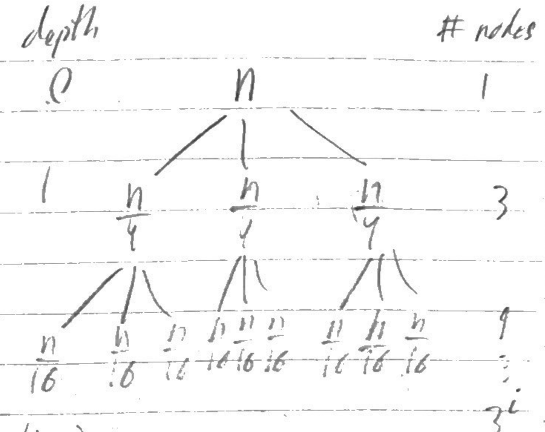
Lab 3



Tree Method

a) $W(n) = \begin{cases} c_0 & \text{if } n=1 \\ 3W(n/4)+c_1n^2+c_2 & \text{otherwise}\end{cases}$

level 0: $c_1\cdot n^2 + c_2$

level 1: $3(c_1(\frac{n}{4})^2+c_2) = 3c_1\frac{n^2}{16}+3c_2 = \frac{3}{16}c_1n^2+3c_2$

level 2: $9(c_1(\frac{n}{16})^2+c_2) = 9c_1\frac{n^2}{256}+9c_2=\frac{9}{256}c_1n^2+9c_2$

$\frac{3^i}{16^i}c_1n^2+3^ic_2$

root: n^2

level 1: $3(\frac{n}{4})^2 = 3\cdot\frac{n^2}{16}=\frac{3}{16}n^2$

$n^2 > \frac{3}{16}n^2 \Rightarrow$ root dom; $O(n^2)$

geom decay

Level i has 3^i nodes

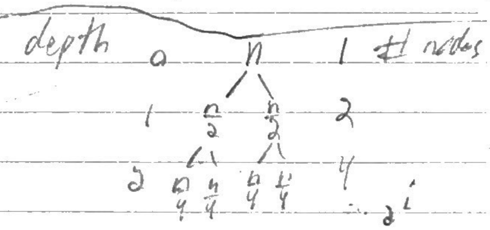
cost of each node: $c_1\cdot\frac{n^2}{16^i}+c_2$

Each level costs $\frac{3^i}{16^i}c_1n^2+3^ic_2$

$$W(n)=c_1\sum_{i=0}^{\lg n}\left(\frac{3}{16}\right)^i n^2 + c_2\sum_{i=0}^{\lg n}3^i \Rightarrow c_1\cdot n^2\sum_{i=0}^{\lg n}\left(\frac{3}{16}\right)^i + c_2\sum_{i=0}^{\lg n}3^i$$

$$\Rightarrow < c_1\cdot n^2\cdot\left(\frac{1}{1-\frac{3}{16}}\right) + c_2\cdot\left(\frac{3}{3-1}\cdot 3^{\lg n}\right)$$

$$\Rightarrow n^2+n \Rightarrow O(n^2)$$



b) $W(n)=\begin{cases} c_0 & \text{if } n=1\\ 2W(n/2)+c_1\cdot\frac{n}{\lg n}+c_2 & \text{otherwise}\end{cases}$

level 0: $(c_1\cdot\frac{n}{\lg n}+c_2)\cdot 2^i$

level 1: $(c_1\cdot\frac{n/2}{\lg\frac{n}{2}}+c_2)\cdot 2^i$

level 2: $(c_1\cdot\frac{n/4}{\lg\frac{n}{4}}+c_2)\cdot 2^i$

level i has 2^i nodes

cost of each node: $c_1\cdot\frac{\frac{n}{2^i}}{\lg\frac{n}{2^i}}+c_2$

Each level costs $2^i\cdot(c_1\cdot\frac{n/2^i}{\lg(n/2^i)}+c_2)$

lg n levels

$$\frac{\frac{n}{2^i}}{\lg\frac{n}{2^i}} = \frac{n}{2^i\cdot\lg\frac{n}{2^i}} = \frac{n}{2^i(\lg n-\lg 2^i)} = \frac{n}{2^i\lg n - i2^i} = \frac{n}{2^i}\cdot\frac{1}{\lg n - i}$$

