

# The approximation ratio of $UM_{OM\_1}$ to $k \times OM_1$ , is unbounded

To prove the theorem, we only need to find one example that satisfies the unbounded situation. To simplify the complex situation, we assume that the Owner has only two items to submit. Then we compare the Profit outcomes of different mechanisms for the Collector. We assess the collector objective of two individually submitted items using  $2 \times OM_1$ , and compare it to the collector objective of two items submitted together using  $UM_{OM\_1}$ .

$t=0.5$

$v=[0,0.333,0.666,1]$

$d(v): [0.2645, 0.5386, 0.1861, 0.0109]$

The matrix  $r(v,s)$  is constructed as follows:

$r(v,s)$ :

$[[0.749 \ 0.128 \ 0.074 \ 0.049]$

$[0.057 \ 0.737 \ 0.19 \ 0.016]$

$[0.018 \ 0.086 \ 0.834 \ 0.062]$

$[0.144 \ 0.147 \ 0.209 \ 0.5 \ ]]$

In the example result, the objective value for  $2 \times OM_1$  is 0.0, while the objective value for  $UM_{OM\_1}$  is 0.0248746. Consequently, we can conclude that there is no multiplicative approximation relationship between  $k \times OM_1$  and  $UM_{OM\_1}$ .

## The result for $UM_{OM\_1}$ : Obj=0.0248746

### The first paper $X_{UM\{OM\_1\}}(v_1,v_2,s_1,s_2)$ :

paper: 1

$v\_1$  value: 0.0

$v\_2$  value: 0.0

$[s_1,s_2]$  value:

0.0,	0.17801,	0.0,	0.17801,
0.0,	0.2769,	0.0,	0.2769,
0.0,	0.2769,	0.0,	0.2769,
0.0,	0.2769,	0.0,	0.2769,

paper: 1

$v\_1$  value: 0.0

$v\_2$  value: 0.3333333333333333

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 1

v\_1 value: 0.0

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 1

v\_1 value: 0.0

v\_2 value: 1.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 1

v\_1 value: 0.3333333333333333

v\_2 value: 0.0

[s1,s2] value:

0.17801,	0.17801,	0.17801,	0.17801,
0.2769,	0.2769,	0.2769,	0.2769,
0.2769,	0.2769,	0.2769,	0.2769,
0.2769,	0.2769,	0.2769,	0.2769,

paper: 1

v\_1 value: 0.3333333333333333

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.15319,	0.25208,	0.0,	0.25208,
0.25208,	0.0,	0.35097,	0.0,
0.0,	0.0,	0.0,	0.35097,
0.0,	0.35097,	0.35097,	0.0,

paper: 1

v\_1 value: 0.3333333333333333

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 1

v\_1 value: 0.3333333333333333

v\_2 value: 1.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 1

v\_1 value: 0.6666666666666666

v\_2 value: 0.0

[s1,s2] value:

0.10142,	0.10142,	0.10142,	0.10142,
0.10142,	0.10142,	0.10142,	0.10142,
0.92593,	0.92593,	0.92593,	0.92593,
0.92593,	0.92593,	0.92593,	0.92593,

paper: 1

v\_1 value: 0.6666666666666666

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.07659,	0.17549,	0.17549,	0.17549,
0.07659,	0.17549,	0.17549,	0.17549,
0.90111,	1.0,	1.0,	1.0,
0.90111,	1.0,	1.0,	1.0,

paper: 1

v\_1 value: 0.6666666666666666

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.0,	0.0,	0.82451,	0.0,
0.0,	0.0,	0.82451,	0.82451,
0.0,	0.0,	1.0,	1.0,
0.82451,	0.82451,	0.64903,	1.0,

paper: 1

v\_1 value: 0.6666666666666666

v\_2 value: 1.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.64903,	0.64903,
0.0,	0.0,	0.64903,	0.64903,

paper: 1

v\_1 value: 1.0

v\_2 value: 0.0

[s1,s2] value:

0.10142,	0.10142,	0.10142,	0.10142,
0.10142,	0.10142,	0.10142,	0.10142,
0.92593,	0.92593,	0.92593,	0.92593,
0.92593,	0.92593,	0.92593,	0.92593,

paper: 1

v\_1 value: 1.0

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.07659,	0.17549,	0.17549,	0.17549,
0.07659,	0.17549,	0.17549,	0.17549,
0.90111,	1.0,	1.0,	1.0,
0.90111,	1.0,	1.0,	1.0,

paper: 1

v\_1 value: 1.0

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.0,	0.0,	0.82451,	0.82451,
0.0,	0.0,	0.82451,	0.82451,
0.82451,	0.82451,	1.0,	1.0,
0.82451,	0.82451,	1.0,	1.0,

paper: 1

v\_1 value: 1.0

v\_2 value: 1.0

[s1,s2] value:

0.0,	0.0,	0.82451,	0.82451,
0.0,	0.0,	0.0,	0.0,
0.82451,	0.82451,	0.64903,	0.64903,
0.0,	0.0,	0.64903,	0.64903,

## The second paper $X_{UM_{\{OM_1\}}}(v_1, v_2, s_1, s_2)$ :

paper: 2

v\_1 value: 0.0

v\_2 value: 0.0

[s1,s2] value:

0.17801,	0.0,	0.17801,	0.0,
0.2769,	0.0,	0.2769,	0.0,
0.2769,	0.0,	0.2769,	0.0,
0.2769,	0.0,	0.2769,	0.0,

paper: 2

v\_1 value: 0.0

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.15319,	0.25208,	0.25208,	0.25208,
0.25208,	0.35097,	0.35097,	0.35097,
0.25208,	0.35097,	0.35097,	0.35097,
0.25208,	0.35097,	0.35097,	0.35097,

paper: 2

v\_1 value: 0.0

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.07659,	0.07659,	0.90111,	0.90111,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,

paper: 2

v\_1 value: 0.0

v\_2 value: 1.0

[s1,s2] value:

0.07659,	0.07659,	0.90111,	0.90111,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,

paper: 2

v\_1 value: 0.3333333333333333

v\_2 value: 0.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 2

v\_1 value: 0.3333333333333333

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.0,	0.0,	0.25208,	0.0,
0.0,	0.35097,	0.0,	0.35097,
0.25208,	0.35097,	0.35097,	0.0,
0.25208,	0.0,	0.0,	0.35097,

paper: 2

v\_1 value: 0.3333333333333333

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.07659,	0.07659,	0.90111,	0.90111,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,

paper: 2

v\_1 value: 0.3333333333333333

v\_2 value: 1.0

[s1,s2] value:

0.07659,	0.07659,	0.90111,	0.90111,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,
0.17549,	0.17549,	1.0,	1.0,

paper: 2

v\_1 value: 0.6666666666666666

v\_2 value: 0.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 2

v\_1 value: 0.6666666666666666

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 2

v\_1 value: 0.6666666666666666

v\_2 value: 0.6666666666666666

[s1,s2] value:

0.0,	0.0,	0.0,	0.82451,
0.0,	0.0,	0.0,	0.0,
0.82451,	0.82451,	0.64903,	0.64903,
0.0,	0.0,	1.0,	0.64903,

paper: 2

v\_1 value: 0.6666666666666666

v\_2 value: 1.0

[s1,s2] value:

0.0,	0.0,	0.82451,	0.82451,
0.0,	0.0,	0.82451,	0.82451,
0.82451,	0.82451,	1.0,	1.0,
0.82451,	0.82451,	1.0,	1.0,

paper: 2

v\_1 value: 1.0

v\_2 value: 0.0

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 2

v\_1 value: 1.0

v\_2 value: 0.3333333333333333

[s1,s2] value:

0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,
0.0,	0.0,	0.0,	0.0,

paper: 2

v\_1 value: 1.0

[illegible]