Final Exam

Artificial Intelligence
Fall 2021 CS47100-AI

Student name: ______________________ Student PUID: ______________
Note: You are free to use your intuition to find the steps in the proof. But, make sure you do not use your intuition to justify steps in your proofs. [1]

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1. Variables H, F, D, E and W denote the event of being health conscious, having free time, following a healthy diet, exercising and having a normal body weight, respectively. If an event does occur, we denote it with a $+$, otherwise $-$, e.g., $+e$ denotes an action of exercising and $-e$ denotes not exercising.

   - A person is health conscious with probability 0.8.
   - A person has free time with probability 0.4.
   - If someone is health conscious, they will follow a healthy diet with probability 0.9.
   - If someone is health conscious and has free time, then they will exercise with probability 0.9.
   - If someone is health conscious, but does not have free time, they will exercise with probability 0.4.
   - If someone is not health conscious, but they do have free time, they will exercise with probability 0.3.
   - If someone is neither health conscious, nor they have free time, then they will exercise with probability 0.1.
   - If someone follows both a healthy diet and exercises, they will have a normal body weight with probability 0.9.
   - If someone only follows a healthy diet and does not exercise, or vice versa, they will have a normal body weight with probability 0.5.
   - If someone neither exercises nor has a healthy diet, they will have a normal body weight with probability 0.2.

   (a) Select the minimal set of edges that needs to be added to complete the following Bayesian network

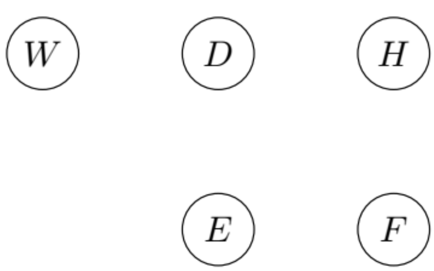


Figure 1: Incomplete Bayesian Network

[1] *The contents of this problem set is based on the AI course CS221 taught at Stanford University.*

A. $D \to H$ B. $F \to E$ C. $D \to E$ D. $D \to W$ E. $H \to D$ F. $D \to W$ G. $F \to D$
H. $W \to D$ I. $H \to F$ J. $H \to W$ K. $H \to E$ L. $E \to W$

Solution:

(b) Suppose we want to estimate the probability of a person being normal body weight given that they exercise (i.e. $P(+w| + e)$), and we want to use **likelihood weighting**.

i. We observe the following sample: $(-w, -d, +e, +f, -h)$. What is our estimate of $P(+w| + e)$ given this one sample? Express your answer in decimal notation rounded to the second decimal point, or express it as a fraction simplified to the lowest terms.

ii. Now, suppose that we observe another sample: $(+w, +d, +e, +f, +h)$. What is our new estimate for $P(+w|+e)$? Express your answer in decimal notation rounded to the second decimal point, or express it as a fraction simplified to the lowest terms.

Solution:

2. Pacman has developed a hobby of fishing. Over the years, he has learned that a day can be considered fit or unfit for fishing **Y** which results in three features: whether or not Ms. Pacman can show up **M**, the temperature of the day **T**, and how high the water level is **W**. Pacman models it as the following Naive Bayes classification problem, shown below:

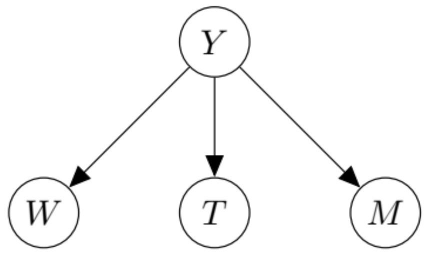


Figure 2: Pacman Model

(a) We wish to calculate the probability a day is fit for fishing given features of the day. Consider the conditional probability tables that Pacman has estimated over the years:

Y	P(Y)
yes	0.1
no	0.9

M	Y	P(M|Y)
yes	*yes*	0.5
no	*yes*	0.5
yes	*no*	0.2
no	*no*	0.8

W	Y	P(W|Y)
high	*yes*	0.1
low	*yes*	0.9
high	*no*	0.5
low	*no*	0.5

T	Y	P(T|Y)
cold	*yes*	0.2
warm	*yes*	0.2
hot	*yes*	0.5
cold	*no*	0.1
warm	*no*	0.2
hot	*no*	0.6

i. Using the method of Naive Bayes, what are these conditional probabilities, calculated from the conditional probability tables above? Fill in your final, decimal answer in the boxes below.

$$P(Y = yes|M = yes, T = cold, W = high) = \boxed{?}$$

$$P(Y = no|M = yes, T = cold, W = high) = \boxed{?}$$

ii. Using the method of Naive Bayes, do we predict that the day is fit for fishing if Ms. Pacman is available, the weather is cold, and the water level is high?