$$2^i\cdot\left(\frac{n}{2^i}\cdot\frac{1}{\lg n-i}\right) = \frac{n}{\lg n - i} \Rightarrow W(n)=c_1 n\sum_{i=0}^{\lg n}\frac{1}{\lg n-i}+c_2\sum_{i=0}^{\lg n}2^i$$

$$< c_1 n\cdot\sum_{i=0}^{\lg n}\frac{1}{\lg n-(\lg n-1)} + c_2\sum_{i=0}^{\lg n}2^i = c_1\cdot n\cdot\sum_{i=0}^{\lg n}1 + c_2\sum_{i=0}^{\lg n}2^i$$

$$= c_1\cdot n\cdot\lg n + c_2\cdot\frac{2}{2-1}2^{\lg n} = c_1\cdot n\cdot\lg n + c_2\cdot 2\cdot n \Rightarrow O(n\lg n)$$

Brick Method

a. $W(n) = 2W(.49n) + 1.01n$

$\boxed{\text{Root dominated}}$

$C(\text{root}) = 1.01n$
$C(\text{level } 1) = 1.01(.49n) + 1.01(.49n) = .9898n$

- The cost is greater in the root than it is in level 1. It has decreased by a factor of 1.02.
- Therefore, we only need to consider the cost of the root.

$$1.01n \in \boxed{O(n)}$$

→ Find $a > 1$:

$$1.01 > a(2(1.01 \cdot .49))$$

$$\frac{1.01}{.9898} > a \Rightarrow a < \frac{1.01}{.9898} \Rightarrow a \approx< 1.02$$

$\boxed{\text{Let } a = 1.01}$

b. $W(n) = W(\frac{n}{2}) + W(\frac{n}{4}) + .999n$

- $C(\text{root}) = .999n$
- $C(\text{level } 1) = .999(\frac{n}{2}) + .999(\frac{n}{4}) = .999n(\frac{1}{2} + \frac{1}{4}) = .999n(\frac{3}{4}) = .74925n$

Find $a > 1$:

$.999 > a \cdot .74925 \Rightarrow a < \frac{.999}{.7425} \Rightarrow a < 1.\overline{3}$

Let $\boxed{a = 1.2}$

- Cost of root is greater than cost of level 1, so it is $\boxed{\text{root dominated}}$. We only need to consider cost of root.

$$\boxed{.999n \in O(n)}$$

$$\frac{1}{\cdot}\sum_{i=0}^{n}(n-i)^{-1}$$

4. $W(n) = \sqrt{n}\, W(\sqrt{n}) + \sqrt{n}$

- $C(\text{root}) = \sqrt{n}$
- $C(\text{Level } 1) = \sqrt{n}\left(\sqrt{\sqrt{n}}\right) = n^{\frac{1}{2}}\left(n^{\frac{1}{2}}\right)^{1/2} = n^{1/2} \cdot n^{1/4} = n^{3/4}$
- I suspect leaf-dominated:

Find $a > 1$:

$$C(v) \leq \frac{1}{a} \sum_{u \in D(v)} C(u)$$

☆ Let $\boxed{a = 2}$

$$\sqrt{n} \leq \frac{1}{a} n^{3/4}$$
$$\sqrt{n} \leq \frac{1}{2} n^{3/4}$$
$$n^{1/2} \leq \frac{1}{2} n^{3/4}$$
$$\forall n > 16,\ n^{1/2} \leq \frac{1}{2} n^{3/4} \quad \checkmark$$

☆ We are interested in end behavior

<u>Solve for i</u>

$$2 = n^{1/2^i}$$
$$\log_2(2) = \log_2(n^{1/2^i})$$
$$1 = \frac{1}{2^i}\log_2(n)$$
$$2^i = \log_2(n)$$
$$i = \log_2(\log_2(n))$$

☆ At level i, we have nodes of length $n^{1/2^i}$ and the node size is $n^{(1-\frac{1}{2^i})}$

- Plug in i:

$$n^{\left(1 - \frac{1}{2^{\log_2(\log_2(n))}}\right)} = n^{\left(1 - \frac{1}{\log_2(n)}\right)}$$

→ As $n \to \infty$, $n^{\left(1 - \frac{1}{\log_2 n}\right)} \to n$.

Thus $W(n) \in \boxed{O(n)}$