c_i$. Instead, we use the approximation

$$\left\lfloor \frac{T/2 - \sigma_1(i)t^1_{\text{com}}}{t_i} \right\rfloor + \left\lfloor \frac{T/2 - \sigma_2(i)t^2_{\text{com}}}{t_i} \right\rfloor.$$

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# A word on complexity

Is **MaxTasks2(**$T$**)** NP-complete?

It is as difficult as the following open (polynomial vs. NP-complete) problem in combinatorial optimization (see [Hed]):

**Permutation Sums:** Let $a_1 \leq a_2 \leq \ldots \leq a_p$ be $p$ positive integers satisfying $\sum_{i=1}^{p} a_i = p(p+1)$. Do there exist two permutations $\sigma_1$ and $\sigma_2$ of $\{1, 2, \ldots, n\}$ such that

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With communications before each task processing

# Problem statement

Each task has its own description file:

- communications are required between the master and the slave before the processing of each task $\leadsto$ each reception costs $t_{\text{com}}$ units of time
- slave $P_i$ process one task in $t_i$ units of time

**MaxTasks3(T):** Given a time bound $T$, determine the best allocation of tasks to slaves, i.e. three functions $f_{\text{startcomm}}$, $f_{\text{startcomp}}$ and $f_{\text{proc}}$ s.t. all processors complete their execution within $T$ units of time and the total number of tasks is maximized:

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# Asymptotically optimal algorithm

Since MaxTasks3(T) is probably NP, we slightly change the problem and look for the maximum steady-state throughput of our platform.

- determining a pattern for communications and computations, that will be reproduced periodically throughout the execution
- two different cases whether the communication network is the limiting resource or not
- $\frac{t_{\text{com}}}{t_{\text{com}}+t_i}$ represent the ratio of the communication involved by processor $P_i$
- the value of $\sum_{i=1}^{p} \frac{t_{\text{com}}}{t_{\text{com}}+t_i}$ enable to know in which situation we are

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# Network is not limiting: $\sum_{i=1}^{p} \frac{t_{com}}{t_{com}+t_i} \leq 1$

- All processors have to be fully used.
- $\tau_i = t_{com} + t_i$, for $1 \leq i \leq p$
- Let $T^{pattern}$ be the least common multiple of these $p$ values $\tau_i$: $T^{pattern}$ determines the length of the pattern.
- Let $\nu_i^{pattern} = \frac{T^{pattern}}{\tau_i}$

$t_{com} = 1$
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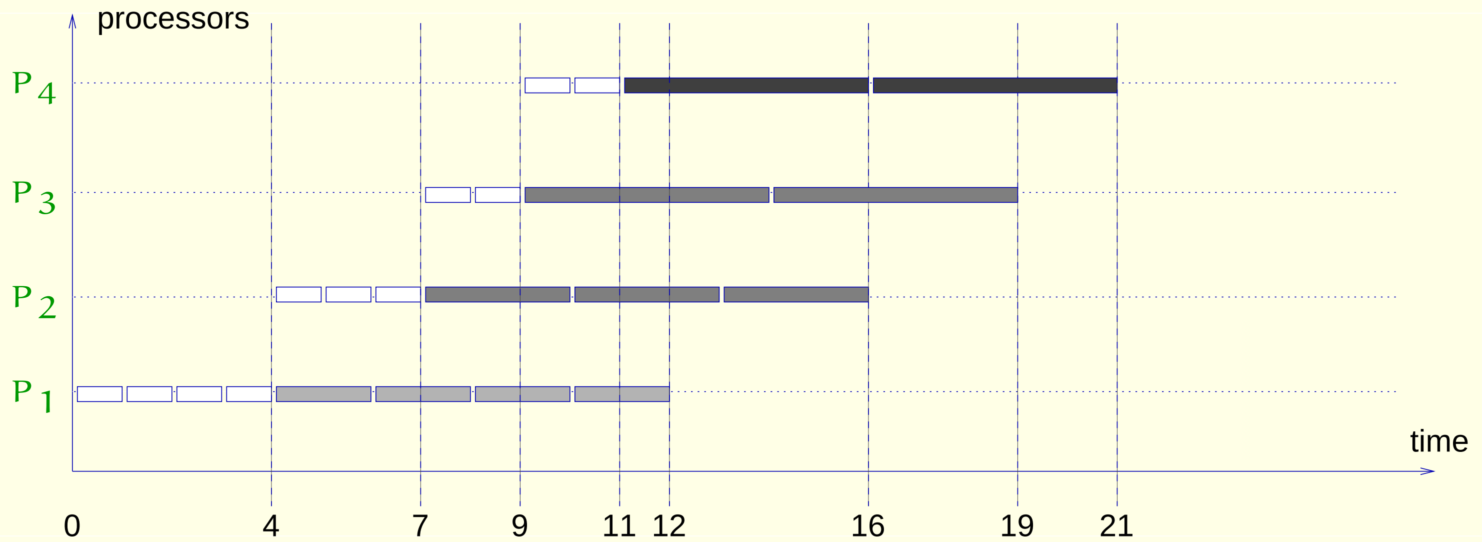