○ Fit for fishing ○ Not Fit for fishing

Solution:

(b) Assume for this problem we do not have estimates for the conditional probability tables, and that Pacman is still using a Naive Bayes model. Write down an expression for each of the following queries.

Express your solution using the conditional probabilities $P(M|Y)$, $P(T|Y)$, $P(W|Y)$, and $P(Y)$ from the Naive Bayes model.

i. Pacman now wishes to find the probability of Ms. Pacman being available or not given that the temperature is hot and the water level is low. Select all expressions that are equal to $P(M|T, W)$.

A. $\frac{\sum_f P(f)P(M|f)P(T|f)P(W|f)}{\sum_f P(f)P(W|f)P(T|f)}$ B. $\sum_f P(M|f)$ C. $\sum_f \frac{P(M|f)P(T|f)P(W|f)}{P(T|f)P(W|f)}$ D. None

ii. Pacman now wants to now choose the class that gives the maximum $P(features|class)$, that is, choosing the class that maximizes the probability of seeing the features. Write an expression that is equal to $P(M, T, W|Y)$.

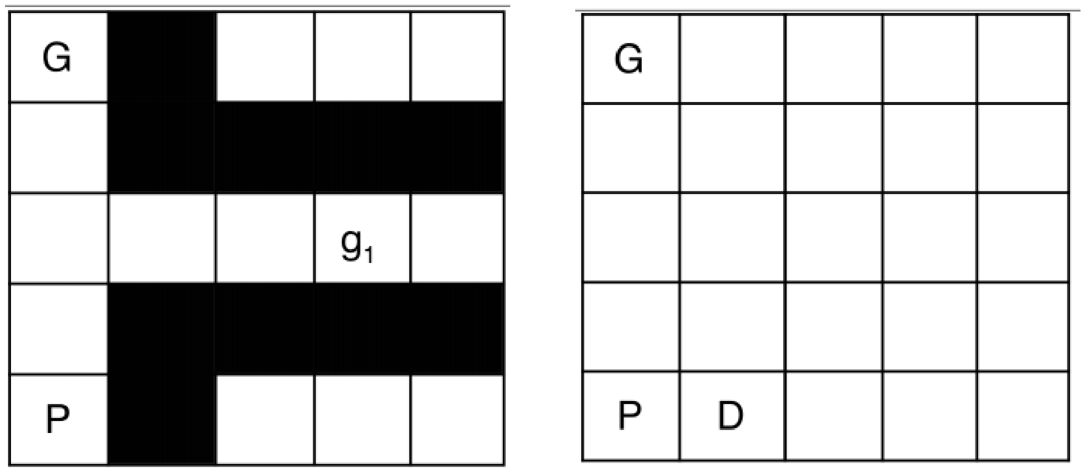
$$P(M, T, W|Y) = \boxed{\quad ? \quad}$$

iii. Assume that Pacman is equally likely to go fishing as he is to not, i.e. $P(Y = yes) = P(Y = no)$. Which method would give the correct Naive Bayes classification of whether a day is a good day for fishing if Pacman observes values for M, T, and W?

A. $\arg\max_y P(M, T, W|Y = y)$ B. $\arg\max_y P(Y = y|M, T, W)$ C. None of the Above

Solution:

3. In each below Gridworld, Pacman's starting position is denoted as P. G denotes the goal. At the goal, Pacman can only "Exit", which will cause Pacman to exit the Gridworld and gain reward +100. The Gridworld also has K gold nuggets $g_1, \ldots, g_k$ which will have the properties listed below in each part. Pacman automatically picks up a gold nugget if he enters a square containing one. Finally, we define P_0 as Pacman's initial state, where he is in position P and has no gold nuggets.



(a) (b)

Figure 3: Gridworlds (a) and (b)

(a) Pacman now ventures into Gridworld (a). Each gold nugget Pacman holds when he exits will add +100 to his reward, and he will receive +100 when he exits from the goal G.

i. When conducting value iteration, what is the first iteration at which $V(P_0)$ is nonzero?

ii. Assume Pacman will act optimally. What nonzero discount factor γ ensures that the policy of picking up g_1 before going to goal G and the policy of going straight to G yield the same reward?

