# Lab 2

ii. $W(n) = a \cdot W(n/b) + n$ : simple_work_calc

s_w_c(10, 2, 2) ; $n=10, a=2, b=2$

$W(10) = 2 \cdot W(5) + 10 = 2 \cdot 13 + 10 = 36$
$W(5) = 2 \cdot W(2) + 5 = 2 \cdot 4 + 5 = 13$
$W(2) = 2 \cdot W(1) + 2 = 2 \cdot 1 + 2 = 2 + 2 = 4$
$W(1) = 1$

$n = 30, a = 4, b = 2$
$W(30) = 4 \cdot W(15) + 30 = 4 \cdot 155 + 30 = 650$
$W(15) = 4 \cdot W(7) + 15 = 4 \cdot 35 + 15 = 155$
$W(7) = 4 \cdot W(3) + 7 = 4 \cdot 7 + 7 = 35$
$W(3) = 4 \cdot W(1) + 3 = 4 \cdot 1 + 3 = 7$
$W(1) = 4$ ?

iii. $W(n) = a \cdot W(n/b) + f(n)$
$f(n) = 1, n = 10, a = 2, b = 2$
$W(10) = 2 \cdot W(5) + 1 = 2 \cdot 7 + 1 = 15$
$W(5) = 2 \cdot W(2) + 1 = 2 \cdot 3 + 1 = 7$
$W(2) = 2 \cdot W(1) + 1 = 2 \cdot 1 + 1 = 3$
$W(1) = 1$

$f(n) = n^2, n = 20, a = 1, b = 2$
$W(20) = 1 \cdot W(10) + 20^2 = 129 + 400 = 530$
$W(10) = 1 \cdot W(5) + 10^2 = 29 + 100 = 130$
$W(5) = 1 \cdot W(2) + 5^2 = 1 \cdot 5 + 25 = 30$
$W(2) = 1 \cdot W(1) + 2^2 = 1 \cdot 1 + 4 = 5$
$W(1) = 1$

$f(n) = n, n = 30, a = 3, b = 2$
$W(30) = 3 \cdot W(15) + 30 = 3 \cdot 90 + 30 = 300$
$W(15) = 3 \cdot W(7) + 15 = 3 \cdot 25 + 15 = 90$
$W(7) = 3 \cdot W(3) + 7 = 3 \cdot 6 + 7 = 25$
$W(3) = 3 \cdot W(1) + 3 = 3 \cdot 1 + 3 = 6$
$W(1) = 1$

$n = 27, a = 4, b = 3, f(n) = 2n$
$W(27) = 4 \cdot W(9) + 2(27) = 4 \cdot 58 + 54 = 286$
$W(9) = 4 \cdot W(3) + 2(9) = 4 \cdot 10 + 18 = 58$
$W(3) = 4 \cdot W(1) + 2(3) = 4 + 6 = 10$
$W(1) = 1$

$n = 16, a = 2, b = 4, f(n) = (\frac{n}{2})^2$
$W(16) = 2 \cdot W(4) + (8)^2 = 2 \cdot 6 + 64 = 76$
$W(4) = 2 \cdot W(1) + (2)^2 = 2 \cdot 1 + 4 = 6$
$W(1) = 1$

$W(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2W(n/2) + 1 & \text{otherwise} \end{cases}$

$W(n) = 2W(\frac{n}{2}) + O(1)$
$= W(\frac{n}{2}) + W(\frac{n}{2}) + c_1$

$c_1$
$c_1$
$\vdots$
$c_1$

level $i$ has $2^i$ nodes
cost of each node is $c_1$
each level costs $2^i \cdot c_1$
# levels $\log_2 n$

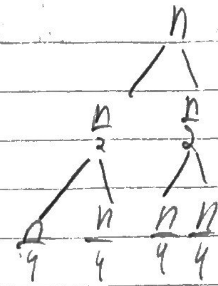
iv. $f(n)=1$

$\sum_{i=0}^{\lg n} 2^i \cdot c_1 = c_1 \sum_{i=0}^{\lg n} 2^i < \frac{2}{2-1} \cdot 2^{\lg n} = 2n \in O(n)$

# Lab 2 Cont.

iv. $W(n) = \begin{cases} 1 & \text{if } n=1 \\ 2W(n/2) + \lg n & \text{otherwise} \end{cases}$

$f(n) = \lg n$



$c_1 \cdot \lg n + c_2$

$2(c_1 \lg \frac{n}{2} + c_2) = 2c_1 \lg \frac{n}{2} + 2c_2$

$4(c_1 \lg \frac{n}{4} + c_2) = 4c_1 \lg \frac{n}{4} + 4c_2$

$W(n) = 2W(n/2) + O(\lg n)$

$\quad = W(n/2) + W(n/2) + c_1 \cdot \lg n + c_2$

$W(n/2) = 2(W/4) + c_1 \lg(\frac{n}{2}) + c_2$

level $i$ has $2^i$ nodes

cost of each node is $c_1 \lg \frac{n}{2^i} + c_2$

Each level costs $2^i \cdot (c_1 \lg \frac{n}{2^i} + c_2) = 2^i c_1 \lg \frac{n}{2^i} + 2^i c_2$

