(57): optimal conditions:

$$\begin{cases} S\,\delta Z\,S + \sum_{i=1}^{m} \delta x_i F_i = -D & (a) \\ \mathrm{Tr}\, F_j\, \delta Z = 0, \quad j=1\ldots m & (b) \end{cases}$$

- (a) $\Rightarrow -\delta Z = S^{-1}\left(D + \sum_{i=1}^{n} \delta x_i F_i\right) S^{-1}$
  - take it into (b)

$$\Rightarrow -\mathrm{Tr}\left[F_j\, S^{-1}\left(D + \sum_{i=1}^{m} \delta x_i F_i\right) S^{-1}\right] = 0. \qquad (57x)$$

- equivalent to solve.

$$\sum_{j=1}^{m} \delta x_j \cdot \mathrm{Tr}\left[F_j\, S^{-1}\left(D + \sum_{i=1}^{m} \delta x_i F_i\right) S^{-1}\right] = 0.$$

(note the variable δx_j as v_j.)

- and rearrange the equation:

$$\mathrm{Tr}\left[\left(\sum_{j=1}^{m} \delta x_j F_j\right) S^{-1}\left(D + \sum_{i=1}^{m} \delta x_i F_i\right) S^{-1}\right] = 0.$$

$D+$

- $\Rightarrow \mathrm{Tr}\left[S^{-1/2}\left(\sum_{j=1}^{m} \delta x_j F_j + D\right) S^{-1/2}\, S^{-1/2}\left(D + \sum_{i=1}^{m} \delta x_i F_i\right) S^{-1/2}\right] = 0.$

$$\Rightarrow \left\| S^{-1/2}\left(D + \sum_{j=1}^{m} \delta x_j F_j\right) S^{-1/2}\right\|_F = 0.$$

- $\Rightarrow \delta x = \underset{v \in \mathbb{R}^m}{\arg\min} \left\| S^{-1/2}\left(D + \sum_{i=1}^{m} v_i F_i\right) S^{-1/2}\right\|_F \qquad (60) \quad \square$

$\left(\|A\|_F = \mathrm{Tr}(A^T A)^{1/2} \right).$

- Also. From (57x)

$$-\mathrm{Tr}\left[F_j S^{-1} D S^{-1}\right] - \sum_{i=1}^{m} \delta x_i\, \mathrm{Tr}\left[F_j S^{-1} F_i S^{-1}\right] = 0. \qquad (61)$$

$$\Rightarrow \sum_{i=1}^{m} \delta x_i\, \mathrm{Tr}\left[(S^{-1/2} F_j S^{-1/2})(S^{-1/2} F_i S^{-1/2})\right] + \mathrm{Tr}\left[(S^{-1/2} F_j S^{-1/2})(S^{-1/2} D S^{-1/2})\right] = 0.$$

LP example: minimize $c^T x$
subject to $Ax + b \geq 0$.

- $F(x) = \mathrm{diag}(Ax+b)$
- take $D = \mathrm{diag}(d)$ and $\delta Z = \mathrm{diag}(\delta z)$

(5.7) to $\Rightarrow$
$$\begin{cases} S\,\mathrm{diag}(\delta z)\,S + A\,\delta x = -\mathrm{diag}(d) \\ \mathrm{Tr}\left(A\,\mathrm{diag}\,\delta z\right) = 0. \end{cases}$$

$$\begin{cases} S^2\,\delta z + A\,\delta x = -d. \\ A^T \delta z = 0. \end{cases}$$

$$\Rightarrow \begin{bmatrix} S^2 & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} \delta z \\ \delta x \end{bmatrix} = \begin{bmatrix} -d \\ 0 \end{bmatrix}$$