We can define $\gamma_F(a,b)$ as below: $\forall a,b \in \mathbf{F}_2^n \gamma_F(a,b) =$

$$\begin{cases} 1 \text{ if } a \neq 0_n \text{ and } F(x)+F(x+a)=b \text{ has solutions} \\ 0 \text{ otherwise} \end{cases}$$

Thus, for every APN (n,n)-function F, we view it as a Boolean function $\frac{|(D_aF)^{-1}(b)|}{2} - 2^{n-1}\delta_0(a,b)$, then we have

$$\widehat{\gamma_F(u,v)} = \frac{1}{2}W_F^2(u,v) - 2^{n-1}.$$

So we confirm that for every u,v:

$$W_{\gamma_F}(u,v) = \begin{cases} 2^n \text{ if } v = 0_n \\ 2^n - W_F^2(u,v) \text{ if } v \neq 0_n. \end{cases}$$

The fourth moment of the Walsh transform of an APN function F:

$$\sum_{u,v\in\mathbf{F}_2^n} W_F^4(u,v) = 3\cdot 2^{4n} - 2^{3n+1}.$$

When apply the Titsworth relation on the γ_F, we have for all $(u_0,v_0) \neq (0_n,0_n)$,

$$\sum_{u,v\in\mathbf{F}_2^n} W_{\gamma_F}(u,v)W_{\gamma_F}(u+u_0,v+v_0) = 0.$$

Then we have:

Theorem 1 *Any APN (n,n)-function F satisfies that $\forall (u_0,v_0)$,*

$$\sum_{\substack{u,v\in\mathbf{F}_2^n \\ v\neq 0_n, v\neq v_0}} W_F^2(u,v)W_F^2(u+u_0,v+v_0) = 2^{4n} - 2^{3n+1} + 2^{4n}\delta_0(u_0,v_0).$$

Corollary 1 *If there exists $(u_0,v_0) \neq (0_n,0_n)$ such that $|W_F(u,v)|$ and $|W_F(u+u_0,v+v_0)|$ both achieve the maximun value of $\{|W_F(u,v)| \mid u,v\in\mathbf{F}_2^n; v\neq 0_n\}$, then we have*

$$nl(F) \geq 2^{n-1} - \frac{1}{2}\sqrt[4]{2^{4n-1} - 2^{3n}}.$$