Solution:

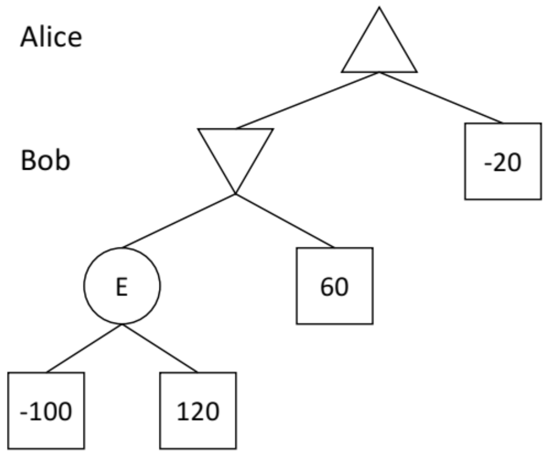
(b) Finally, Pacman finds himself in Gridworld (b). There is now a living reward! He gets the living reward for every action he takes except the Exit action. Pacman receives +0 exiting from the Door (D), and +100 exiting from the Goal (G). Once in the Door, Pacman can only Exit.

i. Suppose $\gamma = 0.5$. For what living reward will Pacman receive the same reward whether he exits via the Door or exits via the goal?

ii. Suppose $\gamma = 0.5$. What is the living reward such that Pacman receives the same reward if he traverses the Gridworld forever or if he goes straight to and exits from the goal?
Hint: $\sum_{t=0}^{\infty} r\gamma^t = \frac{r}{1-\gamma}$

Solution:

4. Alice and Bob are playing an adversarial game as shown in the game tree below. Alice (the MAX player) and Bob (the MIN player) are both rational and they both know that their opponent is also a rational player. The game tree has one chance node E whose outcome can be either $E = -100$ or $E = +120$ with equal 0.5 probability.



Each player's utility is equal to the amount of money he or she has. The value x of each leaf node in the game tree means that Bob will pay Alice x dollars after the game, so that Alice and Bob's utilities will be x and $-x$ respectively.

(a) Suppose neither Alice nor Bob knows the outcome of E before playing. What is Alice's expected utility?
Solution:

(b) Carol, a good friend of Alice's, has access to E and can secretly tell Alice the outcome of E before the game starts (giving Alice the true outcome of E without lying). However, Bob is not aware of any communication between Alice and Carol, so he still assumes that Alice has no access to E.

i. Suppose Carol secretly tells Alice that $E = -100$. What is Alice's expected utility in this case?

ii. Suppose Carol secretly tells Alice that $E = +120$. What is Alice's expected utility in this case?

iii. What is Alice's expected utility if Carol secretly tells Alice the outcome of E before playing?

We define the *value of private information* $V_A^{pri}(X)$ of a random variable X to a player A as the difference in player A's expected utility after the outcome of X becomes a private information to player A, such that A has access to the outcome of X, while other players have no access to X and are not aware of A's access to X.

Using $EU(A)$ as expected utility for A, and $EU(A|X = x)$ is the expected utility for A with X set to value x, we have:

$$V_A^{pri}(X) = \left(\sum_{x \in X} P(X = x) EU(A|X = x) \right) - EU(A)$$

iv. In general, the value of private information $V_A^{pri}(X)$ of a variable X to a player A

A. always satisfies $V_A^{pri}(X) > 0$ in all cases.

B. always satisfies $V_A^{pri}(X) \geq 0$ in all cases.

C. always satisfies $V_A^{pri}(X) = 0$ in all cases.

D. can possibly satisfy $V_A^{pri}(X) < 0$ in certain cases.

v. What is $V_{Alice}^{pri}(E)$, the value of private information of E to Alice in the specific game tree above?

Solution:

(c) David also has access to E, and can make a public announcement of E (announcing the true outcome of E without lying), so that both Alice and Bob will know the outcome of E and are both aware that their opponent also knows the outcome of E. Also, Alice cannot obtain any information from Carol now.

i. Suppose David publicly announces that $E = -100$. What is Alice's expected utility in this case?

ii. Suppose David publicly announces that $E = +120$. What is Alice's expected utility in this case?

iii. What is Alice's expected utility if David makes a public announcement of E before the game starts?

Solution: