

HW4. Q3.1

② Modularity before merging node v

- self edge of C community

$$Q_C = \frac{1}{2m} \sum_j [A_{jj} - \frac{k_j k_j}{2m}] \delta(c_i, c_j)$$

$$= \frac{1}{2m} \sum_j A_{jj} \delta(c_i, c_j) - (\frac{1}{2m})^2 \sum_j k_j k_j \delta(c_i, c_j)$$

$$= \frac{1}{2m} \sum_j A_{jj} - (\frac{1}{2m})^2 \sum_j k_j \sum_j k_j \delta(c_i, c_j)$$

$$= \frac{\sum_{in}}{2m} - (\frac{\sum_{tot}}{2m})^2$$

- self edge of v node:

(community with a single node)

$$Q_v = \frac{1}{2m} \sum_j [A_{jj} - \frac{k_j k_j}{2m}] \delta(c_i, c_j)$$

$$= \frac{1}{2m} \sum_j A_{jj} \delta(c_i, c_j) - \frac{1}{2m} \frac{k_v^2}{2m} = -(\frac{k_v}{2m})^2$$

- the modularity before sums to:

$$Q(\text{before}) = \frac{\sum_{in}}{2m} - (\frac{\sum_{tot}}{2m})^2 - (\frac{k_v}{2m})^2 \quad (a)$$

② Modularity after merging

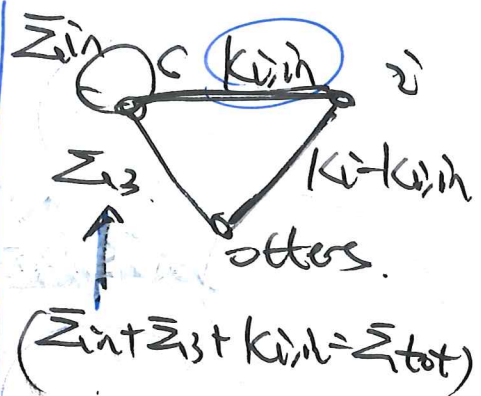
$$Q(C \cup v) = \frac{1}{2m} \sum_j [A_{jj} - \frac{k_j k_j}{2m}] \delta(c_i, c_j)$$

$$= \frac{1}{2m} \sum_j A_{jj} \delta(c_i, c_j) - \frac{1}{2m} \sum_j \frac{k_j k_j}{2m} \delta(c_i, c_j)$$

$$= \frac{1}{2m} (\sum_{in} + k_{vin}) - (\frac{1}{2m})^2 \sum_j k_j \sum_j k_j \delta(c_i, c_j)$$

$$= \frac{\sum_{in} + k_{vin}}{2m} - (\frac{\sum_{tot} + k_{vin} + k_v - k_{vin}}{2m})^2$$

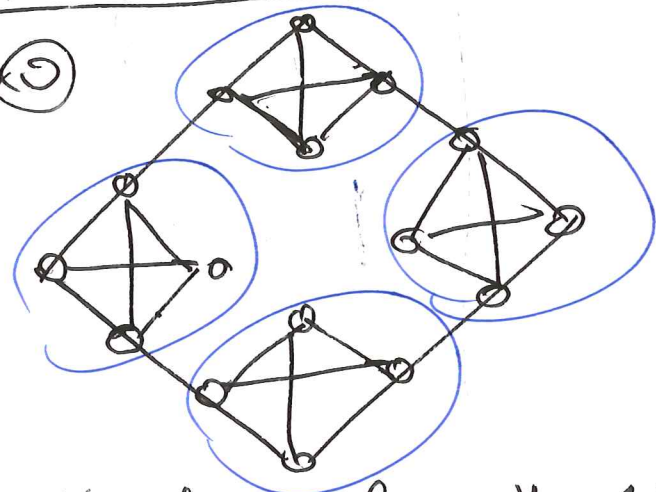
$$\Delta Q(C \cup v) = (b) - (a)$$



here we use a different notation for better meaning

HW1 - Q3.2

(5)



(16 nodes, edge weight = 1).

• the weight of any edge between two distinct nodes in H is 1.

• ... self edge in H is 12.

$$Q(H) = 4 \times Q_C$$

$$= 4 \times \left(\frac{1}{\sum m} \times 12 - \frac{1}{\sum m} \times \frac{1}{\sum m} \times 14^2 \right)$$

$$(m = 12 \times 4 + 1 \times 4 = 52.) \quad \uparrow$$

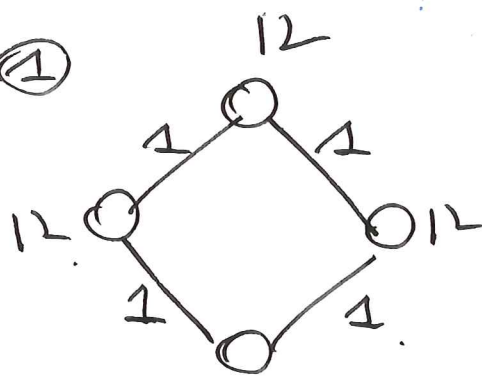
(2) 26

$J: \quad (= 2 \times 1 + 12 + 12)$

$$Q(J) = 2 \times \left(\frac{1}{\sum m} \times 26 - \left(\frac{28}{\sum m} \right)^2 \right)$$

HW1 - Q3.3 see the paper, prof provided.

(1)



$$H^{\uparrow} \quad 12$$

$$(\quad = 2 \times \# \text{edges}(H))$$

the intermediate solutions found by the algo may also be meaningful and that the uncovered hierarchical structure may allow the end-user to zoom in the network and to observe its structure with the desired resolution.