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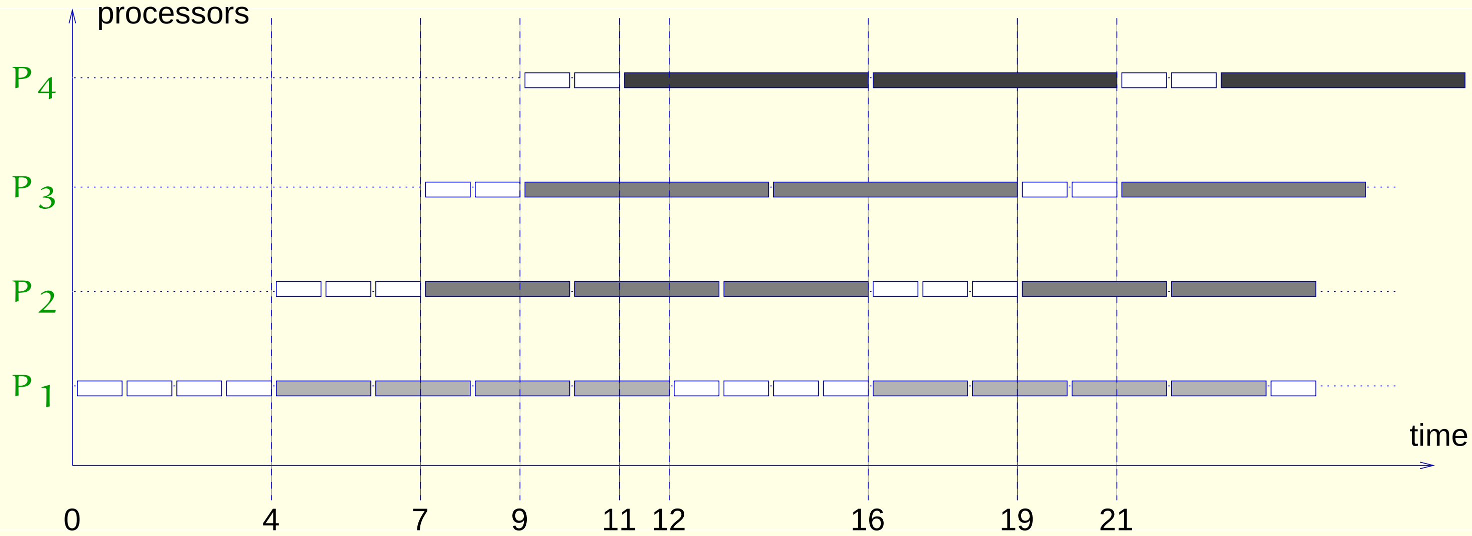


