① take the 1st order approximation of the 1st term, and the 2nd order approximation of the other terms, and only update x for now.

$\varphi(x+v, Z)$ (ignore)

$$= (n+\nu\sqrt{n})\left(\log \mathrm{Tr}(F(x)Z) + \frac{1}{\mathrm{Tr}(F(x)Z)}\sum_{i=1}^{m} \mathrm{Tr}(F_i Z)\, v_i\right)$$

$$- \sum_{i=1}^{m} \mathrm{Tr}\left(F(x)^{-1}F_i\right) v_i + \frac{1}{2}\mathrm{Tr}\left[\left(\sum_{i=1}^{m} v_i F_i\right) F(x)^{-1}\left(\sum_{j=1}^{m} v_j F_j\right) F(x)^{-1}\right]$$

$$- \left(\log\det F(x) - \log\det Z - n\log n\right) \text{ (ignore)}$$

$$= \frac{n+\nu\sqrt{n}}{\mathrm{Tr}(F(x)Z)} \sum_{i} \mathrm{Tr}(F_i Z)\, v_i - \mathrm{Tr}\,F^{-1}\left(\sum_{i=1}^{m} v_i F_i\right)$$

$$+ \frac{1}{2}\mathrm{Tr}\left[\left(\sum_{i=1}^{m} v_i F_i\right) F^{-1}\left(\sum_{j=1}^{m} v_j F_j\right) F^{-1}\right] + \text{const.}$$

as we have the feasible condition:

$$\mathrm{Tr}(F_i Z) = c_i$$

$$\left(\rho \triangleq \frac{n+\nu\sqrt{n}}{\mathrm{Tr}(F(x)Z)} = \frac{n+\nu\sqrt{n}}{c^T x + \mathrm{Tr}\,F_0 Z}\right.$$

and $\sum_i \mathrm{Tr}(F_i Z)\, v_i = c^T v$.

$$\therefore \delta x = \arg\min_{v \in \mathbb{R}^n} \left(\rho\, c^T v + \cdots\right)$$

$$\begin{cases} D = \rho F Z F - F \\ S = F \end{cases} \Rightarrow \text{equivalent to (57)}$$

② similarly, we have the update of Z.

w.r.t. (57): $\begin{cases} D = \rho F - Z^{-1} \\ S = Z^{-1} \end{cases} \Rightarrow$ obtain $\delta x, \delta Z$