$= 2^i c_1 (\lg n - \lg 2^i) + 2^i c_2$

$= 2^i c_1 \lg n - i \cdot 2^i c_1 + 2^i c_2$

$$W(n) = \sum_{i=0}^{\lg n} 2^i c_1 \lg n - i \cdot 2^i c_1 + 2^i c_2$$

$$= \underbrace{c_1 \lg n \sum_{i=0}^{\lg n} 2^i}_{< 2c_1 n \lg n} - c_1 \sum_{i=0}^{\lg n} i \cdot 2^i + \underbrace{c_2 \sum_{i=0}^{\lg n} 2^i}_{< 2c_2 n}$$

$$\sum_{i=0}^{\lg n - 1} i \cdot 2^i = 2 + (\lg n - 2) 2^{\lg n} = 2 + (\lg n - 2)n = 2 + n\lg n - 2n \quad \text{(identity)}$$

$$\sum_{i=0}^{\lg n} i \cdot 2^i = 2 + n\lg n - 2n + \lg n \cdot 2^{\lg n} = 2 + n\lg n - 2n + n\lg n = 2 + 2n\lg n - 2n$$

$\Rightarrow < 2c_1 n\lg n - c_1(2 + 2n\lg n - 2n) + 2c_2 n$

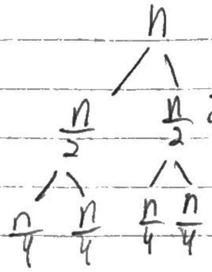
$\Rightarrow 2c_1 n\lg n - 2c_1 - 2c_1 n\lg n + 2c_1 n + 2c_2 n = -2c_1 + 2c_1 n + 2c_2 n$

$\in O(n)$

# Lab 2 Cont.

IV. $W(n) = \begin{cases} c_1 & \text{if } n=1 \\ 2W(n/2) + c_2 n + c_3 & \text{otherwise} \end{cases}$

$f(n)=n$



$c_2 n + c_3$

$2(c_2\frac{n}{2} + c_3) = c_2 n + 2c_3$

$4(c_2\frac{n}{4} + c_3) = c_2 n + 4c_3$

$\vdots$

$c_2 \cdot n$

level $i$ has $2^i$ nodes

cost of each node is $c_2\frac{n}{2^i} + c_3$

Each level costs $2^i \cdot (c_2\frac{n}{2^i} + c_3) = c_2 \cdot n + c_3 2^i$

$\lg n$ levels

$$W(n) = \sum_{i=0}^{\lg n} (c_2 n + 2^i c_3) \Rightarrow c_2 n \sum_{i=0}^{\lg n} 1 + c_3 \sum_{i=0}^{\lg n} 2^i$$

$$\Rightarrow < c_2 n \lg n + 2c_2 n \in O(n \lg n)$$

VI. $S(n) = \begin{cases} c_a & \text{if } n=1 \\ S(n/2) + c_1 n + c_2 & \text{otherwise} \end{cases}$

$f(n)=n$

cost of each node: $c_1\frac{n}{2^i} + c_2$

Each level has 1 node

cost of each level: $1 \cdot (c_1\frac{n}{2^i} + c_2) = c_1\frac{n}{2^i} + c_2$

$$S(n) = \sum_{i=0}^{\lg n} (c_1\frac{n}{2^i} + c_2) \Rightarrow c_1 n \sum_{i=0}^{\lg n} \frac{1}{2^i} + c_2 \sum_{i=0}^{\lg n} 1$$

$$\Rightarrow c_1 n \sum_{i=0}^{\lg n} (\tfrac{1}{2})^i + c_2 \lg n < c_1 n \cdot 2 + c_2 \lg n \in O(n)$$

$S(n) = \begin{cases} c_a & \text{if } n=1 \\ S(n/2) + c_1 \lg n + c_2 & \text{otherwise} \end{cases}$

$f(n)=\lg n$

cost of each node: $c_1 \lg\frac{n}{2^i} + c_2$

Each lvl. has 1 node

cost of each lvl: $c_1 \lg\frac{n}{2^i} + c_2$

# of lvls: $\lg n$

$$S(n) = \sum_{i=0}^{\lg n} (c_1 \lg\frac{n}{2^i} + c_2) = c_1 \lg n \sum_{i=0}^{\lg n} 1 - c_1 \sum_{i=0}^{\lg n} i + c_2 \sum_{i=0}^{\lg n} 1$$

$$= c_1 \lg n(\lg n + 1) - c_1 \cdot \tfrac{1}{2} \lg n(\lg n + 1) + c_2 \lg n$$

$$\Rightarrow \lg^2 n + \lg n - \tfrac{1}{2}\lg^2 n - \tfrac{1}{2}\lg n + \lg n = \tfrac{1}{2}\lg^2 n + \tfrac{3}{2}\lg n \in O(\lg^2 n)$$

$f(n)=1$ $\quad S(n) = \begin{cases} c_a & \text{if } n=1 \\ S(n/2) + c_1 & \text{otherwise} \end{cases}$

cost of each node: $c_1$ } $\Rightarrow$ cost of each lvl: $c_1$

Each lvl. has 1 node } # of lvls: $\lg n$

$$S(n) = \sum_{i=0}^{\lg n} c_1 = c_1 \sum_{i=0}^{\lg n} 1 = c_1 \lg n \in O(n)$$