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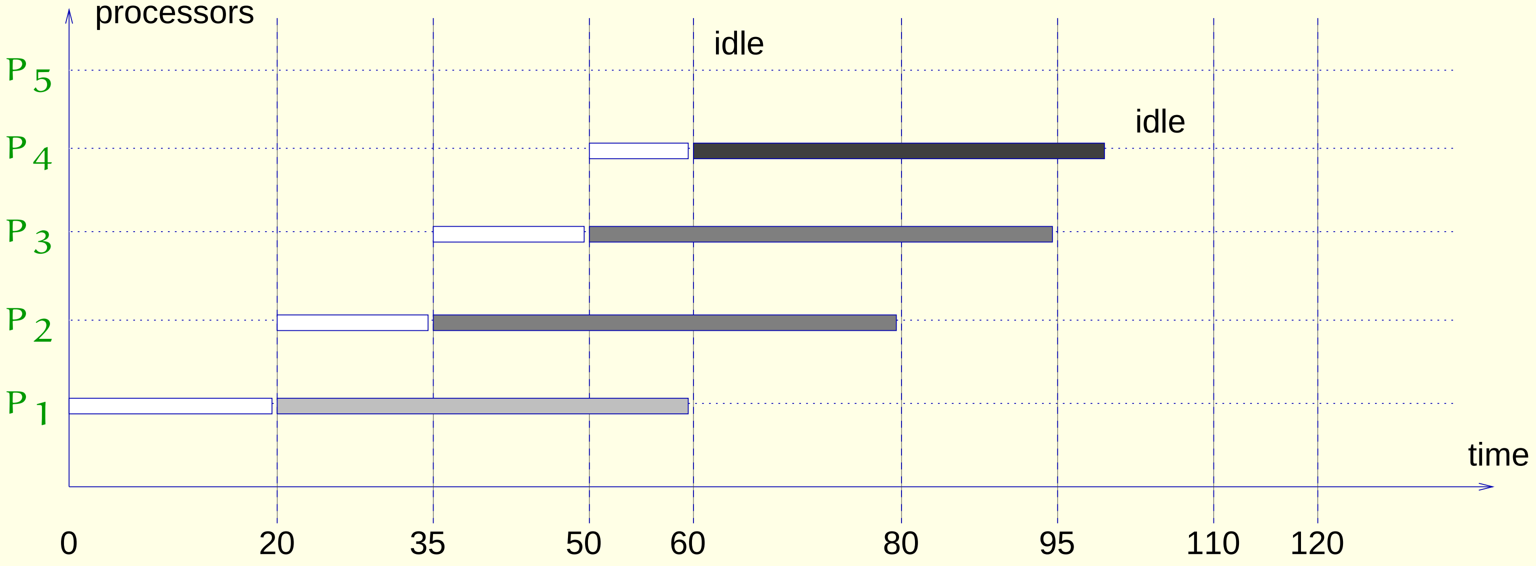


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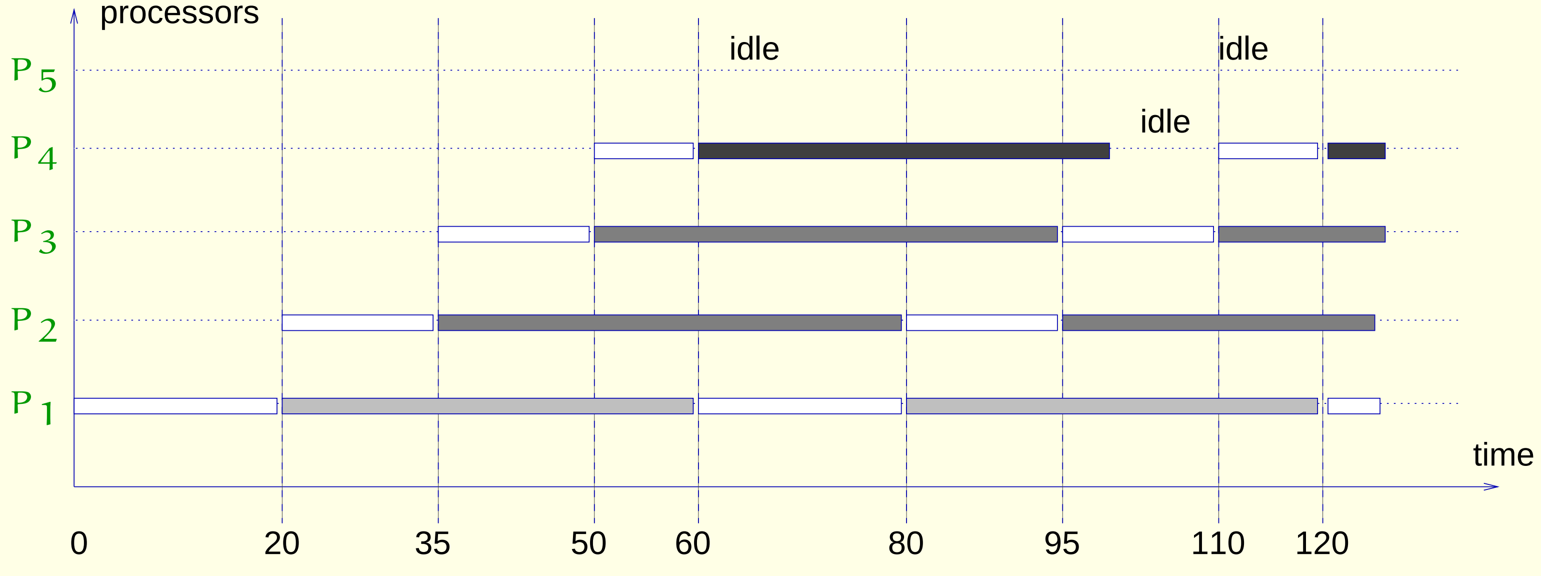


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# Asymptotical optimality

**Theorem 2 ([BLR01]).** *Let* $N_{opt}(T)$ *be the optimal number of tasks that can be executed within* $T$ *time-steps. Let* $N(T)$ *be the number of tasks executed by the first algorithm if* $\sum_{i=1}^{p} \frac{t_{com}}{t_{com}+t_i} \leq 1$*, and by the second algorithm if* $\sum_{i=1}^{p} \frac{t_{com}}{t_{com}+t_i} > 1$*. Then*

$$\lim_{T \to \infty} \frac{N(T)}{N_{opt}(T)} = 1.$$

With communications both before and after each task processing

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**Theorem 3.** *Let $N_{opt}(T)$ be the optimal number of tasks that can be executed within $T$ time-steps, and let $N(T)$ be the number of tasks executed by our algorithm. Then*

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# Related Work

- the most related work is presented in the paper of Andonie et al [ACGG98]
- parallel machine problems with a server : several complexity (NP-completeness) results [HPS00, KW97, BDFK$^+$00] and providing guaranteed approximations [LG96] **MaxTasks1(**T**)** is a very special instance of this class of server problems
- **MaxTasks2(**T**)** is a special instance of the job-shop scheduling problem (see problem SS18 in [GJ91]).Because this instance is very specific, we do not know its complexity (polynomial versus NP-complete).
- **MaxTasks3** and **MaxTasks4** have been extended to trees in [BCF$^+$01]: collaborative work with Larry Carter and Jeanne Ferrante (UCSD)

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# Related Work

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# Related Work

Our four problems differ from those studied in the literature with a server and start-up times in that

- all tasks are identical and independent,
- communication times (the counterpart of the set-up times) are identical too.

The difficulty lies solely in the heterogeneity of the computing resources.

# Conclusion

- Deriving efficient algorithms for the simple master-slave paradigm, in the framework of heterogeneous computing resources, turns out to be surprisingly difficult.
- Optimal polynomial algorithm in the case of an initial scattering of data.
- Guaranteed polynomial approximation algorithm in the case of initial and final communications. We conjecture this problem to be intrinsically difficult even on (intuitively) simple instances.
- Asymptotically optimal algorithms for the case where each task processing must be preceded (and possibly followed) by a communication from (back to) the master

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