Homework 2

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Problem 1

$\eta = 0.5$
Firstly, (-1)pretty bad
$\phi(x) = [1, 0, 1, 0, 0, 0], y = -1, w = [0, 0, 0, 0, 0, 0]$
$1 - w.\phi(x) * y = 1 - [0, 0, 0, 0, 0, 0].[1, 0, 1, 0, 0, 0] * 1 = 1 > 0$
$\nabla Loss(x, y, w) = -\phi(x) * y = -[1, 0, 1, 0, 0, 0] * (-1) = [1, 0, 1, 0, 0, 0]$
w ¡ -[0,0,0,0,0,0] -0.5[1,0,1,0,0,0] ¡ -[-0.5,0,-0.5,0,0,0]
Secondly, (+1) pretty good
$\phi(x) = [0, 1, 0, 1, 0, 0], y = 1, w = [-0.5, 0, -0.5, 0, 0, 0]$
$1 - w.\phi(x) * y = 1 - [-0.5, 0, -0.5, 0, 0, 0].[0, 1, 0, 1, 0, 0] * 1 = 1 > 0$
$\nabla Loss(x, y, w) = -\phi(x) * y = -[0, 1, 0, 1, 0, 0] * 1 = [0, -1, 0, -1, 0, 0]$
w ¡-[-0.5,0,-0.5,0,0,0]-0.5*[0,-1,0,-1,0,0] ¡ -[-0.5,0.5,-0.5,0.5,0,0]
Thirdly, (-1) not good
$\phi(x) = [0, 1, 0, 0, 1, 0], y = -1, w = [-0.5, 0.5, -0.5, 0.5, 0, 0]$
$1 - w.\phi(x) * y = 1 - [-0.5, 0.5, -0.5, 0.5, 0, 0].[0, 1, 0, 1, 0, 0] * -1 = 1.5 > 0$
$\nabla Loss(x, y, w) = -\phi(x) * y = [0, 1, 0, 0, 1, 0]$
w ¡ -[-0.5,0.5,-0.5,0.5,0,0] -0.5[0,1,0,0,1,0] ¡ -[-0.5,0,-0.5,0.5,-0.5,0]
Finally, (+1) pretty scenery
$\phi(x) = [1, 0, 0, 0, 0, 1], y = 1, w = [-0.5, 0, -0.5, 0.5, -0.5, 0]$
$1 - w.\phi(x) * y = 1 - [-0.5, 0, -0.5, 0.5, -0.5, 0].[1, 0, 0, 0, 0, 1] * 1 = 1.5 > 0$
$\nabla Loss(x, y, w) = -\phi(x) * y = [-1, 0, 0, 0, 0, -1]$
w ¡ -[-0.5,0,-0.5,0.5,-0.5,0] -0.5[-1,0,0,0,0,-1] ¡ -[0,0,-0.5,0.5,-0.5,0.5]

Problem 2

1. (-1) 'bad'
2. (+1) 'good'
3. (+1) 'not bad'
4. (-1) 'not good'
5. Proof: $\phi(x) \geq [w_1, w_2, w_3] \geq [”not”,”bad”,”good”]$
   If $w * \phi(x) \geq [k]$, let it be ”+1”. Otherwise, ”-1”.
   Firstly, (+1) good:
   $[w_1, w_2, w_3].[0, 1, 0] \geq [k]$

$w_2 \geq [k]$
Secondly, (-1) bad:
$[w_1, w_2, w_3].[0, 0, 1] < k$
$w_3 < k$
Thirdly, (+1) not bad:
$[w_1, w_2, w_3].[1, 0, 1] \geq [k]$
$w_1 + w_3 \geq [k]$
At Last, (+1) not good:
$[w_1, w_2, w_3].[1, 1, 0] < k$
$w_1 + w_2 < k$
Thus, $w_1 \geq [k]$
Since there is not such value for w_1, there cannot be zero error. However, we could make "not good" an indicator: when two words are all "+1", then the current one will be "+1". i.e. $[w_1, w_2, w_3, w_4] = [''not'', ''good'', ''bad'', ''notgood'']$.

Problem 3

$$Loss(x, y, w) = (\sigma(w * \phi(x)) - y)^2 \tag{1}$$

Problem 4

$$\nabla Loss(x, y, w) = \phi(x)2(p - y)p(1 - p), p = \sigma(w * \phi(x)) \tag{2}$$

Problem 5

In order to minimize the magnitude of gradient, we need to make p tend to 1 or 0 by taking w to be arbitrarily large or small multiple of $\phi(x)$. Thus, the magnitude of the gradient will tend to 0. The magnitude of gradient could not be exact 0, since it only happens when p tends to infinity.

Problem 6

In order to maximize the absolute value of the magnitude of gradient, we need to maximize $p(1-p)^2$. By setting $p = \frac{1}{3}$, we could get the maximum of it, based on the past homework. We could get $\frac{4}{27}$. Therefore, the maximum magnitude is $2 * (\frac{4}{27})||\phi(x)|| = \frac{8}{27}|\phi(x